29/10/2024

"Selected topics in Arithmetic Algebraic Geometry"
Institute of Mathematics of the Vietnam Academy of Science and Technology (VAST).
Course: *Representation of integers by binary forms.*
Michel Waldschmidt

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Fundamental cyclotomic domain

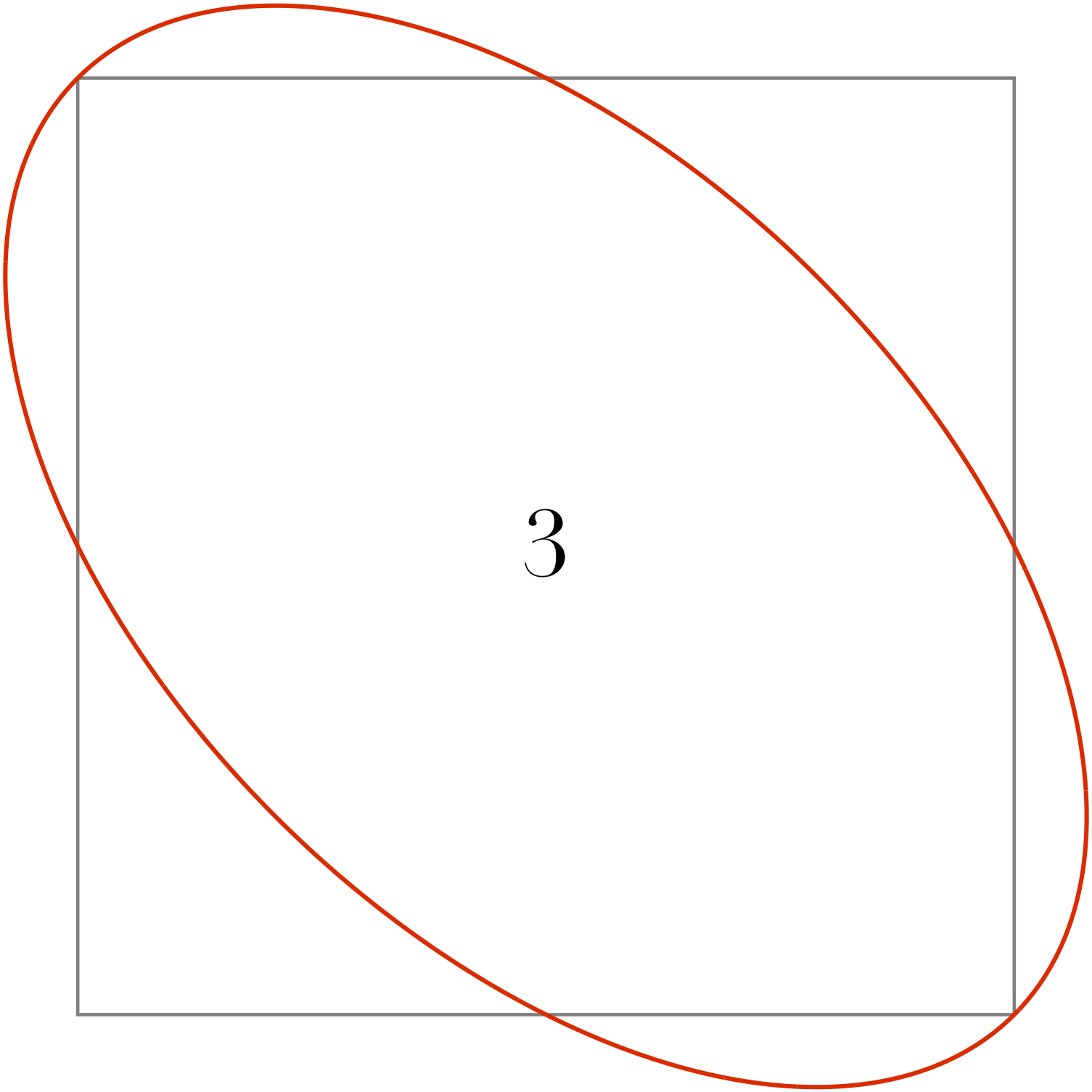
