Table of maximal eigenspaces

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For each irreducible complex reflection coset $W\phi$ and each $w\phi \in W\phi$ which has a non-trivial ζ-eigenspace V for a root of unity ζ of order d which is maximal among such eigenspaces, we give the types of $W_d := N_W(V)/C_W(V)$ and of the coset $C_W(V)w\phi$.

Imprimitive groups

The degrees of $G(ne, e, r)$ are $ne, 2ne, \ldots, (r-1)ne$ and rn. The codegrees are $0, ne, 2ne, \ldots, (r-2)ne$ and $(r-1)ne$ when $n > 1$ or $(r-1)e - r$ when $n = 1$.

The coset ${}^tG(ne, e, r)$, $t|e$ is defined by the automorphism realized by $s_1^{\frac{e}{t}}$ where s_1 is the first generator (of order ne) of $G(ne, 1, r)$.The only generalized reflection degree with a non-trivial factor is (rn, ζ_t^{-1}). There is a generalized codegree with a non-trivial factor only when $n = 1$, which is $((r-1)e - r, \zeta_t)$.

When $n > 1$ the regular ζ are such that $\zeta^{rn} = \zeta_t$, in which case $W_d = G(\operatorname{lcm}(ne, d), e, \gcd(\frac{rn}{d}e, r))$ where $G(ne, e, 1) = G(n, 1, 1) = \mathbb{Z}/n$.

- For general d we give the result when $e = 1$, the case $G(n, 1, r)$. If we set $d' = \frac{d}{\gcd(n,d)}$ then $W_d = G(\operatorname{lcm}(n, d), 1, \lfloor \frac{r}{d'} \rfloor)$ and $C_W(V)w$ is of type $G(n, 1, r \mod d')$ (see [Malle, 3C]).

When $n = 1$, we set $d' = \frac{d}{\gcd(e,d)}$.

- The regular ζ are either such that $\zeta^r = \zeta_t$, then $W_d = G(\operatorname{lcm}(e, d), e, \frac{r}{d'})$, or such that $d|(r-1)e$, then if $\zeta^r \neq \zeta_t$ then $W_d = G(\operatorname{lcm}(e, d), 1, \frac{r-1}{d'})$.
- For general d such that $\zeta^r \neq \zeta_t$ we have $W_d = G(\operatorname{lcm}(e, d), 1, \lfloor \frac{r-1}{d'} \rfloor)$ and if we set $m = 1 + ((r-1) \mod d')$ then $C_W(V)w$ is of type $G(e, e, m)s_1^{p'}$ where $\zeta_e^{p'} = \zeta_t\zeta^{m-r}$ (see [Malle, (5.3), (5.4)]); note that $G(e, e, 1)$ is the trivial group.

Reference

[Malle] G. Malle, Unipotente Grade imprimitiver komplexer Spiegelungsgruppen, J. Algebra **177** (1995), 768–826.

Primitive groups

In the list below, each W_d is given followed by a colon and the list of d for which it is W_d. If ζ_d is not regular the type of the coset $C_W(V)w\phi$ is given in square brackets before d. In this type, we note $G^{(i)}$ a descent of scalars (the product of i copies of G permuted cyclically in the coset).

G_4:$1,2$ Z_6:$3,6$ Z_4:4

G_5:$1,2,3,6$ Z_{12}:$4,12$

G_6:$1,2,4$ Z_{12}:$3,6,12$

G_7:$1,2,3,4,6,12$

G_8:$1,2,4$ Z_{12}:$3,6,12$ Z_8:8

G_9:$1,2,4,8$ Z_{24}:$3,6,12,24$

G_{10}:$1,2,3,4,6,12$ Z_{24}:$8,24$

G_{11}:$1,2,3,4,6,8,12,24$

G_{12}:$1,2$ Z_6:$3,6$ Z_8:$4,8$

G_{13}:$1,2,4$ Z_{12}:$3,6,12$ Z_8:$[A_1\cdot\zeta_8^3]8$

G_{14}:$1,2,3,6$ Z_{24}:$4,8,12,24$

G_{15}:$1,2,3,4,6,12$ Z_{24}:$[A_1\cdot\zeta_8^{-1}]8,[A_1\cdot\zeta_{24}^{19}]24$

