OMNIPRESENT EXCEPTIONAL MODULES FOR HYPERELLIPTIC ALGEBRAS

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A hyperelliptic algebra is a canonical algebra in the sense of Ringel of type $(2, 2, \ldots, 2)$, t entries, $t \geq 5$.

Whereas for domestic canonical algebras explicit descriptions for all indecomposable modules are known and for tubular canonical algebras we have results concerning the coefficients of the matrices of the exceptional modules the situation is much more complicated in the wild case. However a nice class of exceptional modules can be described explicitly by vector spaces and matrices.

It is well known that for a canonical algebra Λ the module category $\mathrm{mod}(\Lambda)$ admits a trisection $(\mathrm{mod}_+(\Lambda), \mathrm{mod}_0(\Lambda), \mathrm{mod}_-(\Lambda))$, where $\mathrm{mod}_0(\Lambda)$ is a separating family of stable tubes. We call a module M from $\mathrm{mod}_+(\Lambda)$ omnipresent if for each $S \in \mathrm{mod}_0(\Lambda)$ there is a non-zero homomorphism from M to S.

Theorem 1 *Let Λ be a canonical algebra of type $(p_1, p_2, \ldots, p_t)$ and M an omnipresent exceptional Λ-module from $\mathrm{mod}_+(\Lambda)$. Then $\mathrm{rk}(M) \geq t - 1$.*

Theorem 2 *Let Λ be a hyperelliptic algebra of type $(2, 2, \ldots, 2)$, t entries. Then, up to duality and "up to shift", there exists a unique omnipresent exceptional module of rank $t - 1$ in $\mathrm{mod}_+(\Lambda)$.*

In the hyperelliptic case we will give explicit matrices for all omnipresent exceptional modules of minimal rank $t - 1$.

The method in the proof is based on the study of universal extensions of line bundles on the corresponding weighted projective line and is useful also in other situations. In particular in the tubular case these techniques provide an algorithmic way for an explicit description of all exceptional modules. In joint work with P. Dowbor we are developing a computer program concerning this problem.