Table-turning to Stable Marriage satisfaction and equity

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***Abstract*—Running a stable marriage algorithm to pairing images in vision, the global satisfaction and sex equality appear as important constraints as the stability itself. In the present paper we outline a novel algorithm based on the rotation of the "marriage table" to align satisfaction and equity with preferences. It turns out that a direct implementation through the lists is doable. Additionally, it shows improved flexibility in balancing constraints. Known algorithms are compared to the present one on 3000 instances of 200 large populations and performances are discussed. Results on real images are displayed in the case of stereo-pairing and motion understanding applications.**

I. Introduction

IN many computer vision applications including motion analysis, stereovision or model fitting, matching is a key step that deserves efficient optimization. In this paper we propose a *stable marriage* algorithm for image matching. It belongs to a bi-partite graph optimization technique based on the so called *marriage table* representation [1]. The BZ algorithm proposed in the latter paper achieves an efficient trade-off between the *global satisfaction*, the *fairness* (or *sex equality*) and the *stability* thanks to this representation. It provides matching results with maximum sex equality and global satisfaction, and with limited instability - about 5% found solutions are unstable. But its complexity is in $O(N^{3/2})$, to be compared to the basic Gale-Shapley (GS) algorithm in O(N). Recently, the *S*-procedure was designed [2] in order to obtain stable matching-results by resolving the BZ oscillating and cycling behaviors in the *marriage table*. The satisfaction and equity are preserved but the complexity grows from $O(N^{3/2})$ to $O(N^2)$. The present paper describes a process for generating intermediate versions between BZ and GS called RZ or RGS. The stress here is put on the algorithmic complexity decreasing from $O(N^{3/2})$ to O(N), while controlling the satisfaction or equality as much as possible..

The paper is organized as follows: we first revisit the stable marriage problem, GS and then BZ based on the marriage table, section 2. Then we explain the RZ algorithm motivations and its variations RGS in section 3. New performances are compared with the GS algorithm and the BZ algorithm in section 4.

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II. Stable marriage revisited

The stable marriage problem was studied by Gale and Shapley [3] who produced the first algorithm and it remains among the popular combinatory problems [4]−[7]. In this problem, two finite sub-sets M and W of two respective populations, say men and women, have to match. Assume $n = \sqrt{N}$ is the number of elements, $M = \{m_i\}_1^n$ and $W = \{w_j\}_1^n$. Each element x creates its preference list $l(x)$ i.e. it sorts all members of the opposite sex from most to less preferred (see example in the next section table 1). A matching M is a one to one correspondence between men and women. If (m, w) is a matched pair in$M$, $M(m) = w$ and $M(w) = m$, and ρ_m is the rank of m in the list of w (resp. ρ_w the rank of w in the list of m. Man m and woman w form a *blocking pair* if (m, w) is not in M but m prefers w to $M(m)$ and w prefers m to $M(w)$. The situation where (m, w) is blocking $(m,\ M(m))$ and $(M(w), w)$ is called *blocking situation*. If there is no blocking pair, then the marriage M is stable.

Gale-Shapley proposed the algorithm to find M with complexity O(N). GS has two different versions: *men-optimal* and *women-optimal*. *GS* with men-optimal can be stated as follows: while there is an unpaired man, pick the unpaired man and the first woman on his list. If she is free, the man and woman are married. If not, she chooses between the challenger and her current partner according to her preferences. The process continues until there is no more unpaired man. However, the stable matching can be such that everybody is unsatisfied. Normally, *Men-optimal* brings a stable matching in which men have the best possible partner and women may have the worst and conversely.

Let us note that Gusfield and Irving [8] quote open problems in conclusion for their extensive study of the algorithms of stable marriage and the derived models of optimization (15 years ago). One of them, problem 11 is the egalitarian stable marriage and can be solved in $O(n^4 \log n)$. Feder [9] has claimed a $O(n^{2.5} \log n)$ and $O(n^3)$ [10]. For them "egalitarian" relates to minimal $\sum(\rho_m + \rho_w)$, however we preferred to call this property "global satisfaction", more evocative of the properties of solutions in our applications. And "sex-equal" mentioned as open problem 6 is translated by Irving into $\sum \rho_m = \sum \rho_w$, here again we preferred "sex equality" to be coded by $\min \sum|\rho_m - \rho_w|$.

