Pseudo reductive groups over $\mathbb{F}_x$?

Michel Broué

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Joint work between Michel Broué, Gunter Malle, and Jean Michel,

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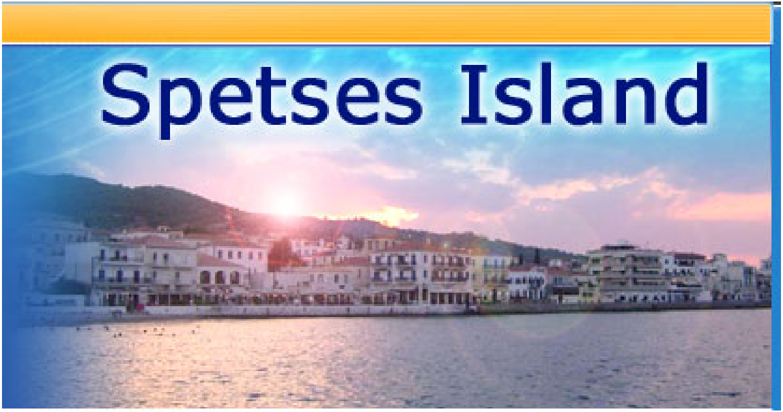

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$\phi_{1,0}$	1	1	1	C_1
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$\phi_{2,1}$	$\frac{1}{6}q\Phi_2^2\Phi_3$	$q\Phi_8$	1	$S_3.(1,1)$
$\phi_{2,2}$	$\frac{1}{2}q\Phi_2^2\Phi_6$	$q^2\Phi_4$	1	$S_3.(g_2,1)$
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$\phi''_{1,3}$	$\frac{1}{3}q\Phi_3\Phi_6$	q^3	1	$S_3.(1,\rho)$
$G_2[1]$	$\frac{1}{6}q\Phi_1^2\Phi_6$	0	1	$S_3.(1,\varepsilon)$
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$G_2[\zeta_3]$	$\frac{1}{3}q\Phi_1^2\Phi_2^2$	0	ζ_3	$S_3.(g_3,\zeta_3)$
$G_2[\zeta_3^2]$	$\frac{1}{3}q\Phi_1^2\Phi_2^2$	0	ζ_3^2	$S_3.(g_3,\zeta_3^2)$

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$\phi_{2,3}$	$\frac{3+\sqrt{-3}}{6} q\Phi_3''\Phi_4\Phi_6'$	$q^3\Phi_4$	1	$X_3.02$
$Z_3 : 2$	$\frac{\sqrt{-3}}{3} q\Phi_1\Phi_2\Phi_4$	0	ζ_3^2	$X_3.12$
$\phi_{3,2}$	$q^2\Phi_3\Phi_6$	$q^2\Phi_3\Phi_6$	1	C_1
$\phi_{1,4}$	$\frac{-\sqrt{-3}}{6} q^4\Phi_3''\Phi_4\Phi_6''$	q^4	1	$X_5.1$
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$Z_3 : 11$	$\frac{\sqrt{-3}}{3} q^4\Phi_1\Phi_2\Phi_4$	0	ζ_3^2	$X_5.4$
G_4	$\frac{1}{2} q^4\Phi_1^2\Phi_3$	0	-1	$X_5.5$

Φ_3', Φ_3'' (resp. Φ_6', Φ_6'') are factors of Φ_3 (resp Φ_6) in $\mathbb{Q}(\zeta_3)$

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- Lusztig's Fourier matrix S := (square) matrix between $\mathrm{UnCh}(G)$ and $\mathrm{UnSh}(G)$.
- The blocks of S correspond to Lusztig's families of unipotent characters.

