



# Differential Equations

## Tutorial #1

**Exercise 1.** Solve the following non-homogeneous linear differential equation:

$$f^{(3)} = 4f^{(2)} - 5f' + 2f + 4e^{3t}$$

with the initial conditions

$$f(0) = 7, \quad f'(0) = 15, \quad f''(0) = 40.$$

**Exercise 2.** Solve the following ordinary differential equation:

$$(\mathcal{E}) : \quad F'(t) = A \cdot F(t) + B(t),$$

where

$$A = \begin{pmatrix} 2 & 3 \\ -1 & -2 \end{pmatrix}, \quad \text{and} \quad B(t) = \begin{pmatrix} 1 \\ b(t) \end{pmatrix}.$$

Hint: first solve the homogeneous equation, then try to find a special solution of  $(\mathcal{E})$  of the form  $F(t) = \exp(tA) \cdot \Lambda(t)$ , with  $\Lambda : \mathbb{R} \rightarrow M_2(\mathbb{R})$  a differentiable map.

**Exercise 3.** Determine the exponential of the following matrix:

$$A = \begin{pmatrix} 2 & 1 & -1 \\ -1 & 0 & 1 \\ 0 & 0 & 2 \end{pmatrix}$$