During the "Mathematics and Narrative" meeting organized at Delphi in July 2007 by Apostolos Doxiadis and Barry Mazur under the sponsorship of Thales and Friends, interviews of each participant by another were organized and the results were then transcribed and edited.
This is the result of my being interviewed by Peter Galison.
The proceedings of the meeting, but not the interviews, will be published by Princeton University Press in late 2011 in a book entitled "Circles disturbed, Mathematics and Narrative". The interviews should be made available on the publisher's web site
With many thanks to the organizers and the other participants of this original and stimulating meeting.

TEISSIER INTERVIEWED BY GALISON

Galison: I thought maybe we'd start by giving a little background. I don't actually know how you came to this complex of questions. Perhaps you could say a little bit about the kind of mathematics that you have been most interested in and then I'll ask about how you got interested in the more philosophical aspects. We'll move into those questions and then we'll look at the intersection of math and narrative.
One of the things which strike me in reading what you've written and in talking to you is that there's something unexpected to me about your trajectory, so let me say what it is.
You have a strong interest and an involvement with Bourbaki and ideas of rigor in the Bourbaki tradition. I'm a complete outsider, but from the point of view of a theoretical Physicist they stand for a certain kind of rigor and for agnosticism about philosophical interpretation and meaning, a deliberate bracketing of issues like: what is the referential structure of these symbols? On the other hand many of your interests seem actually quite sympathetic to Poincaré with his muscular movement of shapes and this Helmholtzian theme of understanding geometry through the manipulation of rigid objects and so on. It is striking as a combination to have these two interests, so I wanted to pursue that a little bit with you.

Teissier : At the time when I began to work for Bourbaki, I already had these preoccupations, but I didn't see things the way I see them now. I was not close to Poincaré in my reflexions. It was rather a very strong unease about all these things which I wanted to make sense. Working with Bourbaki had two aspects which I liked, one is that it was an ascesis. Feeling that need for meaning as I did, I thought that since I was given the opportunity I should really look at what these people, who all were older than me by definition and excellent mathematicians, were doing, and ask myself: what do they get out of this approach of going to the higher structures? why do those guys seem to be perfectly at ease with all this while I am ill at ease?
At the time I was already educated in structuralism. When I was in Classe Préparatoire I used to skip classes to go to the Collège de France to listen to Levi-Strauss, and that was another strong influence. So I was attracted by this ascesis of doing things in a way which

was not natural for me and thus trying to understand what they got out of it. To be in the midst of it and to collaborate is the only way to understand how it works.
There was also another aspect: Bourbaki, as I viewed it and still do, is as a kind of public service, trying to make things as clear and accessible as possible to working mathematicians. I liked that. I didn't teach so I thought I had a social obligation to do something for the community. Working for Bourbaki fulfilled that need in a way which I found interesting.
And I wasn't disappointed, I really got a lot of understanding of this structuralist approach. It is true however that Bourbaki was not at all interested in meaning in the same way that I am.

Galison: Do you think of Bourbaki in its totality, in the volumes as they develop, as being a kind of a narrative about mathematics, a story of a kind?

Teissier: Yes, it claims to be that and I think to some extent it is that. It is a story with many digressions. But it is true that it claims to be a kind of story of origin. The work of Bourbaki begins with set theory and everything that goes with it. And then he said OK from there on let's see if we can move up: algebra is like this, topology is like that, so it is a kind of story of origin. It doesn't tell its name of course but as a matter of fact it is that. But it is not a story of the origin of meaning. If you assume that mathematics flow from set theory from to the ends of a mathematical tree, then it is a description of that flow. At the same time it is logically organized, with proofs, so it is also a foundation of truth of the statements that it contains. And finally it tries to bring to light the "deeper reasons" why the results are true, the structural reasons.

Galison: One of the reasons that I ask. It seems to me very interesting that you have a story that de-emphasizes time, it is a story that in some sense is outside of time, which at first may appear completely paradoxical, but the sequence is a kind of logical one although it is not exactly logical, it is not Russel and Whitehead, it is a story where the flow of time is replaced by a conceptual flow. And did they think about set theory as the only way they could have started or did they think of it as a good way to start?

Teissier: That's an interesting question, I don't know. I do believe that at the time they began they were convinced it was the only way to do it.

Galison: So the idea of a kind of foundational story?

Teissier: Yes. But you see it is the only book that gives a gradient: it says that it goes from the general to the particular. If you think about it it is amazing because it occurs only 30 years after the birth of set theory; in fact it is not even 30 years. Suddenly a group of people decides that this is the point of origin of all mathematics. I think it is a major intellectual statement to say we're going to take set theory as a point of origin and from there everything flows from the general to the particular. I think somewhere Levi Strauss says that genius is remarking on a simple fact and following its consequences to the end. In a way it is a good description of the Bourbaki enterprise. You could say that taking set theory as the foundation was a rather natural step at the time, in view of Russel-

Whitehead, the Hilbertian program and all that, but to decide to follow the consequences to the end, that's something absolutely new.
I think working with Bourbaki stimulated my desire to find foundations of meaning in the "human nature", as one used to say.

Galison: By the move in the other direction.

