Bitcoin Selfish Mining and Dyck Words (with C. Grunspan)

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Breaking Bitcoin 2019

Amsterdam

June 7, 2019

Block withholding strategies

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3 Markov model.

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7 Selfish Mining in Ethereum

Bibliography

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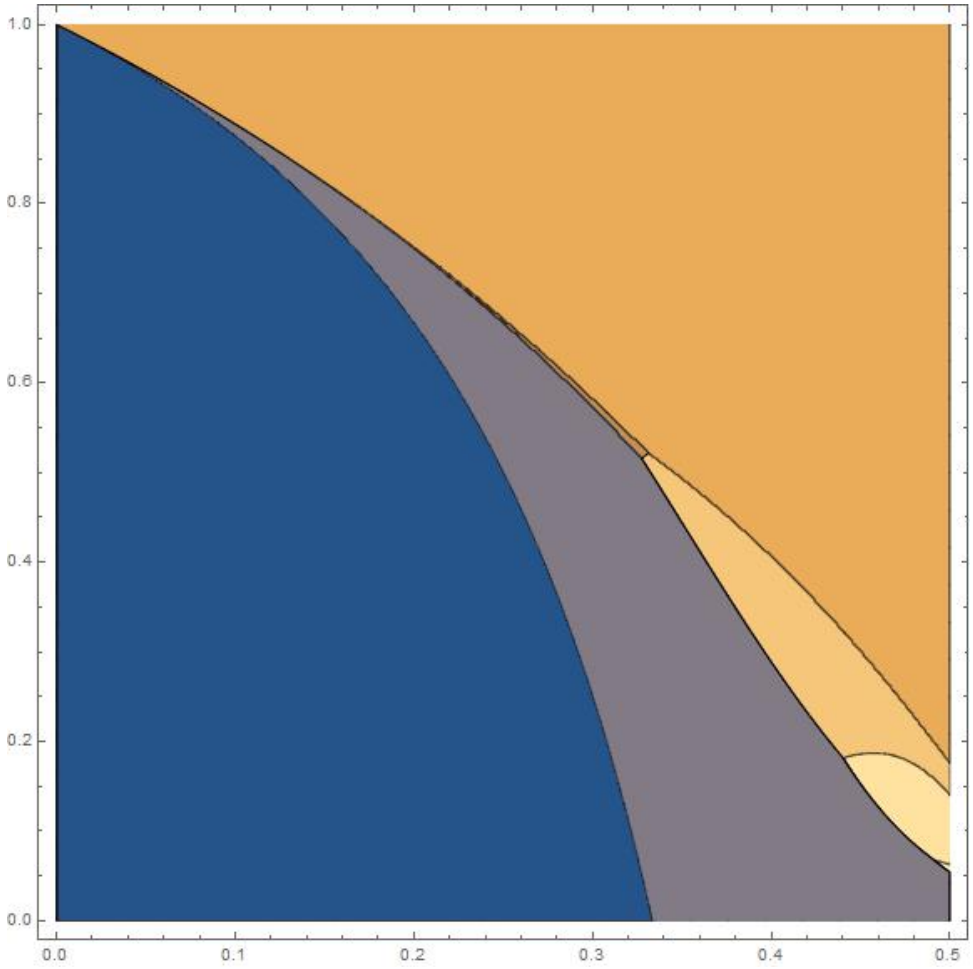
Results



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And the asymptotic $P\&L$ per unit of time

$$P\&LT_\infty = \lim_{T \to +\infty} \frac{R(T) - C(T)}{T}$$

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Proof. Strong law of large numbers.

Comparing repetition games with equal costs

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Definition (Revenue and Cost Ratio)

The Revenue Ratio, resp. Cost Ratio, of a Repetition Strategy ξ is

$$\Gamma(\xi) = \frac{\mathbb{E}[R]}{\mathbb{E}[T]} \quad \textit{resp.} \quad \Upsilon(\xi) = \frac{\mathbb{E}[C]}{\mathbb{E}[T]}$$

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Theorem (Comparing profitabilities of equal cost strategies)

We consider two integrable strategies with $\Upsilon(\xi) = \Upsilon(\xi')$. Then ξ is more profitable than strategy ξ' in the long run if and only if

$$\Gamma(\xi') \leq \Gamma(\xi) \, .$$

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- Key observation:

Block withholding and honest strategies have exactly the same cost

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• The Cost depends on external factors like energy prize, hardware costs, etc These costs are independent of the strategy as long as the strategy is a full time intensive computation.

