

FINITE DIMENSIONAL REPRESENTATIONS OF QUANTUM AFFINE ALGEBRAS

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1. INTRODUCTION

In the case of semi-simple Lie algebras, the structure of the Grothendieck ring of finite dimensional representations of the quantum algebra is well understood : it is analogous to the classic case $q=1$ and we have a ring homomorphism of characters. For the general case of Kac-Moody algebras the picture is less clear. In the affine case, E. Frenkel and N. Reshetikhin have recently introduced an injective ring homomorphism of q -characters. It gives informations about the decomposition in Jordan subspaces for a class of commutative elements. The homomorphism of q -characters has a nice symmetry property analogous to the classic action of the Weyl group : the image is the intersection of the kernels of screening operators. In the ADE case, H. Nakajima, motivated by the geometry of quiver varieties, introduced t -analogs of q -characters. From representation theory point of view, it gives more informations about Jordan subspaces. We define t -analogs of screening operators and give a symmetry property of q, t -characters. Our construction is purely algebraic and is extended to the non-simply laced case : it could help to extend Nakajima's theory in this case.

2. HOMOMORPHISM OF CHARACTERS IN FINITE CASE

\mathfrak{g} : simple Lie algebra of rank n , $I = \{1, \dots, n\}$

$\mathfrak{h} \subset \mathfrak{g}$: Cartan subalgebra

$\Lambda \subset \mathfrak{h}^*$: lattice of weights of \mathfrak{g}

$\omega_i \in \Lambda$: fundamental weights

$\text{Rep}(\mathcal{U}(\mathfrak{g}))$: Grothendieck ring of finite dimensional representations (with \oplus and \otimes)

We have an injective homomorphism of rings :

$$\chi : \text{Rep}(\mathcal{U}(\mathfrak{g})) \rightarrow \mathbb{Z}[y_i^{\pm}]_{i \in I}$$

$$\chi(V) = \sum_{\lambda = \sum_{i \in I} m_i \omega_i \in \Lambda} \dim(V_\lambda) \prod_{i \in I} y_i^{m_i}$$

where V_λ is the weight space of V :

$$V_\lambda = \{x \in V / \forall h \in \mathfrak{h}, h.x = \lambda(h)x\}$$

We have a symmetry property related to the Weyl groups W :

$$\text{Im}(\chi) = \mathbb{Z}[y_i^{\pm}]_{i \in I}^W \simeq \mathbb{Z}[T_1, \dots, T_n]$$

The quantum case is analogueous :

$$\text{Rep}(\mathcal{U}_q(\mathfrak{g})) \simeq \text{Rep}(\mathfrak{g}) \simeq \mathbb{Z}[\Lambda]^W \simeq \mathbb{Z}[T_1, \dots, T_n]$$

where $\text{Rep}(\mathcal{U}_q(\mathfrak{g}))$: finite-dimensional representations of type 1.

In quantum affine case $\text{Rep}(\mathcal{U}_q(\hat{\mathfrak{g}}))$, the picture is less clear.

3. QUANTUM AFFINE ALGEBRA

\mathfrak{g} : simple Lie algebra of rank n , $I = \{1, \dots, n\}$

$(C_{ij})_{1 \leq i, j \leq n} = \left(\frac{2(\alpha_i, \alpha_j)}{(\alpha_i, \alpha_i)} \right)$: Cartan matrix of \mathfrak{g}

$\mathfrak{g} \subset \hat{\mathfrak{g}}$: affine Lie algebra, $(C_{i,j})_{0 \leq i, j \leq n}$ generalized Cartan matrix of $\hat{\mathfrak{g}}$

$r_i = \frac{(\alpha_i, \alpha_i)}{2} \in \mathbb{Z}$, $B_{i,j} = r_i C_{i,j}$ symmetric matrix