Teissier: Precisely. However the manner in which I became interested in cognition to the point of giving it the central role in the foundations of meaning is not at all a deep philosophical evolution. It happened like this:
I was working on a problem related to catastrophe theory. I wanted, in a precise situation, to eliminate critical points of real analytic functions. Then I chanced upon a paper on the mathematical modeling of vision where people used the heat equation to eliminate the critical points of "grey functions". This was an important idea in what is called multiscale analysis, introduced by David Marr and other people in the modelisation of vision in the sixties or the seventies. Perhaps you're familiar with this. You take a grey function g(x,y) encoding a black-and-white picture and consider its two dimensional graph which comes with critical points: maxima, minima, and points of index one like mountain passes. And then you apply the heat question to g(x,y), thinking of it as decribing a distribution of heat in a plane region at an origin of time. And what happens is that under the evolution determined by the heat equation the function gets smoothed, the critical points minima and maxima, points of index one are after some time eliminated. In any case that's the idea. For functions of one variable it works perfectly, for functions of two variables like g(x,y), actually it fails but in a very interesting way: evolution according to the heat equation can create new critical points instead of eliminating them.

Galison: that's interesting.

Teissier: This is exactly the phenomenon which I stumbled upon; it showed that the technique which I was thinking of using could not work but on the other hand it got me interested in the theory of vision. Then I started to read and got interested in why people were thinking of applying the heat equation, but with the time replaced by a "scale" parameter. Somehow it explained that we could see at the same time the (small scale) details of a face and also just the oval of the face. In principle when you apply the heat equation all that remains for large parameters is this big blob, the oval of the face. So I started to really try to understand what's going on and that's how at some point I got introduced to low level vision. Learning what people did for vision led me to apply the same kind of idea to meaning.

Galison: When was this?

Teissier: I'm trying to remember, this was in the late 80's, about twenty years ago. I was working on it very slowly since it is not my main activity of course. But the outcome of it was that I started talking to people who worked on vision, particularly Alain Berthoz at the Collège de France In Paris, a very interesting person, he was very open. About ten years ago he invited me to speak about space in his seminar and at that time I was really angry with Kant because I couldn't understand so I started by thinking I couldn't accept, because for me it is just a term, that space is an a priori of perception. So I started to think about it exactly in the way I've been talking about today. That's more or less the path that lead me there; it started with a problem in mathematics which by chance was connected to a problem with also occupied the people in vision, it moved me to try to understand why and how they were interested in it, and finally I thought I could see the trace of meaning, so I learned about vision and it is only about five years ago that suddenly I realized that the real problem of meaning was: can I give a meaning to the real line? Fortunately by that time I had acquired a certain knowledge, a certain experience from talking with these people and I knew about the definition of the visual line and I knew from the Berthoz seminar about the vestibular line and some of the experiment that had been done. And I thought: ah! but the thing is that they are isomorphic from the view point of the structure and what nature, or evolution, did is to identify them for us, and that's why we can think in several ways about the "same" real line and that's why it is a mathematical object! That was the itinerary.

Galison: So Berthoz himself had not been interested in the real, in the mathematical side of it, he was more at the physical line?

Teissier : Berthoz, I don't exactly know how to say it, he's interested in many things but his real work is really neurophysiology. This isomorphism is called the Poincaré – Berthoz isomorphism in homage to both because Berthoz tried to draw the consequences of Poincaré's idea that the position of a point in space is coded by the muscular tensions corresponding to the finger being positionned at that point. He made experiments inspired by that. For example you might believe that the gesture you make is not unique but in fact there are experiments to prove it is, that the natural gesture you make is severely constrained and in fact it is unique. Of course you can force variations, but the normal gesture will always be the same. He's many experiments in that direction.

Galison: that's interesting

Teissier: He made literally dozens, maybe hundreds of experiments of that nature, on how you perceive symmetry, one of these experiments that struck me early on, the unconscious perceptual system knows that the sum of the angles of a triangle is equal to π. Put someone in a dark room, turn a certain angle, tell them to walk a certain distance and then turn back to its point of origin and people do it rather well….so as an inertial system of navigation the vestibular system is pretty good, pretty efficient. He's not really interested in the meaning of mathematics. Just interested enough that he likes to know how a mathematician thinks about space from this view point and that's how we have all these interesting discussions, but I don't think he's deeply interested in the meaning of mathematics. He's deeply interested in understanding the relationship between vision, the motor system, the vestibular system, the integration of all that, how it works, how the neurophysiology of it works but at the same time as I said he's a very open minded person: at his lectures and seminars there are dancers, actors, because people who master some kind of real expertise in the way the body moves in space interest him. One of the things that I said when we first met, which he liked very much, is that finally I would like to show that explaining a proof is like a dance in some abstract space and that idea struck him. I don't know if I would say exactly the same thing today, but at the time I had this notion, it is dynamic, it is constrained in a given space, a dance in a way is a narrative. So he liked that idea very much and talked to me several times about it and it encouraged me to think about it. To explain the space where we dance is to explain the meaning of the mathematics we are doing, to some extent. It was very helpful to me to talk to people like him.

Galison: Is there any relationship with the tradition of Piaget and his students

Teissier: Well, certainly Berthoz is quite well aware of the work of Piaget and his students, some of Piaget's students come to the seminar. However, there is a strong criticism of the Piaget view of the learning of space and the learning of mathematics and I think it is clear from what I said, I'm m not at all convinced that the sequence that Piaget described is valid.

