CORRECTION TO: HYPER-KÄHLER FOURFOLDS AND GRASSMANN GEOMETRY

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Here are a few comments on [DV, Theorem 2.2(1)] and its proof.

First of all, [DV, Lemma 2.3] is unnecessary: there is an exact sequence ([V, (6.3)])

$$H^{21}(G(3,V_{10}),\mathbf{Q}) \to H^{21}(U,\mathbf{Q}) \xrightarrow{\text{Res}} H^{20}(F_\sigma,\mathbf{Q})_{\text{van}} \to 0$$

from which one deduces immediately, since all odd-degree cohomology groups of $G(3,V_{10})$ vanish, that the residue map is an isomorphism.

Secondly, Laurent Manivel noticed that the claim

$$H^i(G(3,V_{10}),\Omega^j_{G(3,V_{10})}(k)) = 0 \qquad \text{for all } k>0,\ i>0,\ j\geq 0, \tag{1}$$

made on [DV, p. 68, l. 4] is wrong, since $H^{12}(G(3,V_{10}),\Omega^6_{G(3,V_{10})}(3))$ is non-zero. However, to apply Griffiths' theory, we only need that the vanishing (1) hold for $i = n-j \leq n-k$, where $n = \dim(G(3,V_{10})) = 21$, and this vanishing does hold by Bott's theorem.

Finally, Nicholas Addington suggested a way to compute directly (with a computer) the Hodge numbers of F_σ: by the Lefschetz hyperplane theorem, we have $h^{i,j}(F_\sigma) = h^{i,j}(G(3,V_{10}))$ for $i+j<20$, and $h^{i,j}(F_\sigma)=0$ when $i+j\neq 20$ unless $i=j$. For $k<10$, we obtain

$$\begin{aligned}
\chi(F_\sigma,\Omega^k_{F_\sigma}) &= (-1)^k h^{k,k}(F_\sigma) + (-1)^k h^{k,20-k}(F_\sigma)\\
&= (-1)^k h^{k,k}(G(3,V_{10})) + (-1)^k h^{k,20-k}(F_\sigma)\\
&= \chi(G(3,V_{10}),\Omega^k_{G(3,V_{10})}) + (-1)^k h^{k,20-k}(F_\sigma),
\end{aligned}$$

whereas $\chi(F_\sigma,\Omega^{10}_{F_\sigma}) = h^{10,10}(F_\sigma)$ and $\chi(G(3,V_{10}),\Omega^{10}_{G(3,V_{10})}) = h^{10,10}(G(3,V_{10}))$. In particular,

$$h^{k,20-k}(F_\sigma)_{\text{van}} = (-1)^k\big(\chi(F_\sigma,\Omega^k_{F_\sigma}) - \chi(G(3,V_{10}),\Omega^k_{G(3,V_{10})})\big)$$

for all $k\leq 10$. The computer program Macaulay2 computes these Euler characteristics and finds

k	0	1	2	3	4	5	6	7	8	9	10
$\chi(F_\sigma,\Omega^k_{F_\sigma})$	1	-1	2	-3	4	-5	7	-8	9	-11	30
$\chi(G(3,V_{10}),\Omega^k_{G(3,V_{10})})$	1	-1	2	-3	4	-5	7	-8	9	-10	10
$h^{k,20-k}(F_\sigma)_{\text{van}}$	0	0	0	0	0	0	0	0	0	1	20

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References

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