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Cellular algebras

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Abstract

This is a series of 4 lectures on cellular algebras. In the lectures we introduce the theory of cellular algebras which were defined first by Graham and Lehrer in 1996 to deal systematically with parameterizing irreducible representations of algebras and related problems. If a finite-dimensional algebra is given by quiver with relations, the irreducible representations over such an algebra are very easy to describe. But in the real world, algebras, especially those with the interesting applications in mathematics and physics, appear very often not in this form. It turns out that the determination of the irreducible representations of these algebras is a quite hard problem. The difficulty encountered is well-known, for instance, in the representation theory of finite groups. The theory of cellular algebras enables one to solve this problem and study the irreducible representations axiomatically in terms of linear algebra.

Cellular structures of algebras seem to be very common; in the last a few years, a large variety of algebras appearing in both mathematics and physics are proved to have a cellular structure. For example, the Hecke algebras and the q-Schur algebras of type A, Brauer algebras; inverse semigroup algebras; Temperley-Lieb algebras, partition algebras, Birman-Wenzl algebras, and many other diagram algebras. It is worthy to notice that the cellular algebra method can also be used to study algebras of infinite dimension.

The contents of the lectures will include the definitions (original definition of Graham and Lehrer in terms of a basis, and the one which is basis-free), some examples and basic properties of cellular algebras; representation theory of cellular algebras; homological aspects of cellular algebras, and relationship with quasi-hereditary algebras; applications to algebras in mathematics and physics.