Tate Cohomology for Triangulated Categories

Javad Asadollahi

Motivated by the classical structure of Tate cohomology, we develop and study a Tate cohomology theory in a triangulated category $\mathcal{C}$. Let $\mathcal{E}$ be a proper class of triangles. By using $\mathcal{E}$-projective, as well as $\mathcal{E}$-injective objects, we give two alternative approaches to this theory that, in general, are not equivalent. So, we study triangulated categories in which these two theories are equivalent. This leads us to study the categories in which all objects have finite $\mathcal{E}$-$\mathcal{G}$projective as well as finite $\mathcal{E}$-$\mathcal{G}$injective dimension. These categories will be called $\mathcal{E}$-Gorenstein triangulated categories. We give a characterization of these categories in terms of the finiteness of two invariants: $\mathcal{E}$-$silp\mathcal{C}$, the supremum of the $\mathcal{E}$-injective dimension of $\mathcal{E}$-projective objects of $\mathcal{C}$ and $\mathcal{E}$-$spli\mathcal{C}$, the supremum of the $\mathcal{E}$-projective dimension of $\mathcal{E}$-injective objects of $\mathcal{C}$, where finiteness of each of these invariants for a category implies the finiteness of the other. The talk is based on a joint work with Sh. Salarian.