MODEL THEORY OF FIELDS-UMONS 11/18/2013

1. Schedule-(room 3E20, Le Pentagone)

- 9h30-10h *Welcome*

- 10h -11h10: S. Kuhlmann (Konstanz University): The difference rank of a valued difference field (joint work with M. Matusinski and F. Point).

- 11h10-11h30: *Coffee break.*

- 11h30- 12h40 N. Mariaule (Secunda Universita di Napoli): On the decidability of the p-adic exponential ring.

- 12h40-14h00 *Lunch (Forum, Les grands amphitheatres)*

- 14h-15h10 M. Matusinski (Bordeaux 1): The exponential-logarithmic equivalence classes of surreal numbers.

- 15h20-16h30 S. Montenegro (Paris 7): Some model-theoretic properties of Pseudo Real Closed fields.

- 16h30-17h00 *Coffee break*

- 17h-18h10 F. Jahnke (Muenster university): Definable Henselian valuations.

- 19h30 *Social Dinner*

2. Abstracts

S.Kuhlmann (Konstanz university) [*Salma.Kuhlmann@uni-konstanz.de*]

Title: **The difference rank of a valued difference field** (joint work with M. Matusinski and F. Point).

Abstract: There are several equivalent characterizations of the valuation rank of an ordered field (endowed with its natural valuation). We extend the theory to the case of a difference field (respectively of an ordered difference field) and introduce the notion of difference rank. We characterize the difference rank as the quotient modulo the equivalence relation naturally induced by the automorphism (which encodes its growth rate). In analogy to the theory of convex valuations, we prove that any linearly ordered set can be realized as the difference rank of an ordered difference field.

N. Mariaule (Secunda Universita di Napoli) [*mariaule_nathanael@hotmail.com*]

Title: **On the decidability of the p-adic exponential ring**

Abstract: Let exp(x) denote the power series of the usual exponential function. This series is not convergent on the whole p-adic field $\mathbb{Q}_p$. Yet, the series $exp(px)$ determines a function on the ring $\mathbb{Z}_p$ of p-adic integers. In this talk, I will present recent results on the model theory of the p-adic exponential ring, namely the ring $\mathbb{Z}_p$ in the language of rings expanded by a function symbol for $exp(px)$. The main theorem of the talk is that if a p-adic version of Schanuel's conjecture is true then the theory of $(\mathbb{Z}_p, exp(px))$ is decidable. The proof is in two parts: first, a result of effective model-completeness in some nice expansion of the language. Then, assuming Schanuel's conjecture, we can show that the existential part of the theory (in the expansion of the language) is decidable. I will give more precise statements of the above results in the talk and sketch some of the proofs.

M. Matusinski (Bordeaux 1) [*mickael.matusinski@math.u-bordeaux1.fr*]

Title: **The exponential-logarithmic equivalence classes of surreal numbers.**

Abstract: In his monograph, H. Gonshor showed that Conway's real closed field of surreal numbers carries an exponential and logarithmic map. Subsequently, L. van den Dries and P. Ehrlich showed that it is a model of the theory of $\mathbb{R}_{exp}$. After giving a quick overview of these results, we will present in this talk the description of what we call the exponential equivalence classes of surreal numbers. This notion takes place naturally between Conway's omega map and generalized epsilon numbers. This is a first step toward proving that surreal numbers form a field of exp-log series and transseries. This would help us, applying previous results of ours, to endow the field of surreal numbers with a natural derivation.

This is a joint work with Salma Kuhlmann.

S. Montenegro (Paris 7) [*samypi@yahoo.com*]
Title: **Some model-theoretic properties of Pseudo Real Closed fields.**

Abstract: The theory of PRC (and n-PRC) fields was extensively studied by L. van den Dries, A. Prestel, M. Jarden, S. Basarab, D. Haran, and others. In the first part of this talk I will present a short summary of the principal results in the model theory of PRC fields. Let K be a field endowed with n distinct orderings: $P_1, \cdots, P_n$. Then $(K, P_1, \ldots, P_n)$ is a n-PRC field if and only if every absolutely irreducible variety defined over K that has a rational point in every real closure of K, has a K-rational point.

The theory of n-PRC fields is not NIP, and in the second part of this talk I will give you a formula that has the independance property.

The main theorem of this talk is a positive answer to the conjecture by A. Chernikov, I. Kaplan and P. Simon that says: A PRC field is NTP_2 if and only if it is bounded. After introducing the property NTP_2, if time permits, I will give a sketch of the proof.

F. Jahnke (Muenster university) [*franziska.jahnke@uni-muenster.de*]
Title: **Definable Henselian valuations.**

Abstract: Starting from the definition of a henselian valuation, we will discuss the role of henselian valuations in model theory of fields. The main topic will be how, when and why henselian valuations are encoded in the arithmetic of a field, i.e. 0-definable in the theory of the field considered in the language of rings.