CIMPA School

Homological Methods, Representation Theory and Cluster Algebras

Courses and abstracts.

Courses

FIRST WEEK

Advanced homological algebra, by Mariano Suárez-Alvarez (Universidad de Buenos Aires, Argentina)

This course aims at introducing the students to the basic notions of homological algebra, approached first from an elementary viewpoint using derived functors and then using the notion of triangulated category and derived category. We start with a review of functors and categories including several examples and emphasizing the definitions of kernel and cokernel, product and coproduct, as well as the notions of right and left exact functors. Then we will talk about additive categories, the category of complexes of an additive category and the corresponding homotopy category. This will give rise to the notion of derived category.

Introduction to the representation theory of algebras, by Maria Ines Platzeck (Universidad Nacional del Sur, Argentina)

The objective of the course is to introduce basic notions and results of the representation theory of algebras, with particular emphasis on the functorial techniques introduced by Maurice Auslander. This approach has played a fundamental role in the representation theory of finite dimensional algebras and, more generally, of artin algebras.

Auslander-Reiten theory of finite dimensional algebras, by Piotr Malicki (Nicolaus Copernicus University)

In the first lecture we shall present some important facts from homological algebra. In the second lecture we introduce some fundamental concepts such as irreducible morphisms and almost split sequences. The third lecture will include the following concepts: the Auslander-Reiten translation, the Auslander-Reiten formula and the Auslander-Reiten quiver which is an important combinatorial and homological invariant of the module category of finitely generated modules of an algebra. In the last lecture we shall present old and new results on the number of terms in the middle of an almost split sequence in the module category of a finite dimensional algebra. The intention of these lectures is to serve as a source of motivation and information on the main concepts, techniques and results on the Auslander-Reiten theory.

Cluster algebras arising from surfaces, by Ralf Schiffler (University of Connecticut, USA)

Cluster algebras are commutative algebras with a special combinatorial structure. A cluster algebra is a subalgebra of a field of rational functions in several variables, which is defined by constructing a specific set of generators in a recursive way. This course will focus on an important class of cluster algebras, those that are associated to surfaces with boundary and marked points. The generators of these cluster algebra are in bijection with certain curves in the surface and the combinatorial structure of the cluster algebra can be explained in terms of triangulations of the surface. In the simplest example, a regular polygon with n vertices, the generators of the cluster algebra correspond to the diagonals of the polygon. The first class will be on general cluster algebras, and the following three classes on cluster algebras from surfaces. I plan to cover the following list of topics. Cluster algebras, definition and results, cluster algebras from surfaces, definition, examples, relation to representation theory, combinatorial expansion formulas, canonical bases, and if time permits upper cluster algebra, skein relations, snake graph calculus.

SECOND WEEK

Cluster tilted algebras, by Ibrahim Assem (Université de Sherbrooke, Québec, Canada)

Definition and elementary properties. The module category of a cluster tilted algebra. Homological properties. Quiver mutation and the ordinary quiver of a cluster tilted algebra. Relation extensions and the bound quiver of a cluster tilted algebra. The cluster repetitive algebra. The Auslander-Reiten quiver of a cluster tilted algebra: local slices and reflexions. Gentle cluster tilted algebras. Modules determined by their composition factors. Hochschild cohomology of cluster tilted algebras.

Cluster characters, by Pierre-Guy Plamondon (Université Paris-Sud, France)

This course's aim is to present cluster characters for 2-Calabi-Yau triangulated categories and their main properties. The original motivation for the study of these notions resides in their application to Fomin-Zelevinsky's cluster algebras; however, we will focus on the theory itself. First, we shall define triangulated 2-Calabi-Yau categories; our main examples will be orbit categories of derived categories of Dynkin quivers. Then, we will define cluster characters, which are kinds of generating series for "counting" submodules of a given modules according to their dimension. Finally, we will see how these cluster characters interact, giving rise to various multiplication formulas.

Introduction to K-theory, by Guillermo Cortiñas (Universidad de Buenos Aires, Argentina)

The course will be introductory and treat of the following topics.

