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    |\^/|     Maple 9 (IBM INTEL LINUX)
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      |       Type ? for help.
>  interface(screenwidth=120);
>  with(LinearAlgebra):
>  # Construction de l'exemple: on veut une reponse de ce type:
>  f:=(i,j)->if(i=j-1) then 1 else 0 fi;
               f := proc(i, j) option operator, arrow; if i = j - 1 then 1 else 0 end if end proc

>  J:=Matrix(8,8,f);#forme classique d'ordre 8.
                                          [0    1    0    0    0    0    0    0]
                                          [                                   ]
                                          [0    0    1    0    0    0    0    0]
                                          [                                   ]
                                          [0    0    0    1    0    0    0    0]
                                          [                                   ]
                                          [0    0    0    0    1    0    0    0]
                                     J := [                                   ]
                                          [0    0    0    0    0    1    0    0]
                                          [                                   ]
                                          [0    0    0    0    0    0    1    0]
                                          [                                   ]
                                          [0    0    0    0    0    0    0    1]
                                          [                                   ]
                                          [0    0    0    0    0    0    0    0]

>  J[3,4]:=0:J[6,7]:=0:J;#2 blocs d'ordre 3 et un d'ordre 2.
                                        [0    1    0    0    0    0    0    0]
                                        [                                   ]
                                        [0    0    1    0    0    0    0    0]
                                        [                                   ]
                                        [0    0    0    0    0    0    0    0]
                                        [                                   ]
                                        [0    0    0    0    1    0    0    0]
                                        [                                   ]
                                        [0    0    0    0    0    1    0    0]
                                        [                                   ]
                                        [0    0    0    0    0    0    0    0]
                                        [                                   ]
                                        [0    0    0    0    0    0    0    1]
                                        [                                   ]
                                        [0    0    0    0    0    0    0    0]

>  #On veut faire un changement de base simple. Ex det=1 pour garder  des coeffs entiers.
>  #on cree une transvection:
>  f:=(i,j)->if(i=j) then 1 else 0 fi;
                 f := proc(i, j) option operator, arrow; if i = j then 1 else 0 end if end proc

>  T:=proc(i,j,a)
>  A:=Matrix(8,8,f):A[i,j]:=a;A;
>  end proc;
                   T := proc(i, j, a) local A; A := Matrix(8, 8, f); A[i, j] := a; A end proc

>  B:=Matrix(8,8,b);
                 [b(1, 1)    b(1, 2)    b(1, 3)    b(1, 4)    b(1, 5)    b(1, 6)    b(1, 7)    b(1, 8)]
                 [                                                                                  ]
                 [b(2, 1)    b(2, 2)    b(2, 3)    b(2, 4)    b(2, 5)    b(2, 6)    b(2, 7)    b(2, 8)]
                 [                                                                                  ]
                 [b(3, 1)    b(3, 2)    b(3, 3)    b(3, 4)    b(3, 5)    b(3, 6)    b(3, 7)    b(3, 8)]
                 [                                                                                  ]
                 [b(4, 1)    b(4, 2)    b(4, 3)    b(4, 4)    b(4, 5)    b(4, 6)    b(4, 7)    b(4, 8)]
            B := [                                                                                  ]
                 [b(5, 1)    b(5, 2)    b(5, 3)    b(5, 4)    b(5, 5)    b(5, 6)    b(5, 7)    b(5, 8)]
                 [                                                                                  ]
                 [b(6, 1)    b(6, 2)    b(6, 3)    b(6, 4)    b(6, 5)    b(6, 6)    b(6, 7)    b(6, 8)]
                 [                                                                                  ]
                 [b(7, 1)    b(7, 2)    b(7, 3)    b(7, 4)    b(7, 5)    b(7, 6)    b(7, 7)    b(7, 8)]
                 [                                                                                  ]
                 [b(8, 1)    b(8, 2)    b(8, 3)    b(8, 4)    b(8, 5)    b(8, 6)    b(8, 7)    b(8, 8)]

