```
    |\^/|     Maple 9 (IBM INTEL LINUX)
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 \  MAPLE  /  All rights reserved. Maple is a trademark of
 <____ ____>  Waterloo Maple Inc.
      |       Type ? for help.
>  interface(screenwidth=120);
# pour la matrice de passage: Q:=JordanForm(A,output='Q');
>  # conjugaison et unicite de la decomp de Dunford
>  with(LinearAlgebra):
>  C1:=CompanionMatrix((X^3-2)^2);
                                                  [0    0    0    0    0    -4]
                                                  [                           ]
                                                  [1    0    0    0    0     0]
                                                  [                           ]
                                                  [0    1    0    0    0     0]
                                            C1 := [                           ]
                                                  [0    0    1    0    0     4]
                                                  [                           ]
                                                  [0    0    0    1    0     0]
                                                  [                           ]
                                                  [0    0    0    0    1     0]

>  C2:=CompanionMatrix((X-sqrt[3](2))^2);
                                                        [0      -2  ]
                                                  C2 := [         ]
                                                        [        1/2]
                                                        [1    2 2   ]

>  L2:=Matrix([Matrix(2,6),C2]);
                                        [0    0    0    0    0    0    0      -2  ]
                                  L2 := [                                        ]
                                        [                                     1/2]
                                        [0    0    0    0    0    0    1    2 2   ]

>  L1:=Matrix(6,8,C1);
                                          [0    0    0    0    0    -4    0    0]
                                          [                                     ]
                                          [1    0    0    0    0     0    0    0]
                                          [                                     ]
                                          [0    1    0    0    0     0    0    0]
                                    L1 := [                                     ]
                                          [0    0    1    0    0     4    0    0]
                                          [                                     ]
                                          [0    0    0    1    0     0    0    0]
                                          [                                     ]
                                          [0    0    0    0    1     0    0    0]

>  A:=Matrix([[L1],[L2]]);
                                        [0    0    0    0    0    -4    0      0   ]
                                        [                                         ]
                                        [1    0    0    0    0     0    0      0   ]
                                        [                                         ]
                                        [0    1    0    0    0     0    0      0   ]
                                        [                                         ]
                                        [0    0    1    0    0     4    0      0   ]
                                  A := [                                         ]
                                        [0    0    0    1    0     0    0      0   ]
                                        [                                         ]
                                        [0    0    0    0    1     0    0      0   ]
                                        [                                         ]
                                        [0    0    0    0    0     0    0     -2   ]
                                        [                                         ]
                                        [                                      1/2]
                                        [0    0    0    0    0     0    1    2 2   ]

>  A:=C1;#on prend cet exemple plus simple et plus leger.
                                                  [0    0    0    0    0    -4]
                                                  [                           ]
                                                  [1    0    0    0    0     0]
                                                  [                           ]
                                                  [0    1    0    0    0     0]
                                            A := [                           ]
                                                  [0    0    1    0    0     4]
                                                  [                           ]
                                                  [0    0    0    1    0     0]
                                                  [                           ]
                                                  [0    0    0    0    1     0]

>  J:=JordanForm(A);
                                        [   1/3                                   ]
                                        [  2                                      ]
                                        [- ---- + 1/2 I 3    2      , 1 , 0 , 0 , 0 , 0]
                                        [   2                                     ]
                                        [                                         ]
                                        [       1/3                               ]
                                        [      2                                  ]
                                        [0 , - ---- + 1/2 I 3    2      , 0 , 0 , 0 , 0]
                                        [       2                                 ]
                                        [                                         ]
                                        [           1/3                           ]
                                        [          2                              ]
                                  J := [0 , 0 , - ---- - 1/2 I 3    2      , 1 , 0 , 0]
                                        [           2                             ]
                                        [                                         ]
                                        [              1/3                        ]
                                        [             2                           ]
                                        [0 , 0 , 0 , - ---- - 1/2 I 3    2      , 0 , 0]
                                        [              2                          ]
                                        [                                         ]
                                        [                                  (1/3)  ]
                                        [0 ,     0 ,     0 ,     0 ,     2      ,     1]
                                        [                                         ]