I. The K-theory of exact categories.
I.1. Quillen's Q-construction.
I.2. Quillen's fundamental theorems: Localization, Dévissage, Additivity and Resolution.
I.3. Negative K-theory.
II. Algebraic K-theory and triangulated categories.
II.1 The bounded derived category of an exact category.
II.2 Exact sequences of triangulated categories and the Thomason-Waldhausen localization theorem.
II.3 Quillen's fundamental theorems revisited.

Brauer graph algebras and applications to cluster algebras, by Sibylle Schroll (University of Leicester, United Kingdom)

Brauer graph algebras are symmetric algebras of tame representation type. They first appeared, as Brauer tree algebras, in the modular representation theory of finite groups. Since then they have been studied in their own right and much of their representation theory is well understood. It has recently been shown that the class of Brauer graph algebras coincides with the class of symmetric special biserial algebras, which is another class of tame algebras, that has been well-studied. These two presentations, as Brauer graph algebras and as symmetric special biserial algebras, provide two different approaches to the subject. In this course we will define Brauer graph algebras as well as symmetric special biserial algebras and give an overview of their representation theory based on the two approaches. In particular, we will show how Brauer graph algebras have recently been connected to Jacobian algebras associated to triangulations of surfaces.

Talks

ON PROPERTIES OF LEIBNIZ ALGEBRAS AND THEIR ASSOCIATED GRAPHS.

R.M. Aquino

We study the relation between algebraic structures and Graph Theory. We have defined three different weighted digraphs associated to a finite dimensional algebra over a field. We applied those definitions for the case of finite dimensional Lie and Leibniz algebras to describe some properties of those associated graphs and to study the nilpotency and solvability of that algebras. joint work with L.M. Camacho, E.M. Canete, C. Cavalcante, A. Marquez (2015)

UNISTRUCTURALITY OF CLUSTER ALGEBRAS OF TYPE $\tilde{\mathbb{A}}$

Veronique Bazier-Matte

It is conjectured by Ibrahim Assem, Ralf Schiffler and Vasilisa Shramchenko in *Cluster Automorphisms and Compatibility of Cluster Variables* that every cluster algebra is unistructural, that is to say, that the set of cluster variables determines uniquely the cluster algebra structure. In other words, there exists a unique decomposition of the set of cluster variables into clusters. This conjecture has been proven to hold true for algebras of type Dynkin or rank 2 by Assem, Schiffler and Shramchenko. The aim of this talk is to prove it for algebras of type $\tilde{\mathbb{A}}$. We use triangulations of annuli and algebraic independence of clusters to prove unistructurality for algebras arising from annuli, which are of type $\tilde{\mathbb{A}}$.

EXTENSIONS IN GENTLE ALGEBRAS

Ilke Canakci

In this talk, we will introduce a basis for the extension space between indecomposable modules over gentle algebras given in terms of string combinatorics. The first part of this talk will discuss a class of gentle algebras associated to triangulations of unpunctured marked surfaces. This case, joint work with Sibylle Schroll, requires an interaction in the associated cluster category. In the general case of a gentle algebra, we will use its derived category and examine the extension space by developing a mapping cone calculus of homotopy strings. This is a report on joint work with David Pauksztello and Sibylle Schroll

A GENERALIZATION OF QUASITILTED AND ALMOST HEREDITARY ALGEBRAS

Tanise Carnieri Pierin

Motivated by the connection between tilting theory and derived categories, Happel, Reiten and Smalø presented a generalization of the classical tilting process, made by starting with a torsion pair in an abelian category. More precisely, given $(\mathcal{T}, \mathcal{F})$ a torsion pair in an abelian category, one can define another abelian category obtained as the heart of a t-structure, and where $(\mathcal{F}[1], \mathcal{T})$ defines a torsion pair. This construction led to the introduction of the class of quasitilted algebras, which contains the classes of tilted and canonical algebras. By definition, a quasitilted algebra is the endomorphism algebra of a tilting object in an abelian hereditary category. Since hereditary categories are well known and have some good homological properties, one can carry in some way this information to quasitilted algebras. As a result of applying this technique, it is possible to show that a quasitilted algebra has (i) global dimension at most two and (ii) each indecomposable of its module category has projective or injective dimension at most one. Another interesting fact is that algebras which satisfy (i) and (ii), the so-called almost hereditary, are quasitilted algebras, then establishing a simpler characterization to such class of algebras. We propose a generalization to these classes of algebras, which we call (m, n)-quasitilted and (m, n)-almost hereditary, defined in a natural way, and discuss some of the results which can be obtained.

