```
1 restart;maple_mode(1);cas_setup(0,0,0,1,0,1e-10,10,[1,50,0,25],0,0,0); #radians,pas de cmplx, pas de  Sqrt
```

Warning: some commands like subs might change arguments order 0 0 0 1 0 1e-10 10 [1 50 0 25] 0 0

```
2 ---------------------------Exercice--------------------------------------------
```

```
3 P:=x^4+x+1;
```

$$x^4+x+1$$

```
4 A:=1/d*add(a[i]*x^i,i=0..degree(P)-1);
```

$$\left(\frac{1}{d}\right)\cdot\left(a[0]+(a[1])\cdot x+(a[2])\cdot x^2+(a[3])\cdot x^3\right)$$

```
5 H:=d*A;
```

$$d\cdot\left(\frac{1}{d}\right)\cdot\left(a[0]+(a[1])\cdot x+(a[2])\cdot x^2+(a[3])\cdot x^3\right)$$

```
6 M:=matrix([seq([seq(coeff(rem(A*x^i,P,x),x,j),i=0..degree(P)-1)],j=0..degree(P)-1)]);
```

$$\begin{bmatrix} \frac{a[0]}{d} & \frac{-(a[3])}{d} & \frac{-(a[2])}{d} & \frac{-(a[1])}{d} \\ \frac{a[1]}{d} & \frac{a[0]-(a[3])}{d} & \frac{-(a[3])-(a[2])}{d} & \frac{-(a[2])-(a[1])}{d} \\ \frac{a[2]}{d} & \frac{a[1]}{d} & \frac{a[0]-(a[3])}{d} & \frac{-(a[3])-(a[2])}{d} \\ \frac{a[3]}{d} & \frac{a[2]}{d} & \frac{a[1]}{d} & \frac{a[0]-(a[3])}{d} \end{bmatrix}$$

```
7 M:=matrix(4,4,(i,j)->coeff(rem(A*x^(j-1),P),x,i-1)); #c'est plus simple
```

// Warning: A x P declared as global variable(s)

$$\begin{bmatrix} \frac{a[0]}{d} & \frac{-(a[3])}{d} & \frac{-(a[2])}{d} & \frac{-(a[1])}{d} \\ \frac{a[1]}{d} & \frac{a[0]-(a[3])}{d} & \frac{-(a[3])-(a[2])}{d} & \frac{-(a[2])-(a[1])}{d} \\ \frac{a[2]}{d} & \frac{a[1]}{d} & \frac{a[0]-(a[3])}{d} & \frac{-(a[3])-(a[2])}{d} \\ \frac{a[3]}{d} & \frac{a[2]}{d} & \frac{a[1]}{d} & \frac{a[0]-(a[3])}{d} \end{bmatrix}$$

```
8 cp:=charpoly(M,x):
```

Done

```
9 res:=resultant(subs(x=y,P),d*x-subs(x=y,H),y): #attention re est un mot reserve
```

Done

```
10 le poly caract est 1/d^(deg P) * resultant: verification:
```

```
11 normal(d^(degree(P))*cp-res); #ils sont bien egaux
```

0

```
12 P:=x^4+1;
```

$$x^4+1$$

```
13 A:=x^2:H:=A;
```

$$(\text{Done},\ x^2)$$

```
14 M:=matrix([seq([seq(coeff(rem(A*x^i,P,x),x,j),i=0..degree(P)-1)],j=0..degree(P)-1)]);
```

$$\begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

```
15 cp:=charpoly(M,x):
```

Done

```
16 res:=resultant(subs(x=y,P),x-subs(x=y,H),y):
```

Done

```
17 le poly caract est 1/d^(deg P) * resultant: verification:
```

```
18 expand(cp-res);
```