In [1], the *marriage table* representation is proposed for the marriage optimization to meet the three objectives of stability, sex equality and global satisfaction. It is a table with (n+1) lines and (n+1) columns. Lines (resp. columns) frame the preference orders of men, $\{1 \ldots p \ldots N\infty\}$ (resp. women, $\{1 \ldots q \ldots N\infty\}$). The cell (p,q) contains pairs (m,w) such that w is the p^{th} choice of m, and m is the q^{th} choice of w. Cells can thus contain more than one pair or none. The cell (p,∞) (resp.(∞,q)) contains the pairs (m,w), if any, where w is the p^{th} choice of m (resp m the choice of w but m does not exist in her preference list (resp. w is not in his preference list). Figure 1 shows the *marriage table*.

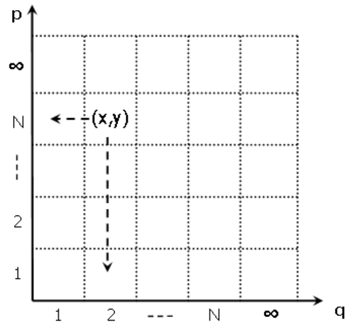


Fig. 1. Marriage table: x is the 2nd choice of y and y is N th for x.

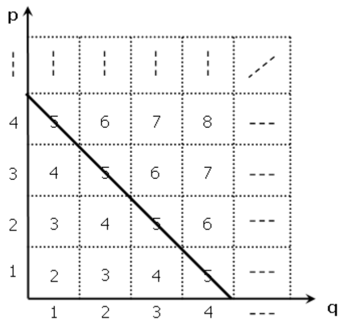


Fig. 2. Satisfaction.

The global satisfaction of matching can be measured by $\overline{S} = \sum_{(m,w)\in M}(\rho_m + \rho_w)$. Note that a solution with maximum global satisfaction would get matched pairs around the origin of the table (bottom-left), as shown figure 2. Conversely, sex equality tends to fit the diagonal of the marriage table. It is defined as $\overline{E} = \sum_{(m,w)\in M}|\rho_m - \rho_w|$, figure 3.

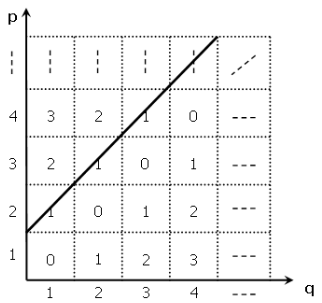


Fig. 3. Sex equality.

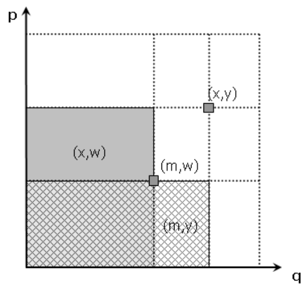


Fig. 4. Blocking situation.

Stability can be translated into the marriage table too, a blocking situation is represented figure 4. Assuming (x,y) and (m,w) were paired, then (x,w) cannot be in the grey rectangle and (m,y) cannot be in the dashed one.

BZ is an algorithm based on that representation. It consists of scanning the marriage table cells in order to first maximize both criteria concurrently. It scans anti-diagonals forward from maximum to minimum global satisfaction while each one is read in swinging from center to sides meaning maximum to minimum sex equality. In each cell, pairs are married if both partners are free. As it is easily proven that no predefined scan warrants stability (see next section), after all cells have been visited the table is scanned again to remove blocking situations: a blocking pair gets married and corresponding blocked pairs are released. The process repeats until there is no more blocking situation or the iteration number is greater than the population size.