G_{16}:$1,2,5,10$ Z_{30}:$3,6,15,30$ Z_{20}:$4,20$

G_{17}:$1,2,4,5,10,20$ Z_{60}:$3,6,12,15,30,60$

G_{18}:$1,2,3,5,6,10,15,30$ Z_{60}:$4,12,20,60$

G_{19}:$1,2,3,4,5,6,10,12,15,20,30,60$

G_{20}:$1,2,3,6$ Z_{12}:$4,12$ Z_{30}:$5,10,15,30$

G_{21}:$1,2,3,4,6,12$ Z_{60}:$5,10,15,20,30,60$

G_{22}:$1,2,4$ Z_{12}:$3,6,12$ Z_{20}:$5,10,20$

H_3:$1,2$ Z_6:$3,6$ Z_{10}:$5,10$

G_{24}:$1,2$ Z_6:$3,6$ Z_{14}:$7,14$ Z_4:$[A_1]4$

G_{25}:$1,3$ G_5:$2,6$ Z_{12}:$4,12$ Z_9:9

G_{26}:$1,2,3,6$ Z_{18}:$9,18$ Z_{12}:$[A_1]4,[A_1\cdot\zeta_3]12$

G_{27}:$1,2,3,6$ Z_{30}:$5,10,15,30$ Z_{12}:$[A_1]4,[A_1\cdot\zeta_3]12$

${}^3D_4$:G_2:$1,2$ G_4:$3,6$ Z_4:12

${}^{3\prime}G_{3,3,3}$:$G_{3,1,2}$:$1,3$ Z_6:$2,6$ Z_3:9

F_4:$1,2$ G_5:$3,6$ G_8:4 Z_8:8 Z_{12}:12

${}^2F_4$: $I_2(8)$:$1,2$ G_{12}:4 G_8:8 Z_6:12 Z_{12}:24

G_{29}:$1,2,4$ Z_{20}:$5,10,20$ Z_{12}:$[A_1]3,[A_1]6,[A_1\cdot\zeta_4^{-1}]12$ Z_8:$[{}^2B_2\cdot\zeta_8^3]8$

H_4:$1,2$ G_{20}:$3,6$ G_{22}:4 G_{16}:$5,10$ Z_{12}:12 Z_{30}:$15,30$ Z_{20}:20

G_{31}:$1,2,4$ G_{10}:$3,6,12$ Z_{20}:$5,10,20$ G_9:8 Z_{24}:24

G_{32}:$1,2,3,6$ G_{10}:$4,12$ Z_{30}:$5,10,15,30$ Z_{24}:$8,24$ Z_{18}:$[Z_3]9,[Z_3\cdot -1]18$

G_{33}:$1,2$ G_{26}:$3,6$ Z_{10}:$5,10$ Z_{18}:$9,18$ G_6:$[A_1]4$ Z_{12}:$[A_1\cdot\zeta_3]12$

G_{34}:$1,2,3,6$ Z_{42}:$7,14,21,42$ G_{10}:$[A_1^2]4,[(A_1\cdot\zeta_3)^2]12$ Z_{30}:$[A_1]5,[A_1]10,[A_1\cdot\zeta_3^2]15,[A_1\cdot\zeta_3]30$ Z_{24}:$[A_1^{(2)}]8,[A_1^{(2)}\cdot\zeta_3]24$ Z_{18}:$[{}^3G_{3,3,3}]9,[{}^{3\prime}G_{3,3,3}\cdot -1]18$

E_6:1 F_4:2 G_{25}:3 G_8:4 G_5:6 Z_8:8 Z_9:9 Z_{12}:12 Z_5:$[A_1]5$

${}^2E_6$:2 F_4:1 G_{25}:6 G_8:4 G_5:3 Z_8:8 Z_9:18 Z_{12}:12 Z_5:$[A_1]10$

E_7:$1,2$ G_{26}:$3,6$ Z_{14}:$7,14$ Z_{18}:$9,18$ G_8:$[A_1^3]4$ Z_{10}:$[A_2]5,10$ Z_8:$[A_1\times A_1^{(2)}]8$ Z_{12}:$[A_1^{(3)}]12$

E_8:$1,2$ G_{32}:$3,6$ G_{31}:4 G_{16}:$5,10$ G_9:8 G_{10}:12 Z_{30}:$15,30$ Z_{20}:20 Z_{24}:24 $Z_{14}[A_1]$:$7,14$ Z_{18}:$[A_2]9,[{}^2A_2]18$

An observation on the table is that in every split case all regular numbers divide a regular degree. Is this clear *a priori*?