

```

1 restart;maple_mode(1);cas_setup(0,0,0,1,0,1e-10,10,[1,50,0,25],0,0,0); #radians,pas de cmplx, pas de Sqrt
2 -----calcul du determinant par interpolation-----
3
monlagrange:= proc (A)
local n,i,j,P,L,M;
n:=dim(A)[1];P:=0;
L:=seq(i,i=0..n);
for i from 0 to n do
M:=1;
for j in L minus [i] do M:=M*(x-j)/(i-j) od;
P:=P+det(i*identity(n)-A)*M;
od;
normal(P);
end_proc ;

proc(A)
local n,i,j,P,L,M;
n:=(dim(A))[1];
P:=0;
L:=seq(i,i=(0 .. n));
for i from 0 to n+1/2 do
M:=1;
for index from 1 to nops(L minus [i])+1 do
j:=(L minus [i])[index];
M:=M*(x-j)/(i-j);
od;;
P:=P+det(i*identity(n)-A)*M;
od;;
normal(P);
end;

4 k:=50:A:=matrix(k,k,(i,j)->rand(21)-10);
// Success
( Done, Done )

5 time(monlagrange(A));
Evaluation time: 1.04
1.04

6 time(det(A-x*idn(k)));
Evaluation time: 6.19
6.19

7

8 k:=50:A:=matrix(k,k,(i,j)->rand(21)-10);
// Success
( Done, Done )

9 pari();
All PARI functions are now defined with the pari_ prefix.
PARI functions are also defined without prefix except:
abs acos acosh arg asin asinh atan atanh binomial bitand bitor bitxor ceil charpoly concat conj content cos cosh divis
Note that p-adic numbers must have O argument quoted e.g. 905/7+O('7^3')
Type ?pari for short help
Inside xcas, try Help->Manuals->PARI for HTML help

10 time(pari_charpoly(A,0));
Evaluation time: 0.82
0.82

11 time(pari_charpoly(A,1));
Evaluation time: 0.92
0.92

12 k:=30:A:=matrix(k,k,(i,j)->(rand(21)-10)/(rand(4)^2+rand(1,3)^2));
// Success
( Done, Done )

```

```

13 time(pari_charpoly(A,0));
Evaluation time: 4.4
4.4
Menu

14 time(pari_charpoly(A,1));
Evaluation time: 4.43
4.43
Menu

15 c'est en  $O(n^4)$ , en fait  $n^4/3$ , alors que Faddeev est en  $n^4$ . Mais il y a plus
de fractions chez lagrange, ce qui peut donner l'avantage a faddeev.

16 -----Polynome minimal-----

17 v:=seq(i,i=1..8);
[ 1 2 3 4 5 6 7 8 ]
Menu

18 poly2symb(v,x);
((((((x+2) · x+3) · x+4) · x+5) · x+6) · x+7) · x+8
Menu

19 revlist(v);
[ 8 7 6 5 4 3 2 1 ]
Menu

20 P:=x^2+1;LP:=[1,0,1];
(x2+1, [ 1 0 1 ])
Menu

21 subs(x=sqrt(2),P);
√22+1
Menu

22 unapply(P,x)(sqrt(2));
3
Menu

23 peval(LP,sqrt(2));
√2 · √2+1
Menu

24 n:=8;v:=seq(i,i=1..n); A:=matrix(n,n,(i,j)-> rand(20)-10);
// Success
( Done, [ 1 2 3 4 5 6 7 8 ],


|    |     |    |    |    |    |     |    |
|----|-----|----|----|----|----|-----|----|
| -1 | -10 | -9 | -3 | 8  | -2 | 1   | 8  |
| 9  | 7   | -9 | 3  | 0  | -5 | 2   | -4 |
| -7 | -6  | 0  | -5 | 6  | -7 | 5   | 2  |
| 5  | 6   | 9  | -2 | -6 | 6  | -3  | 9  |
| -6 | -4  | 3  | 1  | 0  | 7  | -10 | -2 |
| 6  | 5   | 4  | 1  | 8  | -7 | -6  | 9  |
| -7 | 2   | 6  | 7  | -4 | -1 | -7  | 5  |
| -9 | -2  | 4  | -3 | -8 | 8  | -8  | -7 |

 )
Menu

25 B:=v; AA:=v
([ 1 2 3 4 5 6 7 8 ], [ 1 2 3 4 5 6 7 8 ])
Menu

26 for i from 1 to n do AA:=A*AA;B:=B,AA; od;
[ 1 2 3 4 5 6 7 8 ], [ 39 -40 0 93 -45 60 8 -117 ], [ -1326 534 -1382 -678 593
Menu

27 N:=nullspace(transpose([B]))[1];
[ 40445766 -22529433 -2029515 -320 7970 2469 -81 -17 -1 ]
Menu

28 Attention, les polyn{°}omes commencent a la puissance maximale, il faut inverser l'ordre

29
undef
Menu

30 P:=poly2symb(revlist(N),x);

