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CHAPTER 68

GEORGE DAVID BIRKHOFF, *DYNAMICAL SYSTEMS* (1927)

David Aubin

The first book to expound the qualitative theory of systems defined by differential equations, Birkhoff's *Dynamical systems* created a new branch of mathematics separate from its roots in celestial mechanics and making broad use of topology. Important for several fields of mathematics, its impact became massive recently with the spread of 'chaos theory'.

First publication. Providence, Rhode Island: American Mathematical Society (Colloquium Publications Series, no. 9), 1927. viii + 295 pages.

Revised edition. Introduction and addendum by Jürgen Moser, preface by Marston Morse. 1966. xii + 305 pages. Tenth printing 1999.

Russian translation. Dinamicheskie sistemy (trans. E.M. Livenson, ed. A.A. Markov, V.V. Nemytskij and V.V. Stepanov), Moscow and Leningrad: 'Gostekhizdat', 1941. [Repr. Izhevsk: Izd. dom 'Udmurtskij Univ.', Nauchno-Izdatel'skij Tsentr 'Regulyarnaya i Khaoticheskaya Dinamika', 1999 (Seriya Regulyarnaya i Khaoticheskaya Dinamika, no. 8).]

Related articles: Poincaré (§48), Lyapunov (§51), Einstein (§63).

1 INTRODUCTION

'History has responded to these pages on Dynamical Systems in an unmistakable way'. When this book by George David Birkhoff (1884–1944) was reissued in 1966, nearly 40 years after its first publication and more than 20 years after its author's death, Marston Morse stressed its historical legacy in his new preface (p. v). A decade later, such a remark would have seemed superfluous. The craze for 'deterministic chaos' was in full swing and scores of scientists were striving to master dynamical systems theory. Undoubtedly rooted in multifaceted work of Henri Poincaré (1854–1912) at the turn of the century, this theory as Birkhoff defined it was a branch of mathematics that dealt with the global qualitative

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behavior of systems governed by deterministic laws (that is, where randomness played no part). In retrospect, *Dynamical systems* (hereafter, '*DS*') stands strangely isolated among the mathematical literature of its time as a fundamental intermediary between Poincaré's perceptive work and the modern theory.

Deterministic chaos and dynamical systems theory have had a perplexing history [Aubin and Dahan-Dalmedico, 2002]. That older results that could have been 'forgotten' for several decades gave rise to widespread puzzlement. Albeit well received by the mathematical press when it was first published in 1927, *DS* was a textbook for a field of mathematics that barely existed for some decades to come. Its main domain of application—celestial mechanics—seemed to have lost some of its urgency now that relativity theory and quantum mechanics were revolutionizing physics. By insisting on considering general problems of dynamics as opposed to particular ones and by looking globally at sets of motions rather than particular orbits, Birkhoff's way of approaching the topic was highly original. Not only was he creating an up-to-date topological apparatus for the task at hand, he also confronted head-on the problem of finding a role for dynamical theory when the fundamental equations of physics were being recast. The striking contrast between conformist subject-matter and innovative mathematical and epistemological frameworks can account for the unusual career of *DS*, both the relative oblivion into which it fell and its later success. More than the results presented in the book, the main reason for its posthumous fame is surely its *style*, which largely derives from the intellectual context in which it was produced, that of American mathematics in the decade following the Great War.

2 CELESTIAL MECHANICS: THE HISTORICAL BACKGROUND

In the 19th century, the understanding of the analytic structure of the equations of motion derived from Newtonian mechanics was greatly advanced with the work of Joseph Louis Lagrange (§16), W.R. Hamilton and Carl Jacobi. In the paradigmatic field of celestial mechanics, Pierre-Simon Laplace had perfected Leonhard Euler's perturbation method which allowed him and his successors to compute planetary orbits very accurately in terms of power series (§18). The discovery in 1846 of Neptune on the basis of computations made by Urbain Leverrier and John C. Adams showed that results could be astonishing. But the so-called three-body problem remained as frustrating as ever. The law of gravitation acting upon three masses—especially the Sun, the Earth, and the Moon—gave rise to a system of differential equations for which no explicit expression of the solution valid for all time could be found.