The following pages display for $n = 3$ to 150 the fundamental domain

$$\mathcal{O}_n := \{(x, y) \in \mathbb{R}^2 \mid \Phi_n(x, y) \leq 1\}$$

of the n-th cyclotomic form $\Phi_n(X, Y)$ defined by induction for $n \geq 1$ by

$$X^n - Y^n = \prod_{n|d} \Phi_d(X, Y).$$

Exercise. For which $n \geq 3$ is $\mathcal{O}_n$ convex?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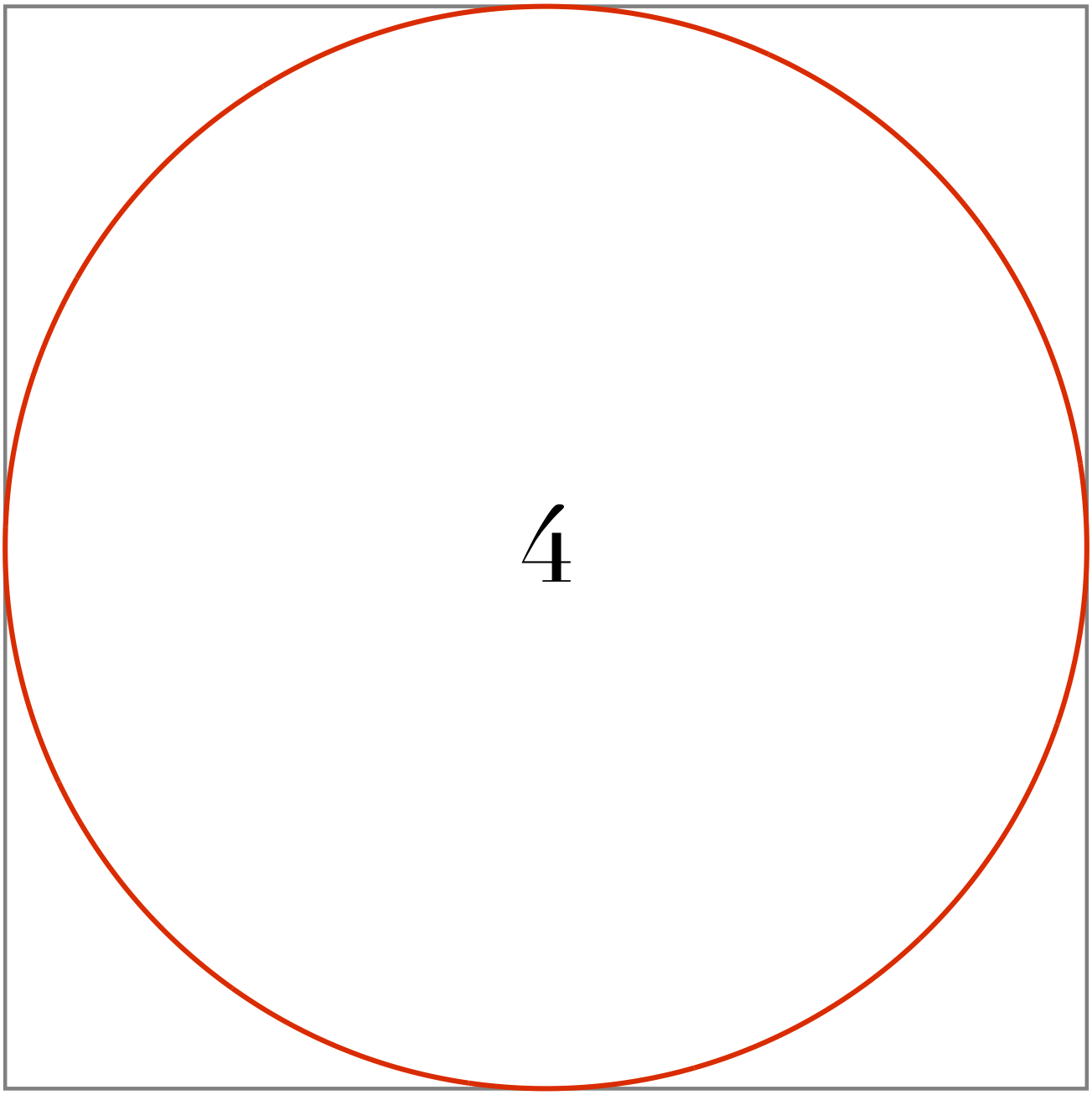
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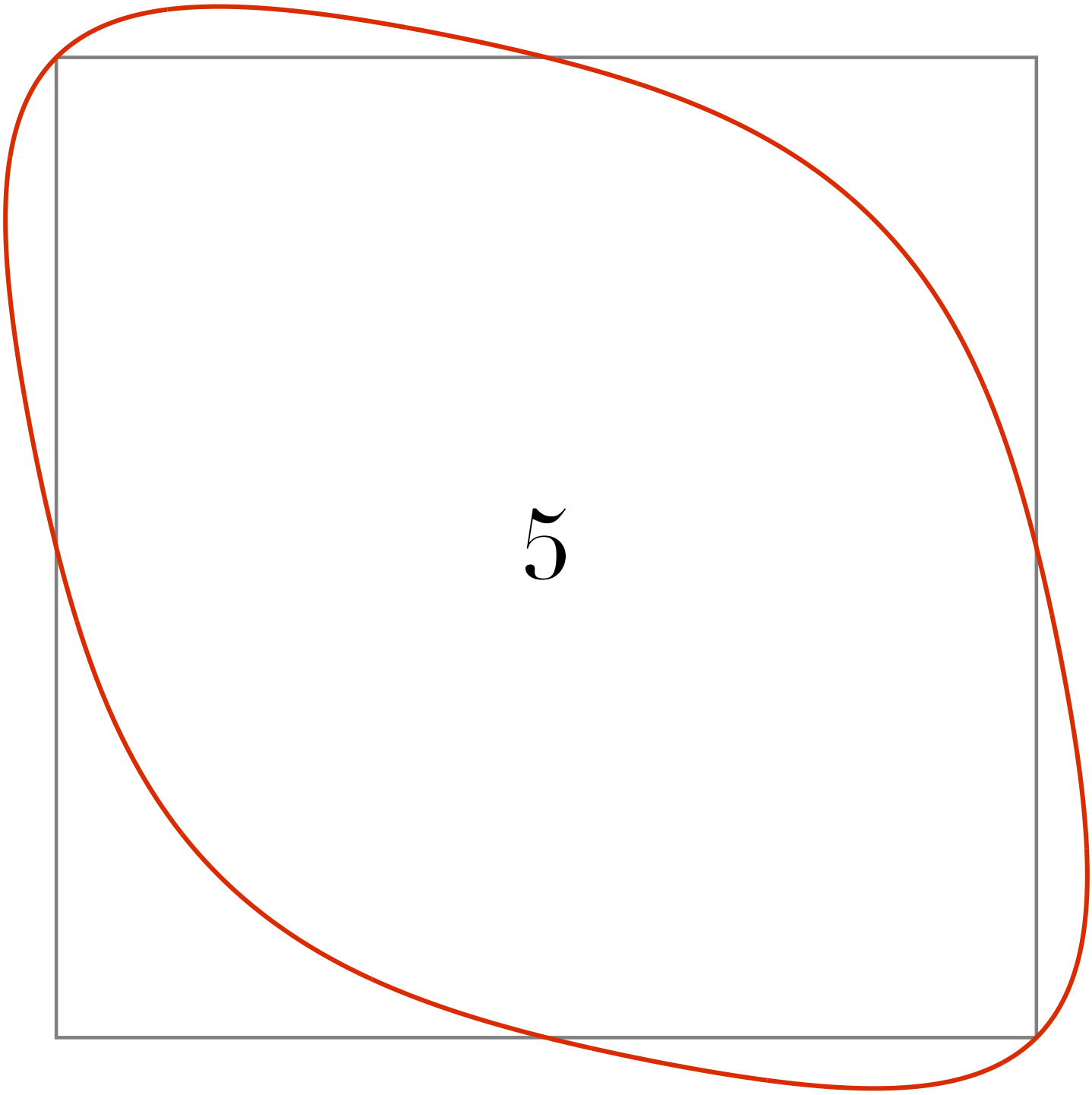
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