$q \in \mathbb{C}^*$ not a root of unity, $q_i = q^{r_i}$

Definition 1. *The quantum affine algebra $U_q(\hat{\mathfrak{g}})$ is the $\mathbb{C}(q)$ -algebra generated by E_i, F_i, K_i, K_i^{-1} ($0 \leq i \leq n$) with relations :*

$$\begin{aligned} K_i K_i^{-1} &= 1 = K_i^{-1} K_i \\ K_i E_j &= q^{B_{ij}} E_j K_i \\ K_i F_j &= q^{-B_{ij}} F_j K_i \\ E_i F_j - F_j E_i &= \delta_{ij} \frac{K_i - K_i^{-1}}{q_i - q_i^{-1}} \\ \sum_{s=0..1-C_{ij}} (-1)^s \begin{bmatrix} 1 - C_{ij} \\ s \end{bmatrix}_{q_i} E_i^{1-C_{ij}-s} E_j E_i^s &= 0 \quad si \ i \neq j \\ \sum_{s=0..1-C_{ij}} (-1)^s \begin{bmatrix} 1 - C_{ij} \\ s \end{bmatrix}_{q_i} F_i^{1-C_{ij}-s} F_j F_i^s &= 0 \quad si \ i \neq j \end{aligned}$$

An Hopf algebra structure is given by formulas :

Comultiplication :

$$\Delta(E_i) = E_i \otimes 1 + K_i \otimes E_i, \Delta(F_i) = F_i \otimes K_i^{-1} + 1 \otimes F_i, \Delta(K_i) = K_i \otimes K_i$$

Counit :

$$\epsilon(E_i) = \epsilon(F_i) = 0, \epsilon(K_i) = 1$$

Antipode :

$$S(E_i) = -K_i^{-1} E_i, S(F_i) = -F_i K_i, S(K_i) = K_i^{-1}$$

4. DRINFELD NEW REALIZATION

Theorem 1. (Drinfeld new realization)

$U_q(\hat{\mathfrak{g}})$ is generated by $x_{i,m}^{\pm}$ ($1 \leq i \leq n, m \in \mathbb{Z}$), k_i^{\pm} ($1 \leq i \leq n$), $h_{i,m}$ ($1 \leq i \leq n, m \in \mathbb{Z}^*$), $c^{\pm \frac{1}{2}}$, with relations :

$$\begin{aligned} k_i k_j &= k_j k_i \\ k_i h_{j,m} &= h_{j,m} k_i \\ k_i x_{j,m}^{\pm} k_i^{-1} &= q^{\pm B_{ij}} x_{j,m}^{\pm} \\ [h_{i,m}, x_{j,m'}^{\pm}] &= \pm \frac{1}{m} [m B_{ij}]_q c^{\mp \frac{|m|}{2}} x_{j,m+m'}^{\pm} \\ x_{i,m+1}^{\pm} x_{j,m'}^{\pm} - q^{\pm B_{ij}} x_{j,m'}^{\pm} x_{i,m+1}^{\pm} &= q^{\pm B_{ij}} x_{i,m}^{\pm} x_{j,m'+1}^{\pm} - x_{j,m'+1}^{\pm} x_{i,m}^{\pm} \\ [h_{i,m}, h_{j,m'}] &= \delta_{m,-m'} \frac{1}{m} [m B_{ij}]_q \frac{c^m - c^{-m}}{q_j - q_j^{-1}} \\ [x_{i,m}^+, x_{j,m'}^-] &= \delta_{ij} \frac{c^{\frac{m-m'}{2}} \phi_{i,m+m'}^+ - c^{-\frac{m-m'}{2}} \phi_{i,m+m'}^-}{q_i - q_i^{-1}} \end{aligned}$$

$$\sum_{\pi \in \Sigma_s} \sum_{k=0..s} (-1)^k \begin{bmatrix} s \\ k \end{bmatrix}_{q_i} x_{i,m_{\pi(1)}}^{\pm} \cdots x_{i,m_{\pi(k)}}^{\pm} x_{j,m'}^{\pm} x_{i,m_{\pi(k+1)}}^{\pm} \cdots x_{i,m_{\pi(s)}}^{\pm} = 0$$

$\phi_{i,m}^{\pm} \in \mathcal{U}_q(\hat{\mathfrak{g}})$ are defined by :

$$\Phi_i^{\pm}(u) = \sum_{m=0..\infty} \phi_{i,\pm m}^{\pm} u^{\pm m} = k_i^{\pm 1} e^{\pm(q-q^{-1}) \sum_{m'=1..\infty} h_{i,\pm m'} u^{\pm m'}} \in \mathcal{U}_q(\hat{\mathfrak{g}})[[\pm u]]$$