Up to that point, in rational mechanics, one mostly tried to find a local trajectory, that is, solve a system of differential equations with given initial conditions without paying much attention to global behaviors. For complicated problems, the influence of each major planet was treated as a perturbation and solutions expressed in forms of power series. In his famous entry to King Oscar of Sweden's prize of 1889, Poincaré showed that such power series were in general divergent (§48.5). Renouncing the idea of obtaining convergent series, several methods—analytic, extremal or topological—were laid out by Poincaré and explored by a few of his followers. His work on curves defined by differential equations, on celestial mechanics whether concerned with astral orbits or shapes of rotating fluid masses, even his path-breaking development of topology, always emphasized the global behavior of solutions to a system.

Among the few astronomers and mathematicians who were inspired by Poincaré's ideas, Birkhoff went the furthest in developing a full-fledged theory of dynamical systems detached from its roots in celestial mechanics, and making a systematic use of topology. Having been exposed to the three most prominent currents of mathematical thought in the United States, Birkhoff was well prepared to tread this topological road. His constant insistence on marrying analysis with topology at the highest level of abstraction possible had no other root.

Born in 1884 in Michigan, Birkhoff was awarded a Ph.D. by the University of Chicago in 1907. He was one of the first leading American mathematicians to be fully trained in the Unites States without having made the trip to Europe. But he had divided his time between the leading centers of American mathematics. At Harvard, William Osgood and Maxime Bôcher introduced him to classical analysis, while at Chicago he learnt the abstract modern ideas of Eliakim Hastings Moore's 'general analysis' [Siegmund-Schültze, 1998]. Through his interactions with Oswald Veblen at Princeton University, where he taught from 1909 to 1912, Birkhoff encountered a third significant current of mathematical thought: *analysis situs,* as the nascent field of topology was then called (compare §76.1).

Shortly after Poincaré's untimely death in 1912, Birkhoff suddenly established his international mathematical stature with a *coup d'éclat* when he published the proof of a conjecture known as 'Poincaré's last geometric theorem'. The theorem stated that continuous, one-to-one, area-preserving maps from the annulus to itself rotating points on the boundaries in opposite directions had at least two fixed points [Poincaré, 1912; Birkhoff, *Papers*, vol. 1,673–681]. As Poincaré had already seen, this theorem has important consequences for dynamics.

3 THE CONTENTS OF BIRKHOFF'S BOOK

DS was published in 1927, when Birkhoff was 43 years old; it is summarised in Table 1. A professor of mathematics at Harvard University since 1912, he was by then a well-respected statesman of the American mathematical community, active in the American Academy of Sciences and the National Research Council, as well as having served as the president of the American Mathematical Society.

Before 1927, the only source on general dynamics had been the three volumes of Poincaré's *Les méthodes nouvelles de la mécanique céleste* (1892–1899), characterized by George Darwin as 'for half a century to come [...] the mine from which humbler investigators will excavate their materials' [Barrow-Green, 1997, 152]. But Poincaré's magisterial treatise contained much that was cumbersome to use, at times obscure, and at times—for those interested in general dynamics—unduly concerned with details of celestial mechanics. For Birkhoff, on the other hand, dynamics ought not to address a single problem, but rather directly tackle the most general class of dynamical systems defined by the differential equations

$$dx_i/dt = X_i(x_1, \dots, x_n), \quad i = 1, \dots, n. \qquad (1)$$

Table 1. Contents by chapters of Birkhoff's book.

Ch.	Page	'Title': other included topics
I	1	'Physical aspects of dynamical systems': general analytic discussion, conservation of energy, Lagrangian equations.
II	33	'Variational principles and applications': Hamiltonian dynamics.
III	59	'Formal aspects of dynamics': formal series.
IV	97	'Stability of periodic motions'.
V	123	'Existence of periodic motions': variational principles (geodesics), analytic continuation.
VI	150	'Application of Poincaré's geometric theorem': Poincaré map.
VII	189	'General theory of dynamical systems': topological definitions.
VIII	209	'The case of two degrees of freedom'.
IX	260	'The problem of three bodies'. [End 295.]

