

```

1 restart;maple_mode(1);cas_setup(0,0,0,1,0,1e-10,10,[1,50,0,25],0,0,0); #radians,pas de cmplx, pas de Sqrt
2 time(p:=nextprime(10^50));
0.0025
3 time(isprime(p));
0.078125
4 time(is_pseudoprime(p));
0.00140625
5 le temps de isprime est trop long pour avoir ete utilise pour
trouver le nombre premier avec nextprime. La doc confirme bien que
nextprime donne un pseudopremier.
6 i:=20;n:=nextprime(10^i)*nextprime(20^i);
( 20, 104857600000000000004089455500000000000000003549 )
7 ifactor(n);
Evaluation time: 2.07812
1000000000000000000039 · 104857600000000000000000091
8 a partir de i=30 ca ne repond plus?
9 p:=nextprime(=nextprime(10^30)*nextprime(20^30));
10737418240000000000000000612032841090000000000000000000008409
10 ifactor(p-1); #ca marche souvent
2^3 · 3 · 389 · 733 · 156904374622257604823879982847602392900751802349981470895277241
11 pari();
All PARI functions are now defined with the pari_ prefix.
PARI functions are also defined without prefix except:
abs acos acosh arg asin asinh atan atanh binomial bitand bitor ceil charpoly concat conj content cos cosh divis
Note that p-adic numbers must have O argument quoted e.g. 905/7+O('7^3')
Type ?pari for short help
Inside xcas, try Help->Manuals->PARI for HTML help
12 pari_isprime(p,1);
Evaluation time: 12.0625
2 7 1
3 2 1
389 2 1
733 2 1
156904374622257604823879982847602392900751802349981470895277241 2 56467
65530849258879746208
13 Les grands nombres premiers apparaissant dans le certificat sont eux
m{^e}me certifi{e}s, ce qui explique les crochets emboit{e}s.
14 p:=nextprime(10^13);i:=0;l=[];

```

```

15 while i<10 do
   if ispseudoprime(4*p+1)>0 then l:=[op(l),4*p+1];i++; fi;
   p:=nextprime(p);
   od:l;
( Done, [ 40000000000733 40000000002629 40000000011053 40000000016309 40000000017917 4000
16 p:= extprime(10^13+100);
100000000000129
17 g:=pari_znprimroot(p);
7 % 10000000000129
18 pari_znlog(3,Mod(g,p));
2552475531946
19 ifactor(p-1);
2^7 · 3 · 98867 · 263401
20 p-1 a de petits facteurs premiers, on peut donc resoudre le probleme
du log discret. Dans le cas suivant ca ne marche plus.
21 p:=[3];
40000000011053
22 g:=pari_znprimroot(p);
2 % 40000000011053
23 pari_znlog(3,Mod(g,p)); #ca depasse la memoire de pari
Error in PARI subsystem
undef
24 -----EXERCICE-----
25 p:=[3];
40000000011053
26 b,e pas trop petits par rapport a p, pour que le modulo p melange
vraiment.
27 message:="ABCD A, Remarquez que des lettres identiques ne sont pas codees de la meme maniere!";asc(message)
( ABCDA, Remarquez que des lettres identiques ne sont pas codees de la meme maniere! , [ 65 66 67 68
28 asc(message)-[seq(65,i=1..length(message))];
[-65 -65 -65 -65 -65 -65 -65 -65 -65 -65 -65 -65 -65 -65 -65 -65 -65 -65 -65 -65
29 b:=12345;e:=54321; igcd(e,p-1); B:=powmod(b,e,p);
( 12345, 54321, 1, 33473253573817 )
30 l:=asc(message);k:=[seq(rand(50),i=1..length(message))];
( [ 65 66 67 68 65 44 32 82 101 109 97 114 113 117 101 122 32 113 117 101 32

```

```

31 C:=[seq(((b^k[i]) mod p,((l[i]*B^k[i]) mod p),i=1..length(message)))];#le message crypte
30090100443913 16716100139370
16088998776349 38338962478444
16140057393540 28774154633666
33604145840536 38170669619125
16140057393540 21090277825400
25616939055286 28970370733378
152399025 35603652517936
26072565355155 24716418195292
28871295273866 36886580726949
12345 2897413490631
16140057393540 29602536281495
18072155013312 11942674603400
9285222567600 3123239293725
8898364078510 30262442773278
2504656109032 5133093648656
18072155013312 21329380798072
10793236508135 35297819144575
8898364078510 39027147015182
22289850177970 22249200168147
2916562363890 4205025507103
6064246205996 10679585368370
10304518854412 19475890601838

```

```

32 ee:=p-1-e;#c'est -e [p-1] qui nous interesse
399999999956731

```

33 L'astuce est que $(B^k) = ((b^e)^k)$ qui vaut par commutativité des la composition des application $B: x \rightarrow x^k$ et $A: x \rightarrow x^e$ $((b^k)^e)$.
 Le principe a retenir, est qu'en crypto on a juste besoin d'operations A,B qui commutent, et que la donnée de A de donne pas $A^{(-1)}$

```

34 decrypt:=[seq(irem(powmod(C[i][1],ee,p)*C[i][2],p),i=1..length(message))];
65 66 67 68 65 44 32 82 101 109 97 114 113 117 101 122 32 113 117 101 32

```

```

35 char(decrypt);
ABCDA, Remarquez que des lettres identiques ne sont pas codees de la meme maniere!