Let us stress upon the bounding result in this process. Given a systematic scan i.e. a permutation of $\mathbb{N}^2, n = s(i,j)$, there exists an instance of population i.e. a set of preference lists $\{l(m), l(w)\}_{M\times W}$ such that (m_i, w_j), (m_k, w_l) and (m_k, w_j) are met along the scan in that order, then (m_i, w_j), and (m_k, w_l) are married while (m_k, w_j) is blocking them. It is trivially enough that

$$s\big(l_j(m_i), l_i(m_j)\big) \leq s(l_l(m_k), l_k(m_l)) \leq s\big(l_j(m_k), l_k(m_j)\big)$$
$$\text{and } l_k(w_j) \leq l_i(w_j) \text{ and } l_j(w_k) \leq l_l(w_k)$$

With $l_j(m_i)$ the ranking order of w_j in $l(m_i)$. Figure 5 shows an example of such a case for the anti-diagonal regular scan.

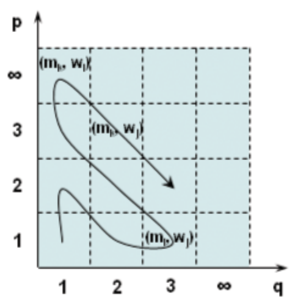


Fig. 5. Bounding result.

In the next section, an alternative algorithm is proposed to get matching results such that everybody is happy (resp. treated fairly) as much as possible while preserving a complexity in O(N).

III. RZ and RGS Algorithms

Here, we propose an algorithm that takes advantage from the lesser complexity of GS and from the coding of *satisfaction* and sex *equality* introduced through the marriage table.

Indeed, if w is the p^{th} choice of m and m is the q^{th} choice of w, then $p+q$ represents the satisfaction and $|p-q|$ shows sex equality between m and w. The primary idea is that scanning the marriage table in a diagonal way is (quasi) equivalent to rotate the same $45°$ before scanning horizontally or vertically. Doing so, at least equivalent results to the first pass in BZ will be obtained. Figure 6(a) shows the rotated version of the marriage table. It is a table with $2n+1$ lines and $n+1$ columns. Lines represent the satisfaction and columns the sex equality. The grey area shows the projected area from the original table. It contains in each cell the couples which have the corresponding satisfaction and sex equality. Thanks to the empty cells after rotation in a sampled space, there is room for expanding couples from one cell over several cells if they concern a same person. A complete order is then recovered following additional constraints. By scanning the latter array left-right and bottom-up, and marrying the occurring free pairs (see figure 6(b)), similar results to the OZ-like algorithm [1] are obtained. Let us call RZ (Rotated Zigzag) the algorithm.

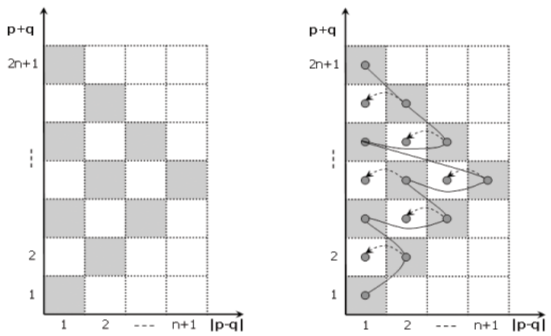


Fig. 6. (a) rotated marriage table, (b) scan and reorder..

Note that OZ is the simplest algorithm based on the marriage table. BZ is improved version of its to target maximum satisfaction and sex equality, and then SBZ [2] reaches the full stability constraint. It means that we can later complete RZ in the $S-$fashion to achieve stability, in contradiction with [11].

