1 art;maple_mode(0);cas_setup(0,0,0,1,0,1e-10,10,[1,50,0,25],0,0,0); //radians,pas de cmplx, pas de Sqrt

[ ], Warning: some commands like subs might change arguments order , 0, 0, 0, 1, 0, 0.1000000000

2 ----------
| Suites de Sturm |
----

3 sign(-5);sign(0);sign(3);//on teste ce que fait sign

(-1, 0, 1)

4 P:=2*x^3+4*x-5;

$$2 \cdot x^3 + 4 \cdot x - 5$$

5 horner(P,sqrt(3));

$$2 \cdot (\sqrt{3} \cdot 3^1) + 4 \cdot \sqrt{3} - 5$$

6

7 Prog Edit Add | 1 | nxt | OK (F9) | Save

```
mysturm:=proc(P,a,b)
local P1,P2,tmp,sa,sb,ho,v;
P1:=P;P2:=diff(P,x);
sa:=sign(horner(P1,a));
sb:=sign(horner(P1,b));
v:=0;//la difference des variations de signe.
while(P2<>0)
{
  tmp:=P2;P2:=-rem(P1,P2,x);P1:=tmp;
  ho:=sign(horner(P1,a));//vaut -1,0,1
  if ((ho<>0)&&(ho<>sa)){sa:=ho;v++;}
  ho:=sign(horner(P1,b));
  //on peut remarquer que (ho<>0)&&(ho<>sb) equivaut a:
  if (ho*sb=-1){sb:=ho;v--;}
}
v;
end;
```

```
(P,a,b)->
{ local P1,P2,tmp,sa,sb,ho,v;
  P1:=P;
  P2:=diff(P,x);
  sa:=sign(horner(P1,a));
  sb:=sign(horner(P1,b));
  v:=0;
  while(P2!=0){
     tmp:=P2;
     P2:=-(rem(P1,P2,x));
     P1:=tmp;
     ho:=sign(horner(P1,a));
     if (((ho!=0) && (ho!=sa))) {
        sa:=ho;
        v++;
      };
     ho:=sign(horner(P1,b));
     if (((ho*sb)==-1)) {
        sb:=ho;
        v--;
```

8 P:=(x+1)^4*(x+2)*(x+4)^5*(x+3);

$$(x+1)^4 \cdot (x+2) \cdot (x+4)^5 \cdot (x+3)$$

9 mysturm(P,-5.5,1.5);

4

10 mysturm(P,-3.5,1.5);

3

11 mysturm(P,-2.5,1.5);

12 sturmab(P,-5.5,1.5);//par xcas seuls les changements de signe sont comptabilises, les zeros pairs ne comptent pas

3

13 ----------

14

undef

15

16 Prog Edit Add | 1 | nxt | OK (F9) | Save

```
ST:=proc(U,n)
local a,aa,b,bb,r,rr,T,S,q;
r:=U;rr:=x^(2*n);
a:=1;b:=0;
aa:=0;bb:=1;
while (degree(r)>=n)
{
  q:=quo(r,rr);
  tmp:=rr;rr:=normal(r-q*rr);r:=tmp;
  tmp:=aa;aa:=normal(a-q*aa);a:=tmp;
  tmp:=bb;bb:=normal(b-q*bb);b:=tmp;
  print("ca doit etre nul",normal(a*U+x^(2*n)*b-r));
};
S:=a;T:=r;
S,T;
end;
```

```
// Warning: x,tmp, declared as global variable(s)
// End defining ST
```

```
(U,n)->
{ local a,aa,b,bb,r,rr,T,S,q;
  r:=U;
  rr:=x^(2*n);
  a:=1;
  b:=0;
  aa:=0;
  bb:=1;
  while((degree(r))>=n){
     q:=quo(r,rr);
     tmp:=rr;
     rr:=normal(r-q*rr);
     r:=tmp;
     tmp:=aa;
```

17 normal((x^3+1)*(x^3-1)*(x^2+x+1));

$$x^8 + x^7 + x^6 - x^2 - x - 1$$

18 on doit donc trouver S=x^3+1; T=-x^2-x-1 a facteur pres

19 ST((x^3-1)*(x^2+x+1),3);

```
"ca doit etre nul",0
"ca doit etre nul",0
"ca doit etre nul",0
"ca doit etre nul",0
```

$$\left(\left(\frac{1}{2}\right) \cdot x^3 + \frac{1}{2},\ -\left(\frac{1}{2}\right) \cdot x^2 - \left(\frac{1}{2}\right) \cdot x + \frac{-1}{2} \right)$$

