```
    |\^/|     Maple 9 (IBM INTEL LINUX)
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 <____ ____>  Waterloo Maple Inc.
      |       Type ? for help.
>  interface(screenwidth=120);
> with(linalg):with(LinearAlgebra):
>  a:='a';coeffs(a^2+2,a);# il n'affiche pas les nuls.
                                                    a := a

                                                     2, 1

>  mescoeffs:=proc(P,a)
>  [seq(coeff(P,a,i),i=0..degree(P,a))];
>  end proc;
                    mescoeffs := proc(P, a) [seq(coeff(P, a, i), i = 0 .. degree(P, a))] end proc

>  #astuce, a retenir
>  M:=Matrix(2,3)+1;N:=Matrix(2,4)+2;OO:=Matrix(3,3)+3;Matrix([M,N]);Matrix([[M],[OO]]);
                                                  [1    0    0]
                                             M := [          ]
                                                  [0    1    0]

                                                  [2    0    0    0]
                                            N := [               ]
                                                  [0    2    0    0]

                                                  [3    0    0]
                                                  [          ]
                                           OO := [0    3    0]
                                                  [          ]
                                                  [0    0    3]

                                       [1    0    0    2    0    0    0]
                                       [                               ]
                                       [0    1    0    0    2    0    0]

                                                  [1    0    0]
                                                  [          ]
                                                  [0    1    0]
                                                  [          ]
                                                  [3    0    0]
                                                  [          ]
                                                  [0    3    0]
                                                  [          ]
                                                  [0    0    3]

>  #A retenir: plot sur un meme dessin...
>  with(plots);with(plottools);
[animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal, conformal3d, contourplot,

    contourplot3d, coordplot, coordplot3d, cylinderplot, densityplot, display, display3d, fieldplot, fieldplot3d,

    gradplot, gradplot3d, graphplot3d, implicitplot, implicitplot3d, inequal, interactive, listcontplot, listcontplot3d,

    listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, odeplot, pareto, plotcompare, pointplot,

    pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot, replot, rootlocus,

    semilogplot, setoptions, setoptions3d, spacecurve, sparsematrixplot, sphereplot, surfdata, textplot, textplot3d,

    tubeplot]

[arc, arrow, circle, cone, cuboid, curve, cutin, cutout, cylinder, disk, dodecahedron, ellipse, ellipticArc, hemisphere,

    hexahedron, homothety, hyperbola, icosahedron, line, octahedron, pieslice, point, polygon, project, rectangle,

    reflect, rotate, scale, semitorus, sphere, stellate, tetrahedron, torus, transform, translate, vrml]

>  l1:= line([0,0], [30,40], color=red, linestyle=3);
                  l1 := CURVES([[0., 0.], [30., 40.]], COLOUR(RGB, 1.00000000, 0., 0.), LINESTYLE(3))

>  l2 := line([0,0], [1,-1], color=blue, linestyle=2);
                   l2 := CURVES([[0., 0.], [1., -1.]], COLOUR(RGB, 0., 0., 1.00000000), LINESTYLE(2))

>  display([l1,l2],view=[-5..5,-5..5]);
                                                        +                                   AAAAA
                                                        +                              AAAA
                                                       4+                          AAAA
                                                        +                      AAAA
                                                        +                  AAAA
                                                        +              AAAAA
                                                       2+          AAAA
                                                        +      AAAA
                                                        +   AAAA
                                                        +  AAAA
   --+----+---+----+---+----+---+----+---+----+---+----****-+----+---+----+---+----+---+----+---+----+---+----+--
            -4                  -2                     0+   DDDDD              2                   4
                                                        +       DDDD
                                                        +
                                                       -2+
                                                        +
                                                        +
                                                        +
                                                       -4+
                                                        +
>  l:=point([1,1],color=red),point([2,3],style=point);
                  l := POINTS([1., 1.], COLOUR(RGB, 1.00000000, 0., 0.)), POINTS([2., 3.], STYLE(POINT))
```

```