In *DS*, Birkhoff summarized more than 15 years of his own research along three main axes: the general theory of dynamical systems; the special case with two degrees of freedom; and the three-body problem in celestial mechanics. These topics form the subject of the last three chapters (VII, VIII, and IX), which have been the most widely admired and studied. In the first two chapters, Birkhoff's treatment was traditional: he gave proofs for existence, uniqueness and continuity theorems, and then discussed Lagrange's equations, Hamiltonian mechanics, and changes of variables. In chapter III, solutions were studied in their formal aspects, that is as power series about which questions of convergence were systematically laid aside as irrelevant to the matter at hand. The next chapter followed Poincaré's idea of investigating the stability of formal solutions near equilibrium or periodic motion. But Birkhoff again went further in considering a vast array of definitions for stability: complete or trigonometric stability, stability of the first order, permanent stability 'for which small displacements from equilibrium remain small over time' (p. 121), semi-permanent stability, unilateral stability (due to Lyapunov: §51), and stability in the sense of Poisson (due to Poincaré).

Chapter V presented four methods by means of which the existence of periodic motions could be established. The first of these made use of the variational principles of dynamics, for example by considering geodesics on a surface and their deformations (a method developed by Jacques Hadamard). A second variational method was called the 'minimax' method, whereby new geodesics were found by considering the lower limit of the length of geodesics stretched by a rotation of the manifold. The minimax method was the original stimulus for Morse theory, which made topological considerations effective for analysis [Morse, 1934, iv]. The third method, due to George W. Hill and Poincaré, looked at the analytic continuation of periodic orbits already known to exist.

The fourth method showing the existence of periodic motion was a generalization of Poincaré's idea of transverse section. Birkhoff's formal theory of Poincaré sections would become an indispensable element of dynamical systems theorists' toolkit (Chapter VI). This gave a dynamical problem a 'striking change of form'. When a continuous dynamical flow was cut transversally by a surface S, each time the continuous dynamical flow was crossing S, the dynamical equation defined a point P_i on the surface. Successive intersections defined a one-to-one analytic transformation T of the section surface S into itself. Poincaré had used the idea to transform the reduced three-body problem into the transformation of the ring into itself. Birkhoff showed that there was a great variety of circumstances where one could use this method. The example of a billiard ball rolling on a flat surface with curved boundaries concretely illustrated the power of the method.

4 BIRKHOFF'S MAIN AMBITION

This was to develop a 'General theory of dynamical systems', which is summarized in Chapter VII of *DS*. 'The final aim of the theory of motion must be directed toward the qualitative determination of all possible types of motions and of the interrelation of these motions' (p. 189). As Koopman [1930] wrote in a review, '[t]he only property made use of in this chapter is the bare fact that the curves are integral curves of analytic differential equations. The treatment has the aspect of a study in point-set theory'. With this work begun in 1912, dynamical systems theory was thoroughly infused with topological ideas.

Birkhoff showed that for arbitrary dynamical systems there always was a closed set of 'central motions' endowed with a certain property of 'recurrence' and towards which all other motions of the system in general tended asymptotically. Considering the equation of motion (1), he looked at states of motion as points in a closed n-dimensional manifold M. To each point P_0 of M (initial conditions), one could associate via (1) a curve of motion lying on M. He then divided the manifold M into two non-intersecting sets: the open set of wandering points, that is, those starting from which the equations of motion would define a trajectory filling open n-dimensional continua in M; and the complementary closed set M_1 of non-wandering points. As time increased or decreased, he showed, every wandering point approached the set M_1 of non-wandering points.

