Master of Science in Mathematics *Michel Waldschmidt* Master Training Program
Royal University of Phnom Penh RUPP URPP - Université Royale de Phnom Penh
Centre International de Mathématiques Pures et Appliquées CIMPA
Coopération Mathématique Interuniversitaire Cambodge France

# MMA105: Discrete Mathematics

**First Assignment, April 24, 2009**

**Exercise 1.**
a) How many subsets of $\{a, b, c, d, e, f\}$ contain neither $b$ nor $d$?
b) How many subsets of $\{a, b, c, d, e, f\}$ contain $b$ but do not contain $d$?
c) How many subsets of $\{a, b, c, d, e, f\}$ contain $b$ and $d$?
d) How many subsets of $\{a, b, c, d, e, f\}$ with 2 elements contain $b$ but do not contain $d$?
e) Given a set $E$ with $n$ elements and two disjoint subsets $F_1$ and $F_2$ of $E$ with $k_1$ and $k_2$ elements respectively,

e1) how many subsets of $E$ contain all elements of $F_1$ but no element of $F_2$?

e2) given also a positive integer $m$, how many subsets of $E$ with $m$ elements contain all elements of $F_1$ but no element of $F_2$?

**Exercise 2.** Let $n$, $k$ be positive integers and $a_1, \ldots, a_k$ be non–negative integers. How many solutions $(x_1, \ldots, x_k) \in \mathbf{Z}^k$ are there to the equation

$$x_1 + \cdots + x_k = n$$

in integers restricted to the conditions $x_i \geq a_i$ for $1 \leq i \leq k$?

This question is the same as the following one: in how many ways can you distribute $n$ pennies to $k$ children in such a way that the first child receives at least $a_1$ pennies, the second at least $a_2$,... and the last one at least $a_k$?

`Hint:` *recall that when* $a_1 = \cdots = a_n = 0$ *the answer is* $\binom{n+k-1}{k-1}$.

**Exercise 3.** Denote by $f : \mathbf{Z} \longrightarrow \mathbf{Z}$ the map $x \longmapsto x^3$. Find $f^{-1}(1200\,\mathbf{Z})$.

**Exercise 4.** Check, for $n \geq 1$,

$$\binom{n}{2} + \binom{n+1}{2} = n^2.$$

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