```
> interface(warnlevel=0,screenwidth=120);
> with(LinearAlgebra):with(plots):
> B:=Matrix(3,3):B[1,1]:=1:B[2,2]:=2:B[3,3]:=3:B[1,3]:=-2:B[3,1]:=
  -2:B;
```

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 2 & 0 \\ -2 & 0 & 3 \end{bmatrix}$$

```
> X:=Vector([x,y,1]);C1:=Transpose(X).B.X;C2:=y^2-x;
```

$$X := \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$C1 := x(x-2)+2y^2-2x+3$$

$$C2 := y^2-x$$

```
> IMPL1:=implicitplot(C1=0,x=0..8,y=-3..3,numpoints=1000,color=blu
  e):
> IMPL2:=implicitplot(C2=0,x=0..8,y=-7..7,numpoints=2000,color=red
  ):
> IMPL:=IMPL1,IMPL2:
> #la matrice de C2dual est l'inverse de celle de C2
> A:=B^(-1);
```

$$A := \begin{bmatrix} -3 & 0 & -2 \\ 0 & \frac{1}{2} & 0 \\ -2 & 0 & -1 \end{bmatrix}$$

```
> #equation de mt mu on simplifie par t-u car mt different de m
> dte:=simplify(Determinant(Matrix([[t^2,t,1],[u^2,u,1],[x,y,1]]))
  /(t-u));
```

$$dte := -u\,y+u\,t+x-t\,y$$

```
> #coordonnees duales de cette droite:
> DTE:=Vector([coeff(dte,x),coeff(dte,y),subs(x=0,y=0,dte)]);
```

$$DTE := \begin{bmatrix} 1 \\ -u-t \\ u\,t \end{bmatrix}$$

```
> #on remarque qu'elles s'expriment facilement avec somme et
  produit
> P:=expand(Transpose(DTE).A.DTE);
```

$$P := -3-3\,u\,t+\frac{1}{2}u^2+\frac{1}{2}t^2-u^2\,t^2$$

```
> a:=coeff(P,t^2);b:=coeff(P,t);c:=subs(t=0,P);
```

$$a := \frac{1}{2}-u^2$$

$$b := -3\,u$$

$$c := -3+\frac{u^2}{2}$$

```
> sol1:=solve(P,u);
```

$$sol1 := \frac{-6\,t+2\sqrt{-4\,t^2+6+2\,t^4}}{2\,(-1+2\,t^2)},\ \frac{-6\,t-2\sqrt{-4\,t^2+6+2\,t^4}}{2\,(-1+2\,t^2)}$$

```
> s1:=simplify(sol1[1]+sol1[2]);
```

$$s1 := -\frac{6\,t}{-1+2\,t^2}$$

```
> p1:=simplify(sol1[1]*sol1[2]);
```

$$p1 := -\frac{-6+t^2}{-1+2\,t^2}$$

```
> D1:=Vector([1,-s1,p1]);
```

$$D1 := \begin{bmatrix} 1 \\ \frac{6\,t}{-1+2\,t^2} \\ -\frac{-6+t^2}{-1+2\,t^2} \end{bmatrix}$$

```
> #D1 decrit donc une conique duale
> simplify(D1*(t^2-25));
```

$$\begin{bmatrix} t^2-25 \\ \frac{6\,(t^2-25)\,t}{-1+2\,t^2} \\ -\frac{(t^2-25)(-6+t^2)}{-1+2\,t^2} \end{bmatrix}$$

```
> #Pour d2 il faut mieux guider les simplifications
> tmp1:=simplify(expand(subs(u=sol1[1],b)*subs(u=sol1[2],a)+subs(u
  =sol1[2],b)*subs(u=sol1[1],a)));
```

$$tmp1 := \frac{9\,t\,(4\,t^2-13)}{(-1+2\,t^2)^2}$$

```
> tmp2:=simplify(expand(subs(u=sol1[1],a)*subs(u=sol1[2],a)));
```

$$tmp2 := -\frac{72\,t^2-121}{4\,(-1+2\,t^2)^2}$$

```
> tmp3:=simplify(expand(subs(u=sol1[1],c)*subs(u=sol1[2],c)));
```

