1
```
restart;maple_mode(0);cas_setup(0,0,0,1,0,1e-10,10,[1,50,0,25],0,0,0); //radians,pas de cmplx, pas de  Sqrt
```

```
Cas_setup vector [0,0,0,1,0,1.0000000000e-10,10,[1,50,0,25],0,0,0]
^M
Warning: some commands like subs might change arguments order
```

2 astuces, a retenir: sous xcas on utilise unapply qui est en general plus sur que la methode suivante avec subs.

3
```
f:=x^2+1; g:=unapply(f,x);expand(g(x+2));
```

$$(x^2+1,\ x\ \to x^2+1,\ x^2+4\cdot x+5)$$

4
```
g:=t->subst(f,x=t);g(x+2);
```

```
// Warning: f,x, declared as global variable(s)
// End defining g
```

$$(t\ \to \mathrm{subst}(f,x=t),\ (x+2)^2+1)$$

5
```
f:=x->x^2-2;
```

```
// Success
// End defining f
```

$$x\ \to x^2-2$$

6
```
df:=x->eval(diff(f(x),x));
```

```
// End defining df
```

$$x\ \to \mathrm{eval}(\mathrm{diff}(f(x),x))$$

7
```
df(x);u:=x^2;df(u);
```

$$(2\cdot x,\ x^2,\ 0)$$

8 ca ne marche pas toujours, mieux vaut utiliser unapply, Ou mieux, deriver une fonction

9
```
df:=unapply(diff(f(x),x),x);df(x);df(u);
```

$$(x\ \to 2\cdot x,\ 2\cdot x,\ 2\cdot x^2)$$

10
```
D(g);fonction_diff(g);D(g)(5);
```

$$(D(t\ \to \mathrm{subst}(f,x=t)),\ \mathrm{fonction\_diff}(t\ \to \mathrm{subst}(f,x=t)),\ D(t\ \to \mathrm{subst}(f,x=t))(5))$$

11 pour changer la valeur par defaut, soit dans le menu, soit avec la variable Digits, mais c'est pris en compte a partir de l'evaluation suivante. Ex: Digits:=100;sqrt(2.0); sur une meme ligne ne marche pas.

12
```
Digits:=1;
```

$$[0,\ 0,\ 0,\ 1,\ 0,\ [1e\text{-}10,\ 1e\text{-}15],\ 1,\ [1,\ 50,\ 0,\ 25],\ 0,\ 0,\ 0]$$

13
```
13*2.;evalf(13*2,3);
```

$$(26.0,\ 26.0)$$

14
```
Digits:=2;
```

$$[0,\ 0,\ 0,\ 1,\ 0,\ [1e\text{-}10,\ 1e\text{-}15],\ 2,\ [1,\ 50,\ 0,\ 25],\ 0,\ 0,\ 0]$$

15
```
13.26*2;
```

$$26.52$$

16
```
Digits:=10000;
```

$$[0,\ 0,\ 0,\ 1,\ 0,\ [1e\text{-}10,\ 1e\text{-}15],\ 10000,\ [1,\ 50,\ 0,\ 25],\ 0,\ 0,\ 0]$$

17
```
U:=1.:;
```

Done

18 On fait 14 iterations de la methode de Newton, et on affiche la difference avec racine de 2. La majoration du reste montre qu'asymptotiquement le nombre de decimales exactes est multiplie par 2 a chaque iteration. Ce qui correspond bien a une majoration du type: |u_{n+1}-l|<C|u_n-l|^2

19
```
normal(x-f(x)/df(x));//On trouve x/2 +1/x c'est donc Heron.
```

$$x^2+2$$

20 r:=evalf(sqrt(2));//on ne le calcule qu'une fois.

