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Weinan Gaoxin Middle School, PRC

Some contributions to number theory by Chinese Mathematicians

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http://www.imj-prg.fr/~michel.waldschmidt/

华罗庚 陈景润 王元

Hua Luogeng

1910 – 1985

Chen Jingrun

1933 – 1996

Wang Yuan

1930 –

Hua Loo Keng 华罗庚 華羅庚

Mathematics Genealogy Project

Loo-Keng Hua (华罗庚)

Dissertation:

Advisor: Unknown

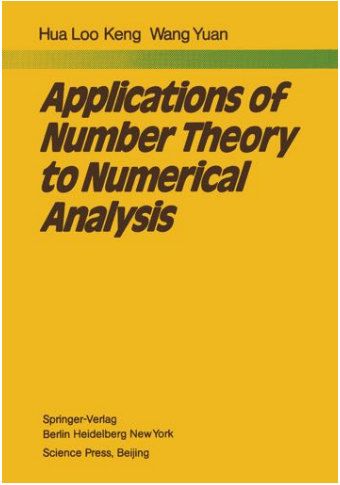
Students:
Click here to see the students listed in chronological order.

Name	School	Year	Descendants
Ayoub, Raymond	University of Illinois at Urbana-Champaign	1950	9
Chen (陈景润), Jingrun	Chinese Academy of Sciences		3
Ding (丁夏畦), Xiaxi (Xiaqi)	Academia Sinica	1956	30
Pan (潘承洞), Chengdong	Peking University	1961	39
Wang (王元), Yuan	Academia Sinica		2
Wan (万哲先), Zhe-Xian	Academia Sinica		17

According to our current on-line database, Loo-Keng Hua (华罗庚) has 6 students and 106 descendants.
We welcome any additional information.

https://genealogy.math.ndsu.nodak.edu/id.php?id=4784

Hua Loo Keng and Wang Yuan 华罗庚 王元



Contents

Hua Loo Keng and Wang Yuan 华罗庚 王元

Chen Jingrun 陈景润 : 陳景潤;

Chen's statue in Xiamen University

Numbers

Numbers = real or complex numbers $\mathbb{R}$, $\mathbb{C}$.

Natural integers : $\mathbb{N} = \{0, 1, 2, \ldots\}$.

Rational integers : $\mathbb{Z} = \{0, \pm 1, \pm 2, \ldots\}$.

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Prime numbers

Numbers with exactly two divisors.

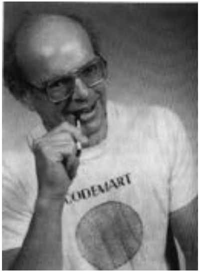
There are 25 prime numbers less than 100 :

$$2,\ 3,\ 5,\ 7,\ 11,\ 13,\ 17,\ 19,\ 23,\ 29,\ 31,\ 37,\ 41,$$

$$43,\ 47,\ 53,\ 59,\ 61,\ 67,\ 71,\ 73,\ 79,\ 83,\ 89,\ 97.$$

The On-Line Encyclopedia of Integer Sequences

http://oeis.org/A000040



Neil J. A. Sloane

The fundamental Theorem of arithmetic

Any positive number is the product, in only one way, of prime numbers.

Prime numbers are related to multiplication, they are the building blocs of the set of integers for the product. One should multiply them, not add them !

But there is no law which would forbid to add prime numbers ! !

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Sums of two primes

Let us list the numbers up to 30 that are sums of two prime numbers : $n = p_1 + p_2$, $p_1 \le p_2$, $n \le 30$.

$p_1 \backslash p_2$	2	3	5	7	11	13	17	19	23
2	4	5	7	9	13	15	19	21	25
3		6	8	10	14	16	20	22	26
5			10	12	16	18	22	24	28
7				14	18	20	24	26	30
11					22	24	28	30	
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The entries on the row with $p_1 = 2$ after the first one are odd, all other entries are even.

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The entries on the row with $p_1 = 2$ after the first one are odd, all other entries are even.

