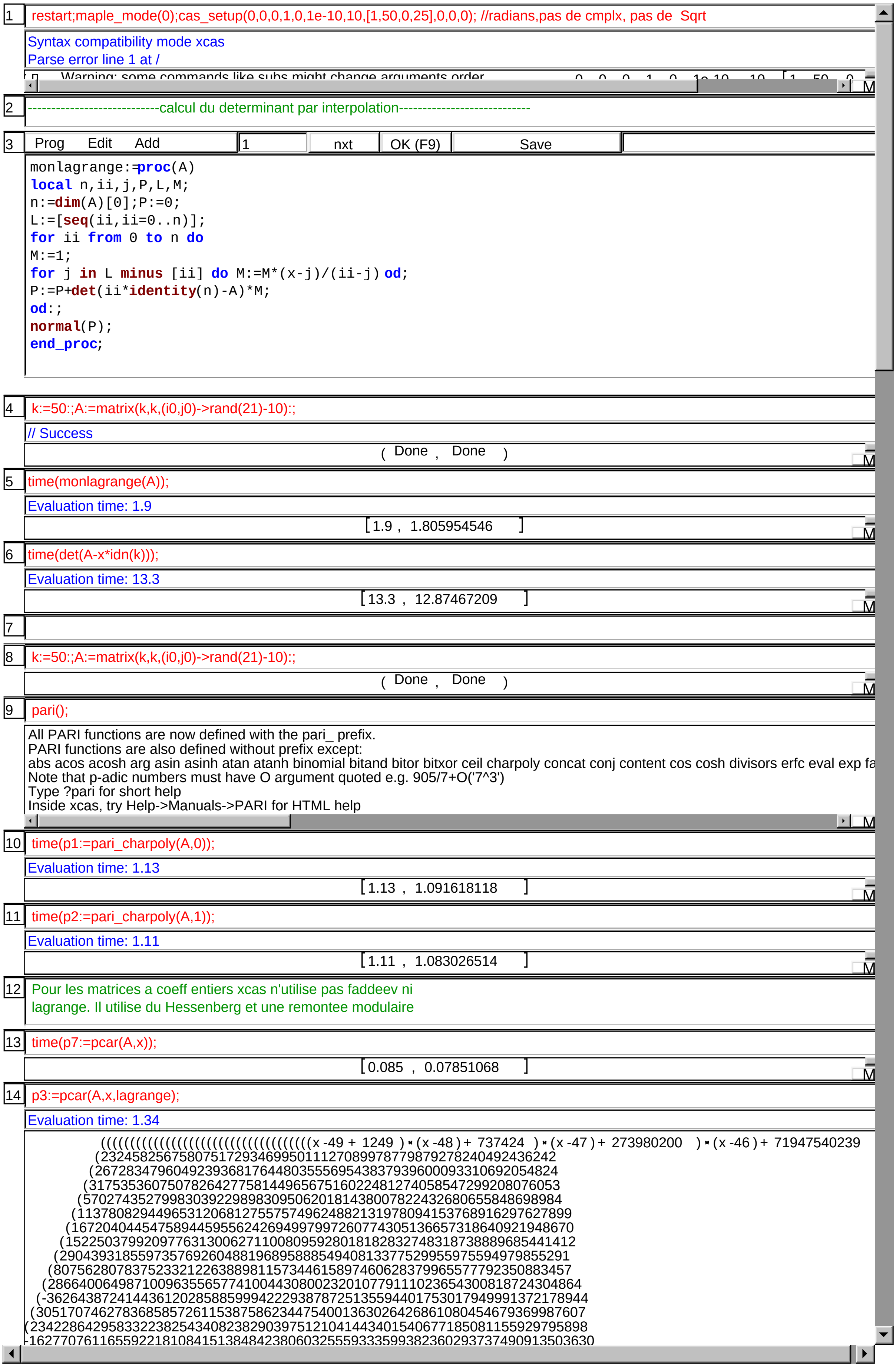

```
1  restart;maple_mode(0);cas_setup(0,0,0,1,0,1e-10,10,[1,50,0,25],0,0,0); //radians,pas de cmplx, pas de  Sqrt
Syntax compatibility mode xcas
Parse error line 1 at /
Warning: some commands like subs might change arguments order
```

```
2  ---------------------------calcul du determinant par interpolation---------------------------
```

```
3
monlagrange:=proc(A)
local n,ii,j,P,L,M;
n:=dim(A)[0];P:=0;
L:=[seq(ii,ii=0..n)];
for ii from 0 to n do
M:=1;
for j in L minus [ii] do M:=M*(x-j)/(ii-j) od;
P:=P+det(ii*identity(n)-A)*M;
od:;
normal(P);
end_proc;
```

```
4  k:=50:;A:=matrix(k,k,(i0,j0)->rand(21)-10):;
// Success
( Done , Done )

5  time(monlagrange(A));
Evaluation time: 1.9
[1.9 , 1.805954546 ]

6  time(det(A-x*idn(k)));
Evaluation time: 13.3
[13.3 , 12.87467209 ]

7

8  k:=50:;A:=matrix(k,k,(i0,j0)->rand(21)-10):;
( Done , Done )

9  pari();
All PARI functions are now defined with the pari_ prefix.
PARI functions are also defined without prefix except:
abs acos acosh arg asin asinh atan atanh binomial bitand bitor bitxor ceil charpoly concat conj content cos cosh divisors erfc eval exp fa
Note that p-adic numbers must have O argument quoted e.g. 905/7+O('7^3')
Type ?pari for short help
Inside xcas, try Help->Manuals->PARI for HTML help

10  time(p1:=pari_charpoly(A,0));
Evaluation time: 1.13
[1.13 , 1.091618118 ]

11  time(p2:=pari_charpoly(A,1));
Evaluation time: 1.11
[1.11 , 1.083026514 ]
```