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$$\mathbb{E}[\tau] = \frac{1}{\alpha - \alpha'}\ , \quad \mathbb{E}[N'(\tau)] = \frac{\alpha'}{\alpha - \alpha'}\ , \quad \mathbb{E}[N(\tau)] = \frac{\alpha}{\alpha - \alpha'}\ .$$

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- It uses Dyck words, Catalan numbers, and Catalan distributions.

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so $C_0 = 1, C_1 = 1, C_2 = 2, C_3 = 5, C_4 = 14, C_5 = 42, C_6 = 132, \ldots$

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- Catalan distributions: For $0 < q < p < 1$ with $p + q = 1$:

$$\sum_{n\geq 0} p(pq)^n C_n = 1$$

$$\sum_{n\geq 0} np(pq)^n C_n = \frac{q}{p-q}$$

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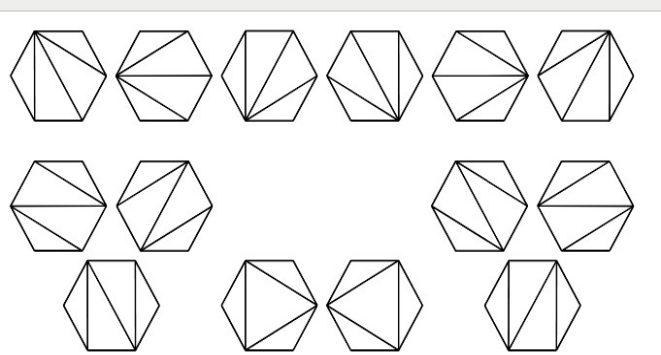
- Number of triangulations of a convex polygon with $n + 2$ sides.



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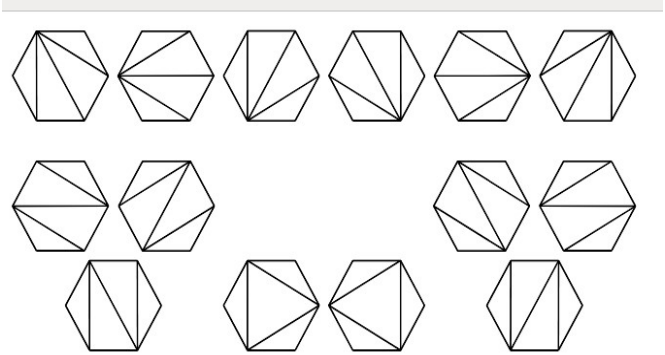
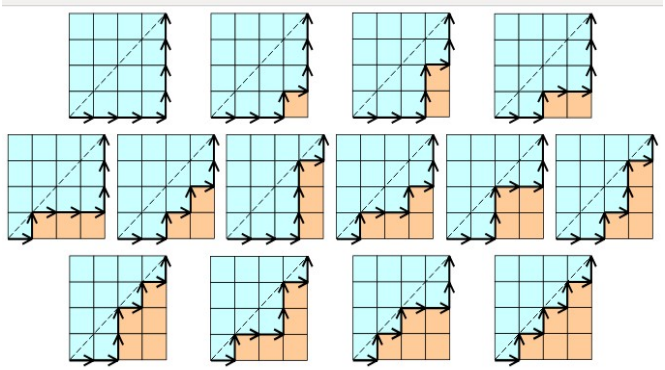


- Number of monotonic lattice paths on a $n \times n$ square grid from one corner to the opposite without crossing the diagonal.