>  display(l);

 3 +
   +
   +
   +
   +
2.5+
   +
   +
   +
   +
 2 +
   +
   +
   +
   +
1.5+
   +
   +
   +
   +
 1 +-*---+----+---+----+---+---+----+---+---+----+---+---+----+---+---+---+----+---+---+----+---+---+----+---+---+---
    1                  1.2                 1.4                 1.6                 1.8
>  cardi:=A->if Determinant(A)=0 then print("cardinal infini") else
>  abs(Determinant(A)) fi;
cardi := proc(A)
option operator, arrow;
    if LinearAlgebra:-Determinant(A) = 0 then print("cardinal infini")
    else abs(LinearAlgebra:-Determinant(A))
    end if
end proc

>  A:=Matrix([[2*4,4,15*4,0],[8,12,18,36],[16,16,32,32],[32,32,32,32]]);
                                                  [ 8     4    60     0]
                                                  [                    ]
                                                  [ 8    12    18    36]
                                             A := [                    ]
                                                  [16    16    32    32]
                                                  [                    ]
                                                  [32    32    32    32]

>  B:=Matrix(ismith(A));cardi(A);
                                                   [2    0     0      0]
                                                   [                    ]
                                                   [0    4     0      0]
                                              B := [                    ]
                                                   [0    0    16      0]
                                                   [                    ]
                                                   [0    0     0    672]

                                                        86016

>  #M/N=M/image(A) il est donc isomorphe a M/B ou B=ismith(A) le
>  #cardinal est donc infini s'il y a un 0 sur la diagnonale de B,
>  #sinon c'est le produit des termes diagonaux de B, donc aussi |detB|=|det A| (dans le cas d'une matrice carree
>  abs(Determinant(B)),abs(Determinant(A));#dans les 2 cas, c'est le cardinal.
                                                      86016, 86016

>  A:=Matrix([[2*4,4,15*4,0],[8,12,18,36],[16,16,32,32],[24,28,50,68]]);
                                                  [ 8     4    60     0]
                                                  [                    ]
                                                  [ 8    12    18    36]
                                             A := [                    ]
                                                  [16    16    32    32]
                                                  [                    ]
                                                  [24    28    50    68]

>  ismith(A);cardi(A);
                                                 [2    0     0    0]
                                                 [                 ]
                                                 [0    4     0    0]
                                                 [                 ]
                                                 [0    0    16    0]
                                                 [                 ]
                                                 [0    0     0    0]

                                                  "cardinal infini"

>  A:=Matrix([[2*4,4,15*4,0,16],[8,12,18,36,28],[16,16,32,32,32],[24,28,50,68,68]]);
                                                 [ 8     4    60     0    16]
                                                 [                         ]
                                                 [ 8    12    18    36    28]
                                            A := [                         ]
                                                 [16    16    32    32    32]
                                                 [                         ]
                                                 [24    28    50    68    68]

>  v1:=Vector([1,2]);v2:=Vector([-1,1]);v3:=Vector([0,2]);
                                                      [1]
                                                 v1 := [ ]
                                                      [2]

                                                      [-1]
                                                 v2 := [  ]
                                                      [ 1]

                                                      [0]
                                                 v3 := [ ]
                                                      [2]
```

```
>  #On fait quelques combinaisons lineaires au hasard, ca serait mieux de trouver une base (on le fera plus dans l'exerc
ice suivant)
>  l:=seq(seq(seq(point(convert(i*v1+j*v2+k*v3,list),color=blue),k=-3..3),j=-4..4),i=-2..2):
bytes used=4000044, alloc=3145152, time=0.12
bytes used=8000208, alloc=5569540, time=0.24
bytes used=12000684, alloc=5635064, time=0.37
bytes used=16000944, alloc=5700588, time=0.50
bytes used=20001400, alloc=5766112, time=0.62
bytes used=24001560, alloc=5766112, time=0.76
bytes used=28001776, alloc=5766112, time=0.88
bytes used=32001992, alloc=5766112, time=1.02
bytes used=36002188, alloc=5766112, time=1.14
bytes used=40002452, alloc=5831636, time=1.27
