On uniform boundedness of Brauer groups (joint with François Charles)

Conference on Algebraic Geometry and Number Theory on the occasion of Jean-Louis Colliot-Thélène's 70th birthday Villa Finaly, Florence, December, 4th-6th 2017

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- k: finitely generated field of characteristic $p \ge 0$
- \overline{k} : separable closure, $\pi_1(k) := Gal(\overline{k}|k)$
- X : smooth proper variety over k

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Lemma (Colliot-Thélène) (Relation with the Tate conjecture for divisors) $p \neq \ell$: prime. The following assertions are equivalent

- $(1) c_1 : \operatorname{Pic}(X_{\overline{k}}) \otimes \mathbb{Q}_{\ell} \twoheadrightarrow \varinjlim_{U \subset \pi_1(k) \text{ open }} H^2(X_{\overline{k}}, \mathbb{Q}_{\ell}(1))^U;$
- (2) $Br(X_{\overline{k}})^U[\ell^{\infty}]$ is finite for every open subgroup $U \subset \pi_1(k)$.

 $B := Br(X_{\overline{k}})$

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 (Kummer)

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$$(2) \Rightarrow (1), \ (1) \Rightarrow (2) :$$

$$\begin{split} B &:= Br(X_{\overline{k}}) \\ T_{\ell} &:= \varprojlim B[\ell^n], \ V_{\ell} := T_{\ell} \otimes \mathbb{Q}_{\ell}, \ M_{\ell} := T_{\ell} \otimes \mathbb{Q}_{\ell} / \mathbb{Z}_{\ell}, \ M_{\ell}[\ell^n] \simeq T_{\ell} / \ell^n \\ & 1 \to \mu_{\ell^n} \to \mathbb{G}_m \stackrel{(-)^{\ell^n}}{\to} \mathbb{G}_m \to 1 \quad \text{(Kummer)} \\ 0 \to NS(X_{\overline{k}}) \otimes \mathbb{Z} / \ell^n \to H^2(X_{\overline{k}}, \mu_{\ell^n}) \to B[\ell^n] \to 0 \\ & (*) \quad 0 \to NS(X_{\overline{k}}) \otimes \mathbb{Q}_{\ell} \to H^2(X_{\overline{k}}, \mathbb{Q}_{\ell}(1)) \to V_{\ell} \to 0 \\ \end{split}$$

$$(2) \Rightarrow (1), \ (1) \Rightarrow (2) : \text{ May assume } U \subset \pi_1(k) \text{ small enough so that} \\ NS(X_{\overline{k}})^U = NS(X_{\overline{k}}) \text{ and } c_1 : \operatorname{Pic}(X_{\overline{k}}) \otimes \mathbb{Q}_{\ell} \to H^2(X_{\overline{k}}, \mathbb{Q}_{\ell}(1))^U \end{split}$$

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$$0 \to M_{\ell} \to B[\ell^{\infty}] \to \mathrm{H}^{3}(X_{\overline{k}}, \mathbb{Z}_{\ell}(1))[\ell^{\infty}] \to 0$$

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S smooth variety over $k, f: X \rightarrow S$ smooth, proper

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Conjecture 1 For every integer $d \ge 1$,

 $\sup\{|Br(X_{\overline{s}})^{\pi_1(s)}[\ell^{\infty}]| \mid s \in S(\leqslant d)\} < +\infty,$

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where $S(\leqslant d) := \{s \in |S| \mid [k(s):k] \leqslant d\}$

• For $d \ge 2$: rather a question (even if p = 0)

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- Special case of a general conjecture on motivic rep. of $\pi_1(S)$

S smooth variety over $k, f: X \to S$ smooth, proper **Assume** $X_s, s \in |S|$ **satisfies the** ℓ -adic **Tate conjecture for divisors** (e.g. families of abelian varieties or K3 surfaces)

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- Relax the condition on the ℓ -adic Tate conjecture for divisors

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$$\begin{array}{cccc} 0 \longrightarrow \mathit{NS}(X_{\overline{\eta}}) \otimes \mathbb{Z}_{\ell} \longrightarrow \mathrm{H}^{2}(X_{\overline{\eta}}, \mathbb{Z}_{\ell}(1)) \longrightarrow \mathit{T}_{\ell}(\mathit{Br}(X_{\overline{\eta}})) \longrightarrow 0 \\ & & & \downarrow & & \downarrow \\ 0 \longrightarrow \mathit{NS}(X_{\overline{s}}) \otimes \mathbb{Z}_{\ell} \longrightarrow \mathrm{H}^{2}(X_{\overline{s}}, \mathbb{Z}_{\ell}(1)) \longrightarrow \mathit{T}_{\ell'}(\mathit{Br}(X_{\overline{s}})) \longrightarrow 0 \end{array}$$

 $\pi_1(s) \to \pi_1(S) \xrightarrow{\rho_\ell} \mathrm{GL}(\mathrm{H}^2(X_{\overline{\eta}}, \mathbb{Q}_\ell(1)))$

 $S_{\ell}^{gen} := \{ s \in S \mid \rho_{\ell}(\pi_1(s)) \subset \rho_{\ell}(\pi_1(S)) \text{ open} \}$

