

```

1 restart;maple_mode(1);cas_setup(0,0,0,1,0,1e-10,10,[1,50,0,25],0,0,0);#radians,pas de cmplx, pas de Sqrt
( [], Warning: some commands like subs might change arguments of procedure 0, 1, 0, 1e-10, 10, [1
2 -----Polynome caracteristique et mineurs principaux-----
3 n:=5; In:={seq(i,i=1..n)}; In minus {2,4};
( 5, [ 1 2 3 4 5 ], [ 1 3 5 ] )
4 extr:=proc(A,II,JJ)
matrix([seq([seq(A[i,j],j=JJ),i=II]));
end_proc;
// Warning: i j declared as global variable(s)
// End defining extr
proc(A,II,JJ)
matrix(seq([seq(A[i,j],j=JJ),i=II]));
end;
5 Attention, diff(f,x,y) derive en x puis y, alors que diff(f,[x,y])
donne la liste des derive en x et en y.
6 diff(x*y,x,y);diff(x*y,[x,y]);
( 1, [ y x ] )
7 purge(a,x);
( a not assigned x not assigned )
8 n:=5;A:=matrix(n,n,(i,j)->a[i,j]);
// Warning: a declared as global variable(s)
( 5,
a[ 1, 1 ] a[ 1, 2 ] a[ 1, 3 ] a[ 1, 4 ] a[ 1, 5 ]
a[ 2, 1 ] a[ 2, 2 ] a[ 2, 3 ] a[ 2, 4 ] a[ 2, 5 ]
a[ 3, 1 ] a[ 3, 2 ] a[ 3, 3 ] a[ 3, 4 ] a[ 3, 5 ]
a[ 4, 1 ] a[ 4, 2 ] a[ 4, 3 ] a[ 4, 4 ] a[ 4, 5 ]
a[ 5, 1 ] a[ 5, 2 ] a[ 5, 3 ] a[ 5, 4 ] a[ 5, 5 ] )
9 d:=diag(seq(x[i],i=1..n));
( x[1] 0 0 0 0
0 x[2] 0 0 0
0 0 x[3] 0 0
0 0 0 x[4] 0
0 0 0 0 x[5] )
10 II:=In minus {1,3};# on essaye (i,j)=(1,3)
[ 2 4 5 ]
11 extr(A,II,II);
a[ 2, 2 ] a[ 2, 4 ] a[ 2, 5 ]
a[ 4, 2 ] a[ 4, 4 ] a[ 4, 5 ]
a[ 5, 2 ] a[ 5, 4 ] a[ 5, 5 ]