Galison: His "staging"

Teissier: I tend to think it is much more complicated than that, you don't learn topology then metric. I think infants have some notion of metric, which is developing, some notion of topology, also developing. They interact for a long time, creating the sensory-motor complex, which has metric and topological aspects. In this topic it is extremely difficult to design experiments which do not incorporate our preconceptions. Piaget's vision I think is not really a myth of origin, it is really a kind of a scenario and I don't think it is very believable.

Galison: But somehow the goal of taking simple operations and trying to enquire into them experimentally in this way is interesting even if he gets a lot of the sequence wrong, the hierarchy.

Teissier: Absolutely, I think in that he was probably a pioneer, making this kind of experiment with children. First of all it was very daring at the time although now people do that all the time with other preoccupations, but it is true that all those who think about this, including Berthoz, including myself, have a great admiration for Piaget.

Galison: Einstein once had a famous exchange with Piaget and Einstein encouraged Piaget to look at the question of simultaneity and to see whether if was innate, as people often claimed in criticizing Einstein. Piaget did those experiments that show that actually simultaneity is a complicated notion and that in children, it is all entangled with the speed for instance, and if you have two trains and one goes fast and one goes slow then you turn off the power to both of them at the same time, the children think that the one that's gone farther stopped later, so it doesn't seem to be strikingly a priori that way.

Teissier: No I think that's typically a notion which is not an elementary construction of our perceptive system; it is something elaborate. And perhaps that's why people had so much trouble with the EPR (Einstein-Podolsky-Rosen) paradox and perhaps that's also why ultimately they don't have so much trouble, simultaneity doesn't really talk to our basic intuition of the world, perhaps because it is important only in very very constrained circumstances. It is an abstract notion for us, we have to become a physicist I suppose to have an intuitive idea of what it is.

Galison: It has often interested me that, in a kind of folk way we often think of things as innate that are rather complex constructions, simultaneity would be one.

Teissier: I think it is a really a major challenge to try to understand what our perceptual system actually says.

Galison: Yes

Teissier: I'm pretty much convinced about what I said about the real line, that's very likely.

Galison: Yes

Teissier: But there are other things like boundaries which are more difficult. Are they really elementary constructions? Are they really low level thinking? I believe that simultaneity is not. To make a list of these low level thoughts is a real challenge.

Galison: So Berthoz had, early on, read and been interested in Poincaré and when you encountered Berthoz had you also been thinking about Poincaré or did you come to that through him?

Teissier: Of course I knew without any great precision the ideas of Poincaré. At that time it certainly had not occurred to me to try to follow them through because the literary style of Poincaré made it very difficult for me to think of this as a basis for building a solution to my preoccupations. So while of course while, like all mathematicians, I am profound admirer of all the things I understand of his work, I was not an admirer of him as a philosopher. I think it was really Berthoz who convinced me that there was something deep because he made those experiments. I was convinced because he showed the neural connections, he conducted the experiments that connected vision with matter. Poincaré's is a nice idea for sure but as I said, for some reason I'm very much allergic to the explanation of words by other words. Perhaps that's due to my original love for the Greek language but anyway wherever I see that in a text I tend to dismiss the text, which is a mistake. But certainly I had that reaction with Poincaré.

Galison: Too many words, not enough logic or experiment.

Teissier: Yes, not enough experiment, not enough meat, too many words like " beauty". But I feel I should have been more humble, and respectful as well. When somebody like Poincaré writes certainly you should take it seriously even if at first it seems it is a bit *facile*. Perhaps what he writes about beauty will turn out to be also a very deep comment at some point, and that would be a typical example of what I think is happening. People like Poincaré and Weyl, they really had this idea that all this has to be somewhere in our perception of the world. That's really, as you said in the beginning, the tradition to which we belong as opposed to the formalistic tradition. And all these people thought that, and they were right to think that but they didn't have the tools, they didn't have neurophysiology, they didn't have the experiments. Now we are beginning to have them, and I think all their ideas are going to be completely transfigured and probably they will become the great visionaries of the Philosophy of Science in the next 20 years. I really believe that. And I fondly hope that the formalistic view of philosophy of science will be reduced to its natural place, which is not at all empty but much more limited than what it has grown to be.

Galison: So in the end you have the view that the meaning-providing vestibular and muscular perception, and in a general sense the low level thinking is quite compatible with the Bourbakian aesthetic more structuralist account they are not the same project but they don't clash, they don't contradict one another.

Teissier: That's true. They don't clash, absolutely not. I lack the vocabulary to explain. Bourbaki's work is for a fairly large part what I would classify as language creation. He tries to find the best possible way to state, to prove the basic facts. Language creation is a fantastic operation. The other project is rather about meaning, as I say it is about creating myths of origin. Of course the two things are not disjoint. You must ultimately give meaning to the words that you've used or invented when you create language. But the two endeavors are clearly distinct. They both answer needs of the human mind, basic pulsions. We need to create language, we need to create these abstract structures but we also need to have meaning. If you look at history, before the non-Euclidian Revolution the meaning of mathematics was not a problem because it was in nature: the principle of Dirichlet was "true" because it is a natural fact. At the scale of scientific thought this was yesterday. Then there was a spell where we had to move away from the roots of mathematics in nature because they had proved to be unreliable; globally we lived without roots for a while, built a reinforced concrete platform on which to base mathematics, and rebuilt them on this by the axiomatic method. The impressive success and richness of this construction caused a certain type of philosophy of mathematics to develop. It was also boosted by the incredibly strong urge we have to find causes and that is, in my language, a low-level urge. But we can't live without roots for very long. Now the time has come to make roots again and we have the tools. I think this will occupy us for the next few decades.

