Cours spécialisé

Panorama of geometry and dynamics of moduli spaces

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Pas de notes de cours prévues.

Présentation

The goal of this course is to present a panorama of concepts and ideas used in contemporary studies of geometry and dynamics of moduli spaces. In particular, I plan to describe celebrated recent results of Alex Eskin and of Maryam Mirzakhani. To cover such a broad material, I would be forced to sacrifice complete proofs focusing on concepts, ideas and ties between them. In this sense, the course might be unusual if not embarrassing for those students who are used to a style, where every fact is justified by a complete and rigorous proof. However, an advantage is that such a course does not need any particular prerequisites.

I am writing this description in English, because it is not excluded that the course would be given in English: the choice of the language would depend on preferences of (majority of) the audience and would be decided at the first lecture.

Part I. Flat world (4 lectures). Billiards in rational polygons, flat surfaces, moduli space of Abelian differentials, Masur-Veech volume element, $GL(2, \mathbb{R})$-action. Magic Wand Theorem of Eskin–Mirzakhani–Mohammadi and Filip.

Interval exchange transformations, measured foliations. Pseudo-Anosov diffeomorphisms of surfaces. Rauzy induction. Idea of renormalization. Lyapunov exponents of the Hodge bundle along the Teichmüller flow. Application to diffusion rate of Ehrenfest-type wind-tree models.

Part II. Hyperbolic world (4 lectures). Basic facts about hyperbolic surfaces. Fenhel-Nilson coordinates in moduli space. Symplectic structure and Weil–Petersson volume form. Integration over moduli space (in the spirit of Mirzakhani).

Measured laminations, train-track coordinates. Mirzakhani's count of simple closed hyperbolic geodesics. Witten-Kontsevich correlators, an idea of symplectic reduction, and an idea of Mirzakhani's proof of Witten's conjecture.

Part III. Bridging flat and hyperbolic worlds (4 lectures). Masur-Veech volume through count of square-tiled surfaces. Count of trivalent integral metric ribbon graphs. Virasoro constraints for Witten-Kontsevich correlators. Large genus asymptotics of Witten-Kontsevich correlators and of Masur-Veech volumes (after Amol Aggarwal).

Random geodesic multicurves on surfaces of large genera and random square-tiled surfaces of large genera. Meanders count.

Contenu

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– Random geodesic multicurves on surfaces of large genera and random square-tiled surfaces of large genera. Meanders count.

Prérequis

aucun

Bibliographie

– Maryam Mirzakhani. Growth of the number of simple closed geodesics on hyperbolic surfaces. *Annals of Math. (2) 168:1 (2008), 97-125* URL

– Benson Farb and Dan Margalit . A primer on mapping class groups. *Chapter 15*

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