Remark: $\{\phi_{i,\pm m}^{\pm}, i \in I, m \in \mathbb{Z}\}$ is commutative

Every finite dimensional representation is a sum

$$V = \bigoplus_{\gamma} V_{\gamma}$$

where V_{γ} is a simultaneous generalized eigenspace :

$$V_{\gamma} = \{x \in V / \exists p \in \mathbb{N}, \forall i \in \{1, \dots, n\}, \forall m \in \mathbb{Z}, (\phi_{i,m}^{\pm} - \gamma_{i,m}^{\pm})^p \cdot x = 0\}$$

5. FINITE DIMENSIONAL REPRESENTATION OF $\mathcal{U}_q(\hat{\mathfrak{g}})$

Theorem 2. (Chari-Pressley) *Every simple $V \in \text{Rep}(\mathcal{U}_q(\hat{\mathfrak{g}}))$ is a highest weight representation V , that is to say $\exists v_0 \in V$ (highest weight vector) :*

$$x_{i,r}^+ \cdot v_0 = 0, V = \mathcal{U}_q(\hat{\mathfrak{g}}) \cdot v_0, c^{\frac{1}{2}} \cdot v_0 = v_0, \phi_{i,m}^{\pm} \cdot v_0 = \gamma_{i,m}^{\pm} v_0$$

Moreover we have an i -uplet $(P_i(u))_{i \in I}$ of (Drinfeld-)polynomials, $P_i(0) = 1$, such that :

$$\gamma_i^{\pm}(u) = \sum_{m \in \mathbb{N}} \gamma_{i,\pm m}^{\pm} u^{\pm} = q_i^{\deg(P_i)} \frac{P_i(uq_i^{-1})}{P_i(uq_i)} \in \mathbb{C}[[u^{\pm}]]$$

and $(P_i)_{i \in I}$ parametrizes simple modules in $\text{Rep}(\mathcal{U}_q(\hat{\mathfrak{g}}))$.

Theorem 3. (Frenkel-Reshetikhin) *The eigenvalues $\gamma_i(u)^{\pm} \in \mathbb{C}[[u]]$ of $V \in \text{Rep}(\mathcal{U}_q(\hat{\mathfrak{g}}))$ have the form :*

$$\gamma_i^{\pm}(u) = q_i^{\deg(Q_i) - \deg(R_i)} \frac{Q_i(uq_i^{-1})R_i(uq_i)}{Q_i(uq_i)R_i(uq_i^{-1})}$$

where $Q_i(u), R_i(u) \in \mathbb{C}[u]$ and $Q_i(0) = R_i(0) = 1$.

Example: $\mathfrak{g} = \mathfrak{sl}_2$, $I = \{1\}$. Explicit calculations with Jimbo's evaluation homomorphism $ev_a : \mathcal{U}_q(\hat{\mathfrak{sl}}_2) \rightarrow \mathcal{U}_q(\mathfrak{sl}_2)$.

For $P(u) = 1 - ua$ we have $M_a = \mathbb{C}v_0 \oplus_{\mathbb{C}} \mathbb{C}v_1$:

$$v_0 \text{ highest weight vector : } \phi^{\pm}(u) \cdot v_0 = q^{\frac{1-uaq^{-1}}{1-uaq}} v_0, P(u) = 1 - ua$$

v_1 simultaneous eigenvector :

$$\phi^{\pm}(u) \cdot v_1 = q^{-1} \frac{1-uaq^3}{1-uaq} v_1, Q(u) = (1-ua), R(u) = (1-uaq^2)(1-ua)$$

