



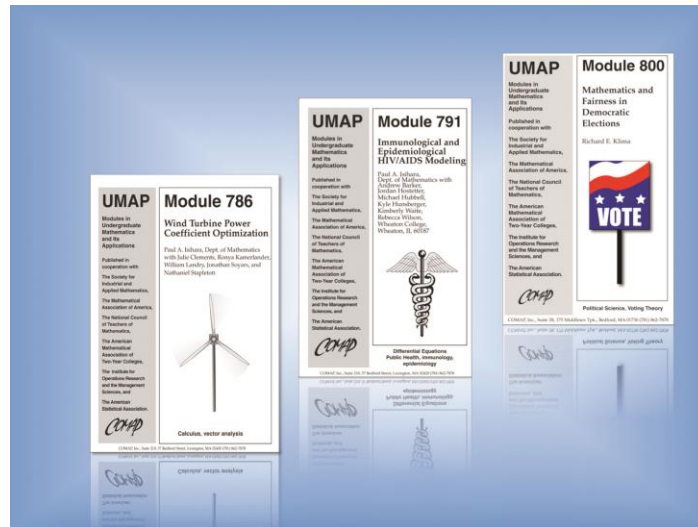
## **Making Cultural Change**

Here is a revisionist personal history. Roughly 50 years ago I decided that U.S. mathematics education needed to undergo a radical change. In my opinion mathematics education needed to morph from being primarily concerned with the education of mathematicians to being primarily concerned with the mathematics education of the general public. Leaving aside the wisdom of that belief, assuming it as a working article of faith – then what?

If you skip the many starts and stops and enthusiasms of youth, I came to another article of faith – that average students would learn more mathematics if they saw a reason to learn that mathematics. – A reason in the real world. And that led me to mathematical modeling. Not having any real experience with modeling and applications (I was a mathematical logician by training) I was fortunate to have Henry Pollak as a mentor. Pollak was the head of mathematics research at Bell Labs for over 28 years and also a past president of the Mathematical Association of America.

But the question remained, how does one go about such a paradigm shift in an educational system so resistant to ANY form of change. One more article of faith – you can't beat something with nothing. In other words, it is not enough (unless you are French) to simply argue a philosophical point of view that things must change, you need to show people, in concrete terms, what you mean by change. And so we began making 'stuff'. We built an organization to create teacher and student materials that embodied our philosophy.

We began small both in the sense of a small working group and in terms of what we produced. We created curriculum modules that were meant to be covered in one hour of undergraduate class time. These modules could range over the entire undergraduate mathematics curriculum and they were meant to be self-contained. Each taught some aspect of mathematics through a real application or model. They were reviewed not only by academic mathematicians, but by teachers and practitioners in the field of application.



In a town with the streets and houses marked, where do you put the fire station; in a long race like a marathon or triathlon how do you group the runners at the start so that they don't interfere with each other yet finish the race before dark, if you read in the newspaper that the rainfall in France last year was so many centimeters, what does that mean, how is it calculated; how deep should you dig a root cellar to keep it at a constant temperature? And on and on.

Of course, there is no complete set of materials that can present all of the uses of all of undergraduate mathematics. And new applications are being discovered all of the time. So, we needed a process that could continue and be self-sustaining. Moreover, we wanted to locate this work squarely within the center of the academy – having it be part of the academic rewards system.

To do both of these things we founded the UMAP Journal. This is a peer reviewed journal containing articles about modeling, new applications of mathematics, and actual student ready materials. It is now in its 37<sup>th</sup> year and just like any research journal authors and reviewers consider it part of their academic responsibilities to work for it. And publication figures into tenure and promotion decisions.



Some more history – the UMAP work was begun in the late 70s with initial funding from the National Science Foundation (NSF). In those days essentially all federal funding for mathematics education came from NSF to colleges and universities. There was little or no work K-12. That all changed on 1983/84 with the publication of a series of reports starting with “A Nation at Risk”. That report lauded the U.S. math and science education at the tertiary level but pointed out deep deficiencies at the school level. This gave rise to new funds being made available for elementary and secondary STEM education.