```

36 -----Exercice-----

```

37 8*2+7 mod 7; (8*2+7) mod 7; #Attention mod est prioritaire sous xcas
( 16, 2 )

```

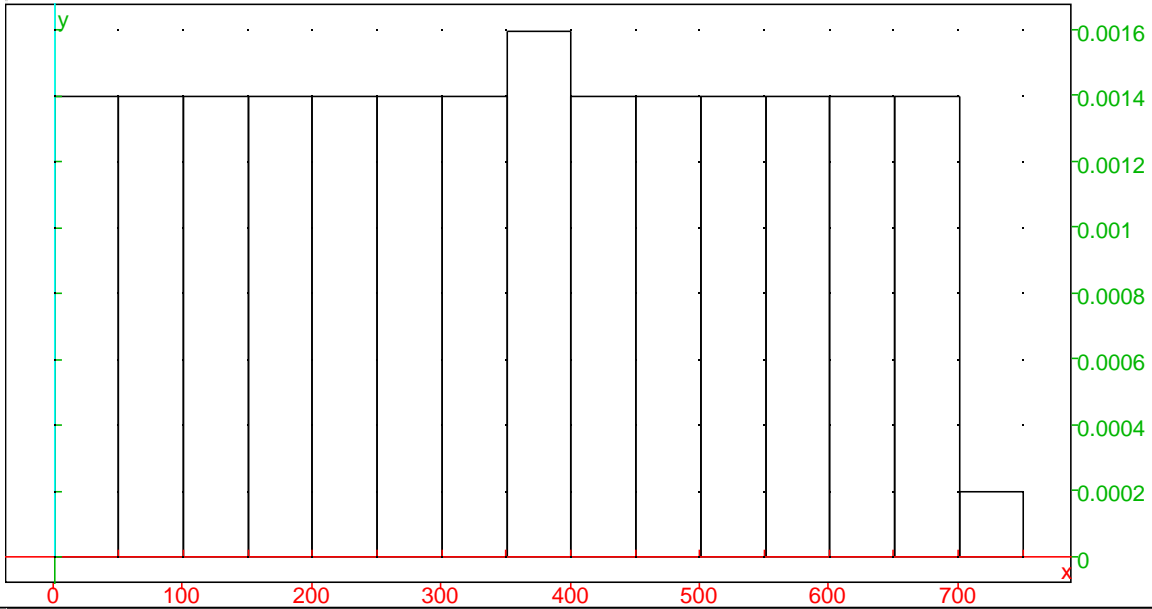
38 Si $a=1$, on a $x_n = x_0 + n.c$ [m], on fait donc par exemple: $c=7, x_0=1$

```

39 l:=[seq((n^7+1) mod 1001,n=1..100)];
8 15 22 29 36 43 50 57 64 71 78 85 92 99 106 113 120 127 134 141 148 155

```

```
40 histogram(classes(l,0,1001/20)); #C'est tout a fait plat. Trop!
```



```
41 Remplacer les crochets par des accolades cree un ensemble, ce qui simplifie  
automatiquement les elements egaux
```

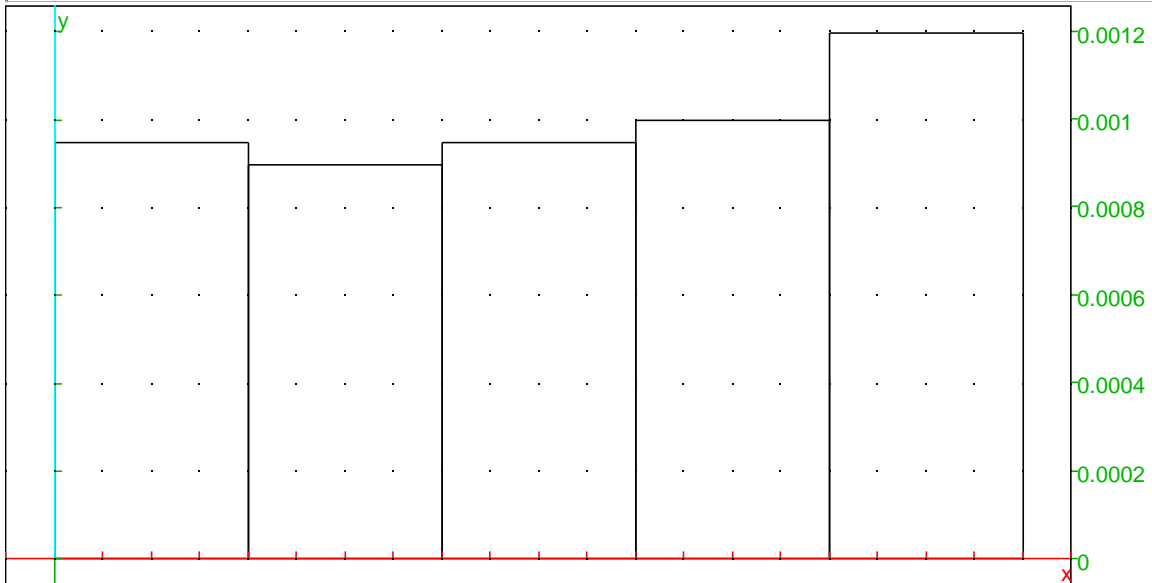
```
42 nops(l);nops({op(l)}); #ils sont tous distincts
```