>  display(l,view=[-5..5,-5..5]);

   D          D           D          D          D          *          D          D          D          D          D
                                                           +
   D          D           D          D          D         4*          D          D          D          D          D
                                                           +
   D          D           D          D          D          *          D          D          D          D          D
                                                           +
   D          D           D          D          D         2*          D          D          D          D          D
                                                           +
   D          D           D          D          D          *          D          D          D          D          D
                                                           +
   *-+----+---*----+---+--*-+---+----*---+----+-*-+----+---*----+----+-*-+----+---*----+---+-*--+---+----*---+----+-*
            -4                     -2                     0+                    2                    4
   D          D           D          D          D          *          D          D          D          D          D
                                                           +
   D          D           D          D          D        -2*          D          D          D          D          D
                                                           +
   D          D           D          D          D          *          D          D          D          D          D
                                                           +
   D          D           D          D          D        -4*          D          D          D          D          D
                                                           +
   D          D           D          D          D          *          D          D          D          D          D

>  ismith(Matrix([v1,v2,v3]));#le reseau engendre est bien Z^2
                                                       [1    0    0]
                                                       [           ]
                                                       [0    1    0]

>  l2:=seq(seq(point(convert(i*v1+j*v2,list),color=red),j=-4..4),i=-3..3):
bytes used=44002732, alloc=5831636, time=1.39
>  display(l2,l,view=[-5..5,-5..5]);# ca ne rempli pas tout

   D          D           D          A          D          *          A          D          D          A          D
                                                           +
   D          A           D          D          A         4*          D          A          D          D          A
                                                           +
   D          D           A          D          D          *          D          D          A          D          D
                                                           +
   A          D           D          A          D         2*          A          D          D          A          D
                                                           +
   D          A           D          D          A          *          D          A          D          D          A
                                                           +
   *-+----+---*----+---+--*-+---+----*---+----+-*-+----+---*----+----+-*-+----+---*----+---+-*--+---+----*---+----+-*
            -4                     -2                     0+                    2                    4
   A          D           D          A          D          *          A          D          D          A          D
                                                           +
   D          A           D          D          A        -2*          D          A          D          D          A
                                                           +
   D          D           A          D          D          *          D          D          A          D          D
                                                           +
   A          D           D          A          D        -4*          A          D          D          A          D
                                                           +
   D          A           D          D          A          *          D          A          D          D          D

>  #aucun des 3 determinants (v1,v2) (v1,v3) (v2,v3) n'est inversible dans Z, on ne peut donc pas en extraire une base.
>  #Famille generatrice de I comme Z module: les generateurs  et leurs multiples par Isqrt(5)
>  M:=Transpose(Matrix([[2,0],[3,3],[0,2],[-15,3]]));
                                                      [2    3    0    -15]
                                                 M := [                  ]
                                                      [0    3    2      3]

>  ismith(M);#Donc N(I)=2
                                                     [1    0    0    0]
                                                     [                ]
                                                     [0    2    0    0]

>  l1:=[seq(line([-10,i*sqrt(5)],[10,i*sqrt(5)],color=red,linestyle=2),i=-10..10)]:#les lignes du reseau.
>  c1:=[seq(line([i,-10],[i,10],color=red,linestyle=2),i=-10..10)]:#les colonnes du reseau.