$$tmp3 := \frac{t^2\,(121\,t^2-216)}{4\,(-1+2\,t^2)^2}$$

```
> s2:=simplify(-2*t-tmp1/tmp2);
```

$$s2 := -\frac{226\,t}{72\,t^2-121}$$

```
> p2:=simplify(tmp3/(tmp2*t^2));
```

$$p2 := -\frac{121\,t^2-216}{72\,t^2-121}$$

```
> D2:=Vector([1,-s2,p2]);
```

$$D2 := \begin{bmatrix} 1 \\ \frac{226\,t}{72\,t^2-121} \\ -\frac{121\,t^2-216}{72\,t^2-121} \end{bmatrix}$$

```
> # c'est encore une conique
> denom(D2[2])*D2;
```

$$\begin{bmatrix} 72\,t^2-121 \\ 226\,t \\ -121\,t^2+216 \end{bmatrix}$$

```
> sol2:=solve(u^2-s2*u+p2=0,u);
```

$$sol2 := \frac{-226\,t+132\sqrt{-4\,t^2+6+2\,t^4}}{2\,(72\,t^2-121)}, \frac{-226\,t-132\sqrt{-4\,t^2+6+2\,t^4}}{2\,(72\,t^2-121)}$$

```
> #d3
> tmp1:=simplify(expand(subs(u=sol2[1],b)*subs(u=sol2[2],a)+subs(u
=sol2[2],b)*subs(u=sol2[1],a)));
```

$$tmp1 := \frac{339\,t\,(314\,t^2-553)}{(72\,t^2-121)^2}$$

```
> tmp2:=simplify(expand(subs(u=sol2[1],a)*subs(u=sol2[2],a)));
```

$$tmp2 := \frac{28900\,t^4-207892\,t^2+96721}{4\,(72\,t^2-121)^2}$$

```
> tmp3:=simplify(expand(subs(u=sol2[1],c)*subs(u=sol2[2],c)));
```

$$tmp3 := \frac{96721\,t^4-623676\,t^2+260100}{4\,(72\,t^2-121)^2}$$

```
> s3:=simplify(-s1-tmp1/tmp2);
```

$$s3 := -\frac{169542\,t}{14450\,t^2-96721}$$

```
> p3:=simplify(tmp3/(tmp2*p1));
```

$$p3 := -\frac{96721\,t^2-43350}{14450\,t^2-96721}$$

```
> D3:=Vector([1,-s3,p3]);
```

$$D3 := \begin{bmatrix} 1 \\ \frac{169542\,t}{14450\,t^2-96721} \\ -\frac{96721\,t^2-43350}{14450\,t^2-96721} \end{bmatrix}$$

```
> # c'est encore une conique dans le plan duale
> denom(D3[2])*D3;
```

$$\begin{bmatrix} 14450\,t^2-96721 \\ 169542\,t \\ -96721\,t^2+43350 \end{bmatrix}$$

```
> ################################################################
> #qui enveloppe la conique d'equation: (le lieu des points du
plan ou
> #les tangentes a cette conique sont identiques).
> gamma3:=discrim((Matrix([[x,y,1]]).(denom(D3[2])*D3))[1],t);
```

$$\gamma 3 := 5590473800\,x^2-39925437364\,x+16771421400+28744489764\,y^2$$

```
> #On verifie que l'equation gamma3 est bien une combi lineaire de
C1 et C2 Donc gamma3 contient C1 inter C2
> expand(gamma3-coeff(gamma3,x^2)*(C1-coeff(C1,y^2)*C2)-coeff(gamm
a3,y^2)*C2);
```