HOPF LINEAR CATEGORIES OVER A GROUP AND THEIR FUNDAMENTAL GROUP

Claude Cibils

Suppose that the objects of a linear category of a finite group G. Then we introduce a Hopf structure on the category, obtaining a so called *Hopf linear category over* G. If G is trivial this corresponds to a Hopf algebra. Alternatively, if there are no morphisms between different objects besides zero, one recovers the Hopf G-coalgebras. We will define the fundamental group *à la Grothendick* of a Hopf linear category over G which is abelian and relate it with the intrinsic fundamental group of the underlying linear category. We will also describe this new invariant for Taft categories. For a cyclic group the fundamental group of the Hopf algebra of maps to a field can be computed for small orders, which provides interesting abelian groups. This is a work in progress with Andrea Solotar.

LINEAR CATEGORIES AND ASSOCIATED RINGS

Guillermo Cortiñas

We will discuss some usually nonunital rings attached to a linear category and how they can be used to derive results about linear categories from results about rings.

MAHLER MEASURE, SPECTRAL RADIUS AND OTHER MEASURES FOR THE COXETER MATRIX OF AN ALGEBRA.

José Antonio de la Peña

REPRESENTATION DIMENSION OF CLUSTER TILTED ALGEBRAS

Alfredo González Chaio

The aim of this talk is to study the representation dimension of the cluster tilted algebras. We say that B is a cluster tilted algebra if B is the endomorphism algebra $End_C(\tilde{T})$ where $\tilde{T}$ is a cluster tilting object over a cluster category $\mathcal{C}$. The cluster tilting object $\tilde{T}$ is always induced by a tilting module T over a hereditary algebra. The above fact allows us to get information of the algebra B by using the torsion pair $(\mathcal{F}, \mathcal{T})$ induced by T . More precisely, we study the case where one of the categories $\mathcal{F}$ or $\mathcal{T}$ is finite (contains a finite number of classes of non isomorphic indecomposable modules). Therefore there are two cases to analyze. $\tilde{T}$ is transjective or the cluster tilted algebra is of tame type. The first case corresponds to the cluster concealed algebras. In this case we prove that the representation dimension of these algebras is less than or equal to three. We show it by constructing an explicit Auslander generator for mod$B$. The second case, we prove that the weak representation dimension of the tame cluster tilted algebras is less than or equal to 3. In the proof of this result we use an explicit description of the torsion theory of the hereditary tame algebras. It is fundamental for this study to use of the shape of the regular components of the Auslander-Reiten quiver. The module that we construct in this case is not always a cogenerator. Thus, the representation dimension can not be always obtained in this way. Finally, we show for which algebras it is possible to get the representation dimension in this way.

m-CLUSTER TILTED ALGEBRAS OF TYPE $\tilde{\mathbb{A}}$

Viviana Gubitosi

In this talk , we characterize all the finite dimensional algebras that are m-cluster tilted of type $\tilde{\mathbb{A}}$. We show that these algebras are gentle and we give an explicit description of their quivers with relations.

CALABI-YAU ALGEBRAS AND THEIR USES

Patrick Le Meur

V. Ginzburg defined Calabi-Yau algebras in 2006 to enhance Calabi-Yau duality on (certain of) their derived categories, in the sense of Kontsevich. In particular, classical Poincaré dualities between (co)homology spaces may sometimes be derived by considering morphism spaces between relevant objects in these categories. From the viewpoint of representation theory, Calabi-Yau algebras have permitted several interactions with other parts of mathematics like resolution of singularities, cluster algebras, quantization and noncommutative geometry, to mention a few. This talk will survey the general properties of Calabi-Yau algebras and their possible uses in connection with representation theory and homological algebra. In particular, the following aspects will be considered: their definition and characterisation, the various means to construct such algebras, their deformations and the resulting connections to Poisson cohomology, their applications to cluster theory and their relevance in the resolution of Gorenstein singularities.

