



# Differential Equations

## Tutorial #3

**Exercise 1.** Redo all quizzes.

**Exercise 2.** Consider the *pendulum equation*:

$$\theta''(t) = \sin(\theta(t))$$

1. Write this as a (non-linear) first-order, vector equation.
2. Is the function  $G$  you wrote Lipschitz? globally Lipschitz?
3. Deduce that for all  $(\theta_0, v_1)$ , there is a unique solution such that  $\theta(0) = \theta_0$  and  $\theta'(0) = v_1$ .

**Exercise 3.** Consider the function  $f : \mathbb{R} \rightarrow \mathbb{R}$  defined as follows:

$$f(x) = \begin{cases} -1 & \text{if } x < -1 \\ \sqrt{1-x^2} & \text{if } x \in [-1, 1) \\ 0 & \text{if } x \geq 1 \end{cases}$$

We consider the Cauchy problem  $x'(t) = f(x(t))$  with initial condition  $x(0) = a$ . Discuss existence and uniqueness depending on  $a$ .

**Exercise 4.** Consider equation  $x'(t) = x^2(t)$  on  $[0, 1]$  with  $x(0) = 1$ . Draw a  $\frac{1}{4}$ -approximate solution.