Galison: Tools like neurophysiology.

Teissier: Yes. Also I think we've learned in this experience with the relationship of mathematics with the real world that one should aproach these things with great humility and not claim to understand the roots immediately. I think it is a very complex process. I fondly hope abstract mathematics will continue to develop pretty much in the way it does and I have no dissatisfaction with that at all. My only dissatisfaction is that I have so many problems understanding the objects that I really need some methods to do it.

Probably some people are more gifted and they don't need to attach meaning to the mathematical objects to be able to be happy with them.

Galison: Do you think that's so?

Teissier: Yes, I can think of some people who don't have that need at all, or at least do not seem to. I was once in a committee promoting one of them, say X, and I asked an extremely respected mathematician, Fields medalist, for a letter. He wrote a beautiful letter about X and his results, and at one point in the letter he says "he's seen what us simple mortals have failed to see", or something like that. This X is an extreme case of somebody who I think is truly a mathematical genius and sees things immediately and doesn't seem to need any meaning. I was there once when some people were explaining why they could not prove some theorem and X was in the audience and he proved the theorem then and there for strictly structural reasons… he said because of what you said this arrow has to be unique and this proves the theorem. He has a sort of structural view of mathematics which is extremely interesting. I think he defended his PhD when he was 16,17.

Galison: Is he still active?

Teissier: He's still active. So I believe there are people like that and it is good. They do not need any kind of meaning, but I do not think they suffice to create all mathematics or determine all interesting directions or research.

Galison: So when you have, in thinking about this meaning, one direction would be pursuing in the laboratory either with neurophysiology or with experimental psychology with children for instance, to try and understand the ways these are constructed. Another question might be: once you begin to think about this low level thinking that gives meaning, you might want to change Pedagogy in some way. Is that something you've thought about ?

Teissier: Yes, I have thought about that, I think it is really important

Galison: It is presumably still early days in the sense we only have a detailed account of low level thinking, but what kind of Pedagogical consequences would there be?

Teissier: Well I think you can think of that on several different levels. One of them is to illustrate, let's say. It is perhaps the easiest way to go at it now. In the early years of mathematical teaching when you introduce a notion, explain to the children that a line is not just an object you define, a line is an object of an environment where you can see lines you can walk on straight lines, make them, use them just as Einstein, make them imagine they are sitting on a ray of light, walking in a straight line, making smaller and smaller steps and approaching a point and stopping, so that they can feel with their bodies a tiny part of the mathematics. But I think it is important because many of them are lost at that stage. There was a famous textbook definition of the real line for the *Classe de Quatrième*, where the kids are about what 13, and the definition that was given was exactly the definition of the line as a principal homogeneous space under the translation group. The definition took five lines and I don't remember it by heart, but if you read it, that is what it said. So you give that to a teen and you hope that he will do something with it, but of course most of them don't… they who have help or are extremely gifted will survive but most of them don't. So I think teaching the teachers that many of these notions the notion of area for example is a very complex notion; it is not part of our elementary intuition of the world; our perceptual system doesn't evaluate the area of a lake in the precise way it evaluates our distance to its shore. Actually the notion of area is a late invention in the history of mankind, probably sixth century BC or something like that in Europe. In Homer, on a few occasions he spoke of the size of the cities and it is determined by the perimeter. So one could explain to the children that there is a problem there and make them realize it is important because of fields and crops, just very simple things like that. I have talked to a kid who was learning about angles, this was in the mountains, and the kid had no idea that the angles of his compass were the same as the angles in the school book. Just nobody told him, how is he supposed to know? Of course he was probably not extremely gifted, not a genius, but to see that the circle he had to draw in his exercise book, marking 90 degrees, 45 degrees and so on, that was the same thing as the circle of his compass had just not occurred to him. So how do you want him to understand the usefulness of adding angles. So this kind of totally elementary thing which sort of makes us understand that we live in space and that our body rotates, etc., we have actually totally eliminated from teaching. Of course this has to stop at some point you can't explain the third degree equation in simple terms like that so at some point you must say now we have seen problems, that's what the Greeks did. They looked at X^2 equals 2, X^2 equals 5, X^2 equals 7, then some genius said: let's look at all equations because then they saw the other problems where there are linear terms in X. So the problem is to solve all possible equations. This I think the kids can understand.

Galison: Like we heard about from Bombelli from Federica

Teissier: Yes exactly. So of course people will say this will take infinite time but I think it is much better to have kids of 16 who really understand what numbers are and really understand what angles are, what length is, what area is, they won't become mathematicians but they will be at ease in life, than to have kids who really…

Galison: Panic

Teissier: Panic exactly, every time they hear the word. What's the best solution?

Galison: One thing I thought I would ask is, once you begin to move from the low level thinking to more abstract and more complex combinations of things, you mentioned in the discussion today that you can go beyond the sort of primate constraints that…

Teissier: Yes

Galison: Are given to us, on neurophysiology, that's interesting maybe you could say a little bit about that.