Lusztig's Fourier matrix for G_2

			$(1,1)$	$(g_2,1)$	$(g_3,1)$	$(1,\rho)$	$(1,\varepsilon)$	(g_2,ε)	(g_3,ζ_3)	(g_3,ζ_3^2)
	1	0	0	0	0	0	0	0	0	0
	0	1	0	0	0	0	0	0	0	0
$(1,1)$	0	0	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{3}$
$(g_2,1)$	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0
$(g_3,1)$	0	0	$\frac{1}{3}$	0	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	0	$-\frac{1}{3}$	$-\frac{1}{3}$
$(1,\rho)$	0	0	$\frac{1}{3}$	0	$-\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	0	$-\frac{1}{3}$	$-\frac{1}{3}$
$(1,\varepsilon)$	0	0	$\frac{1}{6}$	$-\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$	$-\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{3}$
(g_2,ε)	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	0
(g_3,ζ_3)	0	0	$\frac{1}{3}$	0	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	0	$\frac{2}{3}$	$-\frac{1}{3}$
(g_3,ζ_3^2)	0	0	$\frac{1}{3}$	0	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	0	$-\frac{1}{3}$	$\frac{2}{3}$

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	1	.	.	.	.	.	.	.	.	.
	.	1	.	.	.	.	.	.	.	.
$(1,1)$	.	.	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{3}$
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$(1,\rho)$	.	.	$\frac{1}{3}$	.	$-\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	.	$-\frac{1}{3}$	$-\frac{1}{3}$
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(g_3,ζ_3)	.	.	$\frac{1}{3}$	.	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	.	$\frac{2}{3}$	$-\frac{1}{3}$
(g_3,ζ_3^2)	.	.	$\frac{1}{3}$	.	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	.	$-\frac{1}{3}$	$\frac{2}{3}$

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The Fourier matrix for G_4

		01	02	12		1	2	3	4	5
	1	0	0	0	0	0	0	0	0	0
01	0	$\frac{3-\sqrt{-3}}{6}$	$\frac{3+\sqrt{-3}}{6}$	$\frac{\sqrt{-3}}{3}$	0	0	0	0	0	0
02	0	$\frac{3+\sqrt{-3}}{6}$	$\frac{3-\sqrt{-3}}{6}$	$-\frac{\sqrt{-3}}{3}$	0	0	0	0	0	0
12	0	$\frac{\sqrt{-3}}{3}$	$-\frac{\sqrt{-3}}{3}$	$\frac{\sqrt{-3}}{3}$	0	0	0	0	0	0
	0	0	0	0	1	0	0	0	0	0
1	0	0	0	0	0	$-\frac{\sqrt{-3}}{6}$	$\frac{\sqrt{-3}}{6}$	$\frac{1}{2}$	$\frac{\sqrt{-3}}{3}$	$\frac{1}{2}$
2	0	0	0	0	0	$\frac{\sqrt{-3}}{6}$	$-\frac{\sqrt{-3}}{6}$	$\frac{1}{2}$	$-\frac{\sqrt{-3}}{3}$	$\frac{1}{2}$
3	0	0	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$
4	0	0	0	0	0	$\frac{\sqrt{-3}}{3}$	$-\frac{\sqrt{-3}}{3}$	0	$\frac{\sqrt{-3}}{3}$	0
5	0	0	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$

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12	.	$\frac{\sqrt{-3}}{3}$	$-\frac{\sqrt{-3}}{3}$	$\frac{\sqrt{-3}}{3}$	.	.	.	.	.	.
	.	.	.	.	1	.	.	.	.	.
1	.	.	.	.	.	$-\frac{\sqrt{-3}}{6}$	$\frac{\sqrt{-3}}{6}$	$\frac{1}{2}$	$\frac{\sqrt{-3}}{3}$	$\frac{1}{2}$
2	.	.	.	.	.	$\frac{\sqrt{-3}}{6}$	$-\frac{\sqrt{-3}}{6}$	$\frac{1}{2}$	$-\frac{\sqrt{-3}}{3}$	$\frac{1}{2}$
3	.	.	.	.	.	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	.	$-\frac{1}{2}$
4	.	.	.	.	.	$\frac{\sqrt{-3}}{3}$	$-\frac{\sqrt{-3}}{3}$	.	$\frac{\sqrt{-3}}{3}$	.
5	.	.	.	.	.	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	.	$\frac{1}{2}$

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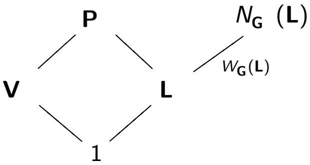