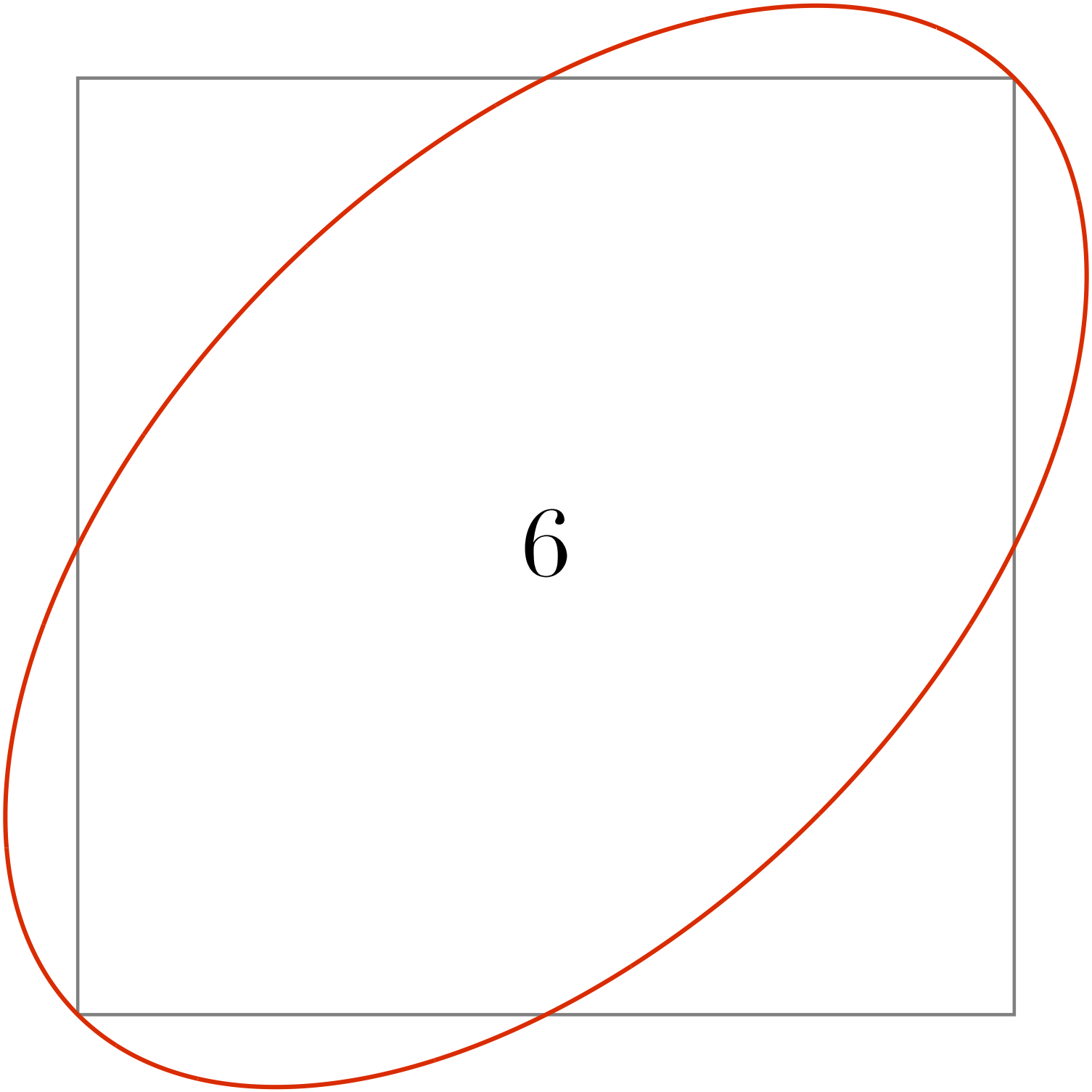
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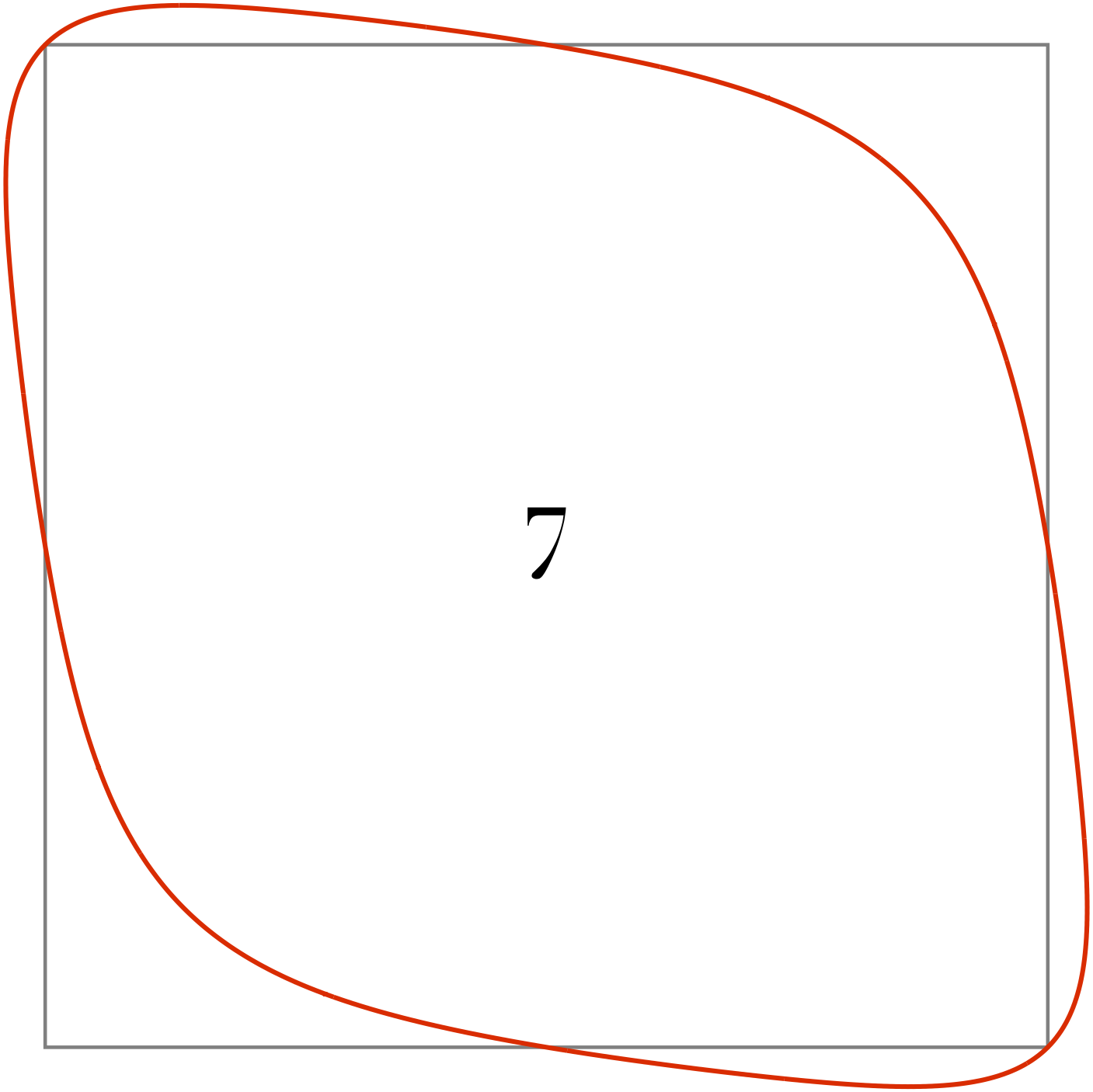
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