>  # faire Li<-Li+aLj c'est multiplier a gauche par T(i,j,a).
>  # Par exemple L3<- L3+aL2 c'est multiplier a GAUCHE par: T(3,2,a);
>  T(3,2,a).B;
   [b(1, 1) , b(1, 2) , b(1, 3) , b(1, 4) , b(1, 5) , b(1, 6) , b(1, 7) , b(1, 8)]

   [b(2, 1) , b(2, 2) , b(2, 3) , b(2, 4) , b(2, 5) , b(2, 6) , b(2, 7) , b(2, 8)]

   [a b(2, 1) + b(3, 1) , a b(2, 2) + b(3, 2) , a b(2, 3) + b(3, 3) , a b(2, 4) + b(3, 4) , a b(2, 5) + b(3, 5) ,

   a b(2, 6) + b(3, 6) , a b(2, 7) + b(3, 7) , a b(2, 8) + b(3, 8)]

   [b(4, 1) , b(4, 2) , b(4, 3) , b(4, 4) , b(4, 5) , b(4, 6) , b(4, 7) , b(4, 8)]

   [b(5, 1) , b(5, 2) , b(5, 3) , b(5, 4) , b(5, 5) , b(5, 6) , b(5, 7) , b(5, 8)]

   [b(6, 1) , b(6, 2) , b(6, 3) , b(6, 4) , b(6, 5) , b(6, 6) , b(6, 7) , b(6, 8)]

   [b(7, 1) , b(7, 2) , b(7, 3) , b(7, 4) , b(7, 5) , b(7, 6) , b(7, 7) , b(7, 8)]

   [b(8, 1) , b(8, 2) , b(8, 3) , b(8, 4) , b(8, 5) , b(8, 6) , b(8, 7) , b(8, 8)]

>  #En revanche: C2<-C2+aC3 c'est multplier a DROITE par T(3,2,a)
>  B.T(3,2,a);
        [b(1, 1)    b(1, 2) + b(1, 3) a    b(1, 3)    b(1, 4)    b(1, 5)    b(1, 6)    b(1, 7)    b(1, 8)]
        [                                                                                              ]
        [b(2, 1)    b(2, 2) + a b(2, 3)    b(2, 3)    b(2, 4)    b(2, 5)    b(2, 6)    b(2, 7)    b(2, 8)]
        [                                                                                              ]
        [b(3, 1)    b(3, 2) + b(3, 3) a    b(3, 3)    b(3, 4)    b(3, 5)    b(3, 6)    b(3, 7)    b(3, 8)]
        [                                                                                              ]
        [b(4, 1)    b(4, 2) + b(4, 3) a    b(4, 3)    b(4, 4)    b(4, 5)    b(4, 6)    b(4, 7)    b(4, 8)]
        [                                                                                              ]
        [b(5, 1)    b(5, 2) + b(5, 3) a    b(5, 3)    b(5, 4)    b(5, 5)    b(5, 6)    b(5, 7)    b(5, 8)]
        [                                                                                              ]
        [b(6, 1)    b(6, 2) + b(6, 3) a    b(6, 3)    b(6, 4)    b(6, 5)    b(6, 6)    b(6, 7)    b(6, 8)]
        [                                                                                              ]
        [b(7, 1)    b(7, 2) + b(7, 3) a    b(7, 3)    b(7, 4)    b(7, 5)    b(7, 6)    b(7, 7)    b(7, 8)]
        [                                                                                              ]
        [b(8, 1)    b(8, 2) + b(8, 3) a    b(8, 3)    b(8, 4)    b(8, 5)    b(8, 6)    b(8, 7)    b(8, 8)]