```

```
12 dij:=diff(det(A-d),x[1],x[3]);
(a[[ 2, 2 ]])·(a[[ 4, 4 ]])·(a[[ 5, 5 ]]) - (a[[ 2, 2 ]])·(a[[ 4, 4 ]])·(x[5]) - (a[[ 2, 2 ]])·(x[4])·(a[[ 5, 5 ]])
- (a[[ 2, 2 ]])·(a[[ 4, 5 ]])·(a[[ 5, 4 ]]) - (x[2])·(a[[ 4, 4 ]])·(a[[ 5, 5 ]]) + (x[2])·(a[[ 4, 4 ]])·(x[4])·(a[[ 5, 5 ]])
- (x[2])·(a[[ 4, 5 ]])·(a[[ 5, 4 ]]) - (a[[ 2, 4 ]])·(a[[ 4, 2 ]])·(a[[ 5, 5 ]]) + (a[[ 2, 4 ]])·(a[[ 4, 2 ]])·(x[4])·(a[[ 5, 5 ]])
+ (a[[ 2, 5 ]])·(a[[ 4, 2 ]])·(a[[ 5, 4 ]]) - (a[[ 2, 5 ]])·(a[[ 4, 4 ]])·(a[[ 5, 2 ]]) + (a[[ 2, 5 ]])·(x[4])·(a[[ 5, 2 ]])
```

```
13 normal(det( extr(A,II,II) ) - subs(x=[0,0,0,0,0],dij));
0
```

```
14 II:=In minus {2,3};# on essaye (i,j)=(2,3)
|| 1 4 5 ||
```

```
15 extr(A,II,II);
| a[[ 1, 1 ] a[[ 1, 4 ] a[[ 1, 5 ] |
| a[[ 4, 1 ] a[[ 4, 4 ] a[[ 4, 5 ] |
| a[[ 5, 1 ] a[[ 5, 4 ] a[[ 5, 5 ] |
```

```
16 dij:=diff(det(A-d),x[2],x[3]);
(a[[ 1, 1 ]])·(a[[ 4, 4 ]])·(a[[ 5, 5 ]]) - (a[[ 1, 1 ]])·(a[[ 4, 4 ]])·(x[5]) - (a[[ 1, 1 ]])·(x[4])·(a[[ 5, 5 ]])
- (a[[ 1, 1 ]])·(a[[ 4, 5 ]])·(a[[ 5, 4 ]]) - (x[1])·(a[[ 4, 4 ]])·(a[[ 5, 5 ]]) + (x[1])·(a[[ 4, 4 ]])·(x[4])·(a[[ 5, 5 ]])
- (x[1])·(a[[ 4, 5 ]])·(a[[ 5, 4 ]]) - (a[[ 1, 4 ]])·(a[[ 4, 1 ]])·(a[[ 5, 5 ]]) + (a[[ 1, 4 ]])·(a[[ 4, 1 ]])·(x[4])·(a[[ 5, 5 ]])
+ (a[[ 1, 5 ]])·(a[[ 4, 1 ]])·(a[[ 5, 4 ]]) - (a[[ 1, 5 ]])·(a[[ 4, 4 ]])·(a[[ 5, 1 ]]) + (a[[ 1, 5 ]])·(x[4])·(a[[ 5, 1 ]])
```

```
17 normal(det( extr(A,II,II) ) - subs(x=[0,0,0,0,0],dij));
0
```

```
18 II:=In minus {4,5};# on essaye (i,j)=(4,5)
|| 1 2 3 ||
```

```
19 extr(A,II,II);
| a[[ 1, 1 ] a[[ 1, 2 ] a[[ 1, 3 ] |
| a[[ 2, 1 ] a[[ 2, 2 ] a[[ 2, 3 ] |
| a[[ 3, 1 ] a[[ 3, 2 ] a[[ 3, 3 ] |
```

```
20 dij:=diff(det(A-d),x[4],x[5]);
(a[[ 1, 1 ]])·(a[[ 2, 2 ]])·(a[[ 3, 3 ]]) - (a[[ 1, 1 ]])·(a[[ 2, 2 ]])·(x[3]) - (a[[ 1, 1 ]])·(x[2])·(a[[ 3, 3 ]])
- (a[[ 1, 1 ]])·(a[[ 2, 3 ]])·(a[[ 3, 2 ]]) - (x[1])·(a[[ 2, 2 ]])·(a[[ 3, 3 ]]) + (x[1])·(a[[ 2, 2 ]])·(x[4])·(a[[ 3, 3 ]])
- (x[1])·(a[[ 2, 3 ]])·(a[[ 3, 2 ]]) - (a[[ 1, 2 ]])·(a[[ 2, 1 ]])·(a[[ 3, 3 ]]) + (a[[ 1, 2 ]])·(a[[ 2, 1 ]])·(x[4])·(a[[ 3, 3 ]])
+ (a[[ 1, 3 ]])·(a[[ 2, 1 ]])·(a[[ 3, 2 ]]) - (a[[ 1, 3 ]])·(a[[ 2, 2 ]])·(a[[ 3, 1 ]]) + (a[[ 1, 3 ]])·(x[2])·(a[[ 3, 1 ]])
```

```
21 normal(det( extr(A,II,II) ) - subs(x=[0,0,0,0,0],dij));
0
```

```
22 B:=A-x*identity(n);
| a[[ 1, 1 ] - x a[[ 1, 2 ] a[[ 1, 3 ] a[[ 1, 4 ] a[[ 1, 5 ] |
| a[[ 2, 1 ] a[[ 2, 2 ] - x a[[ 2, 3 ] a[[ 2, 4 ] a[[ 2, 5 ] |
| a[[ 3, 1 ] a[[ 3, 2 ] a[[ 3, 3 ] - x a[[ 3, 4 ] a[[ 3, 5 ] |
| a[[ 4, 1 ] a[[ 4, 2 ] a[[ 4, 3 ] a[[ 4, 4 ] - x a[[ 4, 5 ] |
| a[[ 5, 1 ] a[[ 5, 2 ] a[[ 5, 3 ] a[[ 5, 4 ] a[[ 5, 5 ] - x |
```

```
23 d:=seq(normal(subs(x=0,diff(det(B),x,i))/i!),i=n..1);
```

```
- (a[1, 1])·(a[2, 2]) - (a  
- (a[1, 1])·(a[4, 4]) - (a  
(a[1, 2])·(a[2, 1])+(a[1,  
(a[1, 4])·(a[4, 1])+(a[1,  
- (a[2, 2])·(a[3, 3]) - (a  
- (a[2, 2])·(a[5, 5]) + (a[  
(a[2, 4])·(a[4, 2])+(a[2,  
- (a[3, 3])·(a[4, 4]) - (a  
(a[3, 4])·(a[4, 3])+(a[3,
```

```
24 P:=charpoly(A);
```