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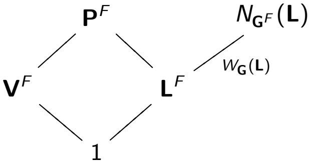

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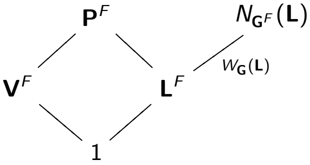

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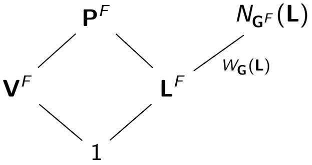

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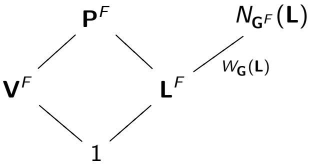

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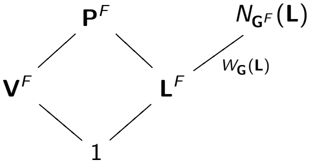

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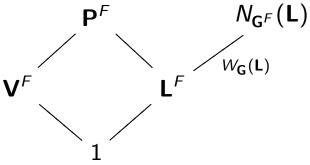

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2. For (L, λ) 1–cuspidal, the relative Weyl group

$$W_G(L, \lambda) := N_G(L, \lambda)/L$$

is a finite Coxeter group.

Definition : cuspidal character

- An irreducible character γ of G is said to be 1–cuspidal if, whenever L is a proper 1–Levi subgroup, we have ${}^*R_L^G(\gamma) = 0$.
- A pair (L, λ) is called 1–cuspidal if L is a 1–Levi subgroup of G and λ a 1–cuspidal irreducible character of L.

Main theorem

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$$W_G(L, \lambda) := N_G(L, \lambda)/L$$
is a finite Coxeter group.
3. The commuting algebra $\mathrm{End}_{KG}\, R_L^G(\lambda)$ is a Hecke algebra for $W_G(L, \lambda)$.

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- The Fourier matrix is independent of q.

In red, the principal series = degrees prime to Φ_1 (*i.e.*, $\mathrm{Deg}_\chi(q)_{q=1} \neq 0$)

Character	Degree	Fake degree	Eigenvalue	Family
$\phi_{1,0}$	1	1	1	C_1
$\phi_{1,6}$	q^6	q^6	1	C_1
$\phi_{2,1}$	$\frac{1}{6}q\Phi_2^2\Phi_3$	$q\Phi_8$	1	$S_3.(1,1)$
$\phi_{2,2}$	$\frac{1}{2}q\Phi_2^2\Phi_6$	$q^2\Phi_4$	1	$S_3.(g_2,1)$
$\phi'_{1,3}$	$\frac{1}{3}q\Phi_3\Phi_6$	q^3	1	$S_3.(g_3,1)$
$\phi''_{1,3}$	$\frac{1}{3}q\Phi_3\Phi_6$	q^3	1	$S_3.(1,\rho)$
$G_2[1]$	$\frac{1}{6}q\Phi_1^2\Phi_6$	0	1	$S_3.(1,\varepsilon)$
$G_2[-1]$	$\frac{1}{2}q\Phi_1^2\Phi_3$	0	-1	$S_3.(g_2,\varepsilon)$
$G_2[\zeta_3]$	$\frac{1}{3}q\Phi_1^2\Phi_2^2$	0	ζ_3	$S_3.(g_3,\zeta_3)$
$G_2[\zeta_3^2]$	$\frac{1}{3}q\Phi_1^2\Phi_2^2$	0	ζ_3^2	$S_3.(g_3,\zeta_3^2)$