                                        [                                  (1/3)  ]
                                        [0 ,     0 ,     0 ,     0 ,     0 ,     2    ]

>  f:=(i,j)->if i=j then J[i,j] else 0 fi;
                    f := proc(i, j) option operator, arrow; if i = j then J[i, j] else 0 end if end proc

>  dia:=Matrix(Dimension(J),f);
                                        [   1/3                                   ]
                                        [  2                                      ]
                                        [- ---- + 1/2 I 3    2      , 0 , 0 , 0 , 0 , 0]
                                        [   2                                     ]
                                        [                                         ]
                                        [       1/3                               ]
                                        [      2                                  ]
                                        [0 , - ---- + 1/2 I 3    2      , 0 , 0 , 0 , 0]
                                        [       2                                 ]
                                        [                                         ]
                                        [           1/3                           ]
                                        [          2                              ]
                                  dia := [0 , 0 , - ---- - 1/2 I 3    2      , 0 , 0 , 0]
                                        [           2                             ]
                                        [                                         ]
                                        [              1/3                        ]
                                        [             2                           ]
                                        [0 , 0 , 0 , - ---- - 1/2 I 3    2      , 0 , 0]
                                        [              2                          ]
                                        [                                         ]
                                        [                                  (1/3)  ]
                                        [0 ,     0 ,     0 ,     0 ,     2      ,     0]
                                        [                                         ]
                                        [                                  (1/3)  ]
                                        [0 ,     0 ,     0 ,     0 ,     0 ,     2    ]

>  Q:=JordanForm(A,output='Q');
bytes used=4000032, alloc=3341724, time=0.14
Q :=

    [ 1/3                                  1/3                                    1/3      ]
    [2                  1/2  (1/3)        2                  1/2  (1/3)          2         ]
    [---- - 1/18 I 3    2      , 1/3 , ---- + 1/18 I 3    2      , 1/3 , - ---- , 1/3]
    [ 18                                   18                                     9        ]

    [          2/3                                    2/3                                  2/3]
    [         2                        1/2  (2/3)    2                        1/2  (2/3)  2 2  ]
    [-1/9 , - ---- - 1/9 I 3    2      , -1/9 , - ---- + 1/9 I 3    2      , -1/9 , ------]
    [          9                                      9                                    9   ]

    [                                                  1/2     1/3                                                                  1/3       1/2
    [-1      1/2                1/2  (2/3)   5 (-1 + 3    I) 2      -1          1/2     1/2  (2/3)    5 2    (1 + 3
    [--- I (3    I - 3) 3    2      , --------------------- , --- I (3 + 3    I) 3    2      , - ------------------
    [108                                        36             108                                          36

       2/3      1/3]
      2      5 2  ]
    - ---- , ------]
       18      18  ]

    [       1/2     1/3          1/3       1/2          1/3    ]
    [(-1 + 3    I) 2           2      (1 + 3    I)       2      ]
    [------------------- , 0 , - ------------------- , 0 , ---- , 0]
    [         36                        36                 18     ]

    [                          2/3                                    2/3                2/3]
    [                         2                        1/2  (2/3)   2                  2  ]
    [1/18 , 1/72 I 3    2      + ---- , 1/18 , -1/72 I 3    2      + ---- , 1/18 , - ----]
    [                            72                                     72                 36 ]

    [   2/3       1/2                1/2     1/3                1/2     2/3     1/3       1/2     2/3      1/3]
    [  2    (1 + 3    I)     (-1 + 3    I) 2      (-1 + 3    I) 2     2    (1 + 3    I)  2     2     2    ]
    [- ----------------- , - ------------------- , ------------------- , ----------------- , ---- , - ----]
    [          72                     36                   72                   36            36      18 ]

>  S:=simplify(Q.dia.Q^(-1));N:=A-S;#on remarque que les coeff sont dansle meme corps que ceux de A
bytes used=8000756, alloc=5176396, time=0.28
bytes used=12001092, alloc=5307444, time=0.39
                                        [ 0       0      2/3     0      0     -8/3]
                                        [                                         ]
                                        [4/3      0       0     2/3     0      0  ]
                                        [                                         ]
                                        [ 0      4/3      0      0     2/3     0  ]
                                  S := [                                         ]
                                        [ 0       0      2/3     0      0     10/3]
                                        [                                         ]
                                        [-1/6     0       0     2/3     0      0  ]
                                        [                                         ]
                                        [ 0      -1/6     0      0     2/3     0  ]

                                        [ 0       0     -2/3     0      0     -4/3]
                                        [                                         ]
                                        [-1/3     0       0     -2/3     0      0  ]
                                        [                                         ]
                                        [ 0      -1/3     0      0     -2/3     0  ]
                                  N := [                                         ]
                                        [ 0       0      1/3     0      0      2/3 ]
                                        [                                         ]
                                        [1/6      0       0     1/3     0      0  ]
                                        [                                         ]
                                        [ 0      1/6      0      0     1/3     0  ]

>  # verification:
```

```
>  simplify(S.N-N.S);
```

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

```
>  #pour obtenir l'inverse de 1+n on regarde la serie: 1/(1+n)
>  #n_u est dans l'ideal N car c'est la somme des u_n-u_{n+1} qui sont tous dans N, donc n est nilpotent.