Actually a horizontal (resp. vertical) scan of the marriage table is also the basis of GS, where the preference list is ordered following p for men and q for women. We should then improve the overall result by merely performing GS on lists transformed by $(p \to p+q;\ q \to\ p-q)$: Instead of transforming the preference lists into the *marriage table* and then finding the matching with maximum satisfaction and sex equality, we introduce the *satisfaction* and *sex equality* into the preference lists then finding the matching result by classical GS. Obviously the stability is no more guaranteed, although GS is run, since it would not be according to the original preferences. But one gets additional variants here as flexible stress can be put on satisfaction, equity or stability in departing more or less from the initial lists. The latter flexibility amounts to the above-mentioned additional constraints supporting the cell expansion up to complete order in the marriage table. In the current version of our algorithm, the man's and woman's preference lists are first reordered by increasing $p+q$, then by increasing $|p-q|$ in case of equal $p+q$, and then by increasing p for man (respectively q for woman) in case of equal $p+q$ and $|p-q|$. Table 1 shows an instance of three men and women with their preference list ordered by p and q respectively.

TABLE I
An instance of 3 men and women and their preference lists

Man				Woman			
p	1	2	3	q	1	2	3
1	C	A	B	A	1	2	3
2	A	C	B	B	2	3	1
3	C	B	A	C	1	2	3

Table 2 shows the value of satisfaction $p+q$ and sex equality $|p-q|$ for each element in their preference list.

TABLE II
Satisfaction and sex equality in the preference lists

Man (p+q , |p-q|)				Woman (p+q , |p-q|)			
1	(2,0)	(3,1)	(6,0)	A	(3,1)	(3,1)	(6,0)
2	(3,1)	(4,0)	(4,2)	B	(4,2)	(4,0)	(6,0)
3	(4,2)	(4,0)	(6,0)	C	(2,0)	(4,0)	(4,2)

Table 3 shows the same instance of men and women with their preference lists reordered by satisfaction, sex equality and initial preference in the order. We can see that the preference list of 3 and B are reordered by $|p-q|$ since there is the conflict on $p+q$. Similarly, the preference list of A is reordered by q since there is a conflict on both $p+q$ and $|p-q|$.

TABLE III
Reordered preference lists

Man				Woman			
1	C	A	B	A	1	2	3
2	A	C	B	B	3	2	1
3	B	C	A	C	1	2	3

Then the marriages result from executing GS on the preference lists of table 3.

Remark: As both man's and woman's lists are shuffled here in stressing the *satisfaction* first, GS with man-optimal and woman-optimal are likely to give the same solution.

The novel algorithm introduced here is then a intermediate version between GS and OZ, let us call RGS this algorithm. Not that as previously mentioned the S −stability applies to RGS as well.

In the next section, we compare RGS with GS and BZ to better understand respective performances.

IV. Algorithm Performance

We study experimentally the global satisfaction, sex equality and stability obtained by RGS, and we compare with GS and BZ. About 3.000 instances are built at random for 200-large populations. Each algorithm is executed and its results are displayed. Figure 7 and figure 8 zoom on 30 instances out of the 3.000 in order to display more in detail the global satisfaction and sex equality respectively. Let us define the number b of instances where RGS is better than the better GS or than BZ as:

$$b = \sum_{all\ instances} \mathcal{H}_{[GS_{m,w}(BZ) - RGS]}$$

With $\mathcal{H}_{[x]} = 1$ if $x > 0$ else $\mathcal{H}_{[x]} = 0$, then

$$\beta = \frac{b \times 100}{\text{number of instances}}$$

We can first note that RGS performs totally better than the better GS for both satisfaction and sex equality, $\beta = 100$. Comparing RGS with BZ, $\beta = 8.13$ and $\beta = 35.77$ repectively for the global satisfaction and sex equality. We note that RGS is comparable to BZ with respect to GS for the satisfaction and sex equality achieved. This is indicated by the following distances: $(S_{RGS} - S_{BZ})/S_{BZ} = 5.75\%$, $(S_{RGS} - S_{GS})/S_{GS} = 22.64\%$, $(E_{RGS} - E_{BZ})/E_{BZ} = 7.24\%$, and $(E_{RGS} - E_{GS})/E_{GS} = 46.96\%$ in average.