Teissier: I think it is partly this process of sedimentation

Galison: What do you mean by that?

Teissier: I mean that we start with this, say, proto-mathematical concepts as I said I would love to have even 5 examples to give you, but I can only give you one, maybe two if I add boundaries and maybe three if I add ordinals. But not many for the moment. And we start doing things with them. Mathematics in some sense talks about the regularity of the world, regularity with respect to time, homogeneity of space. Our friend Meister said I think at some point, that narration is a way to tame the chaos of the world or something to that effect. It seems to me that mathematics originally is a way to talk about or capture the regularities which are observed: the sky, homogeneity of space and things like that, talk about this. The line is a prime example, so we start to capture the regularity of the world, and then according to our low level pulsions when we start to do Euclidean geometry we tend to go all the way we tend to make it complete because I think that one of the basic pulsions of low level thinking to complete what is not complete. So you push what you are doing until you feel there is a kind of completeness to it. So lets say in some sense Euclidean geometry represents a more or less complete state of affairs starting with, say, configurations of lines in the plane. A that point, Euclidean geometry becomes part of our equipment, we think of it as a whole, more or less, as a view of the world which we didn't have a as primate, as a primate we have of course much more elementary views, but once science has acquired that, its there we can use it to build something else. So we can do in different directions. I'm not saying we say ah! let's go to non Euclidean geometry …but for example we can't help to try to use that to work on curved surfaces, which helped to invent the Riemannian geometry, we are led naturally to Riemannian geometry. And for example I think its very striking they try to explain parallel transport is kind of cabled in our brains in some way which is not fully understood, but it is not an exaggeration to say that parallel transport is cabled. Now when Elie Cartan invents parallel transport I'm tempted to argue that he invents it *because* it is cabled in the brain, posibly for evolutionary reasons because its very important to be able to detect angles and parallelism is an extremal situation in the evaluation of angles.

Then, since this parallelism is cabled and becomes crucial in euclidean plane geometry, when we have a space that is no longer the plane of vision, we try to extend that concept to that new space. It may fail or not, in the case of parallel transport it works and if you look at the way Elie Cartan actually describes parallel transport, which is richer than what you find in differential geometry textbooks, it really sounds like: I would like to think of myself following a path and being able to carry this direction with me and all the consequences that it has. So then in turn, that becomes something on which you can apply low level thinking once you have realized that your, lets say primate in fact this case, its probably not primate, its probably man … Primate feeling of or concept or whatever of parallel transport, can be extended then you start thinking you can extend also other concepts of Euclidean Geometry to differentiable manifolds. For example you said OK let's see what happens if I make a loop, then you discover ahh! something goes wrong. So then you invent curvature, or something of that nature. So one way to tell the myth of origin of curvature is this, you take parallel transport, make a loop because that's an operation a primate can understand perfectly well and lo and behold …

Galison: There's a defect

Teissier: There's a defect and then because of your low level thinking, when you find an obstruction, analyze it and give it a name. First, of course, you try to measure it, you see if it is intrinsic, you see if it depends on the loop… All these are in my mind are low level methods of reasoning, first is it independent of the loop? Yes? then let's give it a name because it has something intrinsic, it is an invariant of something, what is it an invariant of? then you discover ahaaa! it is the obstruction to mapping isometrically the surface to a plane, and that's beautiful because it is not at all in the construction you made. You started with parallel transport, you looked at what happens like a child makes some experiments to see what happens, something happened and you named it, then you tried to understand it, and found that it has varied interpretations. So that's when I said: this is a beautiful theory. I feel beauty because I have an element of surprise, because I feel, after I understood the theorem I know that I've used nothing but the Riemannian structure to begin with. Nothing abstract, just that. And I find that I have measured an obstruction to an operation which I find important of course. Then from there you can go on to connections because you try to elaborate a world in which you can play this game but with things more complicated than tangent vectors or whatever. So that's the way I like to think about these things. If I had to make a course of differential geometry I would try to say not begin with definition, rigid answer, but say OK lets look at that and then in the end, you can find you can reinterpret everything with some meaning. This is something which you don't find usually in the books of differential Geometry except maybe for Spivak's or Hermann's books

Galison: Yes

Teissier: For some reason they are not the books which the majority of our math. students look at. But you can sometimes find this approach in physics books.

Galison: Yes, the Wheeler book which I talked about in my paper, that's very much what he does, it is very physical

Teissier: Actually when I was in charge of the Math. library at the Ecole Normale I bought a lot of physics books for the reason that things like connections, Chern classes, are explained much more clearly in physics books, according to my feelings, than in the math books.

Galison: I mean in Wheeler you go from parallel transport you define these things then you get to the Ricci curvature tensor, you go in that direction.

Teissier: Yes but for what reason don't you ever find that in a classical differential geometry book?

Galison: Because you want to go from the general to the specific, I don't know, I mean it is a different strategy of narration.

Teissier: Yes

Galison: Do you have students who are interested in some of these things?

Teissier: Actually yes, I think I've managed to contaminate some. Fortunately they don't view things exactly as I do, but I have one student in particular who likes to teach in that way. He does it almost naturally, I think he is a fantastic teacher. He gave a course this year on topology according to Poincaré in which he really explained topology as Poincaré would. His students are very fortunate. He doesn't have this philosophical aspect of things but as I say spontaneously he's someone who needs to understand mathematics down to the roots. I think that's part of the reasons why he became my student. But of course his thesis is written in the classical style and all his papers are.