$$0$$

```
> ##############################################################
> G3:=implicitplot([gamma3=0],x=0..8,y=-3..3,numpoints=1000,color=
green):#gamma3en vert
> #di
> DI:=proc(tt,i)
> sol:=solve(subs(t=tt,P),u):s0:=2*tt:p0:=tt^2:
> for j from 2 to i do
> tmp1:=simplify(expand(subs(u=sol[1],b)*subs(u=sol[2],a)+subs(u=s
ol[2],b)*subs(u=sol[1],a)));
> tmp2:=simplify(expand(subs(u=sol[1],a)*subs(u=sol[2],a)));
> tmp3:=simplify(expand(subs(u=sol[1],c)*subs(u=sol[2],c)));
> s2:=simplify(-s0-tmp1/tmp2);
> p2:=simplify(tmp3/(tmp2*p0));
> s0:=simplify(sol[1]+sol[2]);
> p0:=simplify(sol[1]*sol[2]);
> sol:=solve(u^2-s2*u+p2=0,u);
> end do;
> denom(p2)*Vector([1,-s2,p2]);
> end proc;
```

DI := **proc**(*tt*, *i*)

local *sol*, *s0*, *p0*, *j*, *tmp1*, *tmp2*, *tmp3*, *s2*, *p2*;

sol := solve(subs(*t* = *tt*, *P*), *u*);

s0 := 2**tt*;

p0 := *tt*^2;

```
      for j from 2 to i do
          tmp1 := simplify(expand(
              subs(u = sol[1], b)*subs(u = sol[2], a) + subs(u = sol[2], b)*subs(u = sol[1], a)));
          tmp2 := simplify(expand(subs(u = sol[1], a)*subs(u = sol[2], a)));
          tmp3 := simplify(expand(subs(u = sol[1], c)*subs(u = sol[2], c)));
          s2 := simplify(-s0 - tmp1 / tmp2);
          p2 := simplify(tmp3 / (tmp2*p0));
          s0 := simplify(sol[1] + sol[2]);
          p0 := simplify(sol[1]*sol[2]);
          sol := solve(u^2 - s2*u + p2 = 0, u)
      end do;
      denom(p2)*Vector([1, -s2, p2])
end proc
```

```
> DI(t,3);#c'est le meme
```

$$\begin{bmatrix} 14450\, t^2 - 96721 \\ 169542\, t \\ -96721\, t^2 + 43350 \end{bmatrix}$$

```
> DI(t,6);#c'est toujours une conique duale.
```

$$\begin{bmatrix} 40173829230002545800\, t^2 - 76187486312803124281 \\ 134534108782574036962\, t \\ 120521487690007637400 - 76187486312803124281\, t^2 \end{bmatrix}$$

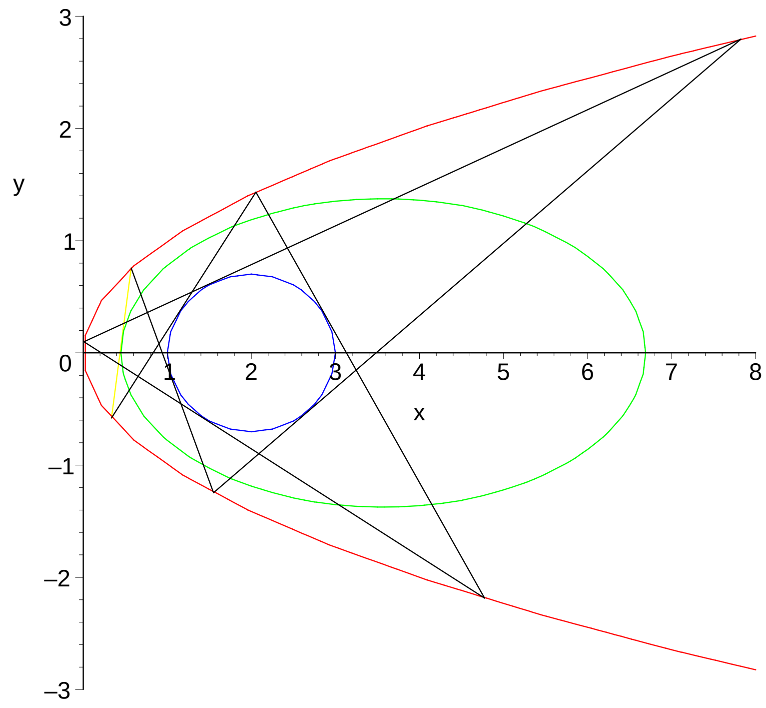
```
> #dessin
> dessin:=proc(tt,i)
> sol:=solve(subs(t=tt,P),u):
> T:=[tt,sol[1]];
> for j from 2 to i do
> newt:=-T[j-1]-subs(u=T[j],b)/subs(u=T[j],a):
> T:=[op(T),newt];
> end do;
> T:=[sol[2],op(T)];
> for j from 2 to i do
> newt:=-T[2]-subs(u=T[1],b)/subs(u=T[1],a):
> T:=[newt,op(T)];
> end do;
> T;end proc;
```

```
dessin := proc(tt, i)
local sol, T, j, newt;
      sol := solve(subs(t = tt, P), u);
      T := [tt, sol[1]];
      for j from 2 to i do
          newt := -T[j - 1] - subs(u = T[j], b) / subs(u = T[j], a); T := [op(T), newt]
      end do;
      T := [sol[2], op(T)];
      for j from 2 to i do newt := -T[2] - subs(u = T[1], b) / subs(u = T[1], a); T := [newt, op(T)]
      end do;
      T
end proc
```

```
> with(plottools):
> imax:=3;
```