TRIANGULATED CATEGORIES WITH AN INFINITE CLUSTER STRUCTURE

Shiping Liu

A cluster category is a 2-Calabi-Yau triangulated category in which the cluster- tilting subcategories form a cluster structure in the sense of Buan-Iyama-Reiten- Scott. We are interested in the problem of constructing this kind of categories. In this talk, we shall only deal with cluster categories of types A_∞, and A_∞^∞. First, we shall show that these two types of cluster categories can be obtained in a canonical way, that is taking the canonical orbit categories of the bounded derived categories of finite dimensional representations of quivers of types A_∞, and A_∞^∞. Then, applying the technique of Galois covering, we shall show that cluster categories of type A_∞^∞ can be realized as bounded derived categories of finite dimensional algebras of finite representation type.

COHERENT SUBCATEGORIES OF FINITELY GENERATED Λ MODULES

Eduardo Marcos

We explore properties of coherent subcategories of the category modΛ of finitely generated Λ modules, for some artin algebra Λ. In particular we look at finitely generated subcategories and give a connection with the class of standard modules and standardly stratified algebras.

RELATIVE COTORSION PAIRS, FROBENUIS AND AUSLANDER-BUCHWEITZ

Octavio Mendoza

Joint work with: V. Becerril, M.A. Pérez y V. Santiago. In this talk we discuss relative cotorsion pairs (in thick subcategories o an abelian category) and the connection with the theory developped by M. Auslander y R.O. Buchweitz. We are going to introduce the notions of Frobenius pair and Auslander- Buchweitz context and we will present a bijective correspondence among relative cotorsion pairs, Frobenius pairs and Auslander-Buchweitz contexts.

CATEGORIFICATION

Agustín Moreno Cañadas

Joint work with: Pedro Fernandez, Isaias Marin, Veronica Cifuentes and Julian Serna. According to Ringel and Fahr *categorification* of an integer sequence means to consider numbers in the sequence as invariants of objects of a given cate- gory. They gave a categorification of Fibonacci numbers by using preprojective, preinjective and regular components of the 3-Kronecker quiver [1,2]. In this talk we describe how Kronecker modules, tiled orders or semimaximal rings (introduced by Zavadskij and Kirichenko) and indecomposable representa- tions of equipped graphs (introduced by Gelfand and Ponomarev) can be used to categorize Catalan numbers and the integer sequences A052558 and A016269 in the OEIS. We recall that such sequences count the number of ways of con- necting n + 1 equally spaced points on a circle with a path of n line segments ignoring reflections, and the number of two-point antichains in the powerset of an n-element set ordered by inclusion respectively [3].

[1] P. Fahr and C.M. Ringel, A partition formula for Fibonacci numbers. Journal of integer sequences 11 (2008)

[2] P. Fahr and C.M. Ringel, Categorification of the Fibonacci numbers. Arxiv 1107.1858v2 (2011)

[3] N.J.A Sloane, On line Encyclopedia of integer sequences, The OESI foundation

GERSTENHABER STRUCTURE OF THE HOCHSCHILD COHOMOLOGY OF A STRING ALGEBRA

María Julia Redondo

This talk will concern about the cup product and the Lie bracket defined in the Hochschild cohomology groups in the particular case of a string algebra. The description of generators of this cohomology allows us to compute these structures in order to get conditions for their non vanishing.

AUSLANDER-REITEN TRIANGLES

Edson Ribeiro Alvares

Auslander-Reiten sequences are important to understand or describe the module category of a finite dimensional algebra and, as one can expect, Auslander-Reiten triangles play a similar role in the bounded derived category. In module categories, an Auslander-Reiten sequence begins with a monomorphism and finishes with an epimorphism. However, in bounded derived categories we have another behavior for irreducible morphisms in an Auslander-Reiten triangle, which we will explain in this exposition.