S smooth variety over $k,\;f:X\to S$ smooth, proper $s\in S,\;\forall\ell\neq\rho$

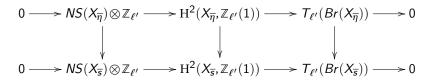
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Lemma Assume p = 0 and $s \in S_{\ell}^{gen}$ for some ℓ . Then one has canonical π_1 -equivariant isomorphisms

$$NS(X_{\overline{\eta}}) \xrightarrow{\sim} NS(X_{\overline{s}}), Br(X_{\overline{\eta}}) \xrightarrow{\sim} Br(X_{\overline{s}})$$

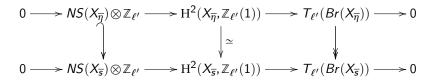
 $\forall \ell'$



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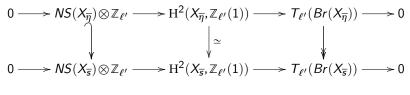
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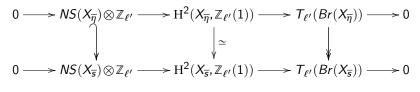
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Fix $c \in NS(X_{\overline{s}})$.

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$$\mathsf{Fix} \ c \in \mathsf{NS}(X_{\overline{s}}). \ s \in S^{gen}_{\ell} \Rightarrow \mathsf{may} \text{ assume } c \in \mathrm{H}^2(X_{\overline{s}}, \mathbb{Z}_{\ell}(1))^{\pi_1(S_{\overline{k}})}$$

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$$\begin{split} & \mathsf{Fix} \ c \in \mathit{NS}(X_{\overline{s}}). \ s \in S_{\ell}^{gen} \Rightarrow \mathsf{may} \ \mathsf{assume} \ c \in \mathrm{H}^2(X_{\overline{s}}, \mathbb{Z}_{\ell}(1))^{\pi_1(S_{\overline{k}})} \\ & \mathsf{Fix} \ \mathsf{a} \ \mathsf{smooth} \ \mathsf{compactification} \ X \hookrightarrow \overline{X}. \end{split}$$

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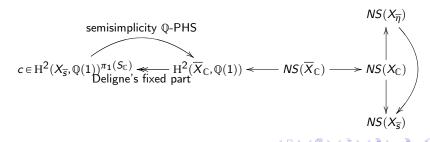
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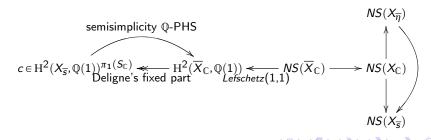
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Fix a smooth compactification $X \hookrightarrow X$. Comparison Betti/ ℓ -adic



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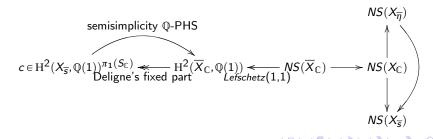
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$$NS(X_{\overline{\eta}}) \cong NS(X_{\overline{s}})$$

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$$NS(X_{\overline{\eta}}) \cong NS(X_{\overline{s}})$$

 $+ Br(X_{\overline{s}}), Br(X_{\overline{\eta}})$ torsion

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Lemma For every $s \in S_{\ell}^{gen}$,

 $NS(X_{\overline{\eta}}) \xrightarrow{\sim} NS(X_{\overline{s}}), Br(X_{\overline{\eta}}) \xrightarrow{\sim} Br(X_{\overline{s}})$

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After possibly replacing S by a connected étale cover, assume (Serre) the Zar-closure of $\rho_{\ell}(\pi_1(S))$ is connected for every $\ell \neq p$

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Conjecture 2 $S_{\ell}^{gen}(\leq d) \subset S$ is not Zariski-dense and $\sup\{[Br(X_{\overline{s}})^{\pi_1(s)}[\ell^{\infty}]: Br(X_{\overline{\eta}})^{\pi_1(S)}[\ell^{\infty}]] \mid s \in (S \setminus S_{\ell}^{gen})(\leq d)\} < +\infty$

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• Conjecture $2 \Rightarrow$ Conjecture 1

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• Conjecture $2 \Rightarrow$ Conjecture 1

Theorem (Ca-Ch, 2016) *If* p = 0 and *S* is a curve then Conjecture 2 (hence Conjecture 1) holds.

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 Special families of (K3, abelian) surfaces over number fields by reducing to questions about rational points on X₀(n) (Manin, Faltings-Frey, Mazur, Merel)
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$$M_{\ell}^{U_d}/M_{\ell}^{\pi_1(S)} \subset \operatorname{im}(\delta_U)/\operatorname{im}(\operatorname{res} \circ \delta_{\Pi}) \leftarrow \operatorname{im}(\delta_U) \subset \operatorname{H}^1(U_{d,\ell}, \mathcal{T}_{\ell})[\ell^{\infty}]$$

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→ Mordell conjecture (Faltings)

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3.5

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Comparison of various categories of *p*-adic coefficients

Bol'shoye Spasibo

Bahut Dhanyavaad

Xie Xie

Merci beaucoup

GRAZIE MILLE

Thank you very much

Vielen Dank

Arigato Gozaimasu

Nagyon Köszönöm

Muchas Gracias

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