1.4142135623730950488016887242096980785696718753769480731766797379907324784621070388503875343276415727350138 4

21 for j from 1 to 14 do U:=U/2+1/U:; a:=U-r:; print(evalf(a,15)) od:;

```
0.8578643762690497e-1
0.2453104293571619e-2
0.2123901414755118e-5
0.1594861824606854e-11
0.8992928321650457e-24
0.2859283843333951e-48
0.2890477193215366e-97
0.2953888516837038e-195
0.3084915037605822e-391
0.3364661831299793e-783
0.4002559988184799e-1567
0.5664097306539410e-3135
0.1134269927524680e-6270
0.0000000000000000
```

Done

22 Digits:=10;

[0, 0, 0, 1, 0, [1e-10 , 1e-15], 10 , [1, 50 , 0, 25], 0, 0, 0]

23 ----------Jordan, Dunford----------

24 La decomposition d'une matrice reelle est reelle: par conjugaison et unicite
de la decomp de Dunford

25 C1:=companion((X^3-8)^2,X);

$$\begin{bmatrix} 0, & 0, & 0, & 0, & 0, & -64 \\ 1, & 0, & 0, & 0, & 0, & 0 \\ 0, & 1, & 0, & 0, & 0, & 0 \\ 0, & 0, & 1, & 0, & 0, & 16 \\ 0, & 0, & 0, & 1, & 0, & 0 \end{bmatrix}$$

26 C2:=companion((X^2-4)^3,X);

$$\begin{bmatrix} 0, & 0, & 0, & 0, & 0, & 64 \\ 1, & 0, & 0, & 0, & 0, & 0 \\ 0, & 1, & 0, & 0, & 0, & -48 \\ 0, & 0, & 1, & 0, & 0, & 0 \\ 0, & 0, & 0, & 1, & 0, & 12 \\ 0, & 0, & 0, & 0, & 1, & 0 \end{bmatrix}$$

27 A:=diag(C1,C2);

$$\begin{bmatrix} 0, & 0, & 0, & 0, & 0, & -64, & 0, & 0, & 0, & 0, & 0, & 0 \\ 1, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 1, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 1, & 0, & 0, & 16, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 1, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 1, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 64 \\ 0, & 0, & 0, & 0, & 0, & 0, & 1, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & 1, & 0, & 0, & 0, & -48 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 1, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 1, & 0, & 12 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 1, & 0 \end{bmatrix}$$

28 cas_setup(0,0,1,1,0,1e-10,10,[1,50,0,25],0,1,0); //mode complexe est racines carrees.

```
( 0, 0, 1, 1, 0, 1e-10 , 10 , [1, 50 , 0, 25 ], 0, 1, 0 )
```

29 rat_jordan(A);

$$\begin{bmatrix} 0, & 0, & 0, & -128, & -96, & 0, & 0, & 0, & 6912, & -13824, & 5760, & 11520 \\ 0, & 0, & 0, & -64, & -16, & 0, & 0, & 0, & 0, & 6912, & -4032, & 5760 \\ 0, & 0, & 0, & -32, & 8, & 0, & 0, & 0, & -1728, & 0, & -288, & -2304 \\ 0, & 0, & 0, & 16, & 36, & 0, & 0, & 0, & -864, & 1728, & -720, & -4032 \\ 0, & 0, & 0, & 8, & 14, & 0, & 0, & 0, & 0, & -864, & 1152, & -720 \\ 0, & 0, & 0, & 4, & 5, & 0, & 0, & 0, & 216, & 0, & -288, & 936 \\ 96, & 144, & 88, & 0, & 0, & 512, & -768, & 672, & 0, & 0, & 0, & 0 \\ 48, & 48, & 20, & 0, & 0, & -256, & 256, & -208, & 0, & 0, & 0, & 0 \\ -48, & -96, & -62, & 0, & 0, & -256, & 512, & -432, & 0, & 0, & 0, & 0 \\ -24, & -36, & -13, & 0, & 0, & 128, & -192, & 120, & 0, & 0, & 0, & 0 \\ 6, & 15, & 16, & 0, & 0, & 32, & -80, & 98, & 0, & 0, & 0, & 0 \\ 3, & 6, & 5, & 0, & 0, & -16, & 32, & -33, & 0, & 0, & 0, & 0 \end{bmatrix}, \begin{bmatrix} 2, & 1, & 0, & 0, & 0, & 0, & 0, & 0, & 0, \\ 0, & 2, & 1, & 0, & 0, & 0, & 0, & 0, & 0, \\ 0, & 0, & 2, & 0, & 0, & 0, & 0, & 0, & 0, \\ 0, & 0, & 0, & 2, & 1, & 0, & 0, & 0, & 0, \\ 0, & 0, & 0, & 0, & 2, & 0, & 0, & 0, & 0, \\ 0, & 0, & 0, & 0, & 0, & -2, & 1, & 0, & 0, \\ 0, & 0, & 0, & 0, & 0, & 0, & -2, & 1, & 0, \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & -2, & 0, \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 1, \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, \end{bmatrix}$$

