

1	art;maple_mode(0);cas_setup(0,0,0,1,0,1e-10,10,[1,50,0,25],0,0,0); //radians,pas de cmplx, pas de Sqrt		
2	l:=%{1,4,5,6,7%};a:=rand(10);	( [ 1, 4, 5, 6, 7 ], Done )	M
3	l:=set[1,4,5,6,7];a:=rand(10);	( [ 1, 4, 5, 6, 7 ], Done )	M
4	member(a,l);	1	M
5	a; //verification.	1	M
6	maple_mode(1); // en mode maple	Warning: some commands like subs might change arguments order	M
7	ln(exp(1));ln(e); //ca n'est que la lettre e, pas le reel.	( 1, ln( e ) )	M
8	maple_mode(0);	Warning: some commands like subs might change arguments order	M
9	evalf(e); //la doc fait reference au mode xcas.	2.718281828	M
10	evalf(exp(1));	2.718281828	M
11	ln(exp(1));	1	M
12	log(exp(1)); // les 2 marchent	1	M
13	\qu Comment obtenir un entier entre 0 et 20? Attention, que donne	Syntax compatibility mode xcas Parse error line 1 at Comment	
		undef	M
14	\verb/rand(2^30);/ et \verb/rand(2^31);/	Syntax compatibility mode xcas Parse error line 1 at /	
		( $\frac{\text{verb}}{741678871}$ , undef && ( $\frac{\text{verb}}{281437032.0}$ ), undef )	M
15	rand(2^30);rand(2^31); //il passe parfois en scientifique quand c'est trop grand?	( 518467323 , 1852766699 )	M
16	bete comme le crible d'eratostene sont en O(p), et bete n'use pas de memoire. donc asymptotiquement, le crible n'est pas avantageux.		

17	Prog Edit Add	1	nxt	OK (F9)	Save
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```

bete:=proc(n)
j:=3;
a:=0;
SQ:=evalf(sqrt(n)); //pour ne le faire qu'une fois. ne pas le mettre dans le while!
while (j<SQ)
{ if (irem(n,j)) <>0 then j:=j+2;
  else a:=j;j:=n;
  fi ;
};
if a<>0 then a; else n fi;
end;

```

// warning: j,a,SQ, declared as global variable(s)  
// End defining bete

```

(n)->
{ local NULL;
j:=3;
a:=0;
SQ:=evalf(sqrt(n));
while(j<SQ){
if ((irem(n,j))!=0) {
j:=j+2;
}
else {
a:=j;
j:=n;
};
};
};

```

18	j:=4;N:=nextprime(10^j+5432)*nextprime(2*10^j+1234);	( 4, 328032433 )	M
19	bete(N);	15439	M
20	j:=5;N:=nextprime(10^j+5432)*nextprime(2*10^j+1234); //j=6 passe encore mais...	( 5, 21218879939 )	M
21	bete(N);	Evaluation time: 0.9 105437	M
22	Attention, pour que la suite r'ecurrente soit bien programme, il ne faut pas recalculer tous les termes jusque u_i ni u_2i a chaque etape!		

23	Prog Edit Add	1	nxt	OK (F9)	Save
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```

pollard := proc(n)
local x,y ;
x:=1; y:=x^2+1;
while (member ( igcd(y-x,n) , set[1,n] ))
{
x:=irem( (x^2+1) , n) ;
y:=irem( (y^2+1)^2+1 , n) ;
};
igcd(y-x,n);
end_proc;