(100, 100)

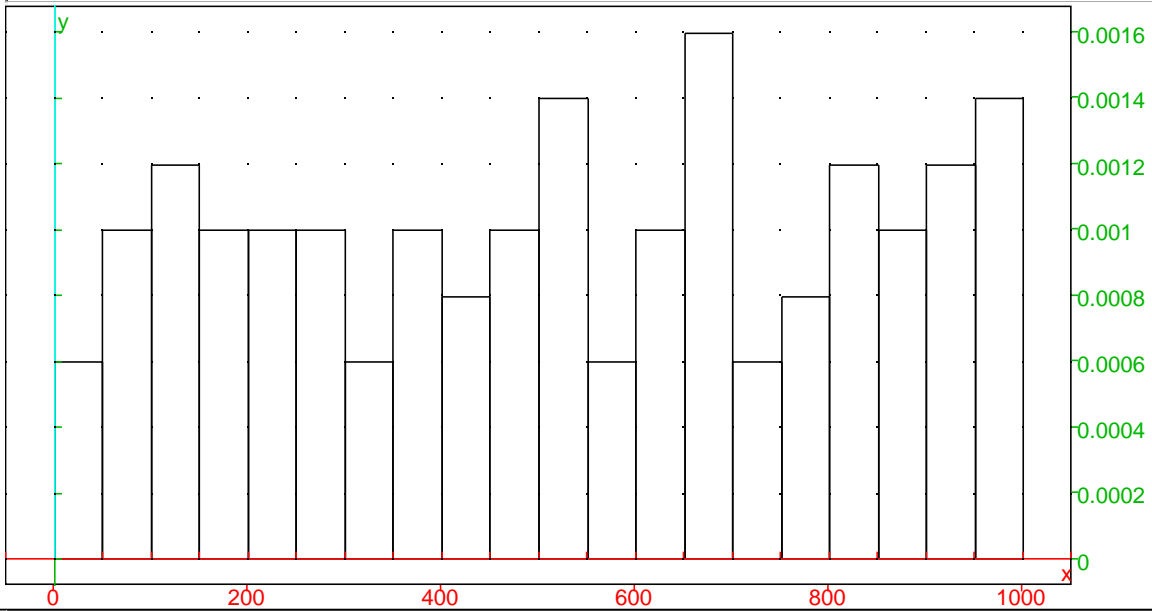
```
43 l:=seq(rand(1001),n=1..100);
```

979 626 186 542 69 189 71 460 540 574 289 231 109 345 255 379 102 373 937

```
44 histogram(classes(l,0,1001/5));
```



45 histogram(classes(l,0,1001/20));



46 nops(l);nops(op(l));#il y a bien des anniversaires identiques

(100, 96)

47 On a en fait 1001^{100} suites possibles, et $1001!/901!$ suites dont tous les termes sont distincts.

48 $1001!/901!/1001^{100}$;

0.00599058972

49 x:=1; a:=237;c:=54321;m:=10^4; l:=[];

(1, 237, 54321, 10000, [])

50 for i from 1 to 500 do x:=(a*x+c) mod m; l:=[op(l),x] ; od:

Done

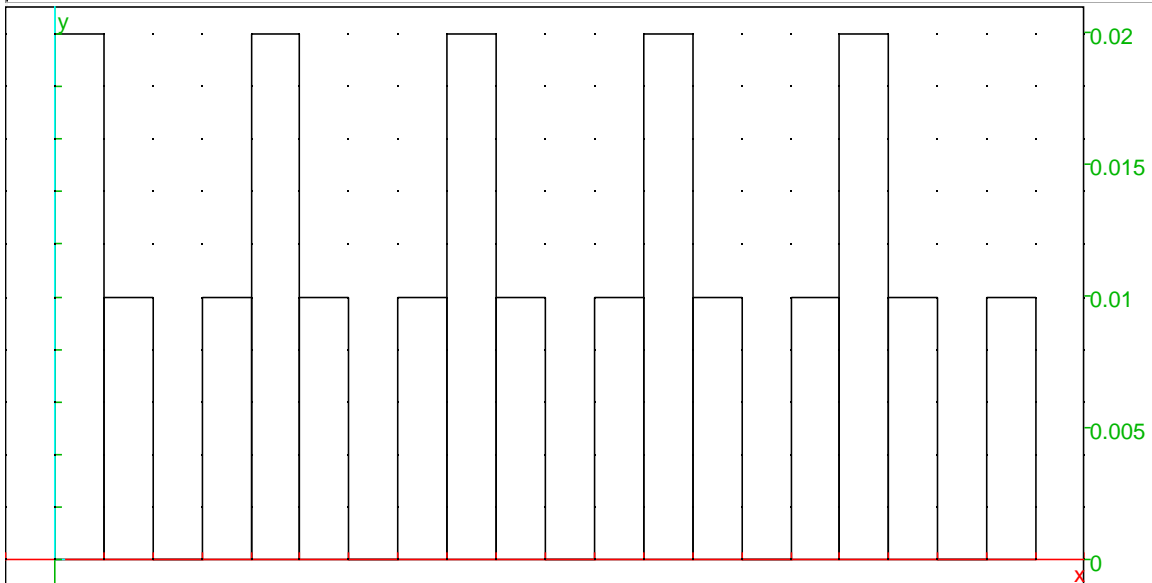
51 l1:=seq(i mod 10^2,i=1);