12 Pour les matrices a coeff entiers xcas n'utilise pas faddeev ni lagrange. Il utilise du Hessenberg et une remontee modulaire

```
13  time(p7:=pcar(A,x));
[0.085 , 0.07851068 ]

14  p3:=pcar(A,x,lagrange);
Evaluation time: 1.34
            ((((((((((((((((((((((((((((((((((((x -49 + 1249 ) * (x -48 ) + 737424 ) * (x -47 ) + 273980200 ) * (x -46 ) + 71947540239
           (2324582567580751729346995011127089978779879278240492436242
          (26728347960492393681764480355569543837939600093310692054824
         (317535360750782642775814496567516022481274058547299208076053
        (5702743527998303922989830950620181438007822432680655848698984
       (113780829449653120681275575749624882131978094153768916297627899
      (1672040445475894459556242694997997260774305136657318640921948670
     (15225037992097763130062711008095928018182832748318738889685441412
    (29043931855973576926048819689588854940813377529955975594979855291
   (807562807837523321226388981157344615897460628379965577792350883457
  (28664006498710096355657741004430800232010779111023654300818724304864
 (-362643872414436120285885999422293878725135594401753017949991372178944
 (3051707462783685857261153875862344754001363026426861080454679369987607
(23422864295833223825434082382903975121041443401540677185081155929795898
-162770761165592218108415138484238060325559333599382360293737490913503630
```

15 Pour obtenir le polynome caract\'eristique sous forme d'un symbole, on passe la variable x en seconde option:

16 p4:=pcar(A,x,fadeev);

Evaluation time: 1.79

x^{50} -59 * x^{49} + 1513 * x^{48} -5152 * x^{47} -3922409 * x^{46} + 180220973 * x^{45} + (-4155689260) * x^{44} + 64387242445 * x^{43} + (-21
329067655887457 * x^{41} + (-9217480768169295) * x^{40} + (-13220201998081770) * x^{39} + 4611156317485845234 * x^{38}
(-778323268671511330816) * x^{37} + 24099168732061456392460 * x^{36} + (-26392114424374794051572 5) * x^{35} +
22654827702287959109217898 * x^{34} + (-56265853149742331371370104 0) * x^{33} + 26351144545951077501413044847
(-42019659822000353774032285244 7) * x^{31} + (-12870895617523736531929248465 18) * x^{30} + 47482502665534416 8
(-48404696290397246094342769800447468) * x^{28} + 15705898287348182015110595526681966 10 * x^{27} +
(-24863291500015319477357600980403232237) * x^{26} + 58067190897701216916510103742932264 3417 * x^{25} +
(-64118896355113005851982024687892251224 35) * x^{24} + (-14649666236533826489267576775459383798 3978
79070962855215822856763773754109647359529 99 * x^{22} + 11289915366613815450760563196703740887111 7634
(-15314853234103004398522497447503493080171981665) * x^{20} + (-24660283193985801044284982647043305966765

17 ?pcar

Liste des coefficients du polynôme caractéristique d'une matrice ou polynôme caractéristique d'une matrice de variable le second argum
Mtrx,[Var]
pcar([[1,2],[3,4]]);pcar([[1,2],[3,4]],x);pcar([[1,2,3],[1,3,6],[2,5,7]]);pcar([[1,2,3],[1,3,6],[2,5,7]],z)
jordan,egv,egvl,companion,rat_jordan,pmin