```

// End defining pollard

```

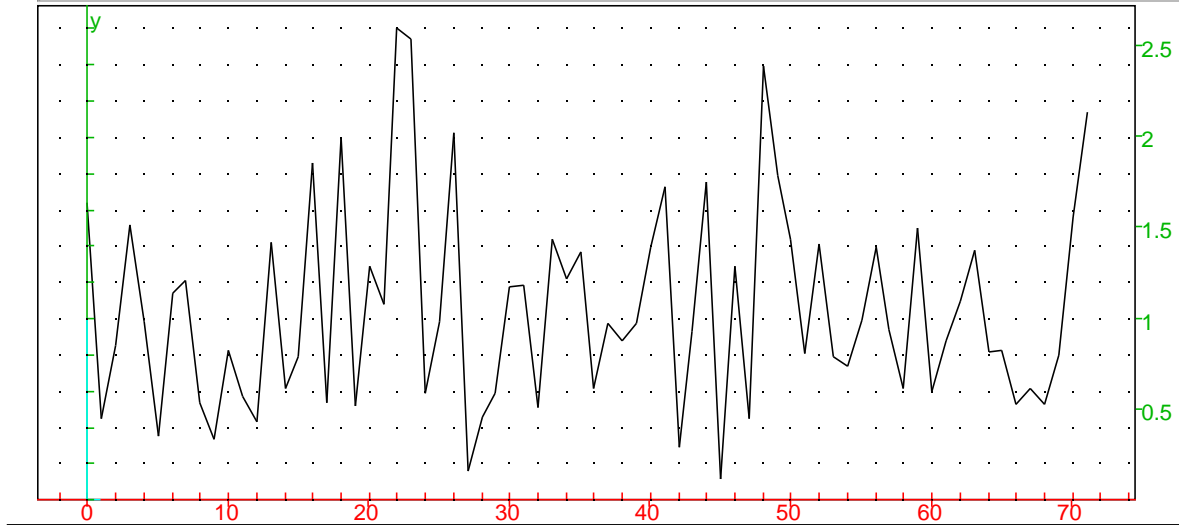
(n)->
{ local x,y;
x:=1;
y:=x^2+1;
while(member(igcd(y-x,n),set[1,n])){
x:=irem(x^2+1,n);

```

24	N:=nextprime(10^6)*nextprime(2*10^6);		
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25	pollard(N);	1000003	M
26	N:=nextprime(10^8+5*rand(1000))*nextprime(2*10^8+11*rand(1000));;	Done	M
27	pollard(N);	100001569	M
28	pollard est en $O(\sqrt{p})$ ssi la fonction suivante est bornee.		
29	<div> <div>Prog Edit Add</div> <div>1</div> <div>nxt</div> <div>OK (F9)</div> <div>Save</div> </div> <pre> comptep:=proc(n) local x,y,j ; x:=1; y:=x^2+1 ;j:=0; while (member ( igcd(y-x,n) , %1,n% ) ) { x:=irem(x^2+1,n) ; y:=irem(y^2+1,n) ; j:=j+1; } evalf(j/sqrt(igcd(y-x,n))) ; end_proc; </pre> <div> <div>// Success</div> <div>// End defining comptep</div> </div> <div> <div>(n)-&gt;</div> <div>{ local x,y,j;</div> <div>x:=1;</div> <div>y:=x^2+1;</div> <div>j:=0;</div> <div>while(member(igcd(y-x,n),set[1,n])){</div> </div>		
30	<p>On cr'ee une liste de valeurs. Il ne faut pas des nombres trop petits pour que pollard ait de bonne chance d'aboutir. On fait imprimer a chaque etape pour voir s'il bloque a une etape.</p> <p>On met des floor pour les grands rand car il bascule en flottant a partir de 2^31</p>		
31	<pre> for j from 1 to 30 do N:=nextprime(floor(alea(3^j)))*nextprime(floor(alea(2^(j+1)))); t:=comptep(N); afficher(j,t); l:=append(l,t); N:=nextprime(floor(alea(3^j)))*nextprime(floor(alea(2^(j+1)))); t:=comptep(N); afficher(j,t); l:=append(l,t); N:=nextprime(floor(alea(3^j)))*nextprime(floor(alea(2^(j+1)))); t:=comptep(N); afficher(j,t); l:=append(l,t); od; </pre> <div> 26,0.6152805577  26,1.497623139  27,0.595442883  27,0.8776031913  27,1.09839073  28,1.379445264  28,0.8187177288  28,0.8248802713  29,0.527697553  29,0.6172450279  29,0.5302144649  30,0.8036918912  30,1.560217665  30,2.136468104  Evaluation time: 15.31 </div> <div> <div>Π, [ 1.641105448 , 0.4527923591 , 0.8516583167 , 1.515985071 , 0.9837827088 , 0.3545937657</div> </div>		

```
32 plotlist(l); //ca a bien l'air borne
```



```
33 P:=i0->product(1-j/p,j=1..i0-1);
```

```
// Warning: j,p, declared as global variable(s)
// End defining P
```

$$i0 \rightarrow \prod_{p \in \mathcal{P}} (1 - \frac{1}{p})_{j=1}^{(i0-1)}$$

```
34 i0:=2;limit(log(P(i0))/(i0*(i0-1)/2/p),p,+infinity);
```

