1
```
restart;maple_mode(1);cas_setup(0,0,0,1,0,1e-10,10,[1,50,0,25],0,0,0);#radians,pas de cmplx, pas de  Sqrt
```

// Warning: some commands like subs might change arguments order ... Menu

2
```
n:=5;A:=matrix(n,n,(i,j)->a[i,j]);M:=A;c:=1;k:=1;
```

// Warning: a declared as global variable(s)

(5,

a[[1, 1]]	a[[1, 2]]	a[[1, 3]]	a[[1, 4]]	a[[1, 5]]
a[[2, 1]]	a[[2, 2]]	a[[2, 3]]	a[[2, 4]]	a[[2, 5]]
a[[3, 1]]	a[[3, 2]]	a[[3, 3]]	a[[3, 4]]	a[[3, 5]]
a[[4, 1]]	a[[4, 2]]	a[[4, 3]]	a[[4, 4]]	a[[4, 5]]

,

a[[1, 1]]	a[[1, 2]]	a[[1, 3]]
a[[2, 1]]	a[[2, 2]]	a[[2, 3]]
a[[3, 1]]	a[[3, 2]]	a[[3, 3]]
a[[4, 1]]	a[[4, 2]]	a[[4, 3]]

Menu

3 Pour M, On peut ecraser car i,j>k

4
```
Lk:=[];
for i from k+1 to n do
for j from k+1 to n do
M[i,j]:=simplify(1/c*(M[k,k]*M[i,j]-M[i,k]*M[k,j]));
II:=[seq(l,l=1..k),i];JJ:=[seq(l,l=1..k),j];
AIIJJ:=matrix([seq([seq(A[u,v],v=JJ)],u=II)])
Lk:=[op(Lk),AIIJJ];
print(simplify(det(AIIJJ)-M[i,j]));
od:od:
c:=M[k,k]:k:=k+1;
Lk;
```

0
0
0
0
0
0
0
0
0
0
0
0
0
0
0

([], Done, Done, 2,

a[[1, 1]]	a[[1, 2]]
a[[2, 1]]	a[[2, 2]]

a[[1, 1]]	a[[1, 3]]
a[[2, 1]]	a[[2, 3]]

a[[1, 1]]	a[[1, 4]]
a[[2, 1]]	a[[2, 4]]

Menu

5
```
Lk:=[];
for i from k+1 to n do
for j from k+1 to n do
M[i,j]:=simplify(1/c*(M[k,k]*M[i,j]-M[i,k]*M[k,j]));
II:=[seq(l,l=1..k),i];JJ:=[seq(l,l=1..k),j];
AIIJJ:=matrix([seq([seq(A[u,v],v=JJ)],u=II)])
Lk:=[op(Lk),AIIJJ];
print(simplify(det(AIIJJ)-M[i,j]));
od:od:
c:=M[k,k]:k:=k+1;
Lk;
```

0
0
0
0
0
0
0
0
0

([], Done, Done, 3,

a[[1, 1]]	a[[1, 2]]	a[[1, 3]]
a[[2, 1]]	a[[2, 2]]	a[[2, 3]]
a[[3, 1]]	a[[3, 2]]	a[[3, 3]]

a[[1, 1]]	a[[1, 2]]	a[[1, 4]]
a[[2, 1]]	a[[2, 2]]	a[[2, 4]]
a[[3, 1]]	a[[3, 2]]	a[[3, 4]]

a
a
a

6
```
Lk:=[];
for i from k+1 to n do
for j from k+1 to n do
M[i,j]:=simplify(1/c*(M[k,k]*M[i,j]-M[i,k]*M[k,j]));
II:=[seq(l,l=1..k),i];JJ:=[seq(l,l=1..k),j];
AIIJJ:=matrix([seq([seq(A[u,v],v=JJ)],u=II)])
Lk:=[op(Lk),AIIJJ];
print(simplify(det(AIIJJ)-M[i,j]));
od:od:
c:=M[k,k]:k:=k+1;
Lk:
```

0
0
0
0

([], Done, Done, 4,

a[[1, 1]]	a[[1, 2]]	a[[1, 3]]	a[[1, 4]]
a[[2, 1]]	a[[2, 2]]	a[[2, 3]]	a[[2, 4]]
a[[3, 1]]	a[[3, 2]]	a[[3, 3]]	a[[3, 4]]
a[[4, 1]]	a[[4, 2]]	a[[4, 3]]	a[[4, 4]]

a[[1, 1]]	a[[1, 2]]	a[[1
a[[2, 1]]	a[[2, 2]]	a[[2
a[[3, 1]]	a[[3, 2]]	a[[3
a[[4, 1]]	a[[4, 2]]	a[[4