Menu

```

(a[ 1, 1 ])·(a[ 2, 2 ])·
(a[ 1, 1 ])·(a[ 4, 4 ])·
- (a[ 1, 2 ])·(a[ 2, 1 ])·
- (a[ 1, 4 ])·(a[ 4, 1 ])·
(a[ 2, 2 ])·(a[ 3, 3 ])·
(a[ 2, 2 ])·(a[ 5, 5 ])·
- (a[ 2, 4 ])·(a[ 4, 2 ])·
(a[ 3, 3 ])·(a[ 4, 4 ])·
- (a[ 3, 4 ])·(a[ 4, 3 ])

```

25 -----calcul du determinant par interpolation-----

26 Prog Edit Add nxt OK Save

```

monlagrange: proc(A)
local n,i,j,P,L,M
n:=dim(A)[1];P:=0
L:=seq(i,i=0..n)
for i from 0 to n do
M:=1;
for j in L minus [i] do M:=M*(x-j)/(i-j) od;
P:=P+det(i*identity(n)-A)*1

```

// Warning: index x declared as global variable(s)
// End defining monlagrange

```

proc(A)
local n,i,j,P,L,M;
n:=(dim(A))[1];
P:=0;
L:=seq(i,i=(0 .. n));
for i from 0 to n+1/2 do
M:=1;
for index from 1 to nops(L minus [i])+1 do
j:=(L minus [i])[index];
M:=M*(x-j)/(i-j);
od;;
P:=P+det(i*identity(n)-A)*M;
od;;
normal(P);

end;

```

27 time(monlagrange(A));

Evaluation time: 0.49

0.49

28

29 k:=50:A:=matrix(k,k,(i,j)->rand(21)-10):

// Success

(Done Done)

```

30 pari();
All PARI functions are now defined with the pari_ prefix.
PARI functions are also defined without prefix except:
abs acos acosh arg asin asinh atan atanh binomial bitand bitor bitxor ceil charpoly concat conj cont
Note that p-adic numbers must have O argument quoted e.g. 905/7+O('7^3')
Type ?pari for short help
Inside xcas, try Help->Manuals->PARI for HTML help
└─ Menu

31 time(pari_charpoly(A,0));
Evaluation time: 0.8
0.8
└─ Menu

32 time(pari_charpoly(A,1));
Evaluation time: 0.82
0.82
└─ Menu

33 k:=30:A:=matrix(k,k,(i,j)->(rand(21)-10)/(rand(4)^2+rand(1,3)^2));
// Success
( Done Done )
└─ Menu

34 time(pari_charpoly(A,0));
print Error: Bad Argument Type
└─ Menu

35 time(pari_charpoly(A,1));
print Error: Bad Argument Type
└─ Menu

36 c'est en  $O(n^4)$ , en fait  $n^4/3$ , alors que Faddeev est en  $n^4$ . Mais il y a plus
de fractions chez lagrange, ce qui peut donner l'avantage a faddeev.

37 uv:=proc(n)
u:=rand(1,n+1):v:=(rand(u,u+n-1) mod n)+1:
[u,v];
end_proc;
// Warning: u v declared as global variable(s)
// End defining uv
proc(n)
u:=rand(1,n+1);
v:=irem(rand(u,u+n-1),n)+1;
[u,v];
end;
└─ Menu

38 uv(10);
[ 7 8 ]
└─ Menu

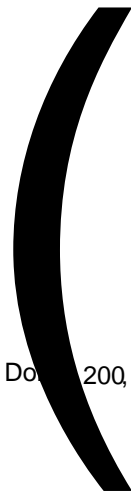
39 On obtient tout element de  $SL_n$  par produit de matrice  $I+aE_{ij}$ 