30 rat_jordan(companion((x^3-2)^2,x));

$$\left(\begin{bmatrix} -8, & 0, & 0, & 0, & 8, & 0 \\ 0, & -8, & 0, & 0, & 0, & 16 \\ 0, & 0, & -8, & 12, & 0, & 0 \\ 4, & 0, & 0, & 0, & 8, & 0 \\ 0, & 4, & 0, & 0, & 0, & 4 \\ 0, & 0, & 4, & 0, & 0, & 0 \end{bmatrix}, \begin{bmatrix} 0, & 0, & 2, & 1, & 0, & 0 \\ 1, & 0, & 0, & 0, & 1, & 0 \\ 0, & 1, & 0, & 0, & 0, & 1 \\ 0, & 0, & 0, & 0, & 0, & 2 \\ 0, & 0, & 0, & 1, & 0, & 0 \\ 0, & 0, & 0, & 0, & 1, & 0 \end{bmatrix} \right)$$

31 J:=jordan(A,'Q');Q;

$$\begin{bmatrix} 2, & 1, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 2, & 1, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 2, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 2, & 1, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 2, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & -2, & 1, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & -2, & 1, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & -2, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\sqrt{3}\cdot i-1, & 1, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\sqrt{3}\cdot i-1, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \sqrt{3}\cdot i-1, & 1 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \sqrt{3}\cdot i-1 \end{bmatrix}, \begin{bmatrix} 0, & 0, & 0, & -128, & -96, & 0, & 0, & 0, \\ 0, & 0, & 0, & -64, & -16, & 0, & 0, & 0, \\ 0, & 0, & 0, & -32, & 8, & 0, & 0, & 0, \\ 0, & 0, & 0, & 16, & 36, & 0, & 0, & 0, \\ 0, & 0, & 0, & 8, & 14, & 0, & 0, & 0, \\ 0, & 0, & 0, & 4, & 5, & 0, & 0, & 0, \\ 96, & 144, & 88, & 0, & 0, & 512, & -768, & 672 \\ 48, & 48, & 20, & 0, & 0, & -256, & 256, & -208 \\ -48, & -96, & -62, & 0, & 0, & -256, & 512, & -432 \\ -24, & -36, & -13, & 0, & 0, & 128, & -192, & 120 \\ 6, & 15, & 16, & 0, & 0, & 32, & -80, & 98, \\ 3, & 6, & 5, & 0, & 0, & -16, & 32, & -33 \end{bmatrix}$$

32 f:=(i0,j0)->if i0=j0 then J[i0,j0] else 0 fi;

```
// Warning: 5, declared as global variable(s)
// End defining f
```