Menu

7
```
Lk:=[];
for i from k+1 to n do
for j from k+1 to n do
M[i,j]:=simplify(1/c*(M[k,k]*M[i,j]-M[i,k]*M[k,j]));
II:=[seq(l,l=1..k),i];JJ:=[seq(l,l=1..k),j];
AIIJJ:=matrix([seq([seq(A[u,v],v=JJ)],u=II)])
print(simplify(det(AIIJJ)-M[i,j]));
od:od:
c:=M[k,k]:k:=k+1;
```

0

([], Done, Done, 5)

Menu

8 On passe de Mk-1 a Mk par: L_i<-(c_k.L_i-a_{i,k}.L_k)/c_{k-1}. Chacune de ces operatons multiplie le determinant par c_k/c_{k-1}, et on ne touche pas a la premiere ligne de M_{k-1}. Donc le determinant de M_{k} fois le premier coefficient de M_{k-1} (qui est c_k) vaut (c_k/c_{k-1})^(dim Mk) fois det M_(k-1). d'ou la formule: det(M_k)=(c^{n-k-1}_k)/(c_{k-1}^{n-k}) *det(M_{k-1}). D'autre part, la matrice M_{n-1} est de taille 1 et vaut c_n. Ce qui donne bien det(M_0)=c_n. Donc c_n est entier. Si l'on applique ce resultat a A_{I,J}, alors le dernier c sera a^(k)_{i,j}, il vaudra donc det A_{I,J} et sera entier. Ce qui prouve que ces divisions sont exactes.

9
```
n:=10;A:=matrix(n,n,(i,j)->rand(21)-10);M:=A;c:=1;k:=1;s:=1;
```

// Success

(10,

10	5	-3	-6	0	7	4	8	3	7
-8	-4	7	4	2	-8	-2	1	-5	1
-2	-9	-4	2	6	-6	1	1	7	7
3	9	-2	8	-10	0	-10	-10	9	3
8	-6	8	10	-2	-4	-1	-2	8	8
-3	-7	3	-9	-7	9	-2	-10	-6	3
10	3	9	6	-5	-8	7	-7	4	-9
-2	4	-8	5	-6	-8	-7	-4	6	9
-2	-1	1	-8	9	-10	5	-10	-5	-9

,

10	5	-3	-6	0	7	4	8	3
-8	-4	7	4	2	-8	-2	1	-5
-2	-9	-4	2	6	-6	1	1	7
3	9	-2	8	-10	0	-10	-10	9
8	-6	8	10	-2	-4	-1	-2	8
-3	-7	3	-9	-7	9	-2	-10	-6
10	3	9	6	-5	-8	7	-7	4
-2	4	-8	5	-6	-8	-7	-4	6
-2	-1	1	-8	9	-10	5	-10	-5

Menu

10
```
for k from 1 to n-1 do
for i from k+1 to n do
for j from k+1 to n do
M[i,j]:=simplify(1/c*(M[k,k]*M[i,j]-M[i,k]*M[k,j]));
od:od:
c:=M[k,k]:k:=k+1;s:=s*c;
od:s;
```

(Done, -94662282052800)

Menu

11 ---

12 A:=matrix(4,4,(i,j)->rand(21)-10);B:=A-x*identity(4);

// Success

$$\left(\begin{bmatrix}6&-9&-7&1\\0&3&-7&-9\\-9&0&-6&0\\10&-9&0&-2\end{bmatrix},\begin{bmatrix}6-x&-9&-7&1\\0&3-x&-7&-9\\-9&0&-6-x&0\\10&-9&0&-2-x\end{bmatrix}\right)$$

Menu

13 P:=normal(poly2symb(charpoly(A,x)));

$$x^4-x^3+(-196)\cdot x^2+(-174)\cdot x+5634$$

Menu

14 for i from 1 to 3 do B:=normal(pivot(B,i,i)); od;

$$\begin{bmatrix}-x^7-(-24)\cdot x^6-126\cdot x^5-1917\cdot x^4-(-31914)\cdot x^3-191484\cdot x^2-(-530712)\cdot x-559872 & 0 & 0\\ 0 & x^6+(-18)\cdot x^5+18\cdot x^4+2025\cdot x^3+(-19764)\cdot x^2+72900\cdot x-93312 & 0\\ 0 & 0 & x^4+(-9)\cdot x^3+(-81)\cdot x^2+1458\cdot\\ 0 & 0 & 0\end{bmatrix}$$

Menu

15 simplify(B[4,4]/P);#le dernier terme diagonal est un multiple du poly carateristique

$$x^4+(-21)\cdot x^3+162\cdot x^2+(-540)\cdot x+648$$

Menu

16 B:=A-x*identity(4);

$$\begin{bmatrix}6-x&-9&-7&1\\0&3-x&-7&-9\\-9&0&-6-x&0\\10&-9&0&-2-x\end{bmatrix}$$