Further, Birkhoff constructed a sequence of sets $M_1, M_2, \ldots,$ where M_2 was the set of non-wandering points with respect to M_1, etc. This process had to end at some point with a set C of *central motions*. This was a generalization of periodic motions to which Poincaré had drawn attention. For Birkhoff, the first problem concerning the properties of dynamical systems was the determination of central motions. The fact that for classical dynamics, central motions were the totality of all motion made amply clear that he was constructing a 'general theory' with a wider range of applicability.

In 1912, Birkhoff also introduced the notions of 'minimal' or 'recurrent' sets of motions. Let α- and *ω-limit points* be points towards which other motions tended as time t approached $-\infty$ or $+\infty$. If Σ was a closed, connected set of limit motions (i.e. trajectories composed of limit points of a motion) and Σ had no proper subset, then Birkhoff defined the members of Σ as recurrent motions and the set itself as minimal. He showed that a motion was recurrent if and only if for any $\varepsilon > 0$, curves of motion would remain for a certain interval of time T within a distance ε of every point of the trajectory. In other word, such motions came back arbitrary close to every point of the curve of motion. They were in the set of central motions but the reverse was not necessarily true.

Based on an astute use of topology, these definitions greatly extended the possibility of classifying the motions generated by dynamical systems. Solomon Lefschetz asked in his review [1929] how classical dynamics was ever able to do without them. Birkhoff's notion of non-wandering points was picked up by the Russian mathematician Aleksandr Andronov, whose roughly contemporary work in this domain ranked equal in importance with Birkhoff's. And it prefigured the concept of 'attractor' that was fundamental for the reconfiguration of dynamical systems theory from the mid-1960s onward. Made famous by René Thom and Steve Smale, an attractor has been succinctly defined as 'an *indecomposable, closed, invariant* set [...] which attracts all orbits starting at points in some neighborhood' [Holmes, 1990].

5 BIRKHOFF'S LECTURE COURSE AND ITS CONTEXT

Though widely admired, the offshoot of the theory was somewhat disappointing. As Birkhoff acknowledged, the remarkable diversity and complexity of behaviors meant that 'rigorously proven qualitative results are rare' [Birkhoff, *Papers*, vol. 2, 246]. As was pointed out by Koopman [1930], what was at stake was the value of a mathematical theory. To reach a better grasp of both Birkhoff's approach to dynamics and the long-term reception of *DS*, one must look at the American postwar context in which it was produced.

Prior to becoming the cornerstone of a branch of mathematics, the American Mathematical Society summer colloquium lectures upon which *DS* was based was the best attended series so far. In September 1920, over 90 mathematicians gathered at the University of Chicago to hear Birkhoff, at the first of these events to take place after the end of the Great War. For many, this was an inspiring return to normalcy. He treated his audience with a review of recent developments in an honored field of mathematical physics crowned with far-ranging philosophical speculations. As was the tradition, the lecturer emphasized his own contributions. In his five lectures, he reviewed traditional approaches to dynamical problems, and summarised his own work on topological tools to describe the various types of motion that could occur in a dynamical system making the crucial distinction between hyperbolic and elliptic motions. His fourth lecture was an application of this 'General analysis' to the three-body problem. Concerned with the 'significance of dynamical systems for general scientific thought', Birkhoff's fifth lecture was not published in *DS*. From their titles, themes broached—'The dynamical model in physics', 'Modern cosmogony and dynamics', 'Dynamics and biological thought', 'Dynamics and philosophical speculation'—are tantalizing in their ambitions [Hurwitz, 1920].

Like other American scientists, mathematicians in the early 20th century were preoccupied by issues of purity [Parshall and Rowe, 1994]: 'Gross utilitarianism is the obvious danger' [Carmichael, 1919, 163]. While astronomers importantly shaped the emerging mathematical community in the United States, a younger generation centered around Chicago 'played a leadership role in defining the mathematical profession on American shores in terms of pure, abstract, rigorous mathematics' [Parshall, 2000, 8]. Their ethos was 'a privileging of pure over applied mathematics, of research over teaching, and of educating future mathematicians over training others who needed advanced mathematical skills' [Butler Feffer, 1997, 66–67]. Birkhoff certainly agreed with a mild version of this credo. Although he always emphasised applications in celestial mechanics, he never computed an orbit.