20
```
MU:=proc(U,n)
local PU;
PU:=[seq(coeff(convert(series(U,x=0,2*n),polynom),x,j),j=0..2*n+1)];
matrix(n+1,n+1,(ii,jj)->PU[ii+jj]);
end;
```

```
// Warning: x,j,ii,jj, declared as global variable(s)
// End defining MU
```

```
(U,n)->
{ local PU;
  PU:=[seq(coeff(convert(series(U,x=0,2*n),polynom),x,j),j=(0 .. (2*n+1)))];
  matrix(n+1,n+1, (ii,jj)->PU[ii+jj]);
}
```

M

21
```
scalU:=proc(p,q,n)
local PU;
([seq(coeff(p,x,ii),ii=0..n)]*MU(U,n)*transpose([seq(coeff(q,x,ii),ii=0..n)]))[0];
end;
```

```
// Warning: x,ii,U, declared as global variable(s)
// End defining scalU
```

```
(p,q,n)->
{ local PU;
  ([seq(coeff(p,x,ii),ii=(0 .. n))]*MU(U,n)*transpose(seq(coeff(q,x,ii),ii=(0 .. n))))[0];
}
```

M

22
```
purge(u):;U:=add(u[ii]*x^ii,ii=0..6);
```

$$(\text{Done},\ u[0]+(u[1])\cdot x+(u[2])\cdot x^2+(u[3])\cdot x^3+(u[4])\cdot x^4+(u[5])\cdot x^5+(u[6])\cdot x^6)$$

M

23
```
scalU(1,x,3);scalU(x^2,x^2,3);
```

$$(u[1],\ u[4])$$

M

24
```
U:=1/(x+1/(x^2+1/(x^3+x+1+1/(x+2+1/x))));
```

$$\cfrac{1}{x+\cfrac{1}{x^2+\cfrac{1}{x^3+x+1+\cfrac{1}{x+2+\cfrac{1}{x}}}}}$$

M

25
```
factor(gramschmidt([1,x,x^2,x^3,x^4],(p,q)->scalU(p,q,4)));
```

```
// Success
```

$$\left[1,\ \frac{\sqrt{3}\cdot x+3\cdot\sqrt{3}}{3},\ \frac{\sqrt{3}\cdot x^2+(3\cdot\sqrt{3})\cdot x-3\cdot\sqrt{3}}{3},\ \frac{((-3*i)\cdot\sqrt{273})\cdot x^3+((-20*i)\cdot\sqrt{273})\cdot x^2+((-21*i)\cdot\sqrt{273}}{273}\right.$$

M

26
```
S:=[seq(pade(U,x,2*ii-1,ii),ii=1..4)]
```

$$\left[\frac{1}{3\cdot x+1},\ -\left(\frac{1}{3\cdot x^2-3\cdot x-1}\right),\ \frac{3\cdot x^2-11\cdot x-3}{42\cdot x^3-21\cdot x^2-20\cdot x-3},\ \frac{626\cdot x^3+192\cdot x^2+375\cdot x+273}{483\cdot x^4+77\cdot x^3+498\cdot x^2+1194\cdot x+273}\right]$$

M

27 On constate bien que les denominateurs des approximants de pade coincident a facteur pres avec les polyn{\^o}mes reciproques des orthonormalises de schmidt (NB le polynome reciproque d'une polynome est le polynome cree a partir de la suite des coefficients pris dans l'ordre inverse. (Le facteur vient de la norme 1)

28
```
recip:=proc(P)
normal(x^degree(P)*subst(P,x=1/x))
end;
```

```
// Warning: x, declared as global variable(s)
// End defining recip
```

```
(P)->
  normal(x^(degree(P))*subst(P,x=(1/x)));
}
```

M

29
```
seq(recip(denom(ii)),ii=S); //donne bien les polyn{\^o}mes obtenus par gramschmidt
```

$$2 \qquad 3 \qquad 2 \qquad 4 \qquad 3 \qquad 2$$