19 le poly min est une puissance de:

```
20 gcd(res,diff(res,x));
```

$$x^2+1$$

```
21 pmin(M,x);#ils sont egaux.
```

$$x^2+1$$

22 --Exercice-----------------------------------

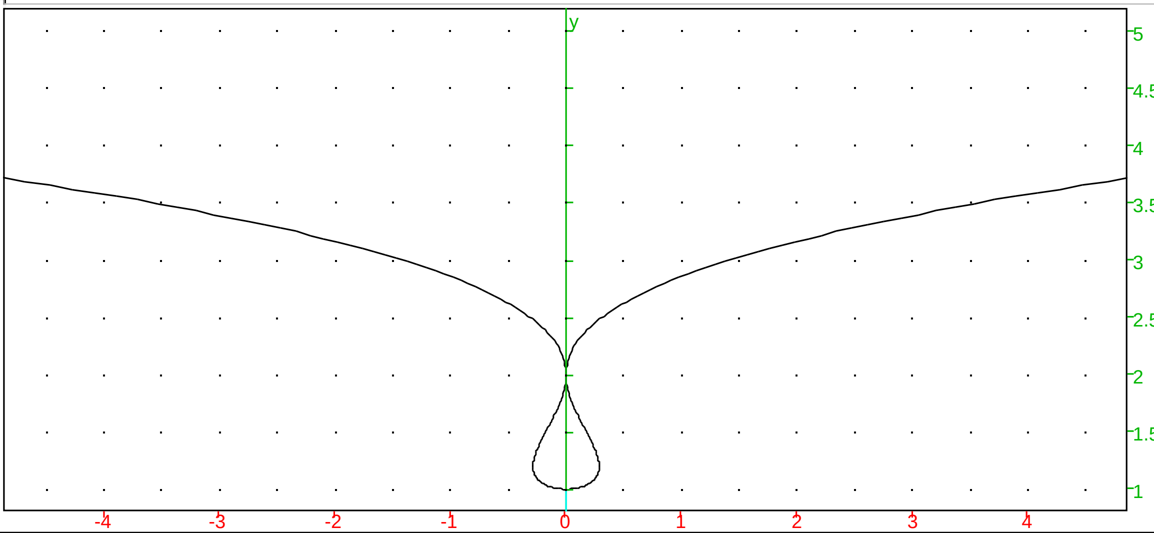
```
23 XT:=t*(t^2-1)^2;YT:=t^2+1;
```

$$(t\cdot(t^2-1)^2,\ t^2+1)$$

```
24 eq:=resultant(XT-x,YT-y,t);
```

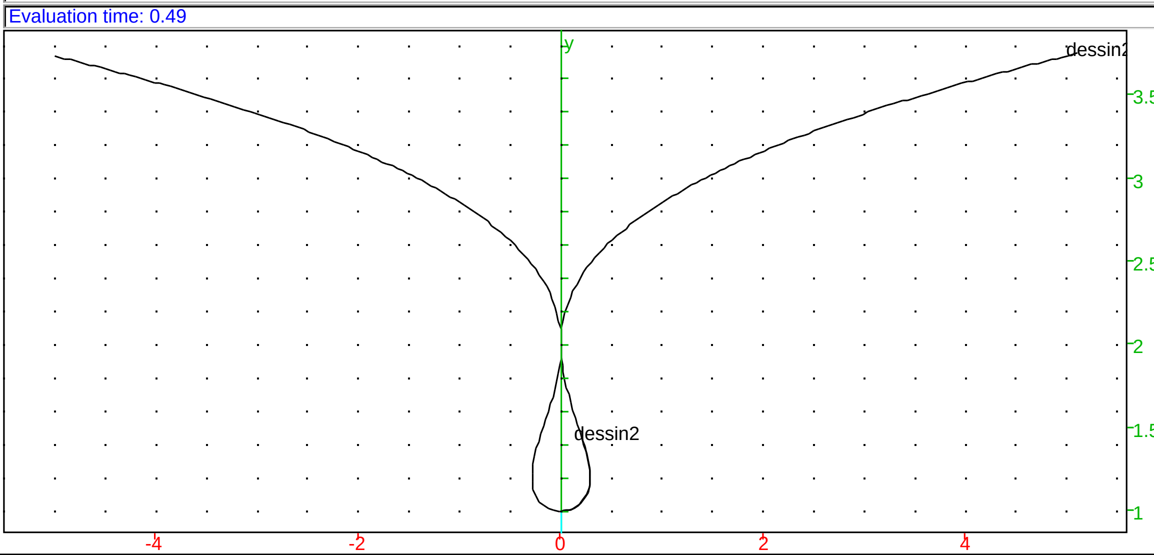
$$x^2-y^5+9\cdot y^4+(-32)\cdot y^3+56\cdot y^2+(-48)\cdot y+16$$

```
25 dessin1:=plotparam(XT+I*YT,t=-2..2);
```




```
26 dessin2:=implicitplot(eq=0,x=-5..5,y=-5..5,xstep=0.01,ystep=0.01);
```