58 67 0 21 98 47 60 41 38 27 20 61 78 7 80 81 18 87 40 1 58 67 0 21 98

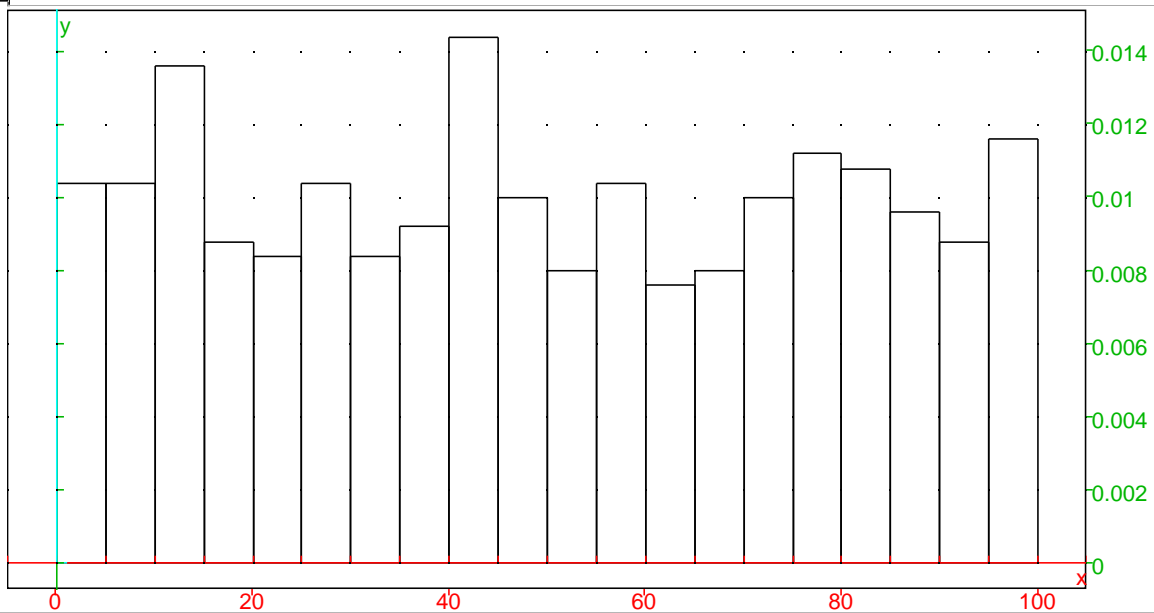
52 l2:=seq(trunc(i/10^2),i=1);

45 45 67 22 6 97 43 76 52 57 16 82 21 5 44 60 55 20 89 31 92 84 10 13 70

53 histogram(classes(l1,0,5));



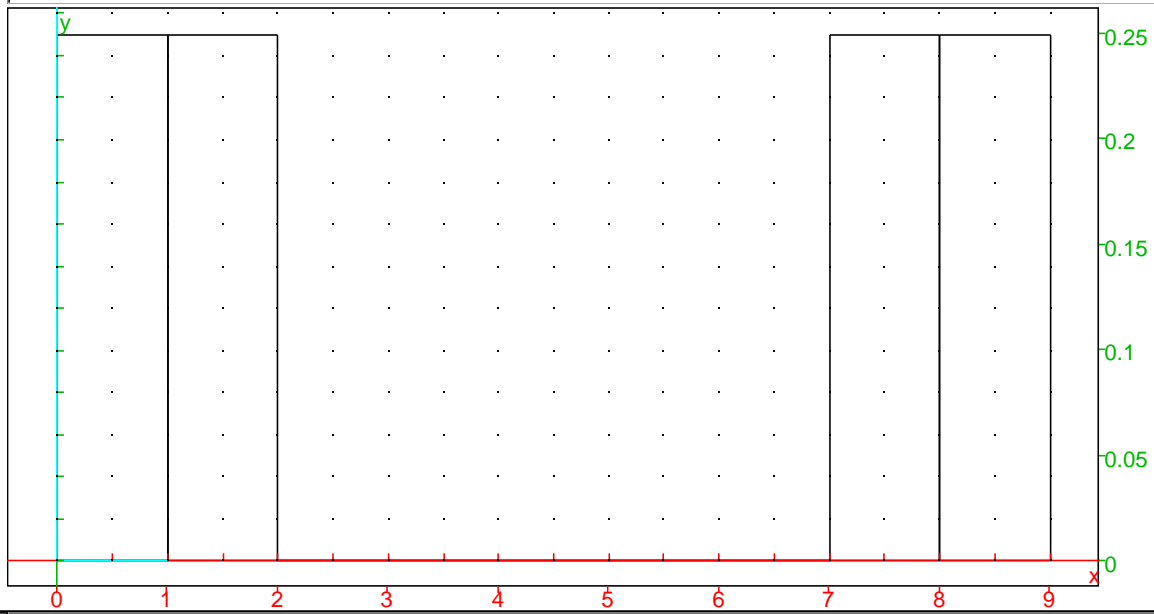
```
54 histogram(classes(l2,0,5));
```



```
55 l1:=seq(i mod 10,i=1);
```

```
8 7 0 1 8 7 0 1 8 7 0 1 8 7 0 1 8 7 0 1 8 7 0 1 8 7 0 1 8 7 0 1 8
```

```
56 histogram(classes(l1,0,1));
```