>  #r(x) est somme de r(s) et d'un element de N, donc r(s)=0 implique
>  #que r(x) est dans N et donc que p divise r par definition de p. donc p est le pol min de s
>  # p = m/D ou D est le pgcd de m et m'
>  # NB:en caract q K=F_q(T), m=X^q-T, alors m'=0, mais p n'est pas 1 car
>  # l'algebre n'a pas de nilpotents.
>  m:=(x^3-2)^5;
```

$$m := (x^3 - 2)^5$$

```
>  Dunford:=proc(A)
>  m:=CharacteristicPolynomial(A,x);
>  p:=y->subs(x=y,simplify(m/(gcd(m,diff(m,x)))));
>  dp:=y->subs(x=y,diff(p(x),x));
>  u:=x;
>  #on itere
>  while rem(p(u),m,x) <> 0 do
>  gcdex(m,dp(u),x,'s','t');
>  invdp:=t;
>  u:=rem(u-p(u)*invdp,m,x);
>  od;
>  S:=0;
>  for i from  0 to degree(u) do S:=S+coeff(u,x,i)*A^i od:
>  S;end proc;
Dunford := proc(A)
local m, p, dp, u, invdp, S, i;
    m := LinearAlgebra:-CharacteristicPolynomial(A, x);
    p := y -> subs(x = y, simplify(m/gcd(m, diff(m, x))));
    dp := y -> subs(x = y, diff(p(x), x));
    u := x;
    while rem(p(u), m, x) <> 0 do gcdex(m, dp(u), x, 's', 't'); invdp := t; u := rem(u - p(u)*invdp, m, x) end do;
    S := 0;
    for i from 0 to degree(u) do S := S + coeff(u, x, i)*A^i end do;
    S
end proc

>  #
>  A:=CompanionMatrix(m);
```

$$A := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 32 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -80 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 80 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -40 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 10 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

```
>  S:=Dunford(A);
bytes used=16003392, alloc=5307444, time=0.50
bytes used=20005192, alloc=5438492, time=0.57
```

$$S := \begin{bmatrix} 0, 0, \frac{70}{243}, 0, 0, \frac{-28}{243}, 0, 0, \frac{40}{243}, 0, 0, \frac{-160}{243}, 0, 0, \frac{3520}{243} \\ \frac{455}{243}, 0, 0, \frac{70}{243}, 0, 0, \frac{-28}{243}, 0, 0, \frac{40}{243}, 0, 0, \frac{-160}{243}, 0, 0 \\ 0, \frac{455}{243}, 0, 0, \frac{70}{243}, 0, 0, \frac{-28}{243}, 0, 0, \frac{40}{243}, 0, 0, \frac{-160}{243}, 0 \\ 0, 0, \frac{280}{243}, 0, 0, \frac{140}{243}, 0, 0, \frac{-128}{243}, 0, 0, \frac{440}{243}, 0, 0, \frac{-8960}{243} \\ \frac{-455}{486}, 0, 0, \frac{280}{243}, 0, 0, \frac{140}{243}, 0, 0, \frac{-128}{243}, 0, 0, \frac{440}{243}, 0, 0 \\ 0, \frac{-455}{486}, 0, 0, \frac{280}{243}, 0, 0, \frac{140}{243}, 0, 0, \frac{-128}{243}, 0, 0, \frac{440}{243}, 0 \\ 0, 0, \frac{-35}{162}, 0, 0, \frac{70}{81}, 0, 0, \frac{80}{81}, 0, 0, \frac{-176}{81}, 0, 0, \frac{3080}{81} \\ \frac{65}{162}, 0, 0, \frac{-35}{162}, 0, 0, \frac{70}{81}, 0, 0, \frac{80}{81}, 0, 0, \frac{-176}{81}, 0, 0 \\ 0, \frac{65}{162}, 0, 0, \frac{-35}{162}, 0, 0, \frac{70}{81}, 0, 0, \frac{80}{81}, 0, 0, \frac{-176}{81}, 0 \\ 0, 0, \frac{10}{243}, 0, 0, \frac{-35}{486}, 0, 0, \frac{160}{243}, 0, 0, \frac{440}{243}, 0, 0, \frac{-4928}{243} \\ \frac{-91}{972}, 0, 0, \frac{10}{243}, 0, 0, \frac{-35}{486}, 0, 0, \frac{160}{243}, 0, 0, \frac{440}{243}, 0, 0 \\ 0, \frac{-91}{972}, 0, 0, \frac{10}{243}, 0, 0, \frac{-35}{486}, 0, 0, \frac{160}{243}, 0, 0, \frac{440}{243}, 0 \\ 0, 0, \frac{-7}{1944}, 0, 0, 5/972, 0, 0, \frac{-5}{243}, 0, 0, \frac{110}{243}, 0, 0, \frac{1540}{243} \\ \frac{35}{3888}, 0, 0, \frac{-7}{1944}, 0, 0, 5/972, 0, 0, \frac{-5}{243}, 0, 0, \frac{110}{243}, 0, 0 \\ 0, \frac{35}{3888}, 0, 0, \frac{-7}{1944}, 0, 0, 5/972, 0, 0, \frac{-5}{243}, 0, 0, \frac{110}{243}, 0 \end{bmatrix}$$

```
>  N:=A-S;
```

$$N := \begin{bmatrix} 0, 0, \frac{-70}{243}, 0, 0, \frac{28}{243}, 0, 0, \frac{-40}{243}, 0, 0, \frac{160}{243}, 0, 0, \frac{4256}{243} \\ \frac{-212}{243}, 0, 0, \frac{-70}{243}, 0, 0, \frac{28}{243}, 0, 0, \frac{-40}{243}, 0, 0, \frac{160}{243}, 0, 0 \\ 0, \frac{-212}{243}, 0, 0, \frac{-70}{243}, 0, 0, \frac{28}{243}, 0, 0, \frac{-40}{243}, 0, 0, \frac{160}{243}, 0 \\ 0, 0, \frac{-37}{243}, 0, 0, \frac{-140}{243}, 0, 0, \frac{128}{243}, 0, 0, \frac{-440}{243}, 0, 0, \frac{-10480}{243} \\ \frac{455}{486}, 0, 0, \frac{-37}{243}, 0, 0, \frac{-140}{243}, 0, 0, \frac{128}{243}, 0, 0, \frac{-440}{243}, 0, 0 \\ 0, \frac{455}{486}, 0, 0, \frac{-37}{243}, 0, 0, \frac{-140}{243}, 0, 0, \frac{128}{243}, 0, 0, \frac{-440}{243}, 0 \\ 0, 0, \frac{35}{162}, 0, 0, \frac{11}{81}, 0, 0, \frac{-80}{81}, 0, 0, \frac{176}{81}, 0, 0, \frac{3400}{81} \\ \frac{-65}{162}, 0, 0, \frac{35}{162}, 0, 0, \frac{11}{81}, 0, 0, \frac{-80}{81}, 0, 0, \frac{176}{81}, 0, 0 \\ 0, \frac{-65}{162}, 0, 0, \frac{35}{162}, 0, 0, \frac{11}{81}, 0, 0, \frac{-80}{81}, 0, 0, \frac{176}{81}, 0 \\ 0, 0, \frac{-10}{243}, 0, 0, \frac{35}{486}, 0, 0, \frac{83}{243}, 0, 0, \frac{-440}{243}, 0, 0, \frac{-4792}{243} \\ \frac{91}{972}, 0, 0, \frac{-10}{243}, 0, 0, \frac{35}{486}, 0, 0, \frac{83}{243}, 0, 0, \frac{-440}{243}, 0, 0 \\ 0, \frac{91}{972}, 0, 0, \frac{-10}{243}, 0, 0, \frac{35}{486}, 0, 0, \frac{83}{243}, 0, 0, \frac{-440}{243}, 0 \\ 0, 0, 7/1944, 0, 0, \frac{-5}{972}, 0, 0, 5/243, 0, 0, \frac{133}{243}, 0, 0, \frac{890}{243} \\ \frac{-35}{3888}, 0, 0, 7/1944, 0, 0, \frac{-5}{972}, 0, 0, 5/243, 0, 0, \frac{133}{243}, 0, 0 \end{bmatrix}$$

