Proper mathematical writing

- ~~Striking something out~~ means that it is not correct.
- Underlining with a wave means that it is not perfect.
- Underlining it regularly means that it is correct.

The purpose of a proof. It is very easy to explain something to someone who already knows it. When you write an assignment, we know precisely what you think even though you do not give all details — we do, because we know more mathematics than you do.
But some day you will have to explain things to people who know *less* mathematics than you: perhaps you'll be an engineer presenting a new device, a scientist writing a research article, or a professor giving a lecture. Saying that you are right will not be sufficient.
A proof will enable you to convince people quickly. Truth is not the only goal of a proof, also clarity, and at times simplicity.

1. Do not write for yourself, do not write for me, write for someone who knows less than you.
2. Always explain your notations, your assumptions, your goals, and your methods.

Words and Symbols.

1. As a rule, symbols are for stating only; not for proving.
2. As a rule, no sentence should mix words and symbols. The following is not correct:

   ~~$\forall x \in \mathbb{R}$, one has~~

3. As a strong recommendation, no sentence should start with a symbol:

   f is differentiable because...

$\therefore$ and $\because$. These symbols should not be used, because symbols are for stating, not for proving. Besides, they tend to make complex arguments unreadable. Here is a nice example: a proof that $1 = 0$.

$S = \{x | x \in x \Rightarrow 1 = 0\}$.

Suppose $S \in S$.

$\because S \in S$
$\because S = \{x | x \in x \Rightarrow 1 = 0\}$
$\therefore S \in S \Rightarrow 1 = 0$
$\therefore 1 = 0$

$\therefore S \in S \Rightarrow 1 = 0$

$\because S = \{x | x \in x \Rightarrow 1 = 0\}$

$\therefore S \in S$

$\because S \in S \Rightarrow 1 = 0$

$\therefore 1 = 0.$

Exercise: find the flaw. Until you do, refrain from using the symbols.

$\Rightarrow$. Extraordinarily few students use $\Rightarrow$ properly.

1. $\Rightarrow$ is a symbol: it is therefore for stating, not for proving.
2. $\Rightarrow$ reads: "implies" or "if ... then... ". The meaning of $x = 0 \Rightarrow x^2 = 0$ is that *if* $x = 0$, then $x^2 = 0$. This implication is *true* for any real number x. It is *not* a deduction that since x^2 is zero, so is x.
3. $\Rightarrow$ may not be used at the beginning or the end of a line: there must be something on its left and right.
4. For a *deduction*, use "so", "therefore", "thus", "hence", or any similar word, but *never* use $\Rightarrow$ (which is for implications, not for deductions).
5. *Conclusion: there is no symbol for "therefore".*

Then. *Then* has two very different uses in English:

1. Then as in "if $x = 0$ then $x^2 = 0$". The whole sentence may be written $x = 0 \Rightarrow x^2 = 0$. Notice that there is an "if".
2. Then as in "I missed the bus, then I was late". There is no "if".

   This is the same as in "Since $x = 0$, one has $x^2 = 0$, and then $x^3 = 0$". Here, "then" expresses *a series of deductions.*

Formulas. As a rule, no two formulas may follow each other:

~~We have $f'(x) = 2f(x) f(x) = \lambda e^{2x}$~~

A comma is not sufficient:

~~We have $f'(x) = 2f(x), f(x) = \lambda e^{2x}$~~

and neither is a line break:

~~We have $f'(x) = 2f(x)$~~
~~$f(x) = \lambda e^{2x}$~~

One should always add at least one word:

We have $f'(x) = 2f(x)$, so $f(x) = \lambda e^{2x}$.

Since and Therefore. In English, *since* and *therefore* are incompatible:

> ~~Since P, therefore Q.~~

One could write:

> Since P, Q.

or:

> P, therefore Q.

But it is not very convenient: in the former case only a comma separates P from Q; in the latter one starts with a symbol. So we recommend the following two:

> Since P, one has Q.

and:

> One has P, therefore Q.

Likewise, the following are not correct: ~~Because P, therefore Q~~, ~~As P, so Q~~, ~~Since P, then Q~~, ...

If and Then. *If* and *Then* may not be separated by a period:

> ~~If $f'(x) = 2f(x)$. Then $f(x) = \lambda e^{2x}$.~~ If $f'(x) = 2f(x)$, then $f(x) = \lambda e^{2x}$.

More generally it is alright to make an assumption using *if*, but only if the conclusion comes in the same sentence. Otherwise one uses *Assume/Suppose*.

Let and $\forall$. *Let* is a word used in proofs, for taking/constructing an object:

> Let $x \in \mathbb{R}$.

or:

> Let f be the following function:...

$\forall$ is a symbol used in statements (not in proofs). The following is highly incorrect:

> ~~$\forall x \in \mathbb{R}$, suppose ...~~

Let and Suppose. *Let* is for taking/constructing an object, as in

> Let $x \in \mathbb{R}$.

Suppose is for making an assumption, as in

> Let $x \in \mathbb{R}$. Suppose $x \in \mathbb{R}_{>0}$.

This can create confusion, because sometimes one directly takes an object with certain properties:

> Let $x \in \mathbb{R}_{>0}$.

But in any case, one may not use *Let* for assumptions:

> Let $x \in \mathbb{R}$ be a real number. There are two cases:
>
> - ~~Let x be positive.~~ Suppose that x is positive. ...
> - Now suppose $x \in \mathbb{R}_{<0}$....

Language. Your text must respect English grammar, spelling, and punctuation. Always make full, correct sentences. And never use abbreviations.