18 p5:=pcar(A,x,pmin);

Evaluation time: 6.37

x^{50} + 24 * x^{49} -1300 * x^{48} -29048 * x^{47} + 3870473 * x^{46} + 107058571 * x^{45} + (-6377211542) * x^{44} + (-254646003583) * x^{43}
632206708762705 * x^{41} + (-3421738392626456) * x^{40} + (-893216124593923169) * x^{39} + 3552013013098120836 * x^{3}
8790351883725098434 61 * x^{37} + 20756105239918272211034 * x^{36} + (-78564803782756379756247 0) * x^{35} +
(-41147890436259152518232373) * x^{34} + 53182861869652583224738 3444 * x^{33} + 27143110855885476923456537848
(-57253571260181300805905114 97 18) * x^{31} + (-36905441075320492173334789605195) * x^{30} + 3703852970214761 8
(-99841048992810896695232093718 43036) * x^{28} + (-12393686991657173040648698816343 43512) * x^{27} +
28518510800566641191024433060177904854 * x^{26} + 59076155330001041330277513383955717 7228 * x^{25} +
(-20505415918156679292222410792943846311359) * x^{24} + (-16246412677876390605033365986113378 3780187
(-60451939770716188165902717599265503850 85642) * x^{22} + 69568010062322610405512554723158738903403 3879
72469813888472671458105447061969741089990 6643 * x^{20} + (-57230900047875611205356128542443731344005214 82
22297503561093289931241665387342616472648972228185 * x^{18} + 25276555882315294141694696135352657424359
(-47287323368560894723508923165255473969525133122812 97) * x^{16} +
(-82162196026846120064621195203415968572187314153051 55) * x^{15} +
13631368678076293090123906283900790534888915273746545 48 * x^{14} +
x^{13}

19 p6:=det(A-x*idn(k));

Evaluation time: 13.9

x^{50} + 24 * x^{49} -1300 * x^{48} -29048 * x^{47} + 3870473 * x^{46} + 107058571 * x^{45} -6377211542 * x^{44} -254646003583 * x^{43} + 5695
632206708762705 * x^{41} -3421738392626456 * x^{40} -893216124593923169 * x^{39} + 3552013013098120836 * x^{38} + 879
20756105239918272211034 * x^{36} -78564803782756379756247 0 * x^{35} -41147890436259152518232373 * x^{34} +
53182861869652583224738 3444 * x^{33} + 27143110855885476923456537848 * x^{32} -57253571260181300805905114 97 1
-36905441075320492173334789605195 * x^{30} + 37038529702147618700077930812 2727 * x^{29} -99841048992810896
-12393686991657173040648698816343 43512 * x^{27} + 28518510800566641191024433060177904854 * x^{26} +
59076155330001041330277513383955717 7228 * x^{25} -20505415918156679292222410792943846311359 * x^{24}
-16246412677876390605033365986113378 3780187 * x^{23} -60451939770716188165902717599265503850 85642
69568010062322610405512554723158738903403 3879 * x^{21} + 72469813888472671458105447061969741089990 6643
19

20 normal([poly2symb(p1-p2,x),poly2symb(p2,x)-p3,p3-p4,p4-p5,p5-p6,p6-p7]);

[0, 0, 0, 0, 0, 0]

21 k:=30:;A:=matrix(k,k,(i0,j0)->(rand(21)-10)/(rand(4)^2+(1+rand(3))^2)):;

// Success

(Done , Done)

22 time(pari_charpoly(A,0));

Evaluation time: 7.75

```
23 time(pari_charpoly(A,1));
```

Evaluation time: 6.99

$$[6.99\ ,\ 6.631146851\ ]$$

```
24 c'est en O(n^4), en fait n^4/3, alors que Faddeev est en n^4. Mais il y a plus
   de fractions chez lagrange, ce qui peut donner l'avantage a faddeev.
25 ------------------------Polynome minimal-----------------------------------------------
26 v:=[seq(ii,ii=1..8)];
```

$$[1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8\ ]$$

```
27 poly2symb(v,x);
```

$$((((((x+2)\cdot x+3)\cdot x+4)\cdot x+5)\cdot x+6)\cdot x+7)\cdot x+8$$

```
28 revlist(v);
```

$$[8,\ 7,\ 6,\ 5,\ 4,\ 3,\ 2,\ 1\ ]$$

```
29 P:=x^2+1;LP:=[1,0,1];
```

$$(x^2+1,\ [1,\ 0,\ 1\ ])$$

```
30 subst(P,x=sqrt(2));
```

$$3$$

```
31 unapply(P,x)(sqrt(2));
```

$$3$$

```
32 peval(LP,sqrt(2));
```

$$\sqrt{2}\cdot\sqrt{2}+1$$

```
33 n:=8:;v:=[seq(ii,ii=1..n)]; A:=matrix(n,n,(i0,j0)-> rand(20)-10);
```

