Applied Representation Theory

July 7 – July 10, 2015

Program

Tuesday

13:30	*Welcome*
14:00 – 15:00	**Bernard Leclerc** (Caen)
	Cluster algebras and canonical bases
15:15 – 16:15	**Jenny Wilson** (Stanford)
	Representation stability and classical Weyl groups
	Coffee Break
17:00 – 18:00	**Emmanuel Breuillard** (Orsay)
	Non-commutative diophantine approximation and representation theory of free Lie algebras.

Wednesday

09:00	*Coffee*
09:30 – 10:30	**Bertrand Patureau** (Vannes)
	The unrolled quantum group $\mathsf{U}_q^H\mathfrak{sl}_2$, from representations to TQFT
10:45 – 11:45	**Eric Marberg** (Stanford)
	Actions of Iwahori-Hecke algebras on involutions
	Lunch : Restaurant "La Bistouille"
14:00 – 15:00	**Kay Magaard** (Birmingham)
	Representations of Unitriangular Groups
15:30 – 16:30	**Joseph Grant** (Norwich)
	Braid groups and quiver mutation
17:00	*Discussion/Session of questions*

Thursday

09:00	*Coffee*
09:30 – 10:30	**Yves de Cornulier** (Orsay)
	Geometry of Lie groups, weights, and 2-cohomology of Lie algebras
10:45 – 11:45	**Philippe Biane** (Marne La Vallée)
	Pitman's theorem and crystals
	Lunch : Restaurant "La Source"
14:00 – 15:00	**Frank Lübeck** (Aachen)
	Kazhdan-Lusztig Polynomials and Applications to Finite Groups
15:15 – 16:15	**Valentin Ovsienko** (Reims)
	Representations of Lie algebras and invariants of differential geometry
16:30	*Discussion/Session of questions*
18:00	*"Hortillonnages"*
20:00	*Conference Dinner : Restaurant* "Le Vert Galant"

Friday

09:00 – 10:00	**Anton Evseev** (Birmingham)
	RoCK blocks, wreath products and KLR algebras
10:30 – 11:30	**Victoria Lebed** (Nantes)
	Representations of braid groups : homological and self-distributivity approaches
11:45 – 12:45	**Emmanuel Wagner** (Dijon)
	Markov traces on the Birman-Murakami-Wenzl algebras

⋆ ⋆ ⋆

Abstracts

Philippe Biane (Marne la Vallée)

Title : Pitman's theorem and crystals

Abstract : Pitman's theorem is a classical result about one dimensional Brownian motion. I will show how it is related to representation theory and how this leads to higher dimensional generalizations.

Emmanuel Breuillard (Orsay)

Title : Non-commutative diophantine approximation and representation theory of free Lie algebras.

Abstract : We start with the following basic diophantine approximation problem : given k elements in a real Lie group G, how fast can one approach the identity by products of increasing length of these elements ? When $k = 2$, G is the real line and the two elements are 1 and x, this is the classical question of how well x can be approximated by rationals. It is well-known that, for almost all x, x cannot be better approximated than what the pigeonhole principle already forces for every x. Surprisingly this is no longer true in general when G is non-commutative. The so-called non-diophantine Lie groups arise as certain nilpotent Lie groups of class bigger than 5. Showing their existence requires a understanding of the multiplicities of simple submodules of the free Lie algebra viewed as a GL_k module. By contrast, diophantine nilpotent Lie groups admit a diophantine exponent that can be shown to be rational when k is large (an instance of representation stability) and can be computed in terms of the dimensions of the simple GL_k-submodules of the relatively free Lie algebra defined by G.

Yves de Cornulier (Orsay)

Title : Geometry of Lie groups, weights, and 2-cohomology of Lie algebras

Abstract : The motivation of the talk is a geometric problem : given a simply connected solvable Lie group G with a left invariant Riemannian metric, how can we homotope a given loop to the trivial loop by sweeping a minimal area ? The Dehn function of G is the smallest function $\delta(r)$ such that every loop of length r can be homotoped with area $\leq \delta(r)$.
The Dehn function of such a Lie group always grows at most exponentially, and in joint work with Romain Tessera we prove that it grows either exponentially or at most polynomially. The dichotomy relies on the geometry of weights and on cohomology in degree 2. Surprisingly, the existence of wild central extensions (that is, defined on the underlying discrete group) plays a role. I'll try to describe these phenomena, with an emphasis on the algebraic side.