POLYNOMIAL RECOGNITION OF CLUSTER ALGEBRAS OF FINITE TYPE

Elisângela Silva Dias

Joint work with Daiane Castonguay. Cluster algebras are a recent topic of study and have been shown to be a usefull tool to characterize structures in several knowledge fields. an important problem is to establish whether or not a given cluster algebra is of finite type. Using the standar definition the problem is infeasible since it uses mutations and that can lead to an infinite process. Barot Geiss and Zelevinski (2006) presented an easier way to verify if a given algebra is of finite type, by testing that all chordless cycles of the graph related to the algebra are cyclically oriented and that there exist a quasi-Cartan companion of the skew-symmetrizable matrix related to the algebra. We develop an algorithm that verifies these conditions and decides whether or not a cluster algebra is of finite type in polynomial time. The second part of the algorithm is used to prove that the more general problem to decide if a matrix has a positive quasi-Cartan companion is $\mathcal{NP}$.

PIECEWISE HEREDITARY INCIDENCE ALGEBRAS

Marcelo Silva

In this talk we are interested in the class of algebras which we call Phia, that is algebras which are at the same time Piecewise Hereditary and Incidence Algebras. We use the work of Elsa Fernandes to give classifications of special classes of such algebras. In particular we classify the ones which are derived equivalent to Hereditary algebras of Dynkin type. We also classify some other families. For the ones which are derived equivalent to canonical algebras we know that there is at most 3 weights. We also look at the relation between their first Hochschild cohomology group and its simple connectedness.

KOZUL CALCULUS

Andrea Solotar

I will present a calculus which is well-adapted to quadratic algebras. This calculus is defined in Koszul cohomology (homology) by cup products (cap products). Koszul homology and cohomology are interpreted in terms of derived categories. If the algebra is not Koszul, Koszul (co)homology provides different information than Hochschild (co)homology. Koszul homology is related to de Rham cohomology. If the algebra is Koszul, Koszul cohomology is related to Calabi-Yau property. The calculus is made explicit on a non-Koszul example. Quadratic algebras are associative algebras defined by homogeneous quadratic relations. Since their definition by Priddy, Koszul algebras form a widely studied class of quadratic algebras. In his monograph, Manin brings out a general approach of quadratic algebras (not necessarily Koszul), including the fundamental observation that quadratic algebras form a category which should be a relevant framework for a noncommutative analogue of projective algebraic geometry. According this general approach, non-Koszul quadratic algebras deserve certainly more attention. The goal here is to introduce new general tools for studying quadratic algebras. These tools consist in a (co)homology, called Koszul (co)homology, together with products, called Koszul cup and cap products. They are organized in a calculus, called Koszul calculus. If two quadratic algebras are isomorphic in the sense of the Manin category, their Koszul calculus are isomorphic. If the quadratic algebra is Koszul, the Koszul calculus is isomorphic to Hochschild (co)homology endowed with usual cup and cap products – called Hochschild calculus. In this introduction, we would like to describe the main features of the Koszul calculus and how they are involved in the course of the article. The Kozul homology $HK_\bullet(A, M)$ of a quadratic algebra A with coeficients in a bimodule M is defined by applying the functor $M \otimes_{A^e} -$ to the Koszul complex of A, analogously for the Koszul cohomology $HK^\bullet(A, M)$. If A is Koszul, the Koszul complex is a projective resolution of A, so that $HK_\bullet(A, M)$ (resp. $HK^\bullet(A, M)$) is isomorphic to the Hochschild homology $HH_\bullet(A, M)$ (resp. Hochschild cohomology $HH^\bullet(A, M)$). Restricting the Koszul calculus to $M = A$, we present a non-Koszul quadratic algebra A which is such that $HK_\bullet(A) \not\cong HH_\bullet(A)$ and $HK^\bullet(A) \not\cong HH^\bullet(A)$. So $HK_\bullet(A)$ and $HK^\bullet(A)$ provide further invariants associated to the Manin category, besides those provided by Hochschild (co)homology. We prove that Koszul homology (cohomology) is isomorphic to a Hochschild hyperhomology (hypercohomology), showing that this new homology (cohomology) becomes natural in terms of derived categories. For any unital associative algebra A, the Hochschild cohomology of A with coefficients in A itself, endowed with the cup product, has a richer structure provided by Gerstenhaber product $\circ$, called Gerstenhaber calculus [1]. When $\circ$ is replaced in the structure by the graded bracket associated to $\circ$, that is, the Gerstenhaber bracket $[-, -]$, the calculus becomes a Gerstenhaber algebra [1]. Next, the Gerstenhaber algebra and the Hochschild homologyof A, endowed with cap products, are organized in a Tamarkin-Tsygan calculus. In the Tamarkin-Tsygan calculus, the Hochschild differential b is defined from the multiplication μ of A and the Gerstenhaber bracket by $b(f) = [\mu, f]$ for any Hochschild cochain f. The obstruction to see the Koszul calculus as a Tamarkin-Tsygan calculus is the following: the Gerstenhaber product $\circ$ does not make sense on Koszul cochains. However, this negative answer can be bypassed by the fundamental formula of the Koszul calculus $b_K(f) = -[e_A, f]_{\smile_K}$ (*) where b_K is the Koszul differential, e_A is the fundamental 1-cocycle and f is any Koszul cochain. In formula (*), $[-, -]_{\smile_K}$ is the graded bracket associated to the Koszul cup product $\smile_K$. In other words, the Koszul differential may be defined from the Koszul cup product. Therefore, the Koszul calculus is simpler than the Tamarkin-Tsygan calculus, since no additional product such as $\circ$ is required to express the differential by means of a graded bracket. The Koszul calculus is more flexible since the formula (*) is valid for any bimodule M, while the definitions of Gerstenhaber product and bracket are meaningless when considering other bimodules of coefficients [2]; it is also more symmetric since there is an analogue of (*) in homology, where the Koszul cup product is replaced by the Koszul cap product. This is a joint work with Roland Berger and Thierry Lambre.