Unipotent characters for G_4 ③—③

Red $=$ the principal series
Blue $=$ series (L,λ)
Purple $=$ Cuspidal

Character	Degree	FakeDegree	Eigenvalue	Family
$\phi_{1,0}$	1	1	1	C_1
$\phi_{2,1}$	$\frac{3-\sqrt{-3}}{6}q\Phi_3'\Phi_4\Phi_6''$	$q\Phi_4$	1	$X_3.01$
$\phi_{2,3}$	$\frac{3+\sqrt{-3}}{6}q\Phi_3''\Phi_4\Phi_6'$	$q^3\Phi_4$	1	$X_3.02$
$Z_3:2$	$\frac{\sqrt{-3}}{3}q\Phi_1\Phi_2\Phi_4$	0	ζ_3^2	$X_3.12$
$\phi_{3,2}$	$q^2\Phi_3\Phi_6$	$q^2\Phi_3\Phi_6$	1	C_1
$\phi_{1,4}$	$\frac{-\sqrt{-3}}{6}q^4\Phi_3''\Phi_4\Phi_6''$	q^4	1	$X_5.1$
$\phi_{1,8}$	$\frac{\sqrt{-3}}{6}q^4\Phi_3'\Phi_4\Phi_6'$	q^8	1	$X_5.2$
$\phi_{2,5}$	$\frac{1}{2}q^4\Phi_2^2\Phi_6$	$q^5\Phi_4$	1	$X_5.3$
$Z_3:11$	$\frac{\sqrt{-3}}{3}q^4\Phi_1\Phi_2\Phi_4$	0	ζ_3^2	$X_5.4$
G_4	$\frac{1}{2}q^4\Phi_1^2\Phi_3$	0	-1	$X_5.5$

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Definition

λ_ρ is the eigenvalue of Frobenius attached to ρ.

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- Spetses island : there we started to play the same game, replacing the Weyl group W by the complex reflection group of order 3.

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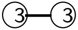
G_2 :  , G_4 : 

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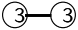
G_2 :  , G_4 : 

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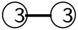
G_2 :  , G_4 : 

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- The associated generic Hecke algebra is defined from such a presentation :

$$\mathcal{H}(G_2) := < S, T \,;\, \begin{cases} STSTST = TSTSTS \\ (S-u_0)(S-u_1) = 0 \\ (T-v_0)(T-v_1) = 0 \end{cases} >$$

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5. The Schur elements of the irreducible characters of W are the elements $s_\chi \in \mathbb{Z}_W[(x_i^{\pm 1}),(y_j^{\pm 1}),\dots]$ defined by

$$t = \sum_{\chi\in\mathrm{Irr}(W)} \frac{1}{s_\chi}\, \chi_{\mathcal{H}}(W) \,.$$

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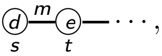
If $G =$ 

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Group G_n	4	5	6	7	8	9	10	11	12	13	14	15	16
Rank	2	2	2	2	2	2	2	2	2	2	2	2	2

Group G_n	17	18	19	20	21	22	23	24	25	26	27
Rank	2	2	2	2	2	2	3	3	3	3	3
Remark							H_3				

Group G_n	28	29	30	31	32	33	34	35	36	37
Rank	4	4	4	4	4	5	6	6	7	8
Remark	F_4		H_4					E_6	E_7	E_8

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- $N_G(\mathbf{T}_w, \mathrm{Id})/T_w \simeq W(w)$.
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This conjecture was prompted by the abelian defect groups conjecture.

For G_2

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Regular ζ	$W(\zeta)$	$\mathcal{H}_W(\zeta)$
1	G_2	$(s-q)(s+1)$
-1	G_2	$(s-q)(s-1)$
ζ_3	C_6	$(s-q^2)(s-q)(s-1)(s^3+q^3)$
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Cyclotomic Hecke algebras

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Regular ζ	$W(\zeta)$	$\mathcal{H}_W(\zeta)$
1	G_4	$(s-q)(s+1)$
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ζ_3	C_6	$(s-q^2)(s-1)(s+1)(s+\zeta_3 q)(s-\zeta_3)(s+q)$
ζ_4	C_4	$(s-q^3)(s-1)(s-q)(s+1)$
ζ_6	C_6	$(s-q^2)(s-q)(s-1)(s-\zeta_3^2 q)(s-\zeta_3^2)(s+1)$

In red = the Φ'_6–series.
● = the Φ_4–series.