Galison: So you said something this morning which I thought was interesting, which was people tend to conflate the foundations of truth and the foundations of meaning. Tell me little what you mean by that.

Teissier: Well it is more or less what we've been discussing …we claim to have foundations of truth when we have a coherent system of axioms inside which we can build the objects of which we are talking. Because in principle then we can prove whenever we know the proof of a theorem, we can translate it into a logical system in which it will be valid. For example the real line has several axiomatic definitions, you can define it with Dedekind cuts, you can define it as the completion of the reals, you can define it with the theory of real-closed fields or something of that nature. That's what I call the foundations of truth. Again you can see that as a story of origin, as the existence of a story of origin to be precise, because once we have stated an axiomatic system we still have to tell the story as a sequence of deductions, which no one will ever be able to do, but in principle it is feasible. And the foundation of meaning is what I described, to say why are we able to think mathematics on the real line? It is because we transport on that object, pairs of intuitions of our perceptual system, vestibular and visual, and if we do that we organize the real line in some way and the mathematical expression of this organization is what we know, an ordered field, but the reason why we really can think about it in a meaningful way, is what I said. The claim I make, which one can certainly discuss, is that both are necessary if we just follow meaning we go into terrible trouble when we are high up in the tree but if we just keep the logic, the truth foundations, we lose meaning at some point.

Galison: We stay high up in tree and we don't touch the ground.

Teissier: Not only that , but we become unable to progress, because to progress you must have desire, to have desire you must have something meaningful. Again some people, perhaps the mathematician X whom I mentioned earlier, get meaning from very high up in the tree, from the structure of the environment, somehow. That is what is meaning for him, knowing the place of the object in some huge category or n-category. and that's enough for him to work with. But for teaching, for perhaps less gifted people and I think also for ontological reasons. I don't think you can stay at that level.

Galison: Ontological meaning…?

Teissier: Yes, I think it is necessary, it is in the nature of mathematics that somehow we must preserve the roots of meaning. That's a philosophical position, I'm convinced that it is true, of course I cannot offer no proof but I'm convinced if we do not pay attention to the foundations of meaning we would run into trouble too.

Galison: What kind of trouble?

Teissier: Lose interest, waste time on meaningless problems, this happens. Make uninteresting mathematics, which will degenerate into something purely academic. I think there is not a real danger of this happening nowadays for most of mathematics. But in the distant future I think it might happen. Also we lose, incidentally, this is not a philosophical statement, we lose students because they are not interested if we give them those abstract definitions and tell them: now go home and come back with a theorem!

Galison: Do you think that even knowing that there are some people who can come up with very interesting mathematics without passing through the connections that are given by low level thinking and some kind of physical or visual or motor understanding. Do you think that understanding in some way requires both the abstract and the meaningful? It is a different question… I'm not asking the question whether it is possible for certain individuals to continue to prove and discover interesting things. I take it as a matter of fact that there are such people; let us just accept that. But do you think that in a broader sense understanding in some important way requires this?

Teissier: Yes that's a very interesting question, yes I do think so.

Galison: I do too

Teissier: It is a very delicate question I think it has to do with the nature of language again. We need the structure of language to understand what we say, just as we need the meaning of words. This morning Meister was talking about semantics and I almost objected. I think that semantics is a loaded word, let's use meaning. I think it is more vague, semantics is too Tarskian or whatever.. But yes I think that's one of the important issues and that why I think we must strive to construct meaning. The syntactic, or to speak like Meisters, the structural aspect is important. If I didn't think that probably I wouldn't be a mathematician I would be doing something else. But it tends to hide meaning after a while and so I think both are necessary and their interaction is necessary, their interaction is something that we can never really understand. It is transcendental, we can't make an analysis of it even if we believe that the quantum harmonic oscillator is at work in the neural connections as Penrose claims I don't think it will give us an explanation of the relationship between, the way our mind structures things and the way it builds meaning. But what I like to think is that to some extent we can approximate that, using those low level pulsions and thinking of them as a kind of cellular automaton: a sequence in which you apply a low level pulsions to some basic intuition then it gives a result then you apply another low level pulsion, is it invariant or something like that, and then perhaps we can produce a new understanding of the way meaning propagates… it will never be perfect but I would be happy to have some kind of vague image of the way it propagates, some good examples. And then perhaps once we have that we can guess a little at the way structure emerges, like structure emerges in cellular automata. You could say this is my program for the next 200 years. That's the way the thing which I'm really interested, to see if it really makes sense to try to do that. And I admit that I start from a very tiny basis, the vision of the real line, and there is an enormous amount of work to do. But fortunately I am part of a large movement: there are now many people working not only on the neurophysiology, but also trying to understand what happens in our brain. It is very diverse. I'm in a group, actually coordinator for a group which contains two mathematicians, Bennequin and me and a number of neurophysiologists. It is called neurogeometry and are trying to see how we can use mathematical ideas to understand what goes on in the functional structure of the visual areas, there are many fascinating comments with each little experiment costs so much time and energy so it is moving slowly. It is not like mathematics: in a way the obstacles are there all the time. Sometimes mathematicians have good ideas and that's it, but there, good idea or not you have to do the lab work and there is no going around it.