57 Si d est un diviseur de m et que $y_n = x_n [d]$, alors $y_{n+1} = ay_n + c [d]$
 Donc si $d = 10^2$, les 2 derniers chiffres de x_n ont p(é)riode d'au plus d.

```
58 x:=1; a:=237;c:=54321;m:=10^4-1; l:=[];
```

```
( 1, 237, 54321, 9999, [] )
```

```
59 for i from 1 to 500 do x:=(a*x+c) mod m; l:=op(l,x); od;
```

Done

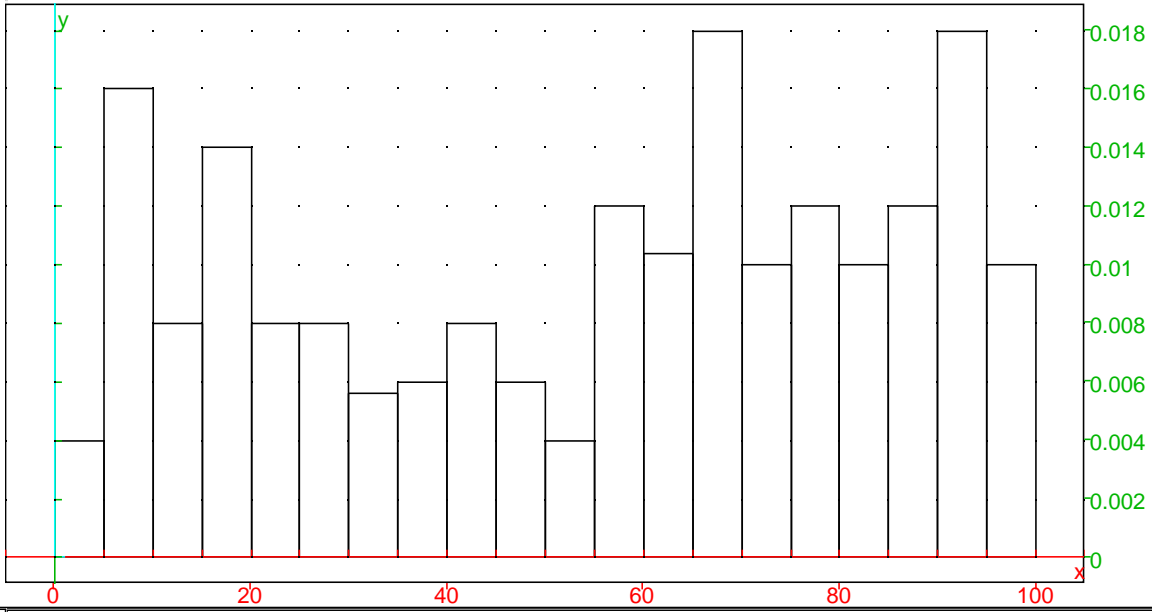
```
60 l1:=seq(i mod 10^2,i=1);
```

```
63 65 70 22 31 78 27 68 10 93 68 17 72 91 56 79 62 81 60 61 25 92 6 69
```