COMAP hoped to build on our success at the college level and soon received a number of grants to produce modular material in applications and modeling at the secondary level.



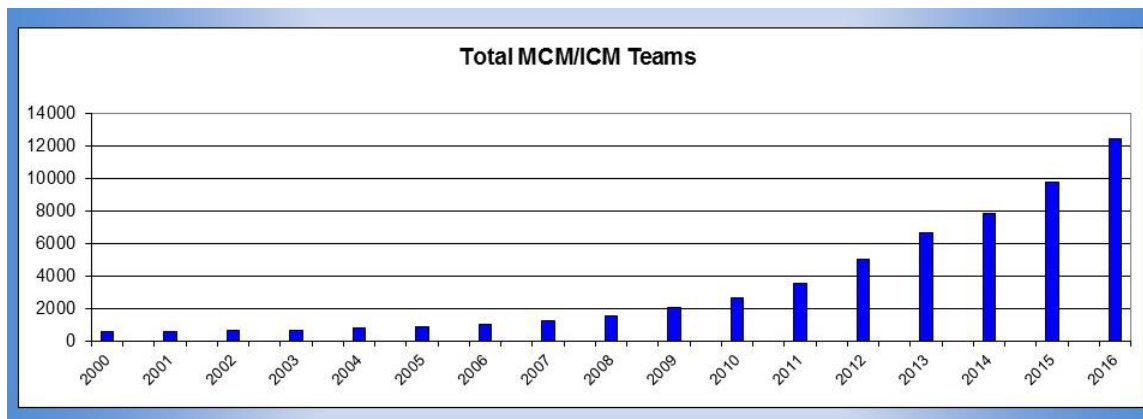
This work helped provide a philosophical foundation for the Standards produced by the National Council for the Teaching of Mathematics (NCTM) in 1989. That document gave rise to many curriculum experiments at the K-12 that have had a profound effect on the introduction of real world problem solving at all academic levels.

I want to be careful here not to oversell. Much of what is taught today and how it is taught is very traditional. But there is no doubt that with technological change and a grudging recognition of the importance of taking a more interdisciplinary approach to math and science curricula things are changing – even for math majors and gifted students.

One last trip back in time: In 1984 COMAP received a small 3 year grant from the Department of Education to found a university modeling contest – the Mathematical Contest in Modeling. In its first full year of operation MCM had 90 teams from 70 U.S. colleges participate.



In 1999 we created a sister contest called ICM with problems of a more interdisciplinary nature, requiring working knowledge of fields outside of mathematics. Up to this point participants were mostly from U.S. universities and the contests experienced modest growth.



In 2016 there were 12,734 teams registered. When we wrote the original grant for the contest we said explicitly that the purpose of the contest was not to reward bright students. Rather, the *raison d'être* was to promote the teaching and learning of mathematical modeling. And there is no question but that this strategy has been successful. These competitions and various clones have been directly responsible for the addition of modeling courses into university math programs in many countries – notably in China and the U.S.

But interestingly, the school curricula have been more resistant to change. In part this is due to a phenomena that is common across many countries. Namely, at the university level

institutions and faculty have a great deal of flexibility in what we teach and when. Often professional mathematics societies can take a leadership role. And while these organizations may be conservative and slow to change they are in no way as political as the school leadership. Because in almost all cases the school program is set by a Ministry of Education which is part of the federal bureaucracy. Moreover, in many countries there is an end of school test which is very very high-stakes. And the curriculum is tied very very tightly to the content of that test.

So, how do we influence that system? One way is to continue to strengthen what we have done. As the MCM/ICM receives greater prestige at the university level, secondary schools become more interested (especially the elite schools). The next obvious move is to institute local high school contests.

But this is not enough. To influence the political establishment at the school level we need more. As a consequence, in 2015 we established the International Mathematical Modeling Challenge (IMMC).