Galison: This is a speculative question, but one of the discussions that take place at the boundary between physics and mathematics is whether the intuitions afforded by what for mathematicians are poorly defined objects or not well defined objects like path integration and so on, are of use in formulating mathematical theorems in algebraic geometry for instance, or Morse theory or knot theory or mirror symmetry. When you think about what's going on when that happens you have to ask what is it that permits a set of intuitions from collisions among these one dimensional physical objects, using methods that were developed to think about point particles, electrons, scattering, to suggest results in algebraic geometry? it is so unexpected that that should have this kind of heuristic role or suggestive intuitive role. What do you think is going on when that happens? And then my follow up question will be is that anything like what is happening at a more fundamental level, with low level thinking? But let's start with the geometrical.

Teissier: I tried to think about that. For example, for a mathematician or a physicist, again part of our sedimentation will contain operations like "evaluate a function" maybe even "integrate a function". For example Feynman diagrams is an idea which I immediately loved, I think essentially because of that, I viewed it the first time I saw it as a sort of low level construction. It is so natural in some sense: you look at all possibilities and then you realize they have some structure and order and then you use that. And you integrate over that and you make a kind of average and then you use a kind of extremal principle and out comes physics. So yes I think this is definitely an operation …I don't want to give the impression that low level thinking will eventually encompass all interesting ideas that's not at all my purpose. But when I see somebody facing a problem which concerns path from A to B, say: let me look at all paths except this is not a thought of a primate, but this is not very far above the thought of a primate. It is not very far above an elementary perception of the world. And now it turns out, for reasons which are coming from the environment that this has a structure so that's the part where the physics kicks in somehow. I think that even in the most abstract realms of thought when you are talking about homogeneous spaces, compact Lie groups, things like that , this kind of reflex: how many ways to go from A to B? Oh, there are many ways to go from A to B but clearly there is one that looks nicer, this kind of thing is part of our being, not as mathematicians, but as primates

Galison: A physicist says: clever primate!

Teissier: It is a clever primate, its an educated primate, the primate of a physicist or of a mathematician is not the same as the primate of a literary person. It has not had the same education so to speak. No, when we learn to be a physicist or a mathematician, we learn to talk to our primate according to a certain code, we explain to it e.g. to see a geodesic when you throw a stone and the primate says OK, and then you teach him to carry this image in more abstarct spaces, but the stone's trajectory is still there. But one advantage of the primate which we must not underestimate, he has these pulsions, maybe not all the pulsions of a human being but many of them, he has this perception of the world. But he has an enormous advantage over us… he's relentless, he never stops, he's infinitely patient he will try and try again. We get tired and say this is boring, but the primate never gets bored and I think that's how eventually we get problems solved in our dreams, we somehow explain, that's compatible with what Poincaré says, we explain the problem to our primate and the primate is good natured, and he works and he tried and he tries and we go to sleep, we go to the movies and at some point somehow the primate, says ahhh! Then he rings a bell to tell our conscious self that he has broken through, and we perceive it as a new idea. This can take the form of a ghost, I knew a mathematician when I was young, who eventually took his own life, he was persuaded that his mathematics was given to him by a goddess, a ghost. He was an excellent mathematician. His mathematics was brought to him by a goddess, and he is not a unique case. One should think about that, there must be a reason why some people have ghosts or goddesses who bring them mathematical statements and we have dreams, dreams of mathematics. Sometimes I have dreams to tell me: what you wrote today is wrong, but sometimes to tell me: this looks good, and sometimes it works. It is not accurate but as I said it is sometimes close to the good idea, and with a little effort you work it through. Poincaré describes it, many mathematicians share that experience, but nobody seems to be interested in finding out why something like that happens, probably because there was no place to begin. But now we have a tiny window into the unconscious. The neurophysiological window is probably a tiny window but we must look through it to see what we can see, so I think that's exciting.

Galison: We've discussed about the relationship of the true to the meaningful and the foundation of the true and the foundation of meaningful. In the non mathematical sciences true is often used in another sense in an empirical sense rather than a logical sense or coherence. But do you think that the empirically true falls on the side of the true in the sense that we've been discussing it or more on the side of the meaningful? Or is it a *tertium quid*?

Teissier: That's a subtle question. I think its probably of the third kind, your tertium quid. Empirically true is meaningful only in so far as the experiment is meaningful. I know there are people in the philosophy of physics,who say that every experiment is meaningful by definition otherwise we wouldn't be making it. But that doesn't teach us much.

Galison: No

Teissier: On the other hand, the empirically true is always relative to certain framework of experiment so it is not true in the sense that we like to think of truth as eternal. It is a kind of local truth

Galison: Contextual

Teissier: Yes, so I think it really falls in that third category: contextually true and not necessarily meaningful. But even in mathematics we can make examples of a special class of variety or whatever, look for something, find it and be tempted to believe that it is true in a much wider class, local experimental truth. But to say that the statement is true in a wider class is obviously a conjecture. But it is an interesting question because precisely you could say that if it is empirically true in the given context of the experiment and so on, can you conjecture that it is absolutely true? Is that something one would like to call a conjecture, I know in mathematics it would be a conjecture or question.

