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Coopération Mathématique Interuniversitaire Cambodge France

MMA105: Discrete Mathematics

# Third Assignment, May 8, 2009

## Exercise 1.

Give the last three decimal digits of  $859^{2001}$ .

## Exercise 2.

How many times do you need to multiply two numbers when you compute  $5^{97}$ ?

## Exercise 3.

For  $k \ge 1$ , let  $N_k$  be the integer 77...77 with k decimal digits, all of which are 7. Equivalently, define  $N_1 = 7$  and by induction on k

$$N_k = 10N_{k-1} + N_1 \quad (k \ge 2).$$

What is the remainder of the division of  $N_k$  by 2? By 3? By 5? By 9? By 11?

## Exercise 4.

Find all  $N \in \mathbf{Z}$  which satisfy

 $N \equiv 2 \pmod{11}$  and  $N \equiv 10 \pmod{13}$ .

What is the smallest such positive N?

## Exercise 5.

Let  $k \ge 1$  be a positive integer and  $p_1, \ldots, p_k$  be distinct primes. Set  $n = p_1 \cdots p_k$ . Assume  $p_j - 1$  divides n - 1 for  $1 \le j \le k$ . Prove

 $a^n \equiv a \pmod{n}$  for all  $a \in \mathbf{Z}$ .

## Exercise 6.

Let G be a graph with n nodes.

a) Show that the following conditions are equivalent.

(i) G is connected.

(ii) G contains a subgraph with n nodes which is a tree.

Deduce that a connected graph with n nodes has at least n-1 edges.

b) Show that the following conditions are equivalent.

(i) G does not contain a cycle.

(ii) G is contained in a graph with n nodes which is a tree.

Deduce that a graph with n nodes which does not contain a cycle has at most n-1 edges.

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