	( 2, -1 )	M
--	-----------	---

35 On devine que  $P(i_0)$  equivaut a:  $-i_0 \cdot (i_0 - 1) / (2p)$ , on l'illustre ainsi:

```
36 l:=limit(log(P(2))/(i0*(i0-1)/2/p),p,+infinity);
```

	-1	M
--	----	---

```
37 for i0 from 3 to 50 do l:=l,limit(log(P(i0))/(i0*(i0-1)/2/p),p,+infinity) od;
```

[illegible]

38 Puisque  $\ln$  est croissante, on se demande quand  $\ln(P(i_0)) > \ln(0.5)$ . On compare donc  $-\ln(0.5)$  avec  $i_0^2/2p$

```
39 sqrt(-2*ln(0.5)); //ca fait a peu pres le 1.18 de l'enonce
```

1.177410023	M
-------------	---

40  $u_{2i} = u_i$  est mauvais pour cette suite recurrent, c'est en  $O(p)$  comme la methode bete.  
En effet  $u_{2i} = a^i u_i + c(a^i - 1)/(a - 1) [p]$ , donc  $u_{2i} = u_i$  ssi  $(a^i - 1)(-u_i + c/(a - 1)) = 0[p]$ . Mais  $a^i = 1[p]$  par ex si  $a$  generateur, alors  $i \geq p - 1$

41

undef	M
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42 illustration: on fait afficher le rapport entre le nombre de tours et  $\sqrt{p}$ .

```
43 a:=54321123;c:=nextprime(10^4);
```

( 54321123 , 10007 ) M

11

45	Prog Edit Add	1	nxt	OK (F9)	Save
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```

comptelin:=proc(n,a,c)
local x,y ;
x:=1; y:=irem(a*x+c, n) ;j:=0;
while (member ( igcd(y-x,n) , {1,n%} ))
{
x:=irem(a*x+c, n) ;
y:=irem( (a*(a*y+c)+c) , n) ;
j:=j+1;
};
evalf(j/sqrt(igcd(y-x,n)));
end proc;

// Warning: j, declared as global variable(s)
// End defining comptelin

(n,a,c)->
{ local x,y;
x:=1;
y:=irem(a*x+c,n);
j:=0;
while(member(igcd(y-x,n),set[1,n])){

```

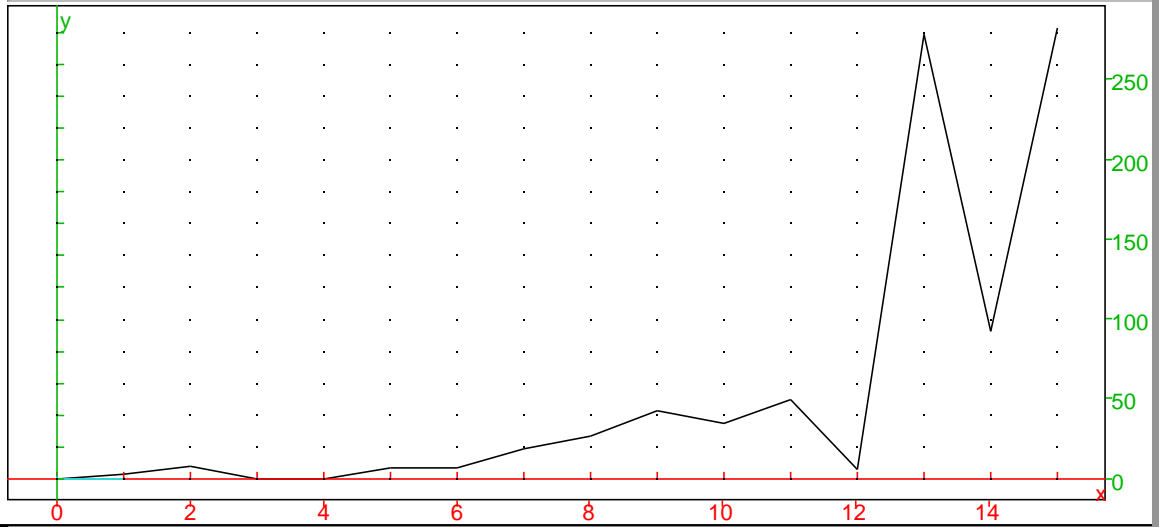
46	l:=[];rand(3^4);	( [], 69 )
----	------------------	------------