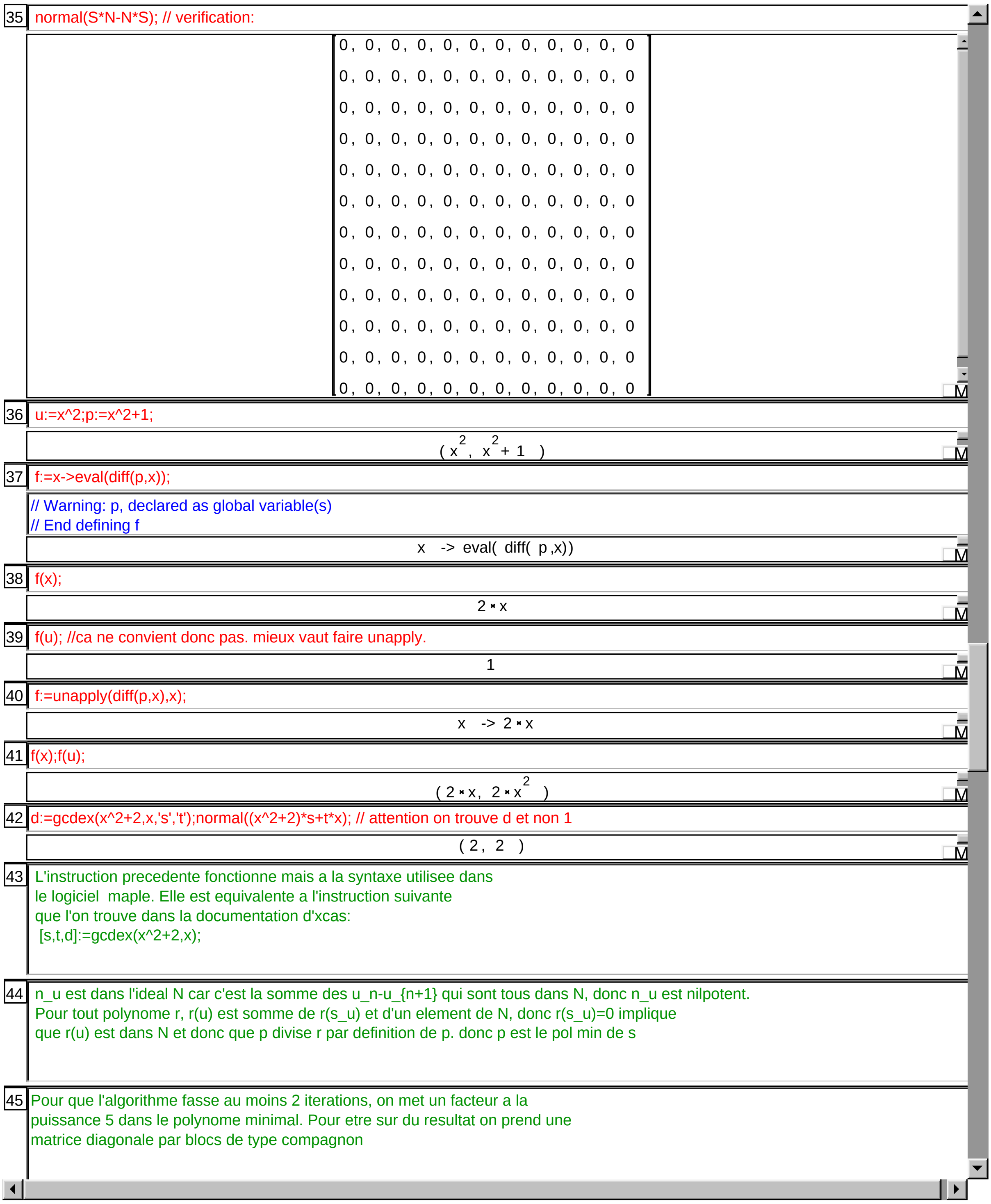
```
if ((i0==j0)) {
  J[i0,j0];
}
else {
  0;
```

33 dia:=matrix(op(dim(J)),f);

$$\begin{bmatrix} 2, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 2, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 2, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 2, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 2, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & -2, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & -2, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & -2, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\sqrt{3}\cdot i-1, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\sqrt{3}\cdot i-1, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \sqrt{3}\cdot i-1, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \sqrt{3}\cdot i-1 \end{bmatrix}$$