```
                        [    -35                                  -5                                  133    ]
                        [0 , ---- , 0 , 0 , 7/1944 , 0 , 0 , --- , 0 , 0 , 5/243 , 0 , 0 , --- , 0]
                        [     3888                                972                                243    ]

>  #verification:
>  N.S-S.N;# S et N commuttent bien
                        [0    0    0    0    0    0    0    0    0    0    0    0    0    0    0]
                        [                                                                        ]
                        [0    0    0    0    0    0    0    0    0    0    0    0    0    0    0]
                        [                                                                        ]
                        [0    0    0    0    0    0    0    0    0    0    0    0    0    0    0]
                        [                                                                        ]
                        [0    0    0    0    0    0    0    0    0    0    0    0    0    0    0]
                        [                                                                        ]
                        [0    0    0    0    0    0    0    0    0    0    0    0    0    0    0]
                        [                                                                        ]
                        [0    0    0    0    0    0    0    0    0    0    0    0    0    0    0]
                        [                                                                        ]
                        [0    0    0    0    0    0    0    0    0    0    0    0    0    0    0]
                        [                                                                        ]
                        [0    0    0    0    0    0    0    0    0    0    0    0    0    0    0]
                        [                                                                        ]
                        [0    0    0    0    0    0    0    0    0    0    0    0    0    0    0]
                        [                                                                        ]
                        [0    0    0    0    0    0    0    0    0    0    0    0    0    0    0]
                        [                                                                        ]
                        [0    0    0    0    0    0    0    0    0    0    0    0    0    0    0]
                        [                                                                        ]
                        [0    0    0    0    0    0    0    0    0    0    0    0    0    0    0]
                        [                                                                        ]
                        [0    0    0    0    0    0    0    0    0    0    0    0    0    0    0]
                        [                                                                        ]
                        [0    0    0    0    0    0    0    0    0    0    0    0    0    0    0]
                        [                                                                        ]
                        [0    0    0    0    0    0    0    0    0    0    0    0    0    0    0]
                        [                                                                        ]
                        [0    0    0    0    0    0    0    0    0    0    0    0    0    0    0]
                        [                                                                        ]
                        [0    0    0    0    0    0    0    0    0    0    0    0    0    0    0]

>  JordanForm(S);
     [%1    0       0       0     0       0       0     0       0       0     0       0       0     0       0   ]