Galison: We've discussed a little bit this morning the possible evolutionary role in picking out some of these primate… you might say I'm putting words in your mouth, a *prima facie* argument for the evolutionary nature of some of these elements of low level thinking is that we share some with primates, and the first order explanation order of that is that we come from the same place. How important is the evolutionary aspect of this account of low level thinking to you? Is that a kind of optional next step in the argument or is it a necessary step to the argument?

Teissier: It is partly a necessary step because if I want low level thinking to play its role in the construction of meaning, then I think we have to share it partly with primates. Again my definition of primate is not the next chimpanzee, it is our ancestral experience of the world which we for a large part share with primates and certainly is unconscious and does not depend on language. So it is important that we share it with primates, because observation shows that they have behaviors that are rather close to us and we have to explain that. My basic view is that to a large extent our experience of the world as primates is the main reservoir of meaning for us.

Of course if you told me there is no continuity between low level thinking of man and low level thinking of chimpanzee it would not destroy my need to understand, but the idea of continuity is so much nicer! You may say it answers a low-level need in me. On the other hand that's a question which I have asked my self repeatedly: low level thinking is probably not a collection of pulsions as I describe it, the need to generalize, the need to categorize, the need to analyze, etc.
I think somehow what we inherit from the primates, in addition to way to perceive the world, is the possibility to create low level thinking rather than the set of low level thoughts. So it is evolutionary in that sense. Perhaps if one could make experiments with babies, which one can certainly not do, one could create babies who would lack some of these pulsions. Perhaps by accident there are such people who lack or have a defective pulsion to organize or to analyze or to synthesize. What is really important for me is that I need a driving force for the creation of meaning to go through mathematics. And these are the driving force: desires. There are driving forces for other activities, sexual desire, desire to command, to be the first, which chimpanzee for example have very strongly. You probably know that chimpanzees are socially very brutal, Bonobos are much nicer, resolve conflicts by sexual activity while chimpanzee resort to violence. So we leave that aside because it doesn't seem to be very closely related to mathematics but it exists.

In so far as mathematics is concerned I think that this low level thinking is part of our equipment to understand the world. We don't inherit our understanding of the world. We inherit the pulsions that ultimately allow us to do that, we inherit the pulsion to form words as babies but we don't inherit language. We form words and then we listen to the answer and by a process of learning we find the way to speak. So I think it is very similar, we have these pulsions we try them on the world we try to organize this and that, perhaps that's a kind of experiment I would love to see. If we can observe infants, trying to organize the world according to certain rules because there are many things which infants do which we consider as totally meaningless but perhaps if we look at it in a different view, as trying to check that's something's is invariant by translation, that objects still exist when we turn around, perhaps we could observe things which gives meaning to some behavior of the infants that have to meaning a priori… I think we inherit that, and as we inherit it it has to have some structure, which I'm totally unable to imagine, and these pulsions are just there, there permanently one another.

Galison: I mean one question would be, with any inherited trait there are multiple possibilities about how it comes about …you could have something that has a selective advantage but you could also have something like what Stephen Jay Gould calls a spandrill, a side effect. For example you would have a trait A which is advantageous for natural selection. It allows me to type, but its not because I survive better if I can type. And another is drift, which concerns certain things that are not selective but turns out to be advantageous. So different mechanisms might be involved, all of which still could be inherited.

Teissier: Yes, I agree. I think I said something about that, about the line, we somehow construct the capacity to perceive curves but probably we don't inherit the capacity to perceive lines. Line is a side effect it is a kind of minimal curve so as we perceive curves, there is a space of curves, there is an energy of perception of each curve and there is a minimal one and that's the line and that's it. It is not because we inherit from our parents the capacity to see straight lines or because detecting straight lines has a direct evolutionary advantage. I don't believe that for a minute. Yes you're right but how can we begin to understand how this kind of thing works? Because we're talking about unconscious pulsions, we must invent experiments, that's one of the things I like to talk about with Berthoz, because he has a great talent for thinking clever experiments. We must invent experiments and first try to identify how these things work and then see if we can find them in the chimpanzees…There is a lot of work to be done, but that's faster than neurophysiology

Galison: The experiments on the individual neuron, that for instance my colleague Hubel did, those beautiful experiments where only a convex object or only a small object moving across the field of vision against the background would fire it, but not the background itself… they're stunning experiments.

Teissier: Yes when I spoke about the neurophysiologists' work on parallel transport, it is a very special case of thousands of very clever experiments on the operations that neurons and their connections are able to do. What they do is absolutely beautiful as an experimental corpus. and probably a lot of it is meaningful for geometry. I mean what I said, concerns maybe one thousandth of what goes on when we view a simple scene: we detect contours, we detect texture, we detect moving things. The neurophysiologists say that 90% of what we perceive does not reach the conscious level.
Perhaps finally what mathematics and narrative have in common is that both are ways to create, using language, coherent pictures of the world, which in some sense <explain> how it works.
We must remember that the first <explanations> of the world in our culture, by mythology, were a sort of modelization of the physical world by characters, the Gods, who were rather human in their pulsions. And we must remember also that just like mathematical objects, and just like the characters in a novel, the Gods acquired a life of their own, well beyond their use as <explanations>. The creation of mathematics and the creation of narrative are both, I think, powered by pulsions that are very close.

Galison: This has been terrific, thank you for now and we will continue our discussions another time and place, in Paris or Harvard.

End of interview.