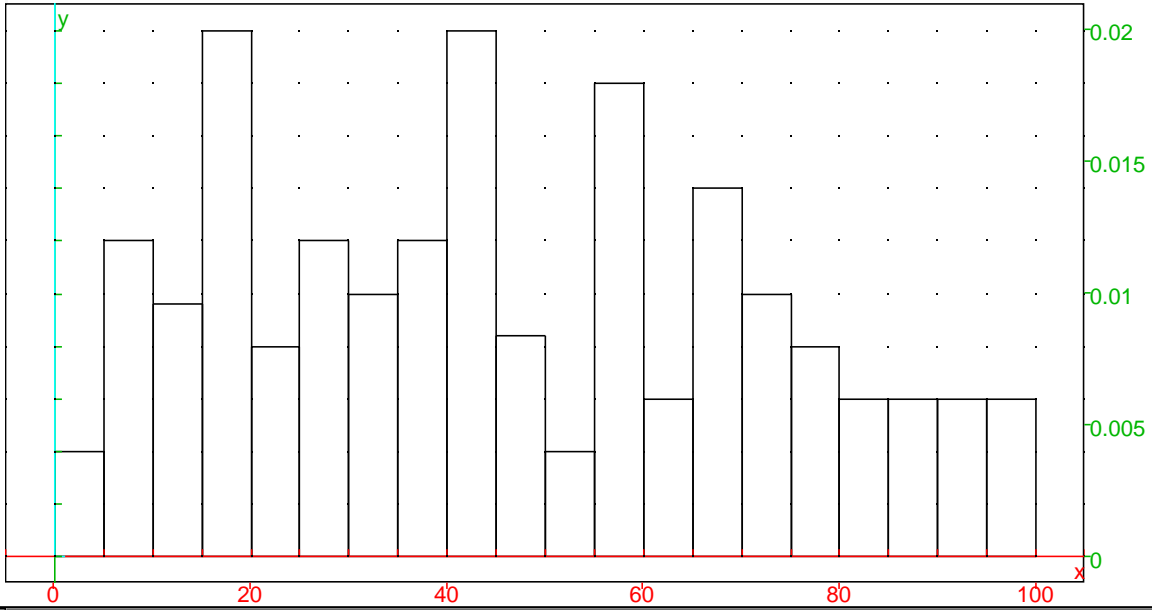
```
61 l2:=seq(trunc(i/10^2),i=1);
```

```
45 58 44 38 2 90 60 28 41 84 73 7 42 68 76 89 25 15 90 17 17 31 9 90 38
```

62 histogram(classes(l1,0,5));



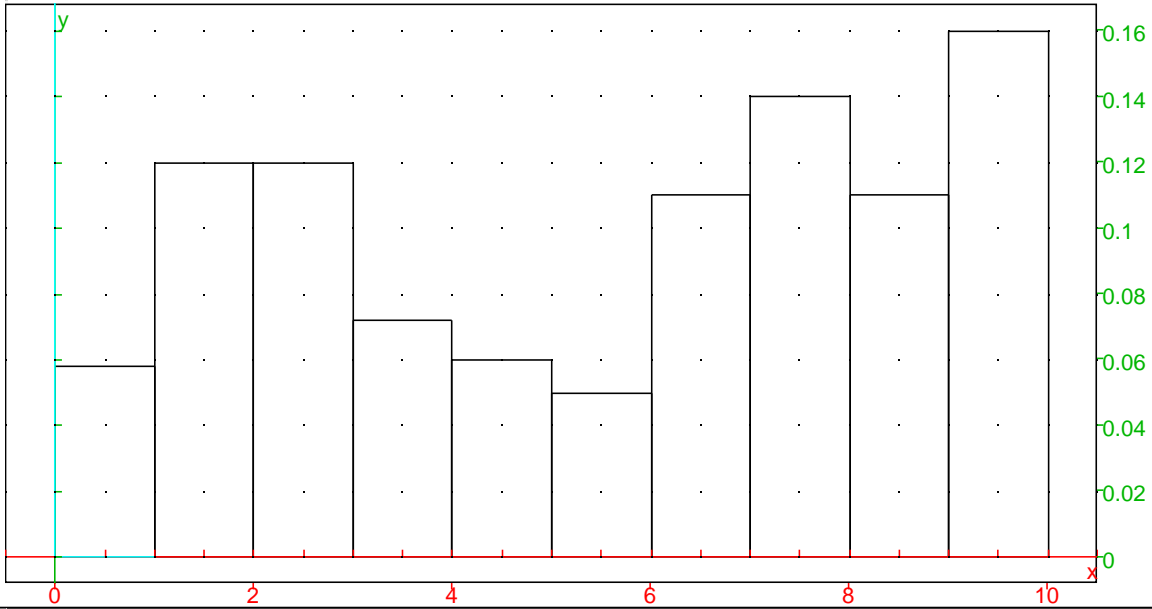
63 histogram(classes(l2,0,5));



64 l1:=seq(i mod 10,i=1);

3 5 0 2 1 8 7 8 0 3 8 7 2 1 6 9 2 1 0 1 5 2 6 9 4 6 1 9 9 5 4 1 8

65 histogram(classes(l1,0,1));



66 x:=1; a:=237;c:=54321;m:=prevprime(10^4); l:=[];

(1, 237, 54321, 9973, [])

67 for i from 1 to 500 do x:=(a*x+c) mod m; l:=[op(l),x] ; od:

Done

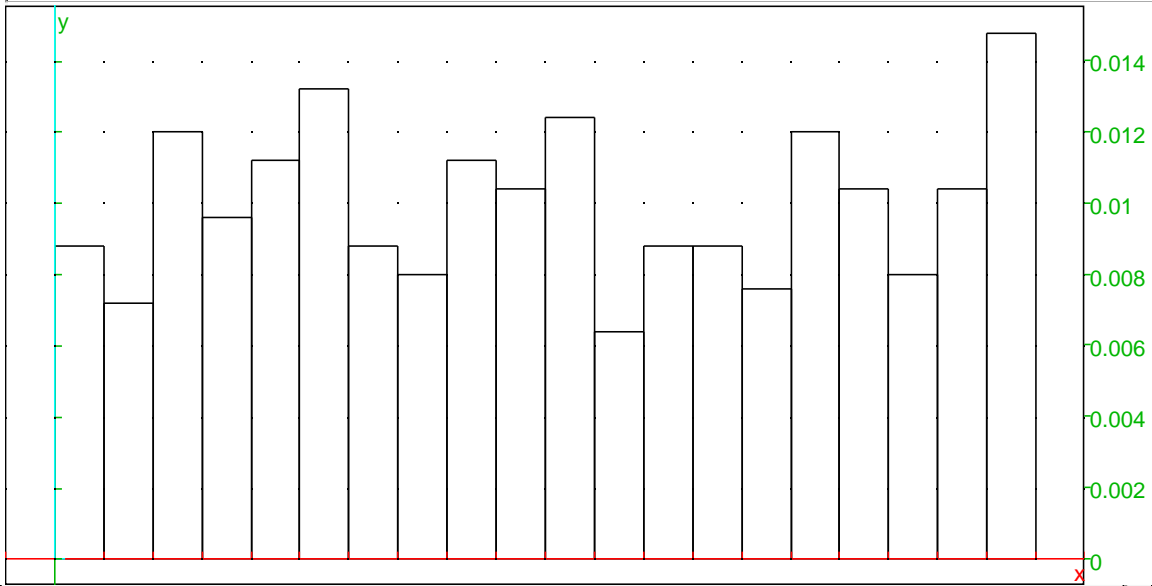
68 l1:=seq(i mod 10^2,i=l);

93 94 44 95 62 27 20 98 29 1 58 77 43 76 26 47 76 89 96 18 57 31 49 96

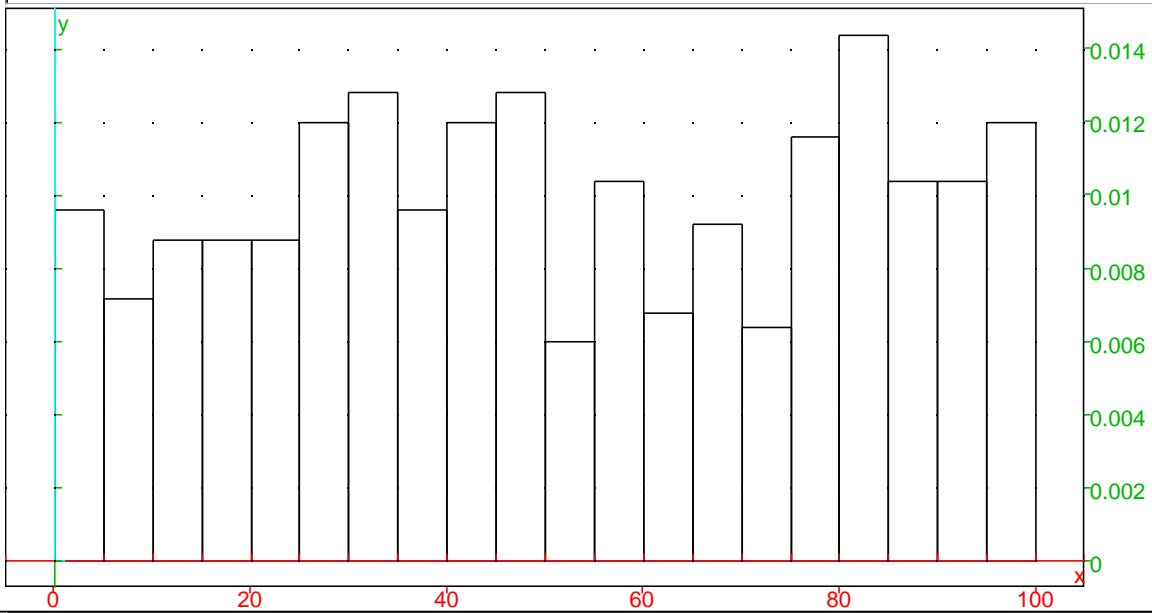
69 l2:=seq(trunc(i/10^2),i=l);

46 96 81 97 21 82 95 67 99 40 52 39 95 22 53 1 93 25 96 86 24 83 42 41

70 histogram(classes(l1,0,5));