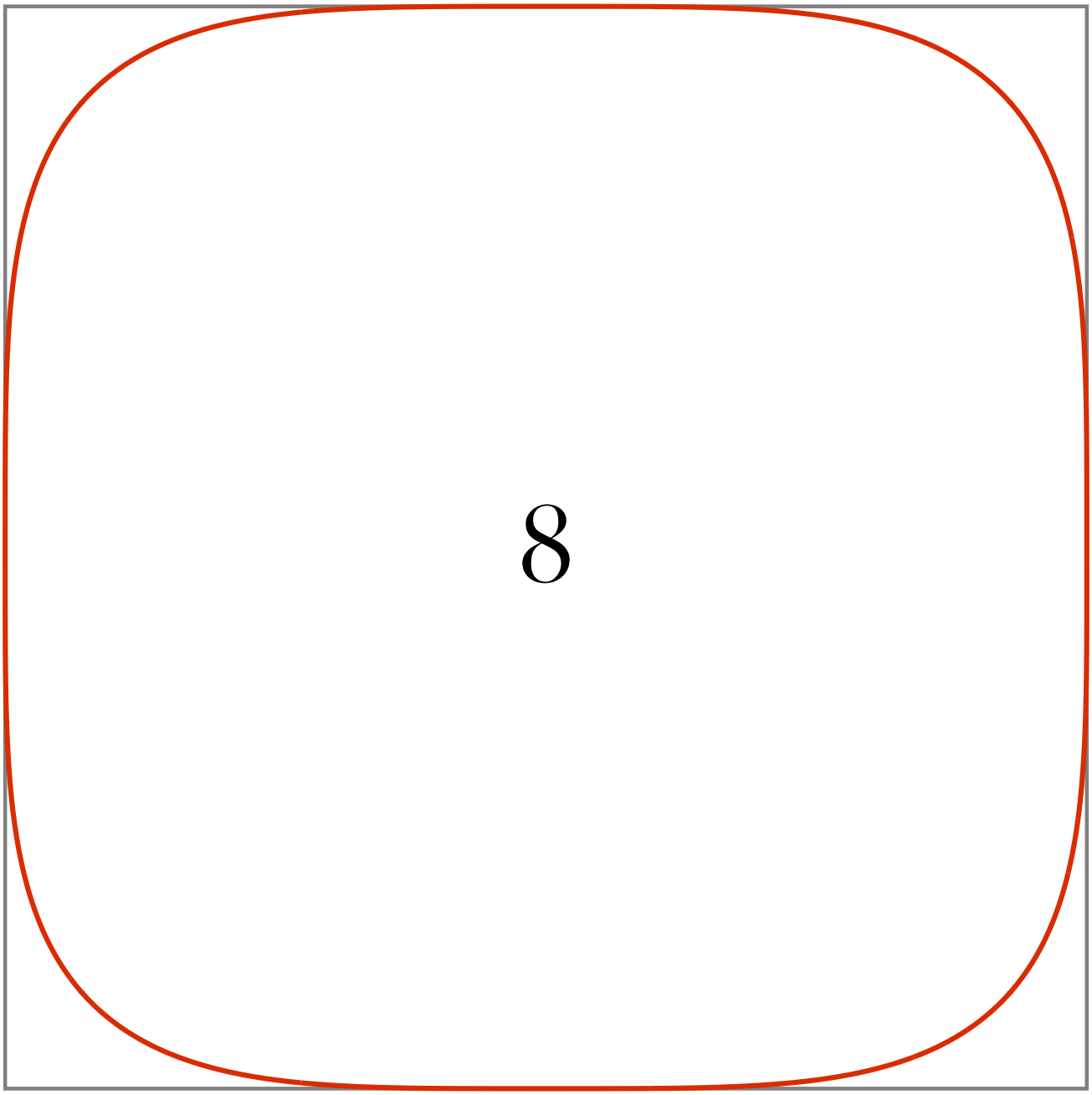
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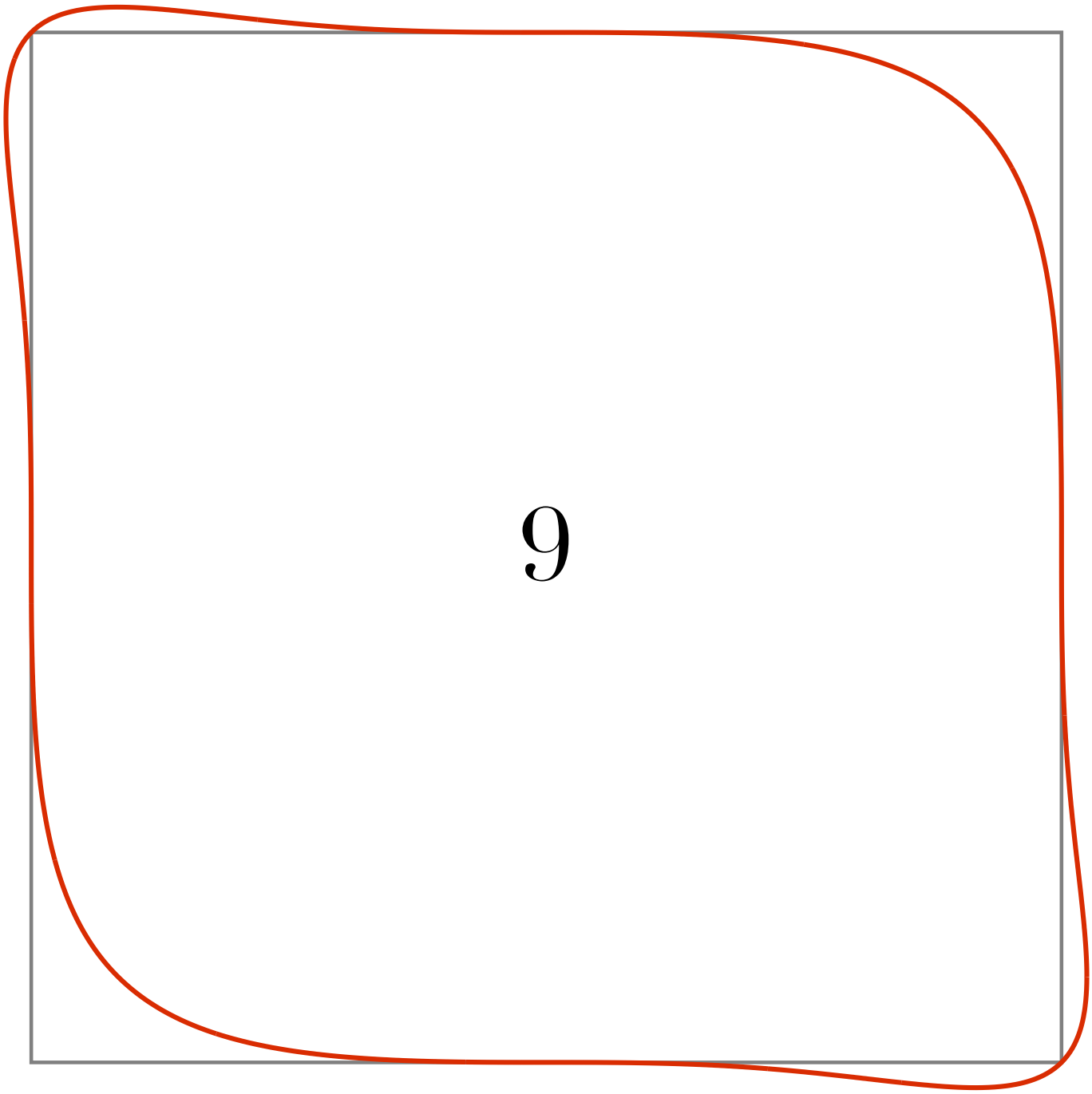
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