Evaluation time: 0.49



27 ---------------------------------------Exercice---------------------------------------

```
28 C1:=x*y-4;C2:=y^2-(x-3)*(x^2-16);
```

$$(x\cdot y-4,\ y^2-(x-3)\cdot(x^2-16))$$

```
29 d1:=implicitplot(C1,x=-7..7,y=-8..8,color=red+line_width_2):
```

Hyperbola of center (0,0)

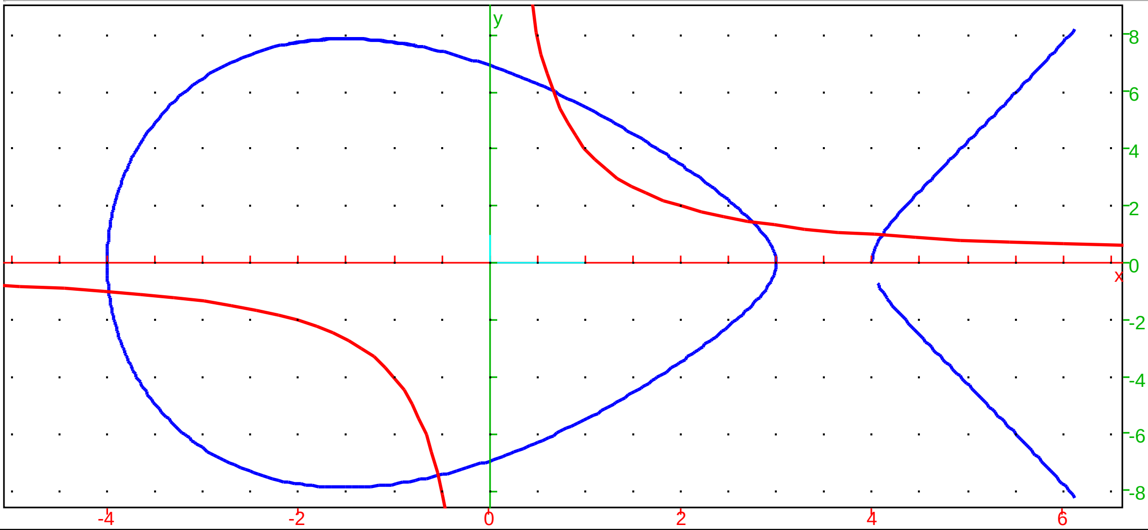
Done

30 d2:=implicitplot(C2,x=-7..7,y=-8..8,color=blue+line_width_2,xstep=0.01,ystep=0.01):

Evaluation time: 0.49

Done

31 (d2,d1);



32 eqx:=resultant(C1,C2,y); #On elimine y

$- x^5 - (-3) \cdot x^4 - (-16) \cdot x^3 - 48 \cdot x^2 + 16$

33 Astuce: pour ne pas passer en flottants via le menu deroulant, on lui donne une extention de corps sous forme d'un flottant.

34 factor(eqx,1.1); # on obtient 5 points reels.

$-1.0 \cdot (x - 4.105876927) \cdot (x - 2.749235551) \cdot (x - 0.6633796231) \cdot (x + 0.5365997443) \cdot (x + 3.981892356)$

35 c'est de degre 5 car le centre de projection (0,1,0) est solution.