[1] M. Gerstenhaber, The cohomology structure of an associative ring, Ann. of Math. 78 (1963), 267-288.

[2] M. Gerstenhaber, S. D. Schack, Simplicial cohomology in Hochschild cohomology, J. of Pure Appl. Algebra 30 (1983).

[3] Yu. I. Manin, Quantum groups and non-commutative geometry, CRM, Université de Montréal, 1988.

[4] A. Polishchuk, L. Positselski, Quadratic algebras, University Lecture Series 37, AMS, 2005.

[5] S. Priddy, Koszul resolutions, Trans. Amer. Math. Soc. 152 (1970), 39-60.

SOME FACTS ON τ-TILTING MODULES

Pamela Suárez

An important class of modules are the tilting modules, introduced more than 30 years ago as a generalization of the Berstein-Gelfand-Ponomarev reflection functors. For any finite dimensional algebra over a field, it is well known that an almost complete tilting module can be completed into a tilting module in exactly one or two ways. Furthermore, it is possible to define a mutation. In general, a limitation of this mutation is that it is not always defined for any tilting module. In fact, this was one of the motivations to introduce the τ-tilting theory. In this talk, we present the concept of support τ-tilting modules and we give some examples and properties concerning them. Moreover, we show some generalizations of well known facts about tilting modules, to the context of support τ-tilting modules. In particular we consider some extension algebras, as one point extensions and split extensions, and we show that τ-tilting modules can be lifted to the extension algebras.

[1] Adachi, T., Iyama, O. and Reiten, I.; τ-tilting theory. Compositio Mathematica, 150(03), 415-452, 2014.

[2] Assem, I., Happel, D. and Trepode, S.; Extending tilting modules to one-point extensions by projectives. Communications in Algebra, 35(10), 2983-3006, 2007.

[3] Assem, I., and Marmaridis, N. (1998). Tilting modules over split-by- nilpotent extensions. Communications in Algebra, 26(5), 1547-1555.