Comparing the stability, the matching results issued by GS are totally stable while around 5% and 87\% of instances are unstable for BZ and RGS respectively. But, more important, the average number of blocking pairs per unstable instance is 10.8 for BZ and 3.3 for RGS, meaning that the expectation of the number of iterations starting from RGS to overcome oscillations towards complete stability is significantly lower than with OZ to BZ and then SBZ.

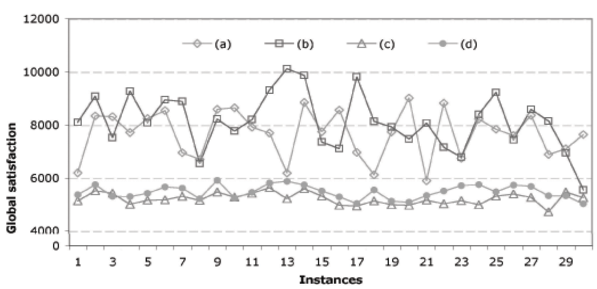


Fig. 7. Comparing global satisfaction between methods : (a) GS man-optimal, (b) GS woman-optimal, (c) BZ, (d) RGS.

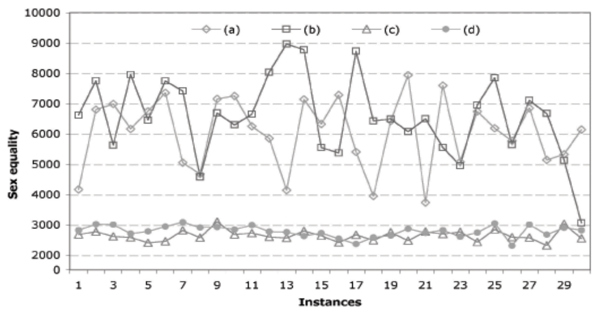


Fig. 8. Comparing sex equality between methods : (a) GS man-optimal, (b) GS woman-optimal, (c) BZ, (d) RGS.

Table 4 shows numeric comparison between RGS and BZ or GS for the stability (number of blocking pairs), global satisfaction and sex equality.

TABLE IV

Comparison of stability, global satisfaction and sex equality between methods

	INB	NBP	DST	S	E
GS	0	0	(BZ, RGS)	280	189
BZ	3.9	11.7	(BZ, RZ_m)	1614	1310
RGS	87.9	3.3	(BZ, RZ_w)	8276	2080
RZ_m	100	23.3	(GS, RGS)	1733	2714
RZ_w	100	73.1	(GS, RZ_m)	854	4108
			(GS, RZ_w)	6277	4881

INB = Instability, NBP = Number of blockages; DST = Distance, S = Satisfaction, and E = Equity.

V. Conclusion

For a quick and temporary conclusion let us show two types of results: (1) comparative results of matching M and M' to compute the transform $\mathbb{T}(\text{affine} + \text{projections})$ between left L and right R images, (2) dominant motion extraction by our algorithms from an image sequence. In the comparison of stereo results, images come from "http://www.gravitram.com/stereoscopic_photography.htm", figure 9. Features to be matched are level line junctions [12]. Figures 10 and 11, we display images of the kind $\sup[|R - \mathbb{T}_M(L)| - |R - \mathbb{T}_{M'}(L)|, 0]$ and dark regions where M is obviously worse than M', the darker the worse. Although several causes − like feature and parameters extraction or transform-computation − exist to explain local imperfections, it appears to the naked eye that stability of GS serves more a local goodness on details while satisfaction and equity would improve results in a wider fashion. That is part of our coming work, to test such conjectures.