34 S:=normal(Q*dia*Q^(-1));N:=A-S; //on remarque que les coeff sont dans le meme corps que ceux de A

$$\begin{bmatrix} 0, & 0, & \frac{8}{3}, & 0, & 0, & \frac{-128}{3}, & 0, & 0, & 0, & 0, & 0, & 0 \\ \frac{4}{3}, & 0, & 0, & \frac{8}{3}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & \frac{4}{3}, & 0, & 0, & \frac{8}{3}, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & \frac{2}{3}, & 0, & 0, & \frac{40}{3}, & 0, & 0, & 0, & 0, & 0, & 0 \\ \frac{-1}{24}, & 0, & 0, & \frac{2}{3}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & \frac{-1}{24}, & 0, & 0, & \frac{2}{3}, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{3}{2}, & 0, & -2, & 0, & 24 \\ 0, & 0, & 0, & 0, & 0, & 0, & \frac{15}{8}, & 0, & \frac{3}{2}, & 0, & -2, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{3}{4}, & 0, & 3, & 0, & -20 \\ 0, & 0, & 0, & 0, & 0, & 0, & \frac{-5}{16}, & 0, & \frac{3}{4}, & 0, & 3, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{-1}{32}, & 0, & \frac{3}{8}, & 0, & \frac{15}{2} \\ 0, & 0, & 0, & 0, & 0, & 0, & \frac{3}{128}, & 0, & \frac{-1}{32}, & 0, & \frac{3}{8}, & 0 \end{bmatrix}, \begin{bmatrix} 0, & 0, & \frac{-8}{3}, & 0, & 0, & \frac{-64}{3}, & 0, & 0, & 0, & 0, & 0, & 0 \\ \frac{-1}{3}, & 0, & 0, & \frac{-8}{3}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & \frac{-1}{3}, & 0, & 0, & \frac{-8}{3}, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & \frac{1}{3}, & 0, & 0, & \frac{8}{3}, & 0, & 0, & 0, & 0, & 0, & 0 \\ \frac{1}{24}, & 0, & 0, & \frac{1}{3}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & \frac{1}{24}, & 0, & 0, & \frac{1}{3}, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{-3}{2}, & 0, & 2, & 0, & 40 \\ 0, & 0, & 0, & 0, & 0, & 0, & \frac{-7}{8}, & 0, & \frac{-3}{2}, & 0, & 2, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{4}, & 0, & -3, & 0, & -28 \\ 0, & 0, & 0, & 0, & 0, & 0, & \frac{5}{16}, & 0, & \frac{1}{4}, & 0, & -3, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{32}, & 0, & \frac{5}{8}, & 0, & \frac{9}{2} \\ 0, & 0, & 0, & 0, & 0, & 0, & \frac{-3}{128}, & 0, & \frac{1}{32}, & 0, & \frac{5}{8}, & 0 \end{bmatrix}$$



35
```
normal(S*N-N*S); // verification:
```

```
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
```

36
```
u:=x^2;p:=x^2+1;
```

$$(x^2, x^2+1)$$

37
```
f:=x->eval(diff(p,x));
```

```
// Warning: p, declared as global variable(s)
// End defining f
```

```
x -> eval( diff( p ,x))
```

38
```
f(x);
```

```
2 * x
```

39
```
f(u); //ca ne convient donc pas. mieux vaut faire unapply.
```

```
1
```

40
```
f:=unapply(diff(p,x),x);
```

```
x -> 2 * x
```

41
```
f(x);f(u);
```

$$(2\cdot x, 2\cdot x^2)$$

42
```
d:=gcdex(x^2+2,x,'s','t');normal((x^2+2)*s+t*x); // attention on trouve d et non 1
```

```
( 2, 2 )
```

43 L'instruction precedente fonctionne mais a la syntaxe utilisee dans
le logiciel maple. Elle est equivalente a l'instruction suivante
que l'on trouve dans la documentation d'xcas:
[s,t,d]:=gcdex(x^2+2,x);

44 n_u est dans l'ideal N car c'est la somme des u_n-u_{n+1} qui sont tous dans N, donc n_u est nilpotent.
Pour tout polynome r, r(u) est somme de r(s_u) et d'un element de N, donc r(s_u)=0 implique
que r(u) est dans N et donc que p divise r par definition de p. donc p est le pol min de s

45 Pour que l'algorithme fasse au moins 2 iterations, on met un facteur a la
puissance 5 dans le polynome minimal. Pour etre sur du resultat on prend une
matrice diagonale par blocs de type compagnon

46 A:=diag([companion((x^3-2)^5,x),companion(x^5-2*x^2,x)]);

$$\begin{bmatrix}
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 32, & 0, & 0, & 0, & 0, & 0 \\
1, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 1, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 1, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -80, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 1, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 1, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 1, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 80, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 1, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 1, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 1, & 0, & 0, & 0, & 0, & 0, & -40, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 1, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 1, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 1, & 0, & 0, & 10, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 1, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 1, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 1, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 1, & 0, & 0, & 2 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 1, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 1, & 0
\end{bmatrix}$$