This challenge is more in the Olympiad mode. In particular, each participating country is allowed to enter up to two teams. Each team consists of four students. The Organizing Committee of IMMC which I chair, does not tell the countries how to choose their representative teams. The problem selection and grading is done by a separate expert panel. Because scheduling at the school level is an issue, teams are permitted 5 consecutive days to work on the problem – but those five days can start anywhere in the contest period which runs from mid-March to mid-May. Again, they may use any inanimate resources. Grading takes place in early June and the Outstanding teams are invited to an awards ceremony. Last year this ceremony was held in conjunction with ICME in Hamburg where the Outstanding teams presented their papers to an international audience. I should mention that we do not rank the winning teams first, second, etc. Rather we have basic categories of which the top is Outstanding. Last year there were three Outstanding teams.

**IM<sup>2</sup>C**

**Organizing Committee**

Solomon Garfunkel, COMAP, USA - Chair  
ANG Keng Cheng (MME), National Institute of Education, Singapore  
Fengshan Bai, Tsinghua University, China  
Alfred Cheung, NeoUnion ESC Organization, Hong Kong (SAR)  
Frederick Leung, University of Hong Kong, Hong Kong (SAR)  
Vladimir Dubrovsky, Moscow State University, Russia  
Henk van der Kooij, Freudenthal Institute, the Netherlands  
Mogens Allan Niss, Roskilde University, Denmark  
Ross Turner, Australian Council for Educational Research, Australia  
Jie "Jed" Wang, University of Massachusetts, Lowell, USA

**IM<sup>2</sup>C**

**Expert Panel**

Frank Giordano, Naval Post Graduate School, USA – Chair  
Konstantin K. Avllov, Institute for Numerical Mathematics, Russia  
Ruud Stolwijk, Cito, The Netherlands  
Liqiang Lu, Fudan University, China  
Jill Brown, Australian Catholic University, Australian  
Yang Wang, Michigan State Univ., USA

In the first year of operation we had 17 teams from 10 countries. In last year's contest we had 40 teams from 23 countries.

**IM<sup>2</sup>C  
Countries**

Australia	Netherlands
Belgium	New Zealand
Bulgaria	Philippines
Canada	Russia
Chile	Singapore
China	Slovakia
Germany	South Korea
Hong Kong (SAR)	Taiwan
Japan	Thailand
Macau (SAR)	United Kingdom
Malaysia	USA
Mexico	

Indications are that in 2017 these numbers will rise significantly.

To give you an idea about the nature of the IMMC problems, the 2015 problem as on planning a movie production and the 2016 problem on insuring a track meet for record setting performance bonuses.



**2015 IM<sup>2</sup>C Problem: Movie Scheduling**

A great deal of preparation must take place before a movie can be filmed. Important sets and scenes need to be identified, resource needs must be calculated, and schedules must be arranged. The issue of the schedule is the focus of the modeling activities. A large studio has contacted your firm, and they wish to have a model to allow for scheduling a movie. You are asked to answer the questions below. You should provide examples and test cases to convince the movie executives that your model is effective and robust.

**Question 1:** Develop a model that will produce a filming schedule given the following constraints:

- \* The availability dates of the stars of the film.
- \* The time required to film at a list of specific sites.
- \* The time required to construct and film on a list of sets.
- \* The availability dates for specific resources. For example a war movie might require helicopters which are available only at specific times.
- \* Some scenes cannot be shot until after certain computer generated content is defined and other physical items are constructed. Your schedule must include extra time to allow for redoing some shots if they turn out to be inadequate after editing and review.

**Question 2:** Develop a model that will take the information and schedule generated from the first question and can adjust them in the event that some delay in one aspect or the availability of some asset changes. For example, if one of the stars has an accident and cannot film for a certain period of time, you should be able to adjust the schedule.

**Question 3:** Use the model developed in the first question to develop a way to determine the most important constraints. That is, identify the constraints that will cause the longest delays if a problem occurs.