[4] Assem, I, and Zacharia, D. (2003). On split-by-nilpotent extensions. In Colloq. Math (Vol. 98, No. 2, pp. 259-275). ISO 690

PREPROJECTIVE ALGEBRAS, STABILITY CONDITIONS AND COXETER GROUPS

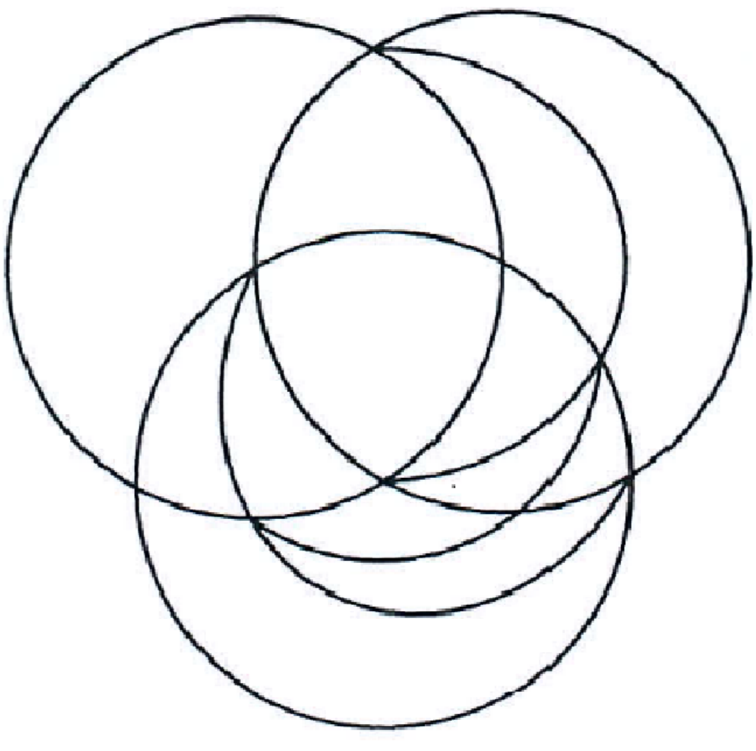
Hugh Thomas

I will recall the definition of stability conditions in the sense of geometric invariant theory (as carried into representation theory of finite dimensional algebras by King). I will give a quick introduction to coxeter groups for understanding the semistable subcategories of finite type prerojective algebras. This is a joint work with David Spayer and will also draw on a previous joint work with Nathan Reading, Idun Reiten and Osamu Iyama.

SEMI-INVARIANT PICTURES, C-VECTORS, MAXIMAL GREEN SEQUENCES

Gordana Todorov

This talk will be about several topics, with main emphasis on the connections between these topics. I will lightly (sometimes only with examples) introduce: quiver representations, their relation to root systems, Auslander-Reiten quivers, derived categories of the representation category, cluster category, cluster tilting objects; I will define semi-invariants, domains of semi-invariants, c-vectors from cluster theory, mutations, green mutations as mutations in the direction of non-negative c-vectors, sequences of green mutations (the last notions are related to some of the notions in BPS states in physics). I will state two conjectures that we proved using semi-invariant pictures and indicate how the semi-invariant pictures were used. Below is the semi-invariant picture for the following quiver of type $\mathbb{A}_3$ $Q = 1 \leftarrow 2 \leftarrow 3$. Each part of a semi-invariant picture (vertex, line segment, triangle, circle, semi-circle) has several interpretations, and I will be adding the following information on the picture: roots of Lie algebra, indecomposable quiver representations, cluster objects, cluster tilting objects, domains of semi-invariants, c-vectors, mutations, green mutations, maximal green sequences.