```

31 normal(subs(x=A,P));

| | | |
|--|-----------------|--|
| | 0 0 0 0 0 0 0 0 | |
| | 0 0 0 0 0 0 0 0 | |
| | 0 0 0 0 0 0 0 0 | |
| | 0 0 0 0 0 0 0 0 | |
| | 0 0 0 0 0 0 0 0 | |
| | 0 0 0 0 0 0 0 0 | |
| | 0 0 0 0 0 0 0 0 | |
| | 0 0 0 0 0 0 0 0 | |
| | 0 0 0 0 0 0 0 0 | |
| | 0 0 0 0 0 0 0 0 | |

32 A:=diag(companion(x^4-1,x),companion((x^2-1)^2,x));

| | | |
|--|------------------|--|
| | 0 0 0 1 0 0 0 0 | |
| | 1 0 0 0 0 0 0 0 | |
| | 0 1 0 0 0 0 0 0 | |
| | 0 0 1 0 0 0 0 0 | |
| | 0 0 0 0 0 0 0 -1 | |
| | 0 0 0 0 1 0 0 0 | |
| | 0 0 0 0 0 1 0 2 | |
| | 0 0 0 0 0 0 1 0 | |

33 B:=v; AA:=v

([1 2 3 4 5 6 7 8], [1 2 3 4 5 6 7 8])

34 for i from 1 to n do AA:=A*AA;B:=B,AA; od;

[1 2 3 4 5 6 7 8], [4 1 2 3 -8 5 22 7], [3 4 1 2 -7 -8 19 22], [2 3 4

35 nullspace(B);

| | | |
|--|---|--|
| | $\frac{9}{5}$ $\frac{4}{5}$ $\frac{9}{5}$ $\frac{4}{5}$ -1 0 -1 0 | |
| | $\frac{4}{5}$ $\frac{9}{5}$ $\frac{4}{5}$ $\frac{9}{5}$ 0 -1 0 -1 | |

36 N:=nullspace(transpose([B]))[1];

| | | |
|--|---|--|
| | 1 1 1 1 $-\frac{43}{5}$ $-\frac{25}{3}$ $\frac{55}{7}$ 8 -1 | |
|--|---|--|

37 P:=poly2symb(revlist(N),x);

((((((- x + 8) · x + 55) · x + -25) · x + -43) · x + 1) · x + 1) · x + 1

38 normal(subs(x=A,P));

| | | | | | | | | |
|--|-----------------|-----------------|-----------------|-----------------|-------------------|-------------------|-------------------|-------------------|
| | $\frac{-43}{5}$ | 9 | $\frac{62}{7}$ | $\frac{-22}{3}$ | 0 | 0 | 0 | 0 |
| | $\frac{-22}{3}$ | $\frac{-43}{5}$ | 9 | $\frac{62}{7}$ | 0 | 0 | 0 | 0 |
| | $\frac{62}{7}$ | $\frac{-22}{3}$ | $\frac{-43}{5}$ | 9 | 0 | 0 | 0 | 0 |
| | 9 | $\frac{62}{7}$ | $\frac{-22}{3}$ | $\frac{-43}{5}$ | 0 | 0 | 0 | 0 |
| | 0 | 0 | 0 | 0 | $\frac{-109}{35}$ | $\frac{-25}{3}$ | $\frac{-118}{35}$ | -10 |
| | 0 | 0 | 0 | 0 | $\frac{-20}{3}$ | $\frac{-109}{35}$ | $\frac{-25}{3}$ | $\frac{-118}{35}$ |
| | 0 | 0 | 0 | 0 | $\frac{118}{35}$ | 10 | $\frac{127}{35}$ | $\frac{35}{3}$ |
| | 0 | 0 | 0 | 0 | $\frac{25}{3}$ | $\frac{118}{35}$ | 10 | $\frac{127}{35}$ |

39 On a calculé les $A^k(v)$ par récurrence. AA est un vecteur donc A^*AA a un coût de n^2 , et l'on fait n tours, donc c'est en $O(n^3)$

40 Il existe toujours un vecteur v tel que $P_-(u,v)=p_{\min}(u)$, les vecteurs à éviter sont dans $\ker(P(u))$ ou P divise p_{\min} , donc nom fini despace vectoriels à éviter.