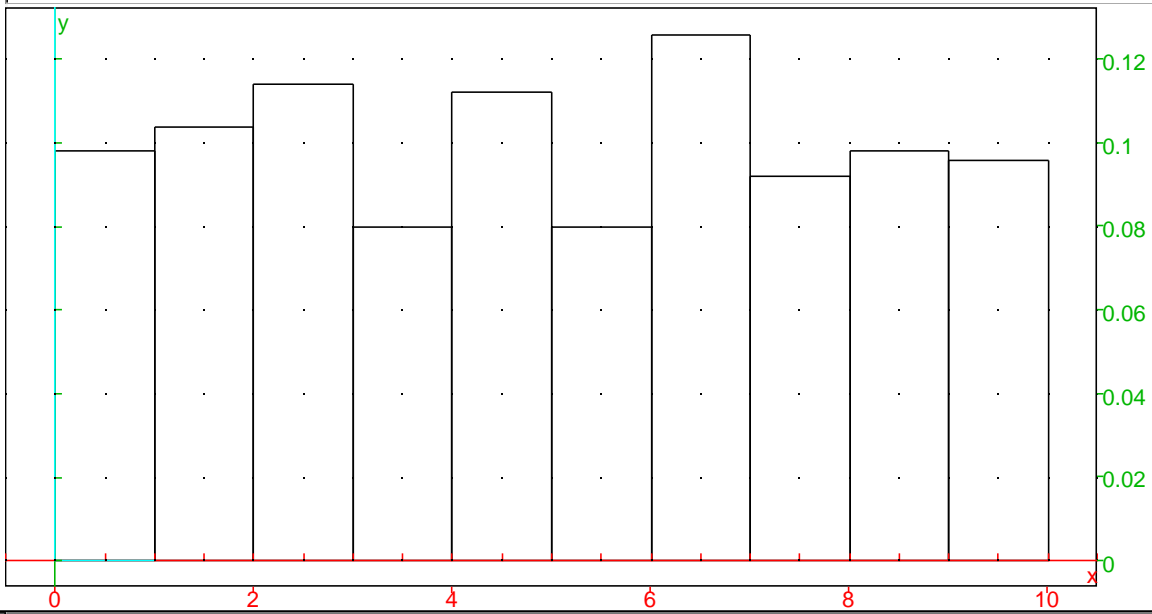

```
71 histogram(classes(l2,0,5));
```



```
72 l1:=seq(i mod 10,i=1);
```

```
3 4 4 5 2 7 0 8 9 1 8 7 3 6 6 7 6 9 6 8 7 1 9 6 8 8 4 8 2 5 6 5 5
```

```
73 histogram(classes(l1,0,1));
```

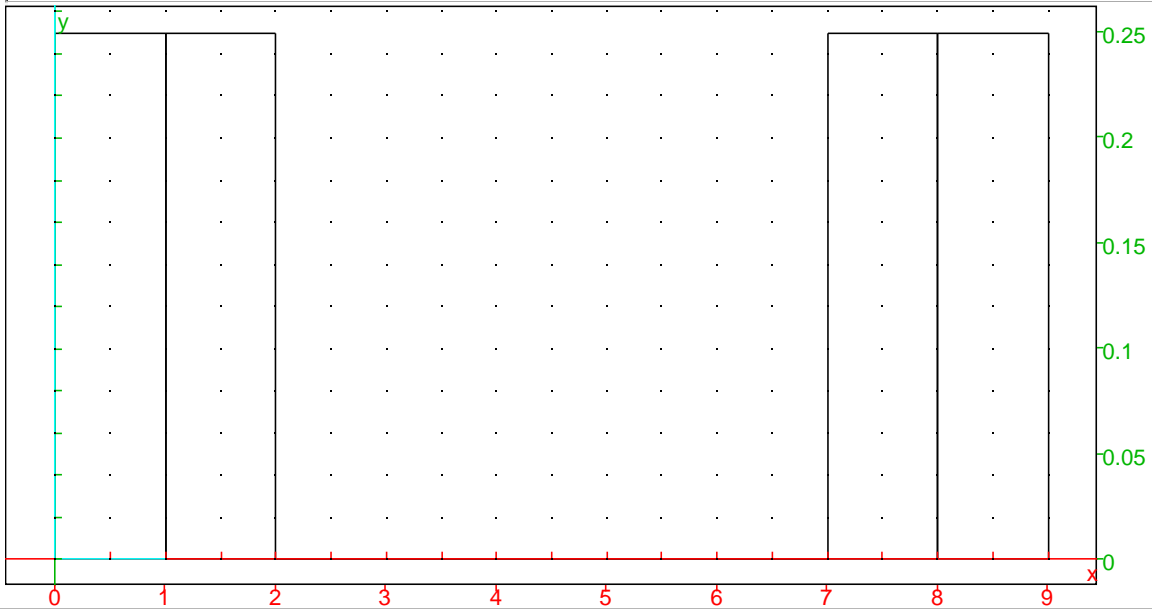


```
74 x:=1; a:=237;c:=54321;m:=10; l:=[];
```

```
( 1, 237, 54321, 10, [] )
```

```
75 for i from 1 to 100 do x:=(a*x+c) mod m; l:=[op(l),x] ; od;
```

76 histogram(classes(l,0,1));#certaines valeurs ne sont pas atteintes.



77

78 Prog Edit Add

nxt

OK

Save

```
periode:= proc (a)
x:=1;p:=1;
if (a mod 5 <> 0) then while x>0 do p:=p+1;x:=(a*x+1) mod 125; od; fi;
p; end proc;
```

```
proc(a)
x:=1;
p:=1;
if (irem(a,5)<>0 then
while x>0 do
p:=p+1;
x:=irem(a*x+1,125);
od;
fi;
p;
```

79 periode(1),periode(5); #verification

(125, 1)

80 l:=seq(periode(i),i=2..124);max(l);

100, 100, 50, 1, 125, 20, 100, 50, 1, 125, 100, 100, 50, 1, 125, 100, 20, 50, 1, 125, 100, 100

81 On a: $x^n = (a^n - 1) / (a - 1) \pmod m$. donc la periode est l'ordre de a.