Some comments on the Undergraduate syllabus in mathematics at Sherubtse (RUB)
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Overall assessment: we feel that the syllabus proposed in mathematics is very strong and complete. It undoubtedly offers the students a large spectrum of the mathematics usually taught at this level. We feel that the syllabus under study is on par with what is offered in leading institutions worldwide and that students who complete the program having mastered the material taught here would advantageously compete with graduates abroad.

General suggestions for improvement: we would suggest that more emphasis is given in relating the various modules to really impress on the students that mathematics should be considered as a whole and that various modules should not be considered as independent subjects from each other. This can be done in several ways, linking through examples, postponed proofs, exercises etc. We shall be more specific below in detailed module comments. We are of the opinion that one fundamental aspect of undergraduate mathematics is nevertheless missing in the present set up: Lebesgue integration theory. We would suggest that the topic be introduced in semester 5, so that it can be used extensively in the later courses and the PG level (semesters 7 and 8). The core content of such a module could be for example the content of Rudin's book.

Module by module discussion:
• MT101. In this module, the first example of our previous general comment could be applied, by announcing the proof of the fundamental theorem of algebra for eg MT819 (complex analysis); simple examples of solving linear differential equations can be also given after the eigen value chapter; also, it might be advisable not to postpone the concept of dimension and basis until semester 7.
• MT202. In this module, it might be useful to introduce and extensive study of (real and complex) sequences with which it is very easy to make the students grasp the notion of limit. A significant part of unit II could be used to show how to practically find truncated power series expansions, as the use of Taylor is not in practice that convenient. A more ambitious reframing would involve moving the multivariate part of the module to MT303 and to send in reverse the one variable integration part in MT202. This would make MT303 more specific to multivariate problems which are conceptually more difficult than one variable integrals. It would be useful to include a specific discussion on the implicit function theorem and the inverse function theorem, with references to applications (eg MT406). Saddle points part is another good point were some parts of MT101 could be recalled to make more fluid the connection between algebra and analysis in the mind of the students.
• MT303. It is not clear where indefinite integrals are studied. Also, results about differentiating parameter dependent integrals could be mentioned somewhere.
• MT304. The purpose of the module is not very clear to us. If the idea is to give a good culture of geometry to future teachers, it would better fit in a 6^{th} (or 8^{th}) semester module, with a slightly more ambitious content. On the other hand, if one wishes to make sure that basic geometric reasoning is mastered by students (both for other mathematics courses and for the physics classes), one could cut its size by half and for example shift the remaining content to MT303 (say unit IV and what is needed about cylinders, spheres etc.).
• MT405 is a solid and probably difficult module. If it turns out that students have a hard time swallowing it, a couple of items could be deleted (eg. Normalizers, properties of R[x]). MT406 and MT507: it might be a good idea to invert these two modules since MT507 is probably easier and enables students to master better the techniques of analysis. This module would certainly gain to be taught as soon as semester 4 and MT406 can probably wait until semester 5 when students get a better grasp of analysis.
• MT508. It would seem more natural to permute the last three topics with the first four ones, that means to start with probability and then go for statistics. By the way, probability would deserve more space in the program.

• MT610. The content of the module could probably be taught at this level in the set up of normed vector spaces as at the 6th semester, the students certainly have a much better grasp of analysis.
• MT611 is another good example were the interlink between algebra and analysis appear clearly (lectures 17-22 could also be part of an algebra class).

• MT712 could probably be designed to be centred on the study of modules over a principal ideal domain, with linear algebra and basic number theory (Chinese remainder theorem, lattices) can be studied together and the Jordan form recovered. If necessary, one could do away completely with field theory and ensure that the students really master both linear algebra and basic number theory in their fourth year. MT714. It might be useful to think about including a small fraction of this module as soon as the second year, as physics needs pretty early to use Green-Riemann's theorem, Stoke's theorem, the notion of potential function for example.
• MT818 could probably be replaced for example by a course on probability or discrete mathematics.

Further comments.

We understand that each module includes 1 hour of exercises for 4 hours of course. In France the standard is 2 hours exercises for 1 hour of course. We consider it is worthwhile to have the students work more by themselves rather than by listening to lessons. Also in assignments and exam papers we often propose sequences of related questions, where they need to use the result of question 1 for solving question 2, with three to four consecutive questions.

The students do not always realize that mathematics is a coherent subject, not a collection of topics like algebra, analysis, geometry and so on. An efficient way of showing them this unity of mathematics is to have each teachers changing of course on a regular basis, by rotating. Also it is useful to stress the connexions between different topics, for instance the use of linear algebra in solving differential equations, the ubiquity of complex numbers as well as of the exponential function for instance.

The implicit function theorem deserves to be given more space in the syllabus. It is a fundamental tool.

Introducing the maximum modulus theorem and the residue calculus at an earlier stage would be better.

At the very beginning of the first year, it might be useful to repeat basic facts which the students should know but may not be sufficiently familiar with. Such topics include logic (contraposition, necessary and sufficient conditions, proofs by contradiction, induction), basic set theory (composition of maps, injection, surjection, bijection, factorisation of maps).

The module MT818 Mathematical Logic could be split into two modules, for instance one with 4 credits including the first part which is indeed mathematical logic, and one with 8 credits which would replace the section on fuzzy sets and fuzzy logic. For instance either Probability or Discrete Mathematics could replace Fuzzy Mathematics. Also more generally for the last semester to split a number of modules into smaller modules with less credits would enable the students to see a larger variety of mathematics.

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