Sums of two primes

Numbers that are sums of two primes :

$$4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 30, \ldots$$

Numbers ≥ 2 that are not sums of two primes :

$$2, 3, 11, 17, 27, 29, \ldots$$

Numbers that are sums of at most two primes :

$$2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29,$$

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Sums of three primes

Notice that 27 is sum of three primes in 7 ways :

$$\begin{aligned}27 &= 2+2+23 = 3+5+19 = 3+7+17 \\ &= 3+11+13 = 5+5+17 = 5+11+11 = 7+7+13.\end{aligned}$$

The number of decompositions of an integer as a sum of three primes is given by the sequence http://oeis.org/A068307, namely

$$0, 0, 0, 0, 0, 1, 1, 1, 2, 1, 2, 2, 2, 1, 3, 2, 4, 2, 3, 2, 5, 2, 5, 3, 5, 3, 7,$$
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$$3,7,2,6,3,9,2,8,4,9,4,10,2,11,3,10,4,12,3,13,4,12,5,15,\ldots$$

Goldbach's Conjecture

Christian Goldbach
(1690 – 1764)

Leonhard Euler
(1707 – 1783)

Letter of Goldbach to Euler, 1742 : *any integer* ≥ 6 *is sum of three primes.*

Euler : Equivalent to :

any even integer ≥ 4 *is sum of two primes.*

Goldbach $\Longleftrightarrow$ Euler

(G) : *any integer* ≥ 6 *is sum of three primes.*
(E) : *any even integer* ≥ 4 *is sum of two primes.*

Proof :
(G)$\Rightarrow$(E)
Let m be an even number ≥ 4. Assuming (G), $m+2$ is sum of three primes, says $m+2=p_1+p_2+p_3$. At least one of them is even, say $p_3=2$, and $m=p_1+p_2$ is sum of two primes.

(E)$\Rightarrow$(G)
Let $m\geq 6$. From (E) is follows that $m-2$ is sum of two primes, $m-2=p_1+p_2$, hence $m=p_1+p_2+2$ is sum of three primes.

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Let $m \geq 6$. From (E) is follows that $m - 2$ is sum of two primes, $m - 2 = p_1 + p_2$, hence $m = p_1 + p_2 + 2$ is sum of three primes.

Number of decompositions of 2n into ordered sums of two odd primes.

http://oeis.org/A002372

$$0, 0, 1, 2, 3, 2, 3, 4, 4, 4, 5, 6, 5, 4, 6, 4, 7, 8, 3, 6, 8, 6, 7, 10, 8, 6, 10, 6, 7, 12, 5, 10, 12, 4, 10, 12, 9, 10, 14, 8, 9, 16, 9, 8, 18, 8, 9, 14, \ldots,$$

Circle method

Srinivasa Ramanujan
(1887 – 1920)

G.H. Hardy
(1877 – 1947)

J.E. Littlewood
(1885 – 1977)

Hardy, ICM Stockholm, 1916
Hardy and Ramanujan (1918) : partitions
Hardy and Littlewood (1920 – 1928) :
Some problems in Partitio Numerorum

Circle method

Hardy and Littlewood

Ivan Matveevich Vinogradov
(1891 – 1983)

Every sufficiently large odd integer is the sum of at most three primes.

Sums of primes

Theorem – I.M. Vinogradov (1937)
Every sufficiently large odd integer is sum of three primes.

Theorem – Chen Jing-Run (1966)
Every sufficiently large even integer is sum of a prime and an integer that is either a prime or a product of two primes.

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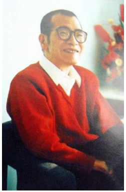


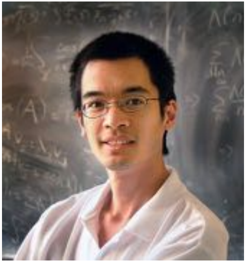
Chen Jing Run
(1933 - 1996)

Sums of primes

In the above proof of the equivalence between Goldbach and Euler, the prime number 2 plays a central role.

- Weak (or ternary) Goldbach Conjecture : *every odd integer* ≥ 7 *is the sum of three odd primes.*

- Terence Tao, February 4, 2012, `arXiv:1201.6656` : *Every odd number greater than 1 is the sum of at most five primes.*



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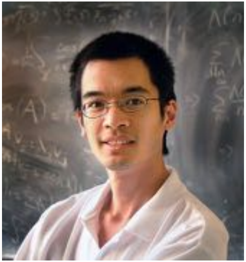

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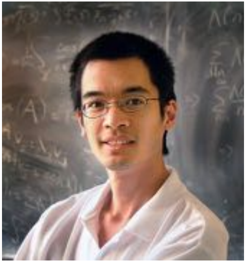

Ternary Goldbach Problem

Theorem – Harald Helfgott (2013).
Every odd number greater than 5 *can be expressed as the sum of three primes.*
Every odd number greater than 7 *can be expressed as the sum of three odd primes.*

Harald Helfgott
(1977-)

Earlier results due to Hardy and Littlewood (1923), Vinogradov (1937), Deshouillers et al. (1997), and more recently Ramaré, Kaniecki, Tao . . .

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