// Success

$$\left(\text{Done}\ ,\ [1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8\ ],\ \begin{bmatrix} 3, & 0, & 8, & 7, & -10, & 0, & -7, & 9 \\ 1, & -3, & 8, & 6, & -1, & -2, & 4, & -8 \\ -10, & -3, & -5, & 3, & 4, & -3, & 0, & -5 \\ 7, & -2, & 6, & 8, & 6, & 0, & -3, & -3 \\ -1, & 6, & -9, & 8, & -10, & 5, & -10, & -4 \\ -1, & -5, & 2, & 8, & 3, & 6, & 8, & -7 \\ -9, & 0, & 2, & -10, & 1, & 9, & 6, & 6 \\ 8, & -1, & 3, & 0, & 1, & 8, & -9, & 4 \end{bmatrix}\right)$$

```
34 B:=v; AA:=v;
```

$$([1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8\ ],\ [1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8\ ])$$

```
35 for ii from 1 to n do AA:=A*AA;B:=B,AA; od;
```

$$[1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8\ ],\ [28,\ -10,\ -57,\ 38,\ -106,\ 78,\ 106,\ 37\ ],\ [545,\ -92,\ -694,\ -887,\ 971,\ 951,\ 708,$$

```
36 N:=nullspace(transpose([B]))[0];
```

$$[308918628\ ,\ -6362942\ ,\ 1858987\ ,\ -662185\ ,\ 34078\ ,\ -148\ ,\ 199\ ,\ 9,\ -1\ ]$$

```
37 Attention, les polyn\^omes commencent a la puissance maximale, il faut inverser l'ordre
38
```

$$\text{undef}$$

```
39 P:=poly2symb(revlist(N),x);
```

$$(((((((-x+9)\cdot x+199)\cdot x-148)\cdot x+34078)\cdot x-662185)\cdot x+1858987)\cdot x-6362942)\cdot x+308918628$$

```
40 normal(subst(P,x=A));
```

$$\begin{bmatrix} 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \end{bmatrix}$$

41 A:=diag(companion(x^4-1,x),companion((x^2-1)^2,x));

$$\begin{bmatrix} 0, & 0, & 0, & 1, & 0, & 0, & 0, & 0 \\ 1, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 1, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 1, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & -1 \\ 0, & 0, & 0, & 0, & 1, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 1, & 0, & 2 \\ 0, & 0, & 0, & 0, & 0, & 0, & 1, & 0 \end{bmatrix}$$

42 B:=v; AA:=v

$$([1, 2, 3, 4, 5, 6, 7, 8], [1, 2, 3, 4, 5, 6, 7, 8])$$

43 for i from 1 to n do AA:=A*AA;B:=B,AA; od;

Warning, i is usually sqrt(-1), I'm using a symbolic variable instead but you should check your input

$$[1, 2, 3, 4, 5, 6, 7, 8], [4, 1, 2, 3, -8, 5, 22, 7], [3, 4, 1, 2, -7, -8, 19, 22], [2, 3, 4, 1, -22, -7,$$

44 nullspace(B);

$$\begin{bmatrix} \frac{9}{5}, & \frac{4}{5}, & \frac{9}{5}, & \frac{4}{5}, & -1, & 0, & -1, & 0 \\ \frac{4}{5}, & \frac{9}{5}, & \frac{4}{5}, & \frac{9}{5}, & 0, & -1, & 0, & -1 \end{bmatrix}$$

45 N:=nullspace(transpose([B]))[0];

$$[-1, 0, 1, 0, 1, 0, -1, 0, 0]$$

46 P:=poly2symb(revlist(N),x);

$$(((-x) \cdot x + 1) \cdot x \cdot x + 1) \cdot x \cdot x - 1$$

47 normal(subst(P,x=A));

$$\begin{bmatrix} 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \end{bmatrix}$$

48 On a calcul\'e les A^k(v) par recurence. AA est un vecteur donc A*AA a un cout de n^2, et l'on fait n tours, donc c'est en O(n^3)

49 Il existe toujours un vecteur v tel que P_(u,v)=pmin(u), les vecteurs a eviter sont dans ker(P(u)) ou P divise pmin, donc nom fini despace vectoriels a eviter.