47

48 Prog Edit Ajouter | 1 | nxt | OK (F9) | Save

```
Dunford:=proc(A)
m:=pcar(A,x,lagrange);
p:=unapply(normal(m/(gcd(m,diff(m,x)))),x);
dp:=unapply(normal(diff(p(x),x)),x);
u:=x;
//on itere
while (rem(p(u),m,x) <> 0 )
 {
   [s,t,d]:=gcdex(m,dp(u),x);
   invdp:=quo(t,d,x); //abcuv permet d'avoir l'inverse plus facilement
   u:=rem(u-p(u)*invdp,m,x);
 };
horner(u,A);
end proc;
```

// End defining Dunford

```
(A)->
{ local NULL;
  m:=pcar(A,x,'lagrange');
  p:=unapply(normal(m/(gcd(m,diff(m,x)))),x);
  dp:=unapply(normal(diff(p(x),x)),x);
  u:=x;
  while((rem(p(u),m,x))!=0){
     [s,t,d]:=gcdex(m,dp(u),x);
     invdp:=quo(t,d,x);
```

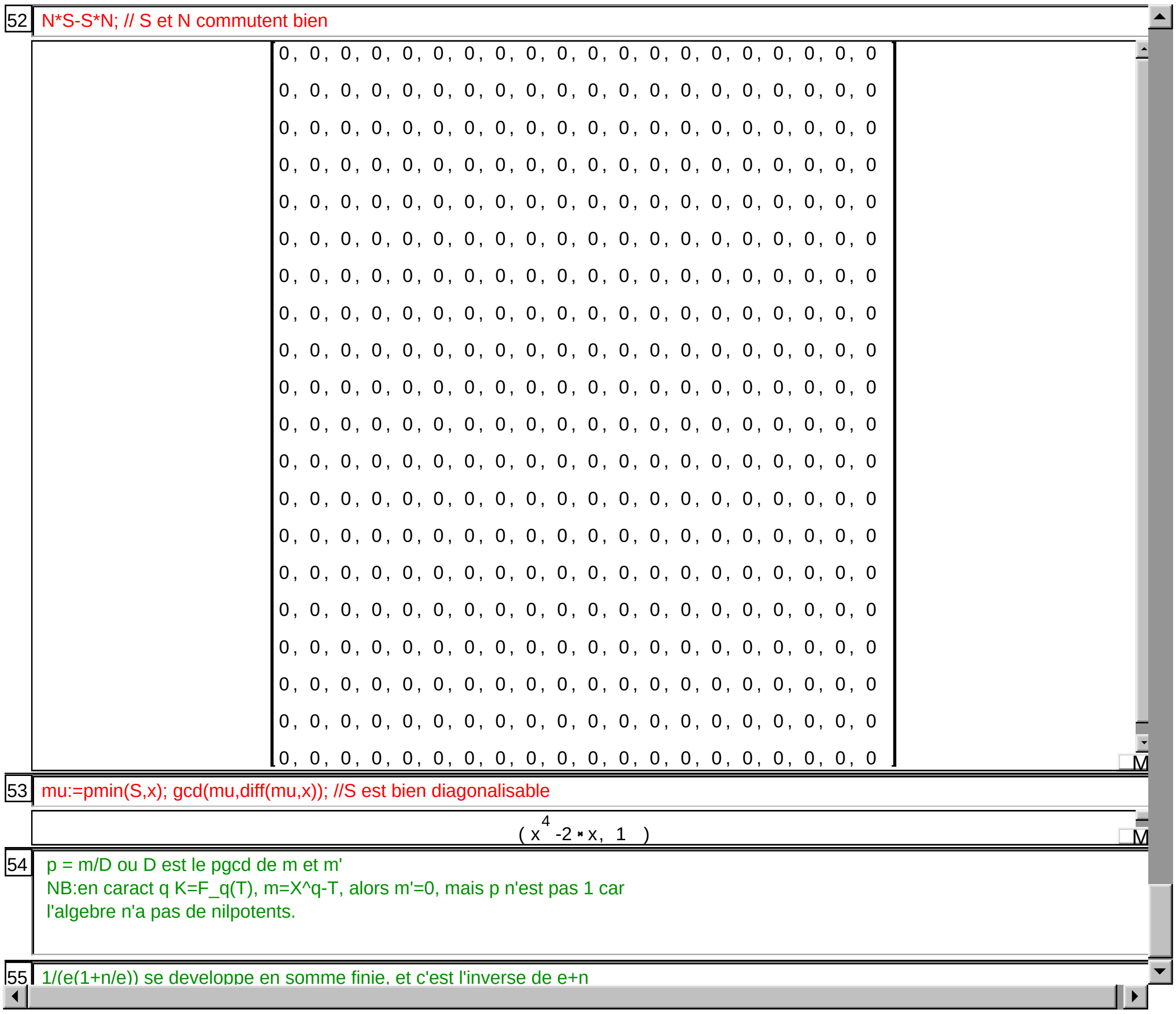
49 Pour avoir au moins deux it\'erations, il faut un facteur de degre au moins 4 dans le polynome minimal, ce que l'on obtient par exemple avec companion((x^3-2)^5,x)