```

47 for i0 from 4 to 19 do
N:=nextprime(alea(3^i0))*nextprime(alea(2^(i0+1)));
t:=comptelin(N,a,c);
afficher(i0,t);
l:=append(l,t);
od;
6,7.941013883
7,0.2457695762
8,0.2560737599
9,7.258741748
10,7.345819086
11,19.24192072
12,26.70622584
13,43.20882014
14,34.73111668
15,49.93885229
16,6.478084885
17,278.2319177
18,93.00836295
19,282.5182393
Evaluation time: 14.97
0.3015113446 , 3.049971407 , 7.941013883 , 0.2457695762 , 0.2560737599 , 7.258741748 , 7.345819086

```

48 `plotlist(l); // ca n'a plus l'air borne`

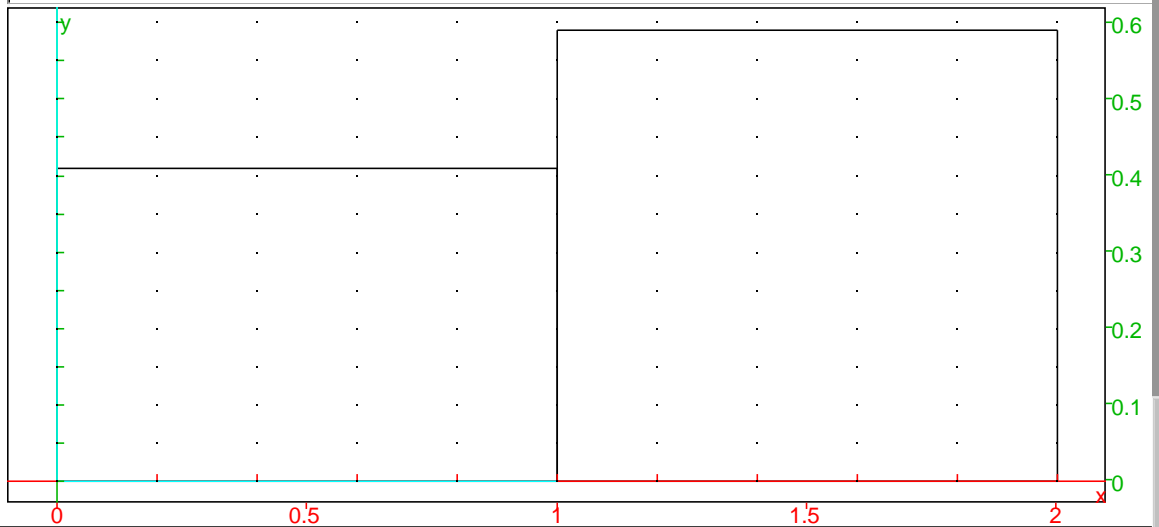


49 `l:=seq(rand(2),j=1..100);;`

Done

M

50 `histogram(classes(l,0,1)); //On commence a 0, largeur constante 1.`

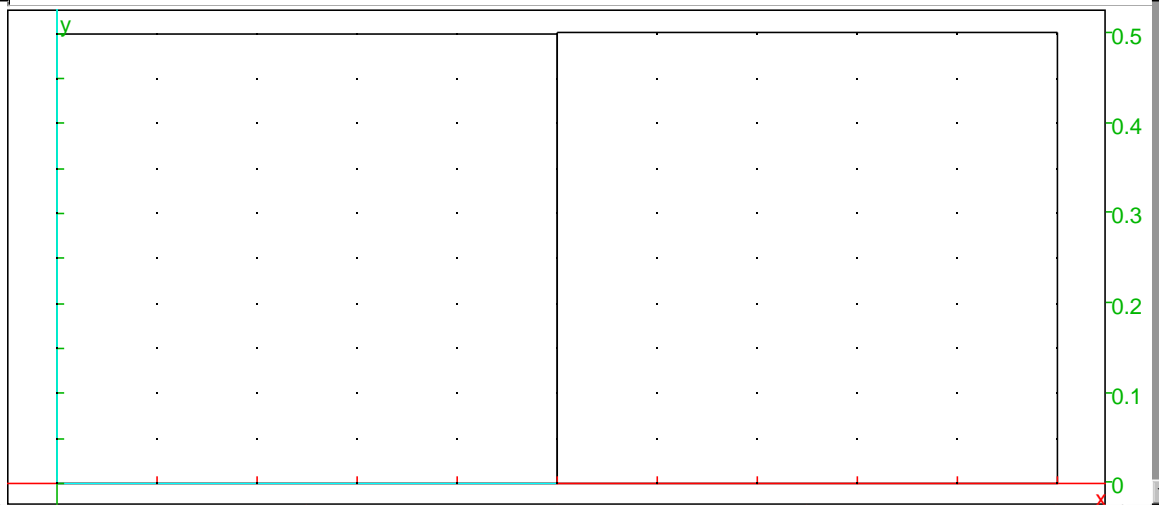


51 `l:=seq(rand(2),j=1..1000);;`

Done

M

52 `histogram(classes(l,0,1));`



53	N:=nextprime(10^6)*nextprime(2*10^6);		
	2000009000009		
54	u:=n->if n=0 then 2 else irem(((u(n-1))^2+1),N) fi;		
	// Warning: u,N, declared as global variable(s) // End defining u u: recursive definition		
	<pre> if ((n==0)) {   2; } else {   irem((u(n-1))^2+1,N); } n -&gt; </pre>		
55	u(10); // u(10000);Error, (in u) too many levels of recursion		