Menu

17 for i from 1 to 3 do B:=normal(pivot(B,5-i,i)); od;

$$\begin{bmatrix}0&0&0&-8100\cdot x^4-(-1587600)\\0&0&-100\cdot x^2-300\cdot x+7470&-90\cdot x^2-(-\\0&8100\cdot x^2+24300\cdot x-605070&0&21870\cdot x+335\\ &2& &-8100\cdot x^3-\end{bmatrix}$$

Menu

18 simplify(B[1,4]/P);#le dernier terme diagonal est un multiple du poly carateristique

-8100

Menu

19 pari()#pour charger pari; sinon faire pari_matdet

All PARI functions are now defined with the pari_ prefix.
PARI functions are also defined without prefix except:
abs acos acosh arg asin asinh atan atanh binomial bitand bitor bitxor ceil charpoly concat conj content cos cosh divis
Note that p-adic numbers must have O argument quoted e.g. 905/7+O('7^3')
Type ?pari for short help
Inside xcas, try Help->Manuals->PARI for HTML help

Menu

20 k:=30;A:=matrix(k,k,(i,j)->rand(21)-10):B:=A-x*identity(k):

// Success

(30, Done, Done)

Menu

21 time(matdet(B,1));#par pivot

Evaluation time: 0.93

22 time(matdet(B,0));#par gauss-bareiss

0.29

23 time(pari_charpoly(A,0));#par faddeev

0.085

24 k:=40;A:=matrix(k,k,(i,j)->rand(21)-10):B:=A-x*identity(k):

// Success

(40, Done, Done)

25 pour k>40 par pivot usuel on depasse la taille apr defaut de pari,
donc on arrete.

26 time(matdet(B,0));#par gauss-bareiss

Evaluation time: 1.3

1.3

27 time(pari_charpoly(A,0));#par faddeev

0.3

28 k:=50;A:=matrix(k,k,(i,j)->rand(21)-10):B:=A-x*identity(k):

// Success

(50, Done, Done)

29 time(matdet(B,0));#par gauss-bareiss

Evaluation time: 4.15

4.15

30 time(pari_charpoly(A,0));#par faddeev

Evaluation time: 0.88

0.88

31 --

32 A := [[0,-2,1,3],[0,0,0,1],[1,1,0,0],[-3,4,1,0]];

$$\begin{bmatrix} 0 & -2 & 1 & 3 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ -3 & 4 & 1 & 0 \end{bmatrix}$$

33 B:=matrix(4,4):B[1,2]:=1:B[2,1]:=1:B[3,3]:=1:B[4,4]:=1:B;

$$\left(\text{Done}, \text{Done}, \text{Done}, \text{Done}, \text{Done}, \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \right)$$

34 B*A;

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & -2 & 1 & 3 \\ 1 & 1 & 0 & 0 \\ -3 & 4 & 1 & 0 \end{bmatrix}$$

35
```
T:=proc(i,j,a)
local TT;
TT:=identity(4):TT[i,j]:=a:TT;
end proc;
```

// Success
// End defining T

```
proc(i,j,a)
  local TT;
  TT:=identity(4);
  TT[i,j]:=a;
  TT;
```

36

undef

37 T(3,1,1/2)*B*A;

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & -2 & 1 & 3 \\ 1 & 1 & 0 & \frac{1}{2} \\ -3 & 4 & 1 & 0 \end{bmatrix}$$

38 T(4,1,3/2)*T(3,1,1/2)*B*A;

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & -2 & 1 & 3 \\ 1 & 1 & 0 & \frac{1}{2} \\ -3 & 4 & 1 & \frac{3}{2} \end{bmatrix}$$

39 F:=matrix(4,4):F[1,1]:=1:F[3,3]:=1:F[2,4]:=1:F[4,2]:=1:F;

$$\left(\text{Done}, \text{Done}, \text{Done}, \text{Done}, \text{Done}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \right)$$

40 F*T(4,1,3/2)*T(3,1,1/2)*B*A;#On compose les transpositions

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ -3 & 4 & 1 & \frac{3}{2} \\ 1 & 1 & 0 & \frac{1}{2} \\ 0 & -2 & 1 & 3 \end{bmatrix}$$

41 LU(A,L,U,P);inv(P)*L*U-A;

$$\left(\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \right)$$

42 il faut deplacer les transpositions: On a S.T(i,j,s)=T(i',j',s).S pour i=sigma(i'),j=sigma(j')

43 U:=T(4,3,-2)*F*T(4,1,3/2)*T(3,1,1/2)*B*A;

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ -3 & 4 & 1 & \frac{3}{2} \\ 1 & 1 & 0 & \frac{1}{2} \\ -2 & -4 & 1 & 2 \end{bmatrix}$$

44 L:=(F*T(3,1,-1/2)*F^(-1))*(F*T(4,1,-3/2)*F^(-1))*T(4,3,2);

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{-3}{2} & 1 & 0 & 0 \\ \frac{-1}{2} & 0 & 1 & 0 \end{bmatrix}$$

45 S:=B*F^(-1);

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Menu

46 S*L*U-A;

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Menu

47