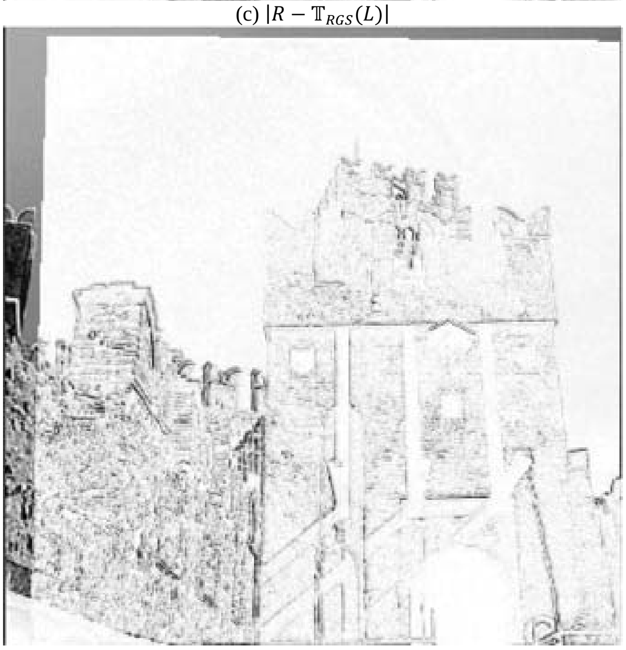
In the motion understanding application, extracted features are again level line junctions. The program runs on a PowerPC G4 /1.3GHz and delivers a dependable result every second. We will soon implement it on a Bi-processor Xeon onboard our autonomous vehicle PICAR [13] and extraction every 200 ms is expected. Figure 12 shows two consecutive images in a sequence. Matching results are displayed as flows in figure 13. Motion identification using optical flow classification separates the dominant mobile object, figure 14(a), and the background, figure 14(b). And figure 15 shows the rebuilt vehicle.

(a) L

(b) R

Fig. 9. Stereo images.

(c) $|R - \mathbb{T}_{RGS}(L)|$



(d) $|R - \mathbb{T}_{RGS}(L)|$

Fig.10. Comparative results.

(e) $\sup[(c) - |R - \mathbb{T}_{GS}(L)|, 0]$

(f) $\sup[(d) - |R - \mathbb{T}_{GS}(L)|, 0]$

Fig.11. Comparative results.

(a)

(b)

Fig. 12. Original images.

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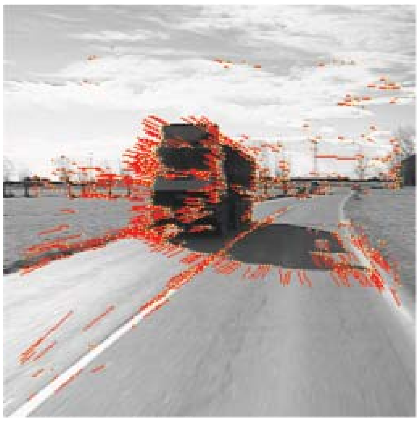
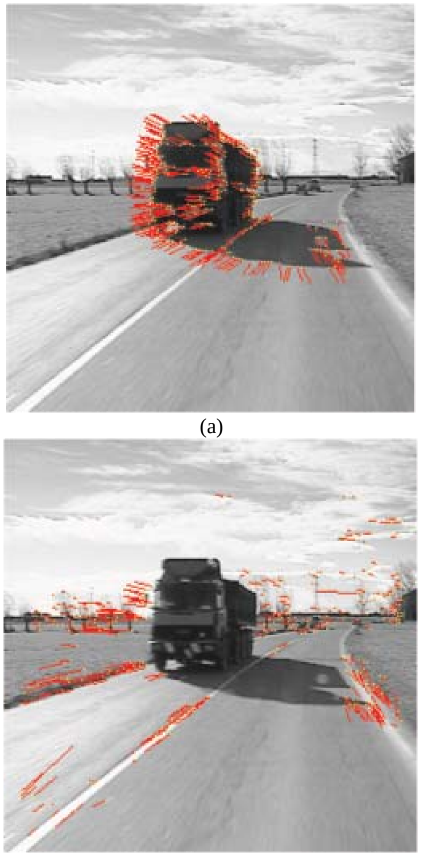


Fig.13. Matching results.



(a)

(b)

Fig.14. Motion identification : (a) object, (b) background.

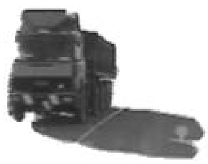


Fig.15. Extracted object.