50 S:=Dunford(A);

0,	0,	$\frac{70}{243}$,	0,	0,	$\frac{-28}{243}$,	0,	0,	$\frac{40}{243}$,	0,	0,	$\frac{-160}{243}$,	0,	0,	$\frac{3520}{243}$,	0,	0,	0,
$\frac{455}{243}$,	0,	0,	$\frac{70}{243}$,	0,	0,	$\frac{-28}{243}$,	0,	0,	$\frac{40}{243}$,	0,	0,	$\frac{-160}{243}$,	0,	0,	0,	0,	0,
0,	$\frac{455}{243}$,	0,	0,	$\frac{70}{243}$,	0,	0,	$\frac{-28}{243}$,	0,	0,	$\frac{40}{243}$,	0,	0,	$\frac{-160}{243}$,	0,	0,	0,	0,
0,	0,	$\frac{280}{243}$,	0,	0,	$\frac{140}{243}$,	0,	0,	$\frac{-128}{243}$,	0,	0,	$\frac{440}{243}$,	0,	0,	$\frac{-8960}{243}$,	0,	0,	0,
$\frac{-455}{486}$,	0,	0,	$\frac{280}{243}$,	0,	0,	$\frac{140}{243}$,	0,	0,	$\frac{-128}{243}$,	0,	0,	$\frac{440}{243}$,	0,	0,	0,	0,	0,
0,	$\frac{-455}{486}$,	0,	0,	$\frac{280}{243}$,	0,	0,	$\frac{140}{243}$,	0,	0,	$\frac{-128}{243}$,	0,	0,	$\frac{440}{243}$,	0,	0,	0,	0,
0,	0,	$\frac{-35}{162}$,	0,	0,	$\frac{70}{81}$,	0,	0,	$\frac{80}{81}$,	0,	0,	$\frac{-176}{81}$,	0,	0,	$\frac{3080}{81}$,	0,	0,	0,
$\frac{65}{162}$,	0,	0,	$\frac{-35}{162}$,	0,	0,	$\frac{70}{81}$,	0,	0,	$\frac{80}{81}$,	0,	0,	$\frac{-176}{81}$,	0,	0,	0,	0,	0,
0,	$\frac{65}{162}$,	0,	0,	$\frac{-35}{162}$,	0,	0,	$\frac{70}{81}$,	0,	0,	$\frac{80}{81}$,	0,	0,	$\frac{-176}{81}$,	0,	0,	0,	0,
0,	0,	$\frac{10}{243}$,	0,	0,	$\frac{-35}{486}$,	0,	0,	$\frac{160}{243}$,	0,	0,	$\frac{440}{243}$,	0,	0,	$\frac{-4928}{243}$,	0,	0,	0,
$\frac{-91}{972}$,	0,	0,	$\frac{10}{243}$,	0,	0,	$\frac{-35}{486}$,	0,	0,	$\frac{160}{243}$,	0,	0,	$\frac{440}{243}$,	0,	0,	0,	0,	0,
0,	$\frac{-91}{972}$,	0,	0,	$\frac{10}{243}$,	0,	0,	$\frac{-35}{486}$,	0,	0,	$\frac{160}{243}$,	0,	0,	$\frac{440}{243}$,	0,	0,	0,	0,