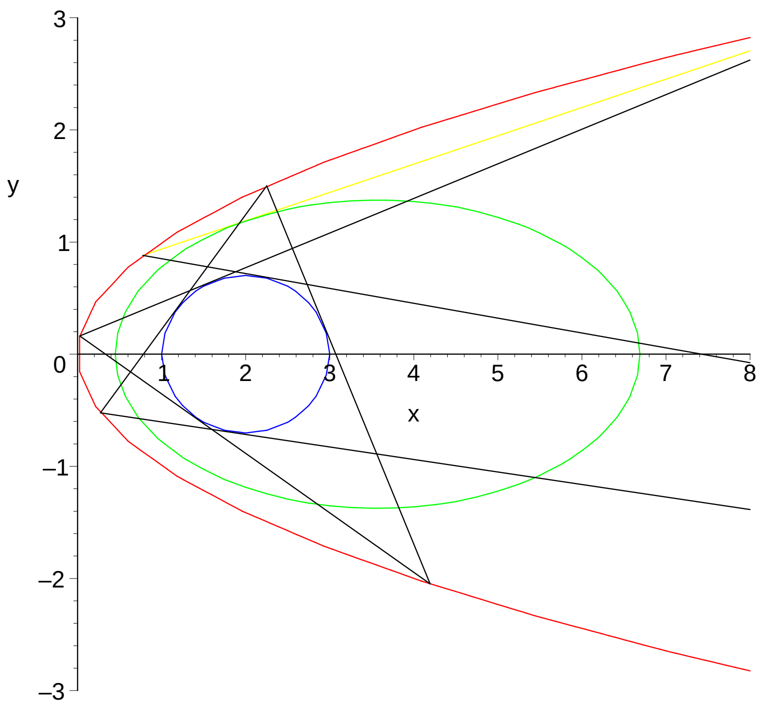
$$imax := 3$$

```
> l:=dessin(.1,imax):#on dessine t0=0.1 jusque timax
> dte:=(u,t)->line([u^2,u],[t^2,t]):
> dtei:=line([l[1]^2,l[1]],[l[2*imax+1]^2,l[2*imax+1]],color=yellow):
> display(seq(dte(l[i],l[i+1]),i=1..2*imax),IMPL,G3,dtei,view=[0..8,-3..3]);
```




```
> #un autre t0
```

```
> l:=dessin(1.5,imax):#on dessine t0=1.7 jusque timax
> dte:=(u,t)->line([u^2,u],[t^2,t]):
> dtei:=line([l[1]^2,l[1]],[l[2*imax+1]^2,l[2*imax+1]],color=yello
  w):
> display(seq(dte(l[i],l[i+1]),i=1..2*imax),IMPL,G3,dtei,view=[0..
  8,-3..3]);
```




```
> #la droite jaune t3t-3 est bien tangente a gamma3
>
```