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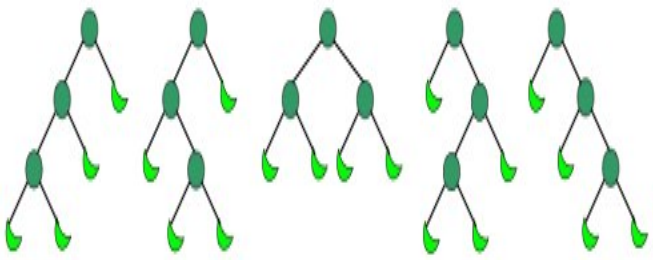

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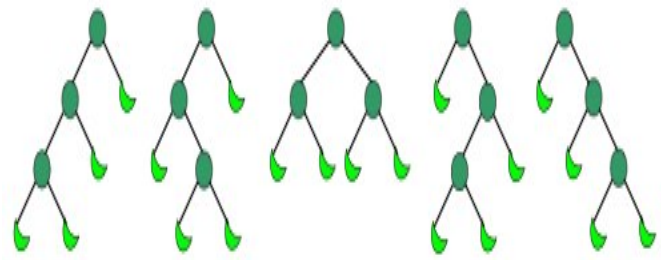

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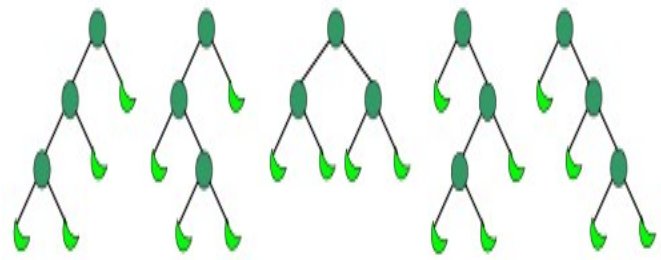

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A Dyck word built on $\{S, H\}$ is a string consisting of n S's and n H's such that no initial segment of the string has more H's than S's.

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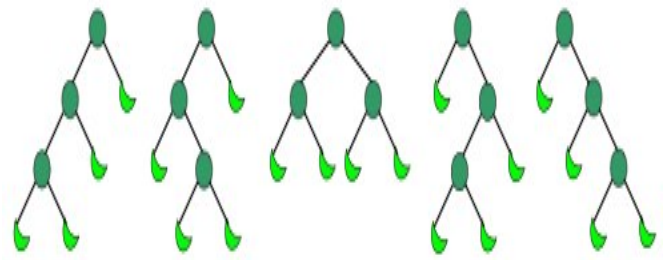

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- Let $\mathcal{D}$ be the space of all Dyck words. Catalan distributions define probability measures on $\mathcal{D}$.

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Corollary

Let L be the number of official blocks added to the blockchain after an attack cycle. Then, $\mathbb{P}[L = 1] = p$, $\mathbb{P}[L = 2] = p + pq^2$ and

$$\mathbb{P}[L = n] = pq^2 (pq)^n C_{n-2}$$

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Sketch of proof.

We have $\mathbb{E}[T] = \mathbb{E}[L]\tau_0$, $\mathbb{E}[Z] = \mathbb{E}[L] - \left(p + (2-\gamma)p^2q\right)$ and $\mathbb{E}[L] = 1 + \frac{p^2q}{p-q}$. □

Other classical block withholding strategies

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Comparison of strategies

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We plot the region $(q, \gamma) \in [0, 0.5] \times [0, 1]$ showing which strategy is more profitable.