6. Q-CHARACTERS

Definition 2. (Frenkel-Reshetikhin's q-characters)

$$\chi_q(V) = \sum_{\gamma} \dim(V_{\gamma}) \prod_{i \in I} \prod_{r=1..k_{\gamma_i}} Y_{i,a_{\gamma_{ir}}} \prod_{s=1..l_{\gamma_i}} Y_{i,b_{\gamma_{is}}}^{-1} \in \mathbb{Z}[Y_{i,a}^{\pm}]_{i \in I, a \in \mathbb{C}^*}$$

where

$$Q_{\gamma_i}(z) = \prod_{r=1..k_{\gamma_i}} (1 - za_{\gamma_{ir}}), R_{\gamma_i}(z) = \prod_{r=1..l_{\gamma_i}} (1 - zb_{\gamma_{ir}})$$

Example:

$$\chi_q(M_a) = Y_a + Y_{aq^2}^{-1}$$

Theorem 4. (Frenkel-Reshetikhin) *The map*

$$\chi_q : \text{Rep}(\mathcal{U}_q(\hat{\mathfrak{g}})) \rightarrow \mathbb{Z}[Y_{i,a}^{\pm}]_{i \in I, a \in \mathbb{C}^*} = \mathcal{Y}$$

is an injective ring homomorphism and the diagram is commutative :

$$\begin{array}{ccc} \text{Rep}(\mathcal{U}_q(\hat{\mathfrak{g}})) & \xrightarrow{\chi_q} & \mathbb{Z}[Y_{i,a}^{\pm}]_{i \in I, a \in \mathbb{C}^*} \\ \downarrow \text{res} & & \downarrow \beta \\ \text{Rep}(\mathcal{U}_q(\mathfrak{g})) & \xrightarrow{\chi} & \mathbb{Z}[y_i^{\pm}]_{i \in I} \end{array}$$

Corollary 1. *The ring $\text{Rep}(\mathcal{U}_q(\hat{\mathfrak{g}}))$ is isomorphic to $\mathbb{Z}[X_{i,a}]_{i \in I, a \in \mathbb{C}^*}$.*

Example:

$$\begin{aligned} \chi_q(M_{aq^2} \otimes M_a) &= (Y_{aq^3} + Y_{aq}^{-1})(Y_{aq} + Y_{aq^{-1}}^{-1}) \\ &= Y_{aq^3} Y_{aq} + Y_{aq^3} Y_{aq^{-1}}^{-1} + Y_{aq}^{-1} Y_{aq^{-1}}^{-1} + 1 = \chi_q(N) + \chi_q(\text{trivial module}) \end{aligned}$$

where N simple module with $\dim(N) = 3$ and $P(u) = (1 - aq^3u)(1 - aqu)$.

Note that $M_{aq^2} \otimes M_a \neq N \oplus \text{triv.}$ is not semi-simple. In fact we have an exact sequence :

$$0 \rightarrow N \rightarrow M_{aq^2} \otimes M_a \rightarrow \text{trivial module} \rightarrow 0$$

Example:

$$\chi_q(M_a \otimes M_a) = Y_a^2 + 2Y_a Y_{aq^2}^{-1} + Y_{aq^2}^{-1}$$

$M_a \otimes M_a$ simple module with $P(u) = (1 - au)^2$.

7. SYMMETRY PROPERTY OF q -CHARACTERS

Let be $A_{i,a}^{\pm} \in \mathcal{Y}$:

$$A_{i,a} = Y_{i,aq_i} Y_{i,aq_i^{-1}} \left(\prod_{j/C_j, i=-1} Y_{j,a}^{-1} \right) \left(\prod_{l/C_l, i=-2} Y_{l,aq}^{-1} Y_{l,aq^{-1}}^{-1} \right) \left(\prod_{m/C_m, i=-3} Y_{m,aq^2}^{-1} Y_{m,a}^{-1} Y_{m,aq^{-2}}^{-1} \right)$$

Definition 3. (Frenkel-Reshetikhin's screening operators)

$$\begin{aligned} S_i : \mathcal{Y} &\rightarrow \bigoplus_{a \in \mathbb{C}^*} \mathcal{Y} \cdot S_{i,a} / \sum_{a \in \mathbb{C}^*} \mathcal{Y} \cdot (S_{i,aq_i^2} - A_{i,aq_i} \cdot S_{i,a}) \\ S_i(Y_{i,a}) &= Y_{i,a} \cdot S_{i,a} \\ \forall U, V \in \mathcal{Y}, S_{i,a}(UV) &= U S_{i,a}(V) + V S_{i,a}(U) \end{aligned}$$