>  # Remarquer que l'inverse de T(i,j,a) est T(i,j,-a)
>  T(3,2,a)^(-1);
                                        [1    0    0    0    0    0    0    0]
                                        [                                   ]
                                        [0    1    0    0    0    0    0    0]
                                        [                                   ]
                                        [0    -a   1    0    0    0    0    0]
                                        [                                   ]
                                        [0    0    0    1    0    0    0    0]
                                        [                                   ]
                                        [0    0    0    0    1    0    0    0]
                                        [                                   ]
                                        [0    0    0    0    0    1    0    0]
                                        [                                   ]
                                        [0    0    0    0    0    0    1    0]
                                        [                                   ]
                                        [0    0    0    0    0    0    0    1]

>  # Donc conjuguer par T(i,j,a) c'est faire:
>  # Li <- Li+aLj et Cj <- Cj-aCi
>  P:=T(6,7,2).T(4,5,1).T(3,2,2).T(1,2,1):
>  P,P^(-1);
         [1    1    0    0    0    0    0    0]  [1    -1    0    0     0    0     0    0]
         [                                   ]  [                                      ]
         [0    1    0    0    0    0    0    0]  [0     1    0    0     0    0     0    0]
         [                                   ]  [                                      ]
         [0    2    1    0    0    0    0    0]  [0    -2    1    0     0    0     0    0]
         [                                   ]  [                                      ]
         [0    0    0    1    1    0    0    0]  [0     0    0    1    -1    0     0    0]
         [                                   ], [                                      ]
         [0    0    0    0    1    0    0    0]  [0     0    0    0     1    0     0    0]
         [                                   ]  [                                      ]
         [0    0    0    0    0    1    2    0]  [0     0    0    0     0    1    -2    0]
         [                                   ]  [                                      ]
         [0    0    0    0    0    0    1    0]  [0     0    0    0     0    0     1    0]
         [                                   ]  [                                      ]
         [0    0    0    0    0    0    0    1]  [0     0    0    0     0    0     0    1]

>  # Donc faire a l'ordinateur:
>  N:=P.J.P^(-1):
>  # est identique a faire a la main a partir de J:
>  # L1 <- L1+L2 ; C2<-C2 - C1 puis
>  # L3 <- L3+2L2 ; C2 <- C2 -2C3
>  # L4 <- L4+L5 ; C5 <- C5 -C4
>  # L6 <- L6 +2 L7; C7 <- C7 -2 C6
>  # On a maintenant trouve un bel exercice: Trouver la forme de
>  # jordan de N et une matrice de passage pour l'obtenir.
>  # On calcule N^2 et son noyau.
>  N,N^2;
         [0    -1    1    0    0    0     0    0]  [0    -2    1    0    0    0     0    0]
         [                                      ]  [                                      ]
         [0    -2    1    0    0    0     0    0]  [0     0    0    0    0    0     0    0]
         [                                      ]  [                                      ]
         [0    -4    2    0    0    0     0    0]  [0     0    0    0    0    0     0    0]
         [                                      ]  [                                      ]
         [0     0    0    0    1    1    -2    0]  [0     0    0    0    0    1    -2    0]
         [                                      ], [                                      ]
         [0     0    0    0    0    1    -2    0]  [0     0    0    0    0    0     0    0]
         [                                      ]  [                                      ]
         [0     0    0    0    0    0     0    2]  [0     0    0    0    0    0     0    0]
         [                                      ]  [                                      ]
         [0     0    0    0    0    0     0    1]  [0     0    0    0    0    0     0    0]
         [                                      ]  [                                      ]
         [0     0    0    0    0    0     0    0]  [0     0    0    0    0    0     0    0]