     [                                                                                                            ]
     [0     %2      0       0     0       0       0     0       0       0     0       0       0     0       0   ]
     [                                                                                                            ]
     [                (1/3)                                                                                       ]
     [0     0     2         0     0       0       0     0       0       0     0       0       0     0       0   ]
     [                                                                                                            ]
     [0     0       0       %1    0       0       0     0       0       0     0       0       0     0       0   ]
     [                                                                                                            ]
     [0     0       0       0     %2      0       0     0       0       0     0       0       0     0       0   ]
     [                                                                                                            ]
     [                                      (1/3)                                                                 ]
     [0     0       0       0     0     2         0     0       0       0     0       0       0     0       0   ]
     [                                                                                                            ]
     [0     0       0       0     0       0       %1    0       0       0     0       0       0     0       0   ]
     [                                                                                                            ]
     [0     0       0       0     0       0       0     %2      0       0     0       0       0     0       0   ]
     [                                                                                                            ]
     [                                                            (1/3)                                           ]
     [0     0       0       0     0       0       0     0     2         0     0       0       0     0       0   ]
     [                                                                                                            ]
     [0     0       0       0     0       0       0     0       0       %1    0       0       0     0       0   ]
     [                                                                                                            ]
     [0     0       0       0     0       0       0     0       0       0     %2      0       0     0       0   ]
     [                                                                                                            ]
     [                                                                                  (1/3)                     ]
     [0     0       0       0     0       0       0     0       0       0     0     2         0     0       0   ]
     [                                                                                                            ]
     [0     0       0       0     0       0       0     0       0       0     0       0       %1    0       0   ]
     [                                                                                                            ]
     [0     0       0       0     0       0       0     0       0       0     0       0       0     %2      0   ]
     [                                                                                                            ]
     [                                                                                                      (1/3) ]
     [0     0       0       0     0       0       0     0       0       0     0       0       0     0     2     ]

                                                   1/3
                                                  2                 1/2  (1/3)
                                        %1 := - ---- + 1/2 I 3    2
                                                   2

                                                   1/3
                                                  2                 1/2  (1/3)
                                        %2 := - ---- - 1/2 I 3    2
                                                   2

> quit
bytes used=22863024, alloc=5438492, time=0.62
```