MMA105: Discrete Mathematics

Third Assignment, May 8, 2009

Exercise 1.
Give the last three decimal digits of 859^{2001}.

Exercise 2.
How many times do you need to multiply two numbers when you compute 5^{97}?

Exercise 3.
For $k \geq 1$, let N_k be the integer $77\ldots77$ with k decimal digits, all of which are 7. Equivalently, define $N_1 = 7$ and by induction on k

$$N_k = 10N_{k-1} + N_1 \quad (k \geq 2).$$

What is the remainder of the division of N_k by 2? By 3? By 5? By 9? By 11?

Exercise 4.
Find all $N \in \mathbf{Z}$ which satisfy

$$N \equiv 2 \pmod{11} \quad \text{and} \quad N \equiv 10 \pmod{13}.$$

What is the smallest such positive N?

Exercise 5.
Let $k \geq 1$ be a positive integer and $p_1, \ldots, p_k$ be distinct primes. Set $n = p_1 \cdots p_k$. Assume $p_j - 1$ divides $n - 1$ for $1 \leq j \leq k$. Prove

$$a^n \equiv a \pmod{n} \quad \text{for all } a \in \mathbf{Z}.$$

Exercise 6.
Let G be a graph with n nodes.
a) Show that the following conditions are equivalent.
(i) G is connected.
(ii) G contains a subgraph with n nodes which is a tree.
Deduce that a connected graph with n nodes has at least $n - 1$ edges.
b) Show that the following conditions are equivalent.
(i) G does not contain a cycle.
(ii) G is contained in a graph with n nodes which is a tree.
Deduce that a graph with n nodes which does not contain a cycle has at most $n - 1$ edges.