51 N:=A-S;

0,	0,	$\frac{-70}{243}$,	0,	0,	$\frac{28}{243}$,	0,	0,	$\frac{-40}{243}$,	0,	0,	$\frac{160}{243}$,	0,	0,	$\frac{4256}{243}$,	0,	0,	0
$\frac{-212}{243}$,	0,	0,	$\frac{-70}{243}$,	0,	0,	$\frac{28}{243}$,	0,	0,	$\frac{-40}{243}$,	0,	0,	$\frac{160}{243}$,	0,	0,	0,	0,	0
0,	$\frac{-212}{243}$,	0,	0,	$\frac{-70}{243}$,	0,	0,	$\frac{28}{243}$,	0,	0,	$\frac{-40}{243}$,	0,	0,	$\frac{160}{243}$,	0,	0,	0,	0
0,	0,	$\frac{-37}{243}$,	0,	0,	$\frac{-140}{243}$,	0,	0,	$\frac{128}{243}$,	0,	0,	$\frac{-440}{243}$,	0,	0,	$\frac{-10480}{243}$,	0,	0,	0
$\frac{455}{486}$,	0,	0,	$\frac{-37}{243}$,	0,	0,	$\frac{-140}{243}$,	0,	0,	$\frac{128}{243}$,	0,	0,	$\frac{-440}{243}$,	0,	0,	0,	0,	0
0,	$\frac{455}{486}$,	0,	0,	$\frac{-37}{243}$,	0,	0,	$\frac{-140}{243}$,	0,	0,	$\frac{128}{243}$,	0,	0,	$\frac{-440}{243}$,	0,	0,	0,	0
0,	0,	$\frac{35}{162}$,	0,	0,	$\frac{11}{81}$,	0,	0,	$\frac{-80}{81}$,	0,	0,	$\frac{176}{81}$,	0,	0,	$\frac{3400}{81}$,	0,	0,	0
$\frac{-65}{162}$,	0,	0,	$\frac{35}{162}$,	0,	0,	$\frac{11}{81}$,	0,	0,	$\frac{-80}{81}$,	0,	0,	$\frac{176}{81}$,	0,	0,	0,	0,	0
0,	$\frac{-65}{162}$,	0,	0,	$\frac{35}{162}$,	0,	0,	$\frac{11}{81}$,	0,	0,	$\frac{-80}{81}$,	0,	0,	$\frac{176}{81}$,	0,	0,	0,	0
0,	0,	$\frac{-10}{243}$,	0,	0,	$\frac{35}{486}$,	0,	0,	$\frac{83}{243}$,	0,	0,	$\frac{-440}{243}$,	0,	0,	$\frac{-4792}{243}$,	0,	0,	0
$\frac{91}{972}$,	0,	0,	$\frac{-10}{243}$,	0,	0,	$\frac{35}{486}$,	0,	0,	$\frac{83}{243}$,	0,	0,	$\frac{-440}{243}$,	0,	0,	0,	0,	0
0,	$\frac{91}{972}$,	0,	0,	$\frac{-10}{243}$,	0,	0,	$\frac{35}{486}$,	0,	0,	$\frac{83}{243}$,	0,	0,	$\frac{-440}{243}$,	0,	0,	0,	0



52
```
N*S-S*N; // S et N commutent bien
```

```
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
```

53
```
mu:=pmin(S,x); gcd(mu,diff(mu,x)); //S est bien diagonalisable
```

$(x^4-2\cdot x,\ 1)$

54

p = m/D ou D est le pgcd de m et m'
NB:en caract q K=F_q(T), m=X^q-T, alors m'=0, mais p n'est pas 1 car
l'algebre n'a pas de nilpotents.

55

1/(e(1+n/e)) se developpe en somme finie, et c'est l'inverse de e+n