Theorem 5. (Frenkel-Reshetikhin-Mukhin)

$$\text{Im}(\chi_q) = \bigcap_{i \in I} \text{Ker}(S_i)$$

and

$$\text{Ker}(S_i) = \mathbb{Z}[Y_{j,a}^{\pm}]_{j \neq i, a \in \mathbb{C}^*} \otimes \mathbb{Z}[Y_{i,b}(1 + A_{i,bq_i}^{-1})]_{b \in \mathbb{C}^*} = \mathfrak{K}_i$$

Example:

$$\begin{aligned} \chi_q(M_a) &= Y_a(1 + A_{aq}^{-1}) \\ \chi_q(N) &= Y_{aq^3} Y_{aq}(1 + A_a^{-1} + A_{aq^2}^{-1} A_a^{-1}) \\ &= Y_{aq^3} Y_{aq}(1 + A_a^{-1} + A_{aq^2}^{-1} A_a^{-1}) \\ &= Y_{aq^3} Y_{aq}(1 + A_a^{-1})(1 + A_{aq^2}^{-1}) - 1 \end{aligned}$$

Remark: It is analogous to the classic symmetry property of χ : $\text{Im}(\chi) = \mathbb{Z}[y_i^{\pm}]_{i \in I}^W$

It gives an algorithm to construct in purely combinatorial way the q -characters.

8. t -ANALOGS OF q -CHARACTERS (ADE CASE)

We assume \mathfrak{g} is of type ADE . Motivated by the geometry of quiver varieties, Nakajima defines : homomorphisms of additive group :

$$\hat{\chi}_{q,t} : \text{Rep}(\mathcal{U}_q(\hat{\mathfrak{g}})) \rightarrow \hat{\mathcal{Y}}_t = \mathbb{Z}[W_{i,a}, V_{i,a}, t^{\pm}]_{i \in I, a \in \mathbb{C}^*}$$

$$\chi_{q,t} : \text{Rep}(\mathcal{U}_q(\mathfrak{g})) \rightarrow \mathcal{Y}_t = \mathbb{Z}[Y_{i,a}^{\pm}, t^{\pm}]_{i \in I, a \in \mathbb{C}^*}$$

a new twisted multiplication $*$ on $\hat{\mathcal{Y}}_t$

$$m_1 * m_2 = t^{2d(m_1, m_2)} m_1 m_2$$

t -analogs of \mathfrak{K}_i :

$$\mathfrak{K}_{t,i} = \mathbb{C} \mathcal{Y}_t, \hat{\mathfrak{K}}_{t,i} \subset \hat{\mathcal{Y}}_t$$

Theorem 6. (Nakajima) *The maps $\chi_{q,t}, \hat{\chi}_{q,t}$ are injective, the diagram :*

$$\begin{array}{ccc} \text{Rep}(\mathcal{U}_q(\hat{\mathfrak{g}})) & \xrightarrow{\hat{\chi}_{q,t}} & \hat{\mathcal{Y}}_t \\ \downarrow & & \downarrow \hat{\Pi}_t \\ \text{Rep}(\mathcal{U}_q(\mathfrak{g})) & \xrightarrow{\chi_{q,t}} & \mathcal{Y}_t \\ \downarrow & & \downarrow \Pi_t \\ \text{Rep}(\mathcal{U}_q(\hat{\mathfrak{g}})) & \xrightarrow{\chi_q} & \mathcal{Y} \end{array}$$

is commutative. We have :

$$\text{Im}(\hat{\chi}_{q,t}) \subset \bigcap_{i \in I} \hat{\mathfrak{K}}_{t,i} = \hat{\mathfrak{K}}_t$$

$$\text{Im}(\chi_{q,t}) = \bigcap_{i \in I} \mathfrak{K}_{t,i} = \mathfrak{K}_t$$

and $\hat{\chi}_{q,t}$ is “almost” a ring homomorphism for the new multiplication $*$.