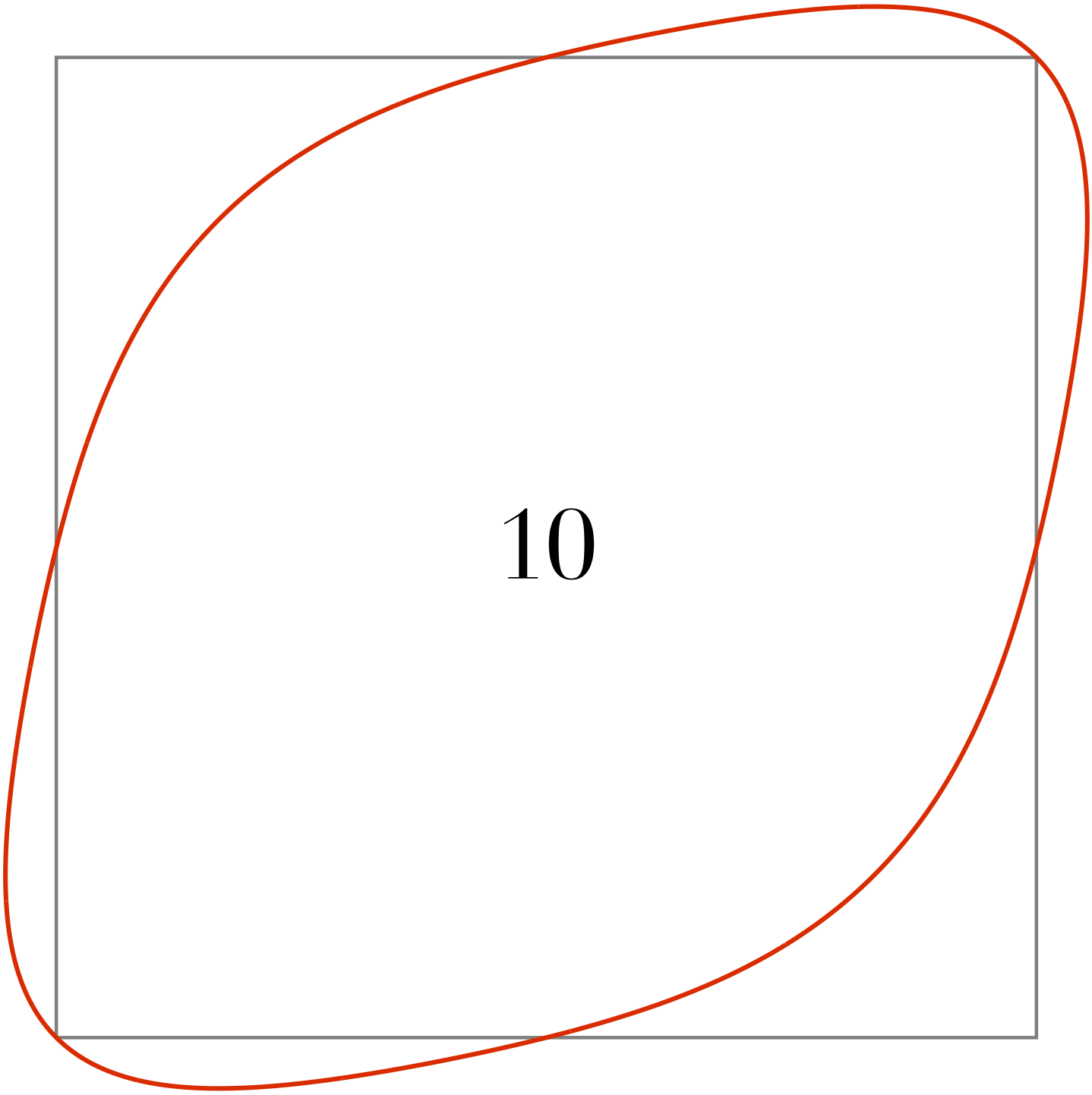
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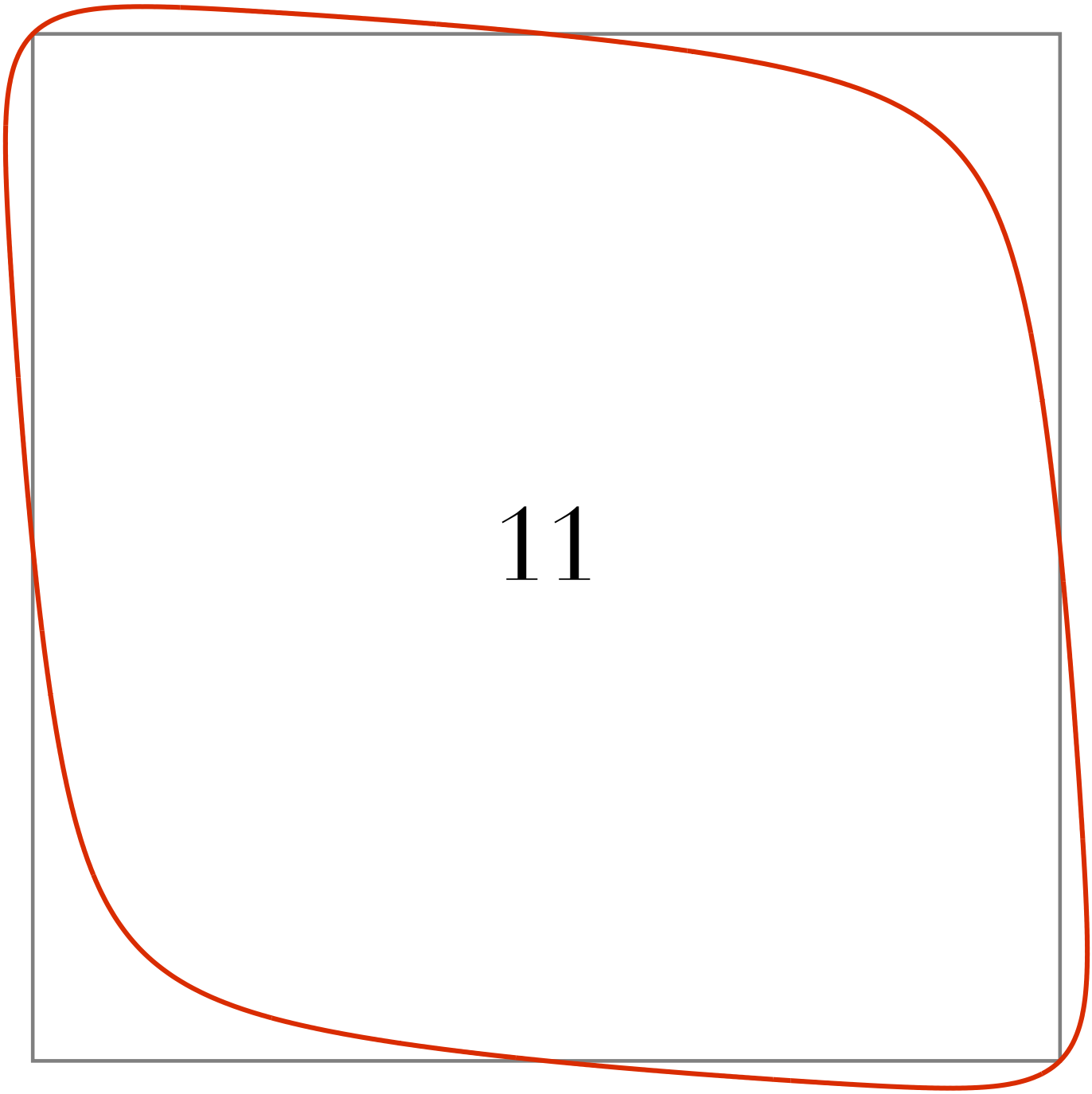
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