Character	Degree	FakeDegree	Eigenvalue	Family
● $\phi_{1,0}$	● 1	1	1	C_1
$\phi_{2,1}$	$\frac{3-\sqrt{-3}}{6}q\Phi'_3\Phi_4\Phi''_6$	$q\Phi_4$	1	$X_3.01$
$\phi_{2,3}$	$\frac{3+\sqrt{-3}}{6}q\Phi''_3\Phi_4\Phi'_6$	$q^3\Phi_4$	1	$X_3.02$
$Z_3 : 2$	$\frac{\sqrt{-3}}{3}q\Phi_1\Phi_2\Phi_4$	0	ζ_3^2	$X_3.12$
● $\phi_{3,2}$	● $q^2\Phi_3\Phi_6$	$q^2\Phi_3\Phi_6$	1	C_1
$\phi_{1,4}$	$\frac{-\sqrt{-3}}{6}q^4\Phi''_3\Phi_4\Phi''_6$	q^4	1	$X_5.1$
$\phi_{1,8}$	$\frac{\sqrt{-3}}{6}q^4\Phi'_3\Phi_4\Phi'_6$	q^8	1	$X_5.2$
● $\phi_{2,5}$	● $\frac{1}{2}q^4\Phi_2^2\Phi_6$	$q^5\Phi_4$	1	$X_5.3$
$Z_3 : 11$	$\frac{\sqrt{-3}}{3}q^4\Phi_1\Phi_2\Phi_4$	0	ζ_3^2	$X_5.4$
● G_4	● $\frac{1}{2}q^4\Phi_1^2\Phi_3$	0	-1	$X_5.5$

Fourier matrices : G_4

		01	02	12		1	2	3	4	5
	1	.	.	.	.	.	.	.	.	.
01	.	$\frac{3-\sqrt{-3}}{6}$	$\frac{3+\sqrt{-3}}{6}$	$\frac{\sqrt{-3}}{3}$	.	.	.	.	.	.
02	.	$\frac{3+\sqrt{-3}}{6}$	$\frac{3-\sqrt{-3}}{6}$	$-\frac{\sqrt{-3}}{3}$	.	.	.	.	.	.
12	.	$\frac{\sqrt{-3}}{3}$	$-\frac{\sqrt{-3}}{3}$	$\frac{\sqrt{-3}}{3}$	.	.	.	.	.	.
	.	.	.	.	1	.	.	.	.	.
1	.	.	.	.	.	$-\frac{\sqrt{-3}}{6}$	$\frac{\sqrt{-3}}{6}$	$\frac{1}{2}$	$\frac{\sqrt{-3}}{3}$	$\frac{1}{2}$
2	.	.	.	.	.	$\frac{\sqrt{-3}}{6}$	$-\frac{\sqrt{-3}}{6}$	$\frac{1}{2}$	$-\frac{\sqrt{-3}}{3}$	$\frac{1}{2}$
3	.	.	.	.	.	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	.	$-\frac{1}{2}$
4	.	.	.	.	.	$\frac{\sqrt{-3}}{3}$	$-\frac{\sqrt{-3}}{3}$	.	$\frac{\sqrt{-3}}{3}$	.
5	.	.	.	.	.	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	.	$\frac{1}{2}$

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(A special character is one whose fake degree has same valuation as the corresponding generic degree)

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- In general, the partition

$$\mathrm{Irr}\, \mathcal{H}_G(\mathbf{L},\lambda) = \dot{\bigcup}_{\mathcal{F}\in\mathrm{Fam}(G)} \mathcal{F} \cap \mathrm{Irr}\, \mathcal{H}_G(\mathbf{L},\lambda)$$

is the partition into Rouquier blocks of the cyclotomic Hecke algebra $\mathcal{H}_G(\mathbf{L},\lambda)$.

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- All Rouquier blocks of all cyclotomic Hecke algebras of all complex reflection groups have been determined (Malle–Rouquier, Broué–Kim, Kim, Chlouveraki).

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- Malle gave a solution for imprimitive Spetsial complex reflection groups in 1995, and also proposed (unpublished) data for many primitive Spetsial groups.
- We can now show that there is a unique solution for all primitive spetsial complex reflection groups.