Since Poincaré had written his *Méthodes nouvelles,* two things had happened that transformed the way dynamics was to be understood: the Great War and relativity theory (§63). If its impact was no way overwhelming, the war effort in 1917–1918 introduced a crucial inflexion in scientists' self-perception [Aubin and Bret, 2003]. A handful of mathematicians, Birkhoff among them, worked on war-related topics (ballistics, sound-ranging, and submarine detection). But as opposed to physicists and chemists, they felt they had a harder time convincing the country that their skills were required for warfare. The war led to a reevaluation of the role played by formal mathematics in the physical sciences and some soul-searching on the mathematicians' part [Servos, 1986]. Some felt that overemphasis on purity had led to a detrimental neglect of applied mathematics. To those concerned with the role of mathematics in science and other human affairs, mathematicians often replied that their inquiries were essential in understanding the deep structures of scientific thought. 'Transcending the flux of the sensuous universe, there exists a stable world of pure thought, a divinely ordered world of ideas, accessible to man, free from the mad dance of time, infinite and eternal' [Keyser, 1915, 679]. Looking for stability in complex flows, dynamical systems theory was Birkhoff's attempt at accommodating two strong, yet antagonistic tendencies of postwar American mathematics: the strive towards purity, if not purism; and the acknowledgment, reinforced by the war, that mathematicians ought to be concerned with applications.

The postwar situation was further complicated by revolutions in physics. Poincaré, it was claimed, 'was depressed when certain recent physical theories seemed to imply that differential equations are not so fundamental to the understanding of phenomena as he had supposed' [Carmichael, 1917, 168]. More than physicists and astronomers, American mathematicians often readily welcomed relativity theory [Goldberg, 1987]. Birkhoff published two books on Einstein's theory (1923, 1925); the first was, with Stanley Eddington's *Mathematical theory of relativity* of 1923, among the earliest books in English explaining relativity with sufficient mathematical sophistication.

Like Veblen, Birkhoff argued that crises in fundamental physics increased the importance of the mathematician who provided a 'rigorous and qualitative background' to the 'more physical, formal, and computational aspects of the sciences' [Birkhoff, *Papers*, vol. 2, 110]. Through the years, his position evolved and it later seemed that dynamical systems theory was for skeptics. 'At a time when no physical theory can properly be termed fundamental—the known theories appear to be merely more or less fundamental in certain directions—it may be asserted with confidence that ordinary differential equations in the real domain, and particularly equations of dynamical origin, will continue to hold a position of the highest importance' (*DS*, iii). 'In view of the many indignities which mechanics has suffered in recent years', a reviewer wrote with a sigh of relief, 'this volume merely illustrates that additional hypotheses are not as yet needed if one wishes to make new discoveries in dynamics' [Bartky, 1928].

6 ON THE IMPACT AND RENAISSANCE OF THE BOOK

DS was not Birkhoff's last word on the topic. In particular, his proof in [Birkhoff, 1931] of the ergodic theorem was deemed as important as his proof of Poincaré's geometric theorem. Introduced by Ludwig Boltzmann, ergodicity has been a cornerstone of statistical mechanics. It described systems such that each particular motion when continued indefinitely passed through every configuration compatible with energy conservation. Allying topological consideration with Henri Lebesgue's theory of integration (§59.3), Birkhoff developed the notion of transitivity introduced in *DS*(that is, the property of a dynamical system whereby small neighborhoods of curves of motion filled the whole manifold up to a set of measure zero) and showed that it was a widespread property for Hamiltonian systems. Birkhoff knew that this property was not generic, but his results prompted further developments in ergodic theory [Dahan-Dalmedico, 1995].