	1997309223146		
56			
57	local x,l; x:=12345;l:=[]; for i0 from 1 to M do x:=irem((x^2+1), n) ; l:=[op(l),x]; od ; l; end;		
	// Warning: i0, declared as global variable(s) // End defining etudesuite		
	<pre> (n,M)-&gt; { local x,l;   x:=12345;   l:=[];   for (i0:=1;i0&lt;=M;i0:=i0+abs(1)) {     x:=irem(x^2+1,n);     l:=[op(l),x];   };   l; } </pre>		
58	donnees:=(etudesuite(N,2000));		
	152399026 , 1358617644169 , 1831711515025 , 190274858284 , 1218880810174 , 893513062655		
59	cldonnees:=classes(donnees,0,N/40); //40 classes		
	1.50000675e+11	.. 2.000009e+11 ,	46
	2.000009e+11	.. 2.50001125e+11 ,	44
	2.50001125e+11	.. 3.0000135e+11 ,	48
	3.0000135e+11	.. 3.50001575e+11 ,	44
	3.50001575e+11	.. 4.000018e+11 ,	50
	4.000018e+11	.. 4.50002025e+11 ,	50
	4.50002025e+11	.. 5.0000225e+11 ,	48
	5.0000225e+11	.. 5.50002475e+11 ,	51
	5.50002475e+11	.. 6.000027e+11 ,	51
	6.000027e+11	.. 6.50002925e+11 ,	53
	6.50002925e+11	.. 7.0000315e+11 ,	48
	7.0000315e+11	.. 7.50003375e+11 ,	52
	7.50003375e+11	.. 8.000036e+11 ,	45
	8.000036e+11	.. 8.50003825e+11 ,	40
	8.50003825e+11	.. 9.0000405e+11 ,	50
	9.0000405e+11	.. 9.50004275e+11 ,	46