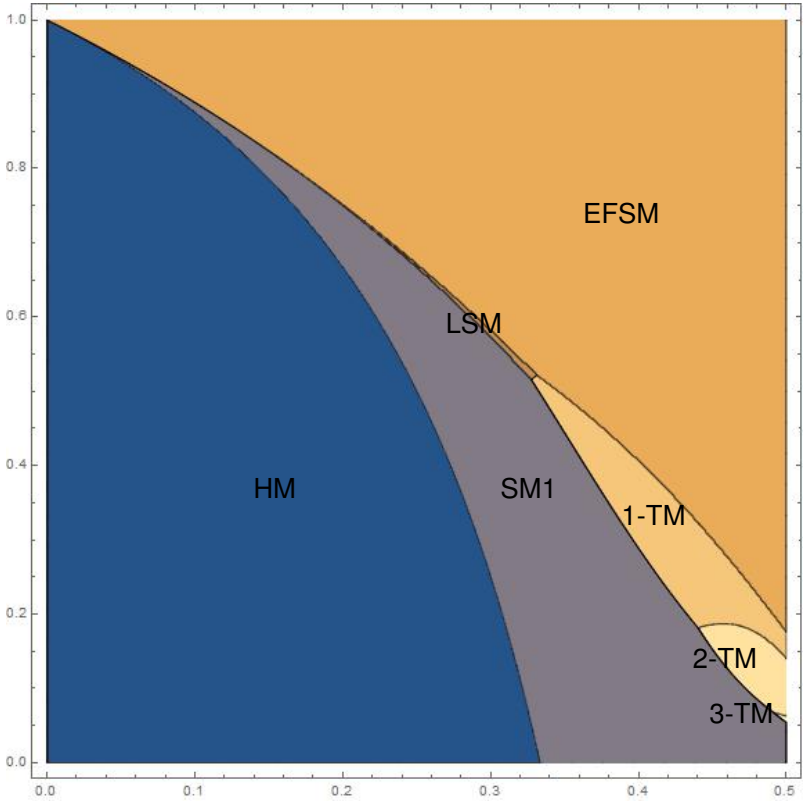

Figure: Most profitables strategies.

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- Different reward system and difficulty adjustment algorithm.

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- Best strategy (for $q > 25\%$): Avoid competitions and ignore uncles.

Selfish Mining in Ethereum

Theorem (Comparison of SM strategies in Ethereum)

We have $\Gamma(SM1) < \Gamma(SM2A)$. *For* $q > 30.1\,\%$ $\Gamma(SM2A) < \Gamma(SM2B)$

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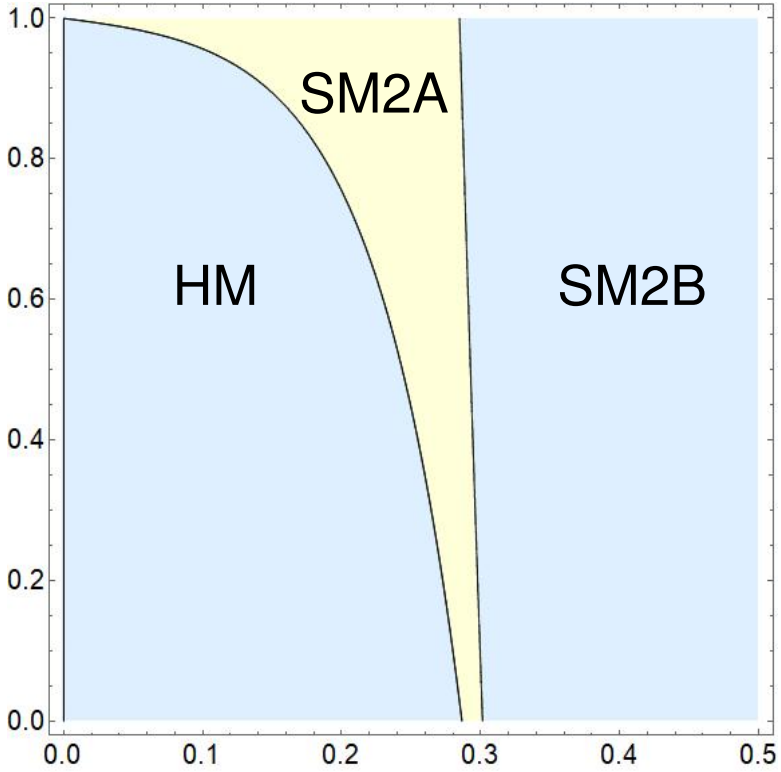

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- For computing only long term profitabilities, the Markov approach can be replaced by the more direct and simple combinatorial approach with Dyck words.
- Ethereum Selfish Mining confirms that SM is an attack to the DA.

Thank you for your attention!