DS also shaped much of the work done by Aleksandr Kolmogorov, Vladimir I. Arnol'd, and Jürgen Moser in the 1950s and 1960s on the celebrated KAM theorem that invalidated Birkhoff's ergodic conjecture [Diacu and Holmes, 1996]. Several other concepts introduced by Birkhoff were later picked up by others. On the notion of recurrent motion, Morse, Walter H. Gottschalk, and Gustav A. Hedlund built an abstract theory of symbolic dynamics in 1955 that is used today in theoretical computer science. Another example is the 'bad' curve studied by Birkhoff in 1932, a complicated state of motion that ultimately formed the basis for Smale's 'horseshoe', a stable, yet chaotic motion [Abraham, 1985].

Very technical, those developments kept the memory of Birkhoff's *DS* alive, but restricted to specialized fields of inquiry until dynamical systems theory was spectacularly revived after the Second World War by Lefschetz [Dahan-Dalmedico, 1994]. But Lefschetz and his collaborators rediscovered the work of Poincaré through their close study of Russian sources rather than in Birkhoff's work. One reason for this was the insistence put on dissipative systems where energy is not conserved, as opposed to conservative ones emphasized by Birkhoff. In *DS*, the section on dissipative systems occupied less than two pages. He acknowledged that '[c]onservative systems are often limiting cases of what is found in nature', but dissipative systems generally tended toward a the motion of a conservative system with fewer degrees of freedom.

The mathematicians' more active participation to the Second World War and the Cold War, as well as concerns with nonlinear oscillations arising from radio-engineering (B. Van der Pol), led to an understanding of dynamical systems different than that stemming from Birkhoff's nearly exclusive concern with celestial mechanics. This crucial difference in emphasis is brought to light by comparing Birkhoff's attitude concerning stability with Andronov's [Dahan-Dalmedico, 2004]. Both dealt with general systems of differential equations using many of the same sources (Poincaré, Lyapunov). They nonetheless ended up with almost opposite views on stability. Inspired by the famous 'problem of stability' of the three-body problem, Birkhoff restricted the study of stability to that of orbits lying near a periodic (or central) motion. He thought one had to dictate, by convention or by a judicious choice of problems to be answered, the kind of stability that one wanted to look at. 'All that stability can mean is that, for the system under consideration, those motions whose curves lie in a certain selected part of phase space from and after a certain instant are *by definition* called stable, and other motions unstable' [Birkhoff, *Papers*, vol. 3, 602]. Concerned with radio systems, Andronov imagined a more general type of stability that applied not only to solutions of a system of differential equations, but to the system itself. The only interesting systems for modeling, he thought, were structurally stable, that is, keeping the same qualitative behavior under small deformations.

In the 1960s, a generation too young to have had to participate to the war effort revived widespread interest in *DS*. Less interested in control than their elders, Mauricio Peixoto and Smale launched a general program of classification of dynamical systems that thrived on the topological approach that was characteristic of Birkhoff's work. Following the publication of Smale's *Differentiable dynamical systems* in 1967, a blooming field was established that had a profound impact on the way that the mathematical modeling of natural phenomena was to be understood. Edward N. Lorenz in 1963 and David Ruelle in 1971 independently exhibited systems governed by simple deterministic laws that nonetheless exhibited complex, apparently erratic behaviors [Aubin, 2001].

All of a sudden, *Dynamical systems* enjoyed a second life. In this book, people interested in chaos found a straightforward style that corresponded to their expectations. Physicists liked to see equations of motion written in a form they recognized. They were comfortable with discussions of Lagrangian and Hamiltonian functions. No fancy Bourbakist abstraction here defaced them [Aubin, 1997]. No more than an elementary topological knowledge was required to grasp the most innovative ideas introduced in the book. Readers also appreciated the self-contained character of the book and the tools presented in all generality, in less than 300 pages of clear English prose. A generation mobilized against the Vietnam war and intend to 'explicitly direct [its] work toward socially-positive goals' [Smale, 1972, 3] found in American struggles with issues of purity after the Great War in the face of new wars and upheavals in physics an epistemological and moral framework with which they felt comfortable.

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Reviews of the book are gathered at the end.

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