Anton Evseev (Birmingham)

Title : RoCK blocks, wreath products and KLR algebras

Abstract : The so-called RoCK (or Rouquier) blocks play an important role in representation theory of symmetric groups over a finite field of characteristic p, as well as of Hecke algebras at roots of unity. Turner has conjectured that a certain idempotent truncation of a RoCK block is Morita equivalent to the principal block of the wreath product $S_p \wr S_d$ of symmetric groups, where d is the "weight" of the block. The talk will outline a proof of this conjecture, which generalizes a result of Chuang-Kessar proved for $d < p$. The proof uses an isomorphism between a Hecke algebra at a root of unity and a cyclotomic Khovanov-Lauda-Rouquier algebra, the resulting grading on the Hecke algebra and the ideas behind a construction of R-matrices for modules over KLR algebras due to Kang-Kashiwara-Kim.

Joseph Grant (Norwich)

Title : Braid groups and quiver mutation

Abstract : To any ADE Dynkin diagram one can associate an Artin braid group, defined by generators and relations, which generalizes the classical braid groups in type A. By orienting the edges in the Dynkin diagram, one obtains a directed graph known as a quiver. Quiver mutation is an important combinatorial ingredient in the theory of cluster algebras which can drastically change a quiver, creating oriented cycles where none existed before. Robert Marsh and I showed that any quiver in the mutation class of a Dynkin quiver gives a new presentation of the braid group. I will explain this theory, and give some further interpretations using the geometry of triangulations and the homological algebra of Ginzburg dg-algebras.

Victoria Lebed (Nantes)

Title : Representations of braid groups : homological and self-distributivity approaches

Abstract : In the representation theory of braid groups, two approaches turn out to be particularly fruitful. The first one revolves around Lawrence's homological representations, and yields the linearity of braid groups, which long remained an open problem. The second one is based on self-distributive structures ; it allowed one to show the orderability of braid groups, and is at the heart of a very efficient construction of braid and knot invariants. The aim of this talk is to interpret Lawrence's representations from the self-distributivity viewpoint, and to discuss potential applications.

Bernard Leclerc (Caen)

Title : Cluster algebras and canonical bases

Abstract : Let N denote a maximal unipotent subgroup of a simple complex algebraic group, and let $C[N]$ be its coordinate ring. In 2001 Fomin and Zelevinsky endowed $C[N]$ with the structure of a cluster algebra, and conjectured that all its cluster monomials belong to Lusztig's dual canonical basis of $C[N]$. A few months ago Kang, Kashiwara, Kim and Oh announced a proof of this long-standing conjecture, based on the representation theory of KLR-algebras. In this lecture I will try to outline the main ideas of their work.

Frank Lübeck (Aachen)

Title : Kazhdan-Lusztig Polynomials and Applications to Finite Groups

Abstract : My interest in Kazhdan-Lusztig polynomials comes from a certain character formula for reductive algebraic groups which was first conjectured by Lusztig.
More generally, parabolic Kazhdan-Lusztig polynomials can be defined for arbitrary Coxeter groups. These polynomials are notoriously difficult to compute. I will report on an implementation of an algorithm to compute them which goes much further than previous programs.
It turned out that some of the coefficients of Kazhdan-Lusztig polynomials which I were able to compute have interesting (and for me unexpected) interpretations in the context of conjectures by Guralnick (on a bound of the dimension of first cohomology groups for finite groups) and Wall (on the number of maximal subgroups of any finite group). I will also explain these conjectures and sketch the connection between these topics.

