Number Theory
African Institute for Mathematical Sciences (AIMS)

Michel Waldschmidt, Sorbonne Université

Quizz 2 (15')   *February 8, 2023*

How can you pay 128 Francs if you only have coins of 15, 21 and 35 Francs ?

(The answer that such coins do not exist is not the right one).

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Solution

There are plenty of solutions.

The gcd of 15 and 21 is 3, the gcd of 21 and 35 is 7, the gcd of 15 and 35 is 5. Since $128 = 2^7$ is relatively prime with 3, 5, 7, we need to use the three coins.

We select one of the coins, say 35, which is prime to the gcd of 15 and 21. Using that coin we pay first 35 Francs.

We still need to pay the difference $128 - 35 = 93 = 3 \cdot 31$, which is a multiple of the gcd of 15 and 21 (this is why we used only one coin of 35).

To pay 93 Francs with coins 15 and 21 is the same as paying 31 with coins 5 and 7 : we can use 2 coins of 5 and 3 coins of 7 for 31, 2 coins of 15 and 3 coins of 21 for 93 This yields

$$128 = 25 + 2(2 \cdot 5 + 3 \cdot 7) = 35 + 2 \cdot 15 + 3 \cdot 21 = 35 + 30 + 63.$$

Finally we use 2 coins of 15, 3 coins of 21 and 1 coin of 35.