36 resultant(subs(y=-2*x,C1),subs(y=-2*x,C2),x); #il n'est pas nul

-4492

37 NB: A=(1,1) n'est pas sur la droite y=2x. Bt=(t,-2t) eq param de la droite (ABt): x=1+l(t-1),y:=1+l(-2t-1)

38 C1t:=subs(x=1+l*(t-1),y=1+l*(-2*t-1),C1);

$(1 + l \cdot (t - 1)) \cdot (1 + l \cdot (-2 \cdot t - 1)) - 4$

39 C2t:=subs(x=1+l*(t-1),y=1+l*(-2*t-1),C2);

$(1 + l \cdot (-2 \cdot t - 1))^2 - (1 + l \cdot (t - 1) - 3) \cdot ((1 + l \cdot (t - 1))^2 - 16)$

40 la projection est donn'ee par les points de coordonn'ee (t,-2t) o'u t racine de:

41 p2:=resultant(C1t,C2t,l);

$- 14699 \cdot t^6 - (-25716) \cdot t^5 - (-1779) \cdot t^4 - 15116 \cdot t^3 - 624 \cdot t^2 - (-2496) \cdot t + 448$

42 car c'est l'intersection de y=2x avec la droite passant par (1,1) et le point a l'infini de Oy qui etait bien dans C1 inter C2

43 sol:=solve(p2*1.0,t);

Warning! Algebraic extension not implemented yet for poly [14699,-11017,-12796,2320,2944,448]

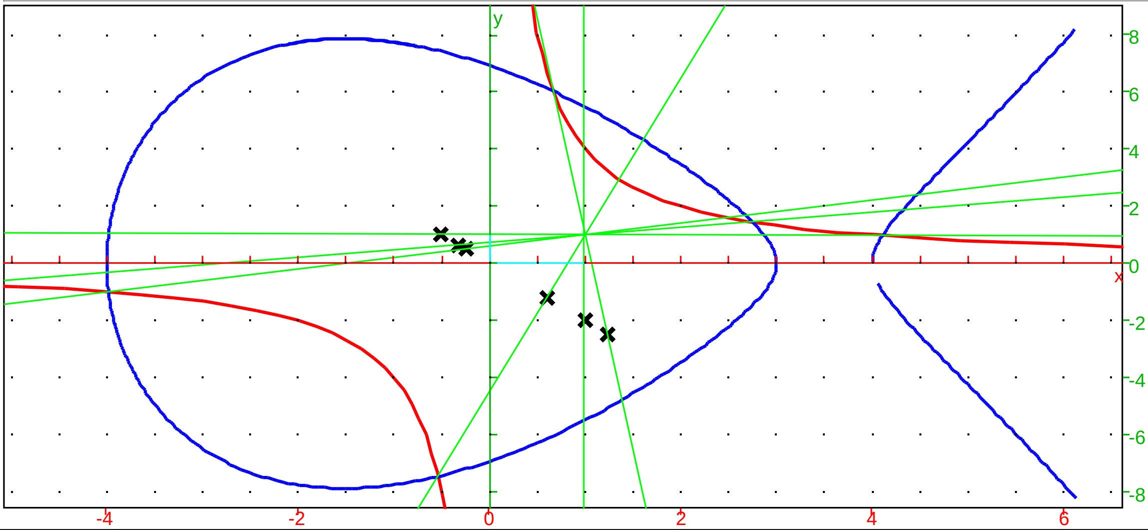
(1, 0.6001057333 , -0.2487685675 , -0.3273837129 , -0.5062528649 , 1.231806181)

44 pi1:=seq(point(sol[i]-2*I*sol[i],color=black+point_width_3),i=1..6): #les points sur la droite y=-2x

Done

45 di1:=seq((droite(1+I,pi1[i],color=green)),i=1..6): # les 6 droites:

46 (d1,d2,pi1,di1); #NB un point d'intersection de C1 et C2 est a l'infini.