HOCHSCHILD COHOMOLOGY OF A SMASH PRODUCT WITHA CYCLIC GROUP

Y. Volkov

This talk is based on joint work with E. Marcos. Let A be a finite-dimensional unital algebra over a field k and G be a finite group acting on A. Then we can define a smash product of A and kG, which is denoted by $A\#kG$. Let M be an $A\#kG$-bimodule with compatible structure of k-algebra. It is well known that there is an isomorphism of graded algebras $HH^*(A\#kG, M) \simeq HH^*(A, M)$ in the case where the group algebra kG is semisimple. On the other hand, there is no familiar good connection between the Hochschild cohomology of A and the Hochschild cohomology of $A\#kG$ in the p-modular case. We consider the case where G is a trivial extension of a cyclic p-group by some p- group, where $p = chark$. We give a definition of (m, h)-degenerated $A\#kG$-bimodule. We prove that $dim_k HH^n(A, M)^G \leq dim_k HH^n(AG, M)$ for any n and $dim_k HH^n(A, M)^G = dim_k HH^n(AG, M)$ for any n iff M is (m, h)-degenerated. For (m, h)-degenerated M we establish some relations between $HH^*(A\#kG, M)$ and $HH^*(A, M)^G$. We give some applications for the case where A is a factor of some repetetive algebra by a power of its Nakayama automorphism and G is a cyclic group generated by a Nakayama automorphism of A.

ON THE REPRESENTATION DIMENSION OF ARTIN ALGEBRAS

Heily Wagner

When studying algebras of finite representation type, Auslander (1971) proved that, given a non semisimple algebra of finite representation type and M the direct sum of all its non-isomorphic indecomposable modules, the global dimension of $EndM$ is two. His findings led to the definition of an invariant, the so-called representation dimension of a non-semisimple artin algebra, as the infimum of the global dimensions of $EndM$, where M is a module which admits each indecomposable injective and projective modules as direct summand. Auslander showed that an artin algebra is representation-finite if and only if its representation dimension equals two, which made him expect that this invariant would give a measure of how far an algebra is from being representation-finite. In the past few years it was proven that many classes of algebras have representation dimension at most three. For instance, tilted (Assem, Platzeck, Trepode - 2006), quasitilted (Oppermann - 2010), cluster-concealed (González Chaio, Trepode - 2013) and ada algebras (Coelho, Wagner - 2014). This show us that the behavior of the module categories of algebras with representation dimension three can be very different, including tame and wild algebras.

LINEAR MODULES AND EXCEPTIONAL SHEAVES ON THE PROJECTIVE n-SPACE

Dan Zacharia

I will present some joint work with Otto Kerner and with Helmut Lenzing. I plan to talk on a representation theoretical approach to studying some problems in algebraic geometry. Let V be an $n+1$-dimensional vector space over an algebraically closed field k, let $R = \wedge V$ be the exterior algebra on V , and let $S = k[x_0, \ldots, x_n]$ be the polynomial algebra in $n + 1$ indeterminates. Finally, let $cohP^n$ denote the category of coherent sheaves on the projective n-space. We will work in the context of these objects. A coherent sheaf E is called rigid if $Ext^1(E, E) = 0$. E is called exceptional if $Ext^i(E, E) = 0$ for all $i > 0$ and, in addition E has an endomorphism ring isomorphic to k. I will talk how to reduce certain problems about rigid and exceptional sheaves to solving problems that involve linear modules over the polynomial algebra and over the exterior algebra.

BEHAVIOUR OF INJECTIVE DIMENSION WITH RESPECT TO REGRADINGS

Pablo Zadunasky

Given a left noetherian k-algebra A graded by a group G, an injective object I in the category of G-graded A-modules and a morphism from G to another group G', we provide bounds for the injective dimension of I as a G'-graded A-module. For this we use three change of grading functors between the categories of G-graded and G'-graded A-modules this functors can be defined at the level of H-comodule algebras, where H is a Hopf algebra, so we introduce them in this general context.

SINGULAR LOCUS OF ORBIT CLOSURES FOR REPRESENTATIONS OF DYNKIN QUIVERS

Grzegortz Zwara

Let M be a finite dimensional representation of a Dynkin quiver Q, and let X be the (Zariski) closure of the orbit corresponding to M. It is an interesting problem to characterize the singular locus of X in module theoretic terms. The answer is relatively easy if Q is of type $\mathbb{A}$. We show a characterization of singular locus for type $\mathbb{D}$ (which is a joint work with Grzegorz Bobiński).