A Hall algebra approach to cluster algebras

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This is joint work with Bernhard Keller, [CK1], [CK2]. The cluster category is a triangulated category which enjoys a nice symmetry of Calabi-Yau type. We show how this symmetry property provides morphisms among varieties of triangles of the category. We then define Hall algebra type multiplication rules between objects of the cluster category, whose coefficients are Euler characteristics of varieties of triangles. On an opposite side, cluster algebras are commutative algebras defined inductively by generators and relations. We show how the multiplication rule connects explicitly the cluster category with an associated cluster algebra. This Hall algebra approach of cluster algebras enables to solve conjectures of Fomin and Zelevinsky, [?].

References

- [CK1] P. Caldero, B. Keller. From triangulated categories to cluster algebras I, math.RT/0506018.
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- [FZ] S. Fomin, A. Zelevinsky. Cluster algebras: Notes for the CDM-03 conference. Math.RT/0311493.