Kay Magaard (Birmingham)

Title : Representations of Unitriangular Groups

Abstract : Let $UY_n(q)$ be a Sylow p-subgroup of an untwisted Chevalley group $Y_n(q)$ of rank n defined over $\mathbb{F}_q$ where q is a power of a prime p. We discuss what is known about the elements of $\mathrm{Irr}(UY_n(q))$, where Y is of exceptional type, and report on recent work with Goodwin, Himstedt, Le and Passolini.

Eric Marberg (Stanford)

Title : Actions of Iwahori-Hecke algebras on involutions

Lusztig and Vogan have shown that the involutions in any Coxeter group carry an essentially unique structure as a module of the group's Iwahori-Hecke algebra H(q). For generic parameters q, this module has a canonical basis analogous to the Hecke algebra's Kazhdan-Lusztig basis, which is already of interest in the theory of the unitary representations of complex reductive groups. On the other hand, the consideration of this module with q specialized to certain real numbers leads to some natural combinatorial and probabilistic questions about involutions in Coxeter groups, whose answers have surprising connections to things like the unipotent characters of Chevalley groups, the geometry of symmetric varieties, and q-analogues of classical orthogonal polynomials. For example, the degenerate specialization q=0 leads to a notion of "reduced words" for involutions, which in type A have a number of particularly nice enumerative properties. On the other hand, when q is specialized to a positive real number less than the inverse of the golden ratio, the action of the Iwahori-Hecke algebra defines a Markov chain on involutions, which can be precisely analyzed using information about the involution module's irreducible decomposition. In this talk I will outline some recent results, joint with Bernstein, Hamaker, Pawlowski, and White, in each of these contexts.

Valentin Ovsienko (Reims)

Title : Representations of Lie algebras and invariants of differential geometry

Abstract : Classical and famous objects of differential geometry and analysis, such as Laplace operators, Schwarzian derivative, and many others, arise originally in purely geometric problems. It turns out that all of them can be understood in the context of representation theory. The goal of this talk is to explain, using these examples, how "algebra helps geometry".

Bertrand Patureau (Vannes)

Title : The unrolled quantum group $\mathsf{U}_q^H\mathfrak{sl}_2$*, from representations to TQFT.*

Abstract : Reshetikhin and Turaev has shown after Witten how modular categories lead to 2+1 TQFT. After recalling briefly the construction, I will present some aspect of the category of representations of the unrolled quantum group $\mathsf{U}_q^H\mathfrak{sl}_2$ which is not modular, and what kind of TQFT can be built from its data. This is a work with Christian Blanchet, Francesco Costantino and Nathan Geer.

Emmanuel Wagner (Dijon)

Title : Markov traces on the Birman-Murakami-Wenzl algebras

Abstract : (joint work with Ivan Marin) In order to classify all Markov traces (and hence link invariants) factoring through the Birman-Murakami-Wenzl algebra we introduce an extension of this algebra which takes simultaneously into account the symplectic and orthogonal incarnation of this algebra. On one hand, for generic enough values of the dening parameters we prove that the only possible Markov traces are the ones providing the HOMFLYPT and the Kauffman link invariant and that the extension is in fact trivial. On the other hand for a family of special parameters, we obtain new algebraic object as well as new Markov traces. This new algebraic structure allows in particular to dene extension of the Temperley-Lieb algebra and of the Hecke algebra for $q = -1$.

Jenny Wilson (Stanford)

Title : Representation stability and classical Weyl groups

Abstract : Over the past few years Church, Ellenberg, Farb, and others have developed machinery for studying sequences of representations of the symmetric groups S_n by encoding these sequences into algebraic objects they call FI-modules. Their work is motivated by patterns and stability phenomena they observed in the rational cohomology of groups and spaces related to the pure braid groups. In this talk I will give an overview of their theory and describe how it generalizes to sequences of representations of the Weyl groups in type B/C and D. In this framework, an elementary finite generation condition implies strong constraints on the structure of the sequence : the characters are polynomial, and the decomposition into irreducible representations stabilizes in a precise sense. I will outline some applications in geometry and topology, including stability results for the cohomology of Coxeter hyperplane complements and generalized (pure) braid groups.