47 C1:=(x-2)^2+y^2-4;

$$(x-2)^2+y^2-4$$

48 C2:=y^2-(x-3)*(x^2-16);

$$y^2-(x-3)\cdot(x^2-16)$$

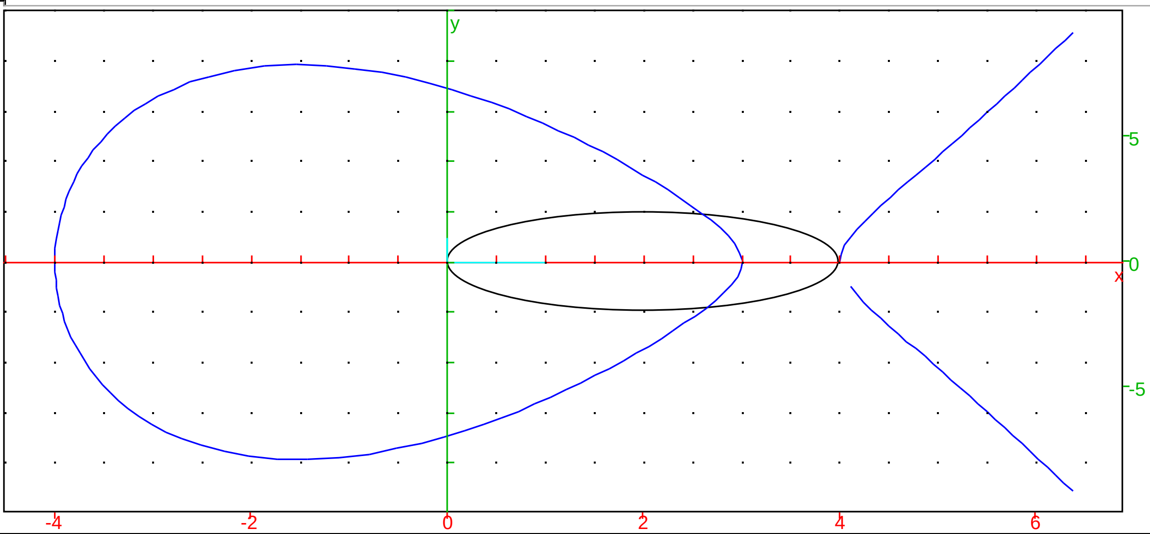
49 d1:=implicitplot(C1,x=-1..5,y=-3..3,color=red):

Done

50 d2:=implicitplot(C2,x=-5..8,y=-8..8,color=blue):

Done

51 (d1,d2);



52 eqx:=resultant(C1,C2,y); #On elimine y

$$x^6+(-4)\cdot x^5+(-36)\cdot x^4+176\cdot x^3+208\cdot x^2+(-1920)\cdot x+2304$$

53 factor(eqx,1.1); # on obtient 5 points reels.

$$(x-4.00000003)\cdot(x-3.99999997)\cdot(x-2.605551347)\cdot(x-2.605551204)\cdot(x+4.605551263)\cdot(x+4.605551288)$$

54 factor(eqx); #On voit qu'il faut introduire le discriminant.

$$(x-4)^2\cdot(x^2+2\cdot x-12)^2$$

55 factor(eqx,sqrt(13));

$$(x-4)^2\cdot(x+-\sqrt{13}+1)^2\cdot(x+\sqrt{13}+1)^2$$

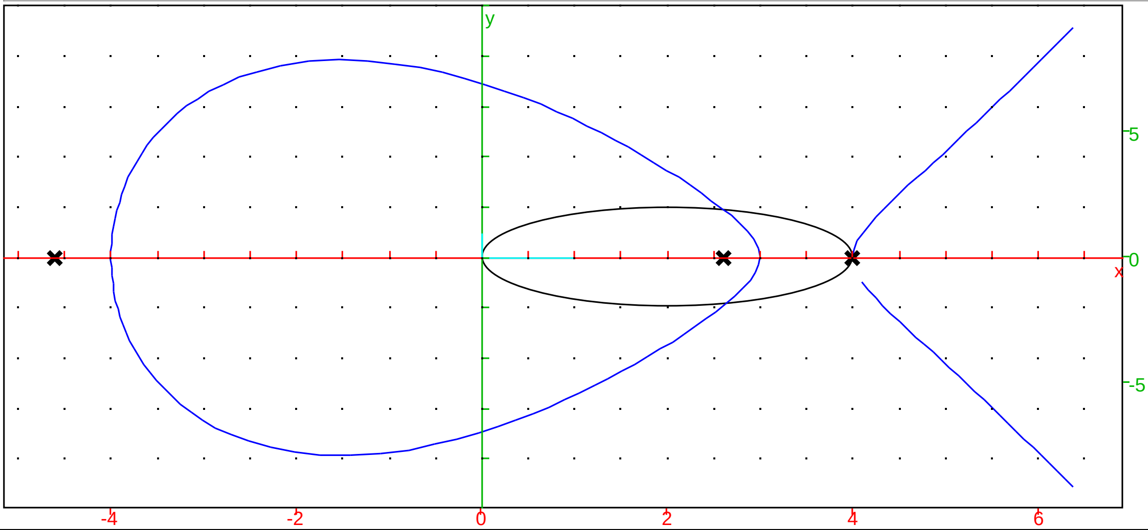
56 eqred:=gcd(eqx,diff(eqx,x));