Remark: Nakajima gives an alternative conjectural definition, for M standard module (tensor product of fundamental representations) :

$$\chi_q(M) = \sum_{\gamma} \dim(M_{\gamma}) m_{\gamma} \Rightarrow \chi_{q,t}(M) = \sum_{\gamma, r} t^{2r} \dim(M_{\gamma}^{r+1}/M_{\gamma}^r) m_{\gamma}$$

where a filtration of generalized eigenspace is used :

$$M_{(\gamma^{\pm})}^r = \{x \in V / \forall i \in \{1, \dots, n\}, \forall m \in \mathbb{Z}, (\phi_{i,m}^{\pm} - \gamma_{i,m}^{\pm})^r . x = 0\}$$

Example:

$$\chi_{q,t}(M_{aq^2} \otimes M_a) = Y_{aq^3} Y_{aq} + Y_{aq^3} Y_{aq^{-1}}^{-1} + Y_{aq^{-1}}^{-1} Y_{aq^{-1}}^{-1} + 1$$

$$\chi_{q,t}(M_a \otimes M_a) = Y_a^2 + (1 + t^2) Y_a Y_{aq^2}^{-1} + Y_{aq^2}^{-1} = Y_a^2 (1 + (1 + t^2) A_{aq}^{-1} + A_{aq}^{-2})$$

$$\hat{\chi}_{q,t}(M_a \otimes M_a) = W_a^2 (1 + (1 + t^2) V_{aq} + V_{aq})$$

$$= (W_a (1 + V_{aq})) * (W_a (1 + V_{aq})) = \hat{\chi}_{q,t}(M_a) * \hat{\chi}_{q,t}(M_a)$$

9. t -ANALOGS OF SCREENING OPERATORS (GENERAL CASE)

Goal : Symmetry property for q, t -characters.

We define a bimodule structure on $\bigoplus_{a \in \mathbb{C}^*} \hat{\mathcal{Y}}_t \cdot S_{i,a}$:

$$S_{i,a} \cdot m = t^{2u_{i,a}(m)} \cdot S_{i,a}$$

Then

$$\sum_{a \in \mathbb{C}^*} \hat{\mathcal{Y}}(V_{i,aq_i} S_{i,aq_i^2} - t^2 S_{i,a})$$

is a subbimodule and we note the quotient $\hat{\mathcal{Y}}_{t,i}$.

Definition 4. (t -analogs of screening operators)

$$\hat{S}_{t,i}(m) = m \left(\sum_{a \in \mathbb{C}^*} (\mathbf{1}_{u_{i,a}(m) \geq 0} (1 + t^2 + \dots + t^{2(u_{i,a}(m)-1)}) - \mathbf{1}_{u_{i,a}(m) < 0} (t^{-2} + \dots + t^{2u_{i,a}(m)})) S_{i,a} \right)$$

Theorem 7. (H.) *The diagram :*

$$\begin{array}{ccccc} \hat{\mathcal{Y}}_t & \xrightarrow{\hat{S}_{t,i}} & \hat{\mathcal{Y}}_{t,i} & & \\ \hat{\Pi}_t \downarrow & & \downarrow & & \hat{\Pi}_{t,i} \\ \mathcal{Y}_t & \xrightarrow{S_{t,i}} & \mathcal{Y}_{t,i} & & \\ \Pi_t \downarrow & & \downarrow & & \Pi_{t,i} \\ \mathcal{Y} & \xrightarrow{S_i} & \mathcal{Y}_i & & \end{array}$$

is commutative and we have :

$$\begin{aligned} \bigcap_{i \in I} \text{Ker}(S_{i,t}) &= \mathfrak{K}_t \\ \bigcap_{i \in I} \text{Ker}(\hat{S}_{i,t}) &= \hat{\mathfrak{K}}_t \end{aligned}$$

More over in the ADE case, $\hat{S}_{t,i}$ is a derivation for $*$:

$$\forall X, Y \in \hat{\mathcal{Y}}_t, \hat{S}_{t,i}(X * Y) = \hat{S}_{t,i}(X) * Y + X * \hat{S}_{t,i}(Y)$$

Remark: The construction is purely algebraic and is extended to the non-simply laced case. Next step : to extend Nakajima's theory...

We hope to understand these operators as quantum differential operators.

10. REFERENCES

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