```

```
40 n:=40;k:=200;R:=identity(n);for i from 1 to k do
T:=identity(n);a:=uv(n);T[a[1],a[2]]:=rand(-5,6);R:=R*T;od;
```

Evaluation time: 1

21	175	0	0	0	0	7420	-75	1000
-40	-13097	300	12	503	10020	27687	909	-66970
0	-82	1	6	5	88	170	9	-422
0	0	0	1	0	0	0	0	0
6	-3692	150	0	1	20	3351	-15	-18626
-5	100	0	0	0	1	475	0	550
40	-9246	375	0	3	0	8283	-36	-46645
-200	46085	-1875	0	0	0	-41282	226	232475
0	-19	0	3	0	5	0	0	-105
0	1845	-75	0	0	0	-987	-15	9315
10	3267	-75	-3	-125	-2505	-6915	-225	16705
140	-53014	1950	12	425	7616	58539	600	-268353
0	0	0	-2	0	4	25	0	0
0	0	0	0	0	0	0	0	0
-400	92172	-3750	0	0	0	-82288	449	464962
8	-1841	75	0	0	0	1651	-9	-9287
0	104	-5	3	0	5	-75	0	516
0	0	0	-1	0	0	-4	0	0
0	-20	0	0	0	21	100	0	-100
-360	136832	-4800	0	-1680	-30000	-174385	-2700	693012
0	40	0	4	0	0	-20	5	200
42	-7538	300	6	0	8	5654	-36	-38138
0	167	-5	0	0	16	160	0	865



Done 200, Done

```
41 det(R);
```

1

```
42 n:=3;N:=matrix(n,n,(i,j)->if (i=j+1) then 1 else 0 fi)
```

// Success

$$\left(\text{Done} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \right)$$

```
43 T:=diag((3*identity(n)+N),(-2*identity(n)+N),(5*identity(n)+N),(-1*identity(n)+N));
```

3	0	0	0	0	0	0	0	0	0	0	0
1	3	0	0	0	0	0	0	0	0	0	0
0	1	3	0	0	0	0	0	0	0	0	0
0	0	0	-2	0	0	0	0	0	0	0	0
0	0	0	1	-2	0	0	0	0	0	0	0
0	0	0	0	1	-2	0	0	0	0	0	0
0	0	0	0	0	0	5	0	0	0	0	0
0	0	0	0	0	0	1	5	0	0	0	0
0	0	0	0	0	0	0	1	5	0	0	0
0	0	0	0	0	0	0	0	0	-1	0	0
0	0	0	0	0	0	0	0	0	1	-1	0
0	0	0	0	0	0	0	0	0	0	1	-1

```
Menu
```

```
44 n:=10;
```

10

```
Menu
```

```
45 T:=diag((3*identity(n)+N),(-2*identity(n)+N),(5*identity(n)+N),(-1*identity(n)+N));
```

Done

```
Menu
```

```
46 A:=R^(-1)*T*R;
```

Done

```
Menu
```

```
47 time(pari_charpoly(A,0));
```

0.31

```
Menu
```

```
48 time(pari_charpoly(A,1));
```

0.3

```
Menu
```

```
49
```

```
50
```

```
51
```