57 rac:=[solve(approx(eqred),x)];

$[2.605551275 \quad 4 \quad -4.605551275]$

58 On constate qu'il y a un point qui ne semble pas correspondre a une projection
d'un point d'instersection. Nous allons l'interpr'eter maintenant.

59 (d1,d2,seq(point(i,couleur=black+point_width_3),i=rac));



60 solx:=[solve(eqred,x)];

$[4 \quad \sqrt{13}-1 \quad -\sqrt{13}-1]$

61 soly:=seq(gcd(subs(x=solx[i],C1),subs(x=solx[i],C2)),i=1..3);

$(y^2, \; y^2+-6\cdot\sqrt{13}+18, \; y^2+6\cdot\sqrt{13}+18)$

62 les ordonn'ees des points d'abscisse solx[i] sont:

63 seq(cSolve(soly[i],y),i=1..3);

$((0, \; 0), (\sqrt{6\cdot\sqrt{13}-18}, \; -\sqrt{6\cdot\sqrt{13}-18}), (i\cdot\sqrt{6\cdot\sqrt{13}+18}, \; (-i)\cdot\sqrt{6\cdot\sqrt{13}+18}))$

64 On constate que pour solx[3] les ordonn'ees des points de C1 inter
C2 ayant cette abscisse sont complexes conjuguees bien que solx[3]
soit reel ce qui explique pourquoi le dessin ne nous donnait que 4 points.

65 ---Exercice-----------------------------------

66 G:=t^2*(x^2+y^2-1)+t*(x*y+y^2-x^2)+(x^2+2*y^2-1)

$t^2\cdot(x^2+y^2-1)+t\cdot(x\cdot y+y^2-x^2)+x^2+2\cdot y^2-1$

67 dG:=diff(G,t)

$2\cdot t\cdot(x^2+y^2-1)+x\cdot y+y^2-x^2$

68 eq:=resultant(G,dG,t)

69 factor(eq);

$(x^2+y^2-1)\cdot(3\cdot x^4+2\cdot x^3\cdot y+13\cdot x^2\cdot y^2+(-8)\cdot x^2+(-2)\cdot x\cdot y^3+7\cdot y^4+(-12)\cdot y^2+4)$

70 c'est divisible par le coefficient de t^2 dans G! En effet, on a calcule un
resultant entre un polyn^ome de degre 2 et un autre de degre 1. Mais en un point
ou x^2+y^2=1, le degre de ces polynomes chute de 1, et l'on aurait donc du
prendre pour un tel point une formule de resultant pour degre inferieur. On a
donc rajoute des solution a notre probleme. l'equation de l'enveloppe est donc:

71 eqenv:=normal(eq/(coeff(G,t^2)))

$3\cdot x^4+2\cdot x^3\cdot y+13\cdot x^2\cdot y^2+(-8)\cdot x^2+(-2)\cdot x\cdot y^3+7\cdot y^4+(-12)\cdot y^2+4$

72 Gt:=unapply(G,t)

$t \;->t^2\cdot(x^2+y^2-1)+t\cdot(x\cdot y+y^2-x^2)+x^2+2\cdot y^2-1$

73 dGt:=unapply(diff(G,t),t)

74 envel:=implicitplot(eqenv,x=-7..7,y=-7..7,xstep=0.01,ystep=0.01,affichage=blue+line_width_3):

Evaluation time: 0.73

Done

75

76 Fig Edit Graphe Frame Mode

<Save figure as text>

1 envel;

Done

2 u:=element(-5..5,0);

parameter(u,-5,5,0,0.1)

3 C1:=implicitplot(Gt(u),x=-7..7,y=-7..7,couleur=red+line_widt

Ellipsis of center (0,0)

plotparam(cos(t)+((I)*sqrt(2)*sin(t))/2,t=0.0..6.283185307)

4