>  #Pour le reseau associe a I on cherche d'abord une base de I comme Z module, soit avec ismith soit de tete par operat
ion sur les colonnes de M
>  M:=Matrix([[2,1],[0,1]]);#est une base de I
                                                            [2    1]
                                                       M := [      ]
                                                            [0    1]

>  l2:=[seq(line([2*i-10,-10*sqrt(5)],[2*i+10,10*sqrt(5)],color=blue,linestyle=3),i=-10..10)]:
>  c2:=[seq(line([-10+i,i*sqrt(5)],[10+i,i*sqrt(5)],color=blue,linestyle=3),i=-10..10)]:
>  display(l2,c2,l1,c1,view=[-5..5,-5..5]);

   A          ADDD        A          ADDD       A          *DDD       A          ADDD       A          *DD        A
   ****************************************************************************************************************
   A      DD  A           A      DDD A          A      DDD4*          A      DDD A          A      DDD  A          A
   A    DD    A           A    DD    A          A    DD    *          A    DD    A          A    DD    A          A
   A  DD      A           A DDD      A          A DDD      *          A DDD      A          A DDD      A          A
   ****************************************************************************************************************
   A          A        DDDA          A        DDA         2*        DDDA          A        D*          A        DDA

   A          A      DD   A          A     DDD  A          *      DD   A          A      DDD A          A     DD
   A          A   DDD     A          A   DD     A          *   DDD     A          A    DD    A          A   DD
   A          ADDD        A          ADDD       A          * DD        A          A DDD      A          ADDD
   ****************************************************************************************************************
   A       DD-4           A      DDD-2          A        D0*           A       DDD2          A       DDD4
   A     DD   A           A    DD    A          A      DDD *           A     DD   A          A     DDD  A
   A  DDD     A           A DDD      A          A   DD     *           A  DDD     A          A   DD     A
   ADD        A           *D         A          ADDD     -2*           ADD        A          ADDD       A
   ****************************************************************************************************************
   A          A       DDD A          A      DDD A          *       DDD A          A      DDD A          A     D
   A          A     DD    A          A    DD    A          *     DD    A          A    DD    A          A    DD
   A          A  DDD      A          A DDD      A        -4*  DDD      A          A DDD      A          A  DD
   ****************************************************************************************************************
   A        DD*           A       DDDA          A        DD*           A       DDDA          A       DDDA

bytes used=48006456, alloc=5831636, time=1.49
>  #base de R: (1,a,a^2,a^3) ou a=i.5^(1/4). Il faut une famille
>  #generatrice de I comme Z-module. I=(2,1-a^2). R=Z[a]/(a^4-5)
> M:=Matrix(4)+2;N:=Matrix([[1,0,-5,0],[0,1,0,-5],[-1,0,1,0],[0,-1,0,1]]);
                                    [2    0    0    0]
                                    [                ]
                                    [0    2    0    0]
                               M := [                ]
                                    [0    0    2    0]
                                    [                ]
                                    [0    0    0    2]

                                   [ 1     0    -5     0]
                                   [                    ]
                                   [ 0     1     0    -5]
                              N := [                    ]
                                   [-1     0     1     0]
                                   [                    ]
                                   [ 0    -1     0     1]

#verification:
> a:='a';
                                          a := a

> Transpose(Matrix(4,1,(i,j)->a^(i-1))).N;
                          [      2          3           2            3]
                          [1 - a     a - a     -5 + a     -5 a + a ]

> seq(rem(a^(i)*(1-a^2),a^4-5,a),i=0..3);
                                 2        3         2            3
                            1 - a , a - a , -5 + a , -5 a + a

> M:=Matrix([M,N]);#est generatrice de I comme Z module
                          [2    0    0    0     1     0    -5     0]
                          [                                        ]
                          [0    2    0    0     0     1     0    -5]
                     M := [                                        ]
                          [0    0    2    0    -1     0     1     0]
                          [                                        ]
                          [0    0    0    2     0    -1     0     1]

> ismith(M);# Donc R/I est de cardinal 4.