>  N2:=NullSpace(N^2);
                          [0]  [0]  [0]  [0]  [ 0 ]  [1]
                          [ ]  [ ]  [ ]  [ ]  [   ]  [ ]
                          [0]  [0]  [0]  [0]  [1/2]  [0]
                          [ ]  [ ]  [ ]  [ ]  [   ]  [ ]
                          [0]  [0]  [0]  [0]  [ 1 ]  [0]
                          [ ]  [ ]  [ ]  [ ]  [   ]  [ ]
                          [0]  [0]  [0]  [1]  [ 0 ]  [0]
                   N2 := {[ ], [ ], [ ], [ ], [   ], [ ]}
                          [0]  [0]  [1]  [0]  [ 0 ]  [0]
                          [ ]  [ ]  [ ]  [ ]  [   ]  [ ]
                          [0]  [2]  [0]  [0]  [ 0 ]  [0]
                          [ ]  [ ]  [ ]  [ ]  [   ]  [ ]
                          [0]  [1]  [0]  [0]  [ 0 ]  [0]
                          [ ]  [ ]  [ ]  [ ]  [   ]  [ ]
                          [1]  [0]  [0]  [0]  [ 0 ]  [0]

>  #on choisit a et b  independants modulo ker N^2 (qui est aussi im
>  # N). (attention a et b hors de ker N^2 est insuffisant).
>  a:=Vector([0,0,1,0,0,0,0,0]):b:=Vector([0,0,0,0,0,1,0,0]):
>  Rank(Matrix([op(N2),a,b]));# doit etre dim ker N^2 +2.
                                                8

>  N1:=NullSpace(N);
                                     [1]  [0]  [0]
                                     [ ]  [ ]  [ ]
```

```
                                   [0]  [0]  [0]
                                   [ ]  [ ]  [ ]
                                   [0]  [0]  [0]
                                   [ ]  [ ]  [ ]
                                   [0]  [1]  [0]
                           N1 := {[ ], [ ], [ ]}
                                   [0]  [0]  [0]
                                   [ ]  [ ]  [ ]
                                   [0]  [0]  [2]
                                   [ ]  [ ]  [ ]
                                   [0]  [0]  [1]
                                   [ ]  [ ]  [ ]
                                   [0]  [0]  [0]

>  #dim ker N^2 -dim ker N= 6-3=2+1 donc N.a,N.b doit etre complete par c
>  # tq N.a,N.b,c indep modulo ker N. Par exemple on prend celui la:
>  c:=Vector([0,0,0,0,0,0,1]):
>  #On verifie qu'il convient:
>  Rank(Matrix([op(N1),N.a,N.b,c]));
bytes used=4000208, alloc=3472772, time=0.10
                                                  6

>  # dim ker N - dim ker N^0=3 c'est donc engendr\'e par
>  # N^2.a,N^2.b,N.c. Il n'y a plus rien a faire, et l'on prend
>  # la base suivante:
>  Q:=Matrix([(N^2).a,N.a,a,(N^2).b,N.b,b,N.c,c]);
                              [1    1    0    0    0    0    0    0]
                              [                                   ]
                              [0    1    0    0    0    0    0    0]
                              [                                   ]
                              [0    2    1    0    0    0    0    0]
                              [                                   ]
                              [0    0    0    1    1    0    0    0]
                         Q := [                                   ]
                              [0    0    0    0    1    0    0    0]
                              [                                   ]
                              [0    0    0    0    0    1    2    0]
                              [                                   ]
                              [0    0    0    0    0    0    1    0]
                              [                                   ]
                              [0    0    0    0    0    0    0    1]

>  #On sait maintenant que Q^(-1).N.Q doit donner J. verification:
>  Q^(-1).N.Q;
                            [0    1    0    0    0    0    0    0]
                            [                                   ]
                            [0    0    1    0    0    0    0    0]
                            [                                   ]
                            [0    0    0    0    0    0    0    0]
                            [                                   ]
                            [0    0    0    0    1    0    0    0]
                            [                                   ]
                            [0    0    0    0    0    1    0    0]
                            [                                   ]
                            [0    0    0    0    0    0    0    0]
                            [                                   ]
                            [0    0    0    0    0    0    0    1]
                            [                                   ]
                            [0    0    0    0    0    0    0    0]

> quit
bytes used=5103340, alloc=3472772, time=0.13
```