On instabilities of the Bitcoin protocol

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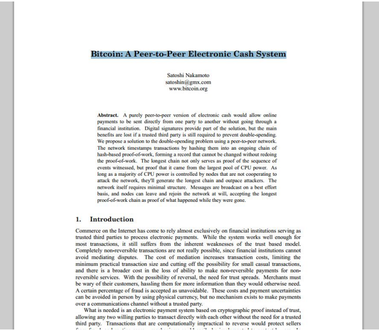

Bitcoin: A Peer-to-Peer Electronic Cash System

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Abstract. A purely peer-to-peer version of electronic cash would allow online payments to be sent directly from one party to another without going through a financial institution. Digital signatures provide part of the solution, but the main benefits are lost if a trusted third party is still required to prevent double-spending. We propose a solution to the double-spending problem using a peer-to-peer network. The network timestamps transactions by hashing them into an ongoing chain of hash-based proof-of-work, forming a record that cannot be changed without redoing the proof-of-work. The longest chain not only serves as proof of the sequence of events witnessed, but proof that it came from the largest pool of CPU power. As long as a majority of CPU power is controlled by nodes that are not cooperating to attack the network, they'll generate the longest chain and outpace attackers. The network itself requires minimal structure. Messages are broadcast on a best effort basis, and nodes can leave and rejoin the network at will, accepting the longest proof-of-work chain as proof of what happened while they were gone.

1. Introduction

Commerce on the Internet has come to rely almost exclusively on financial institutions serving as trusted third parties to process electronic payments. While the system works well enough for most transactions, it still suffers from the inherent weaknesses of the trust based model. Completely non-reversible transactions are not really possible, since financial institutions cannot avoid mediating disputes. The cost of mediation increases transaction costs, limiting the minimum practical transaction size and cutting off the possibility for small casual transactions, and there is a broader cost in the loss of ability to make non-reversible payments for non-reversible services. With the possibility of reversal, the need for trust spreads. Merchants must be wary of their customers, hassling them for more information than they would otherwise need. A certain percentage of fraud is accepted as unavoidable. These costs and payment uncertainties can be avoided in person by using physical currency, but no mechanism exists to make payments over a communications channel without a trusted party.

What is needed is an electronic payment system based on cryptographic proof instead of trust, allowing any two willing parties to transact directly with each other without the need for a trusted third party. Transactions that are computationally impractical to reverse would protect sellers

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Theorem

Transparency Theorem: *An electronic decentralized currency must rely on a public ledger.*

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- The blocks are generated by “miners” that validate current transactions.

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- The mechanism of consensus: “The trust machine”.

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The trust machine
How the technology behind bitcoin could change the world
IN BLOCKCHAIN WE TRUST

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The Byzantine Generals Problem.
The situation can be described as the siege of a city by a group of generals of the Byzantine army. Communicating only by messenger, the generals must agree upon a common battle plan. However, one or more of them may be traitors who will try to confuse the others. The problem is to find an algorithm to ensure that the loyal generals will reach an agreement.

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- The difficulty is adjusted to find a solution in about 10 minutes.
- The miner that solves it receives an award in newly created bitcoins.

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- Big pools are a thread to mining decentralization.
- Monopole position on mining hardware manufacturing is a thread to mining decentralization.

Selfish mining attack

(join work with C. Grunspan)

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• Costs of this strategy are not properly accounted in the literature.

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- Only viable with more than $30 - 40\%$ of the hashrate.

Selfish-mining and Nash equilibrium

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- After all, the protocol is well aligned.

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- There is a unique $v_m^n = (E_m^n)^{-1}(0)$.

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Summation formula

Theorem (Formula with generalized Dyck paths)

$$E_m^n(v) = \sum_{f \in \mathcal{D}_m^n} \pi(f)(f - w(f))_+$$

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Practical application

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Theorem

If $q > 0.43$, $m = 2$, and $b > 0$ is the block reward, then $\lim_{n\to+\infty} E_n^2(3b) > 0$.

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This means that it makes sense to catch-up mining 2 blocks behind if your hashrate is over 43%.

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The bitcoin protocol is unstable with respect to catch-up mining.

Sorry for the formulas...

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...and thank you for your attention!!