                          [1    0    0    0    0    0    0    0]
                          [                                    ]
                          [0    1    0    0    0    0    0    0]
                          [                                    ]
                          [0    0    2    0    0    0    0    0]
                          [                                    ]
                          [0    0    0    2    0    0    0    0]

>  (i,j,k,l):='i','j','k','l';#on libere i,j,k,l
                                i, j, k, l := i, j, k, l

>  z:=i+j*a+k*a^2+l*a^3;
                                                2      3
                               z := i + j a + k a  + l a

>  MM:=Transpose(Matrix([seq(mescoeffs(rem(expand(a^(u)*z),a^4-5,a),a),u=0..3)]));
                                   [i    5 l    5 k    5 j]
                                   [                      ]
                                   [j     i     5 l    5 k]
                             MM := [                      ]
                                   [k     j      i     5 l]
                                   [                      ]
                                   [l     k      j      i ]

>  normez:=det(MM);
                  4       2              2              2       2  2        2  2       4          2          4
     normez := i  - 20 i  l j + 20 i k j  + 100 i k l  - 10 k  i  + 50 l  j  - 5 j  - 100 j k  l + 25 k  - 125 l

>  normez mod 5; # -1=x^4 mod 5 n'a pas de solutions, donc on ne peut pas avoir N(I)=N(z) puisque c'est impossibl
                                          4
                                         i

>  A:=Matrix([[2*4,4,15*4,0],[8,12,18,36],[16,16,32,32],[32,32,32,32]]);
                                   [ 8     4    60     0]
                                   [                    ]
                                   [ 8    12    18    36]
                              A := [                    ]
                                   [16    16    32    32]
                                   [                    ]
                                   [32    32    32    32]

>  B:=ismith(A,U,V);
                                    [2    0     0     0]
                                    [                  ]
```

```
                                                  [0    4     0      0]
                                            B := [                    ]
                                                  [0    0    16      0]
                                                  [                    ]
                                                  [0    0     0    672]

>  evalm(U.A.V-B);#verification.
                                                 [0    0    0    0]
                                                 [                 ]
                                                 [0    0    0    0]
                                                 [                 ]
                                                 [0    0    0    0]
                                                 [                 ]
                                                 [0    0    0    0]

>  evalm(U^(-1)),evalm(U^(-1).B);#est une base de M resp N;
                                  [  2     -1     0    0]  [  4     -4      0      0]
                                  [                     ]  [                         ]
                                  [ -1     -3     1    0]  [ -2    -12     16      0]
                                  [                     ], [                         ]
                                  [  0     -4     1    0]  [  0    -16     16      0]
                                  [                     ]  [                         ]
                                  [496    160    44    1]  [992    640    704    672]

>  M:=Matrix(7,7,(i,j)->if i=j-1 then 1 else 0 fi):M[1,1]:=1:M[2,2]:=1:M[2,3]:=0:M[4,5]:=0:M;
                                         [1    1    0    0    0    0    0]
                                         [                               ]
                                         [0    1    0    0    0    0    0]
                                         [                               ]
                                         [0    0    0    1    0    0    0]
                                         [                               ]
                                         [0    0    0    0    0    0    0]
                                         [                               ]
                                         [0    0    0    0    0    1    0]
                                         [                               ]
                                         [0    0    0    0    0    0    1]
                                         [                               ]
                                         [0    0    0    0    0    0    0]

>  smith(M-x,x);
                                   [1    0    0    0    0    0           0       ]
                                   [                                             ]
                                   [0    1    0    0    0    0           0       ]
                                   [                                             ]
                                   [0    0    1    0    0    0           0       ]
                                   [                                             ]
                                   [0    0    0    1    0    0           0       ]
                                   [                                             ]
                                   [0    0    0    0    1    0           0       ]
                                   [                                             ]
                                   [                              2              ]
                                   [0    0    0    0    0    x           0       ]
                                   [                                             ]
                                   [                                 5      4    3]
                                   [0    0    0    0    0    0     x  - 2 x  + x ]

> quit
bytes used=50667732, alloc=5831636, time=1.54
```