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## MMA105: Discrete Mathematics

Third Assignment, May 8, 2009

## Exercise 1.

Give the last three decimal digits of $859^{2001}$.

## Exercise 2.

How many times do you need to multiply two numbers when you compute $5^{97}$ ?

## Exercise 3.

For $k \geq 1$, let $N_{k}$ be the integer $77 \ldots 77$ with $k$ decimal digits, all of which are 7 .
Equivalently, define $N_{1}=7$ and by induction on $k$

$$
N_{k}=10 N_{k-1}+N_{1} \quad(k \geq 2)
$$

What is the remainder of the division of $N_{k}$ by 2 ? By 3 ? By $5 ?$ By $9 ?$ By $11 ?$
Exercise 4.
Find all $N \in \mathbf{Z}$ which satisfy

$$
N \equiv 2 \quad(\bmod 11) \quad \text { and } \quad N \equiv 10 \quad(\bmod 13)
$$

What is the smallest such positive $N$ ?
Exercise 5.
Let $k \geq 1$ be a positive integer and $p_{1}, \ldots, p_{k}$ be distinct primes. Set $n=p_{1} \cdots p_{k}$. Assume $p_{j}-1$ divides $n-1$ for $1 \leq j \leq k$. Prove

$$
a^{n} \equiv a \quad(\bmod n) \quad \text { for all } a \in \mathbf{Z}
$$

## Exercise 6.

Let $G$ be a graph with $n$ nodes.
a) Show that the following conditions are equivalent.
(i) $G$ is connected.
(ii) $G$ contains a subgraph with $n$ nodes which is a tree.

Deduce that a connected graph with $n$ nodes has at least $n-1$ edges.
b) Show that the following conditions are equivalent.
(i) $G$ does not contain a cycle.
(ii) $G$ is contained in a graph with $n$ nodes which is a tree.

Deduce that a graph with $n$ nodes which does not contain a cycle has at most $n-1$ edges.
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