**2016 IM<sup>2</sup>C Problem: Record Insurance**

In athletics, one of the possible distances to run is 15,000 meters or 15k (in the picture you see the leader in an annual 15k - race in the Netherlands. Please see Wikipedia article below). For this type of run, 15k on a street track, there is a world record, as there are records for all other distances that are run in athletics (e.g. the marathon). In such a race, the organizing committee will usually pay a significant amount of money as a bonus to the winner if he or she succeeds in setting a new world record. These amounts of money can get quite large in order to attract top runners: in the race shown in the picture there was a 25,000 euro bonus if the winner succeeded in improving the 15k world record - which, by the way, he (un)fortunately did not achieve. Had he done so, there would have been a major financial problem for the organizing committee, since they had not purchased any insurance. Usually, insurance will be purchased by the organizing committee for such a running event, since the financial risks can be quite large. The fee they will have to pay for such insurance will be, of course, significantly lower than the bonus they would have to pay for a world record. Let's define the average cost of the bonus as the ratio of the amount of bonus divided by the expected number of times the event is replicated before the current record is broken. For example, if based on our analysis, we currently expect the record to be broken every 25 repetitions under conditions prevailing for a specified event, then the average cost of the bonus is 1000 euro per race. The first question is:

1. For the 15K run described above with a 25,000 euro bonus what is the average cost of the bonus?
 

The insurance company will add an amount to the computed average cost. The amount of the addition may be very reasonable or not. The insurance company expects to cover their costs and realize a profit over a long time period with multiple subscribers. The organizing committee can decide to purchase the insurance or not (that is, "self insure").
2. What criteria should the insurance company use in determining the amount to add to the average cost for the above race? Specifically, how do they weight each factor in determining their decision? For example, begin by considering the case where the insurer will add 20% to cover his operating costs, time value of money, and realize a profit over a period of time.
  3. (a) What criteria should the organizing committee use to determine whether or not they should purchase the insurance? Assume that they intend to sponsor this race many times in the near future. By self insuring, they expect to save the insurance company's added cost over a period of time.
    - (b) But should they take the risk?
 

Now consider that you are a member of the organizing committee of a major track meet with 20 men's and 20 women's athletic events, including field events (long jump, high jump, etc.)
    4. Assume the organizing committee can purchase the insurance or not for each of the 40 events. For example, they may choose to insure 10 of the 40 events. What factors should the organizing committee consider in their decision to purchase insurance or not for each of the events at the meet? Specifically, how do they weight each factor in determining their decision?
    5. Develop a general decision-scheme for the organizing committees to determine for each event whether they should purchase insurance or self insure. This scheme should be written in a form easily understood and implemented by a typical organizing committee.

I mentioned that the organizers of this challenge do not specify how participating countries should choose their two representative teams. Different countries have proceeded differently. We do ask that each country designate a country representative, so that we have one main contact person. In many cases, the first year that a country participates two teams are simply anointed usually from schools where there is a modeling focus. But in subsequent years countries often have put in place some kind of internal national competition to choose their representative teams. Some examples of how this is working:

In the U.S., Australia, and China schools are invited to have teams work on the IMMC problem early in the contest period. Based on their papers, a local committee decides on the two best papers and these are submitted to the Expert Panel which judges the international challenge. In China there is an extra step where top teams come to a central location and present their papers in talks to the local judging group. They also designate regional winners as well as choose their final two teams. In the Netherlands, there is a national contest already in place and the two top teams in their contest are picked to work on IMMC.

At its heart, IMMC represents an attempt through the creation of an international prestigious challenge to influence school programs. But how? Imagine that teachers and even some administrators get the message that modeling is important. What do they do, given the fact that they likely do not have a clear idea of what modeling is and is not. To address precisely that issue we created GAIMME – Guidelines for Assessment and Instruction in Mathematical Modeling Education.



Quoting from the GAIMME Preface:

“A major reason for the creation of GAIMME was the fact that, despite the usefulness and value in demonstrating how mathematics can help analyze and guide decision making for real world messy problems, many people have limited experience with math modeling. We wanted to paint a clearer picture of mathematical modeling (what it is and what it isn’t) as a process and how the teaching of that process can mature as students move through the grade bands, independent of the mathematical knowledge they may bring to bear.”

As I mentioned at the start of this talk – cultural change is difficult and cannot occur overnight. We have begun by increasing the understanding of the importance of mathematical modeling. We have shown by example how mathematical modeling can be introduced at ALL educational levels. And now we need to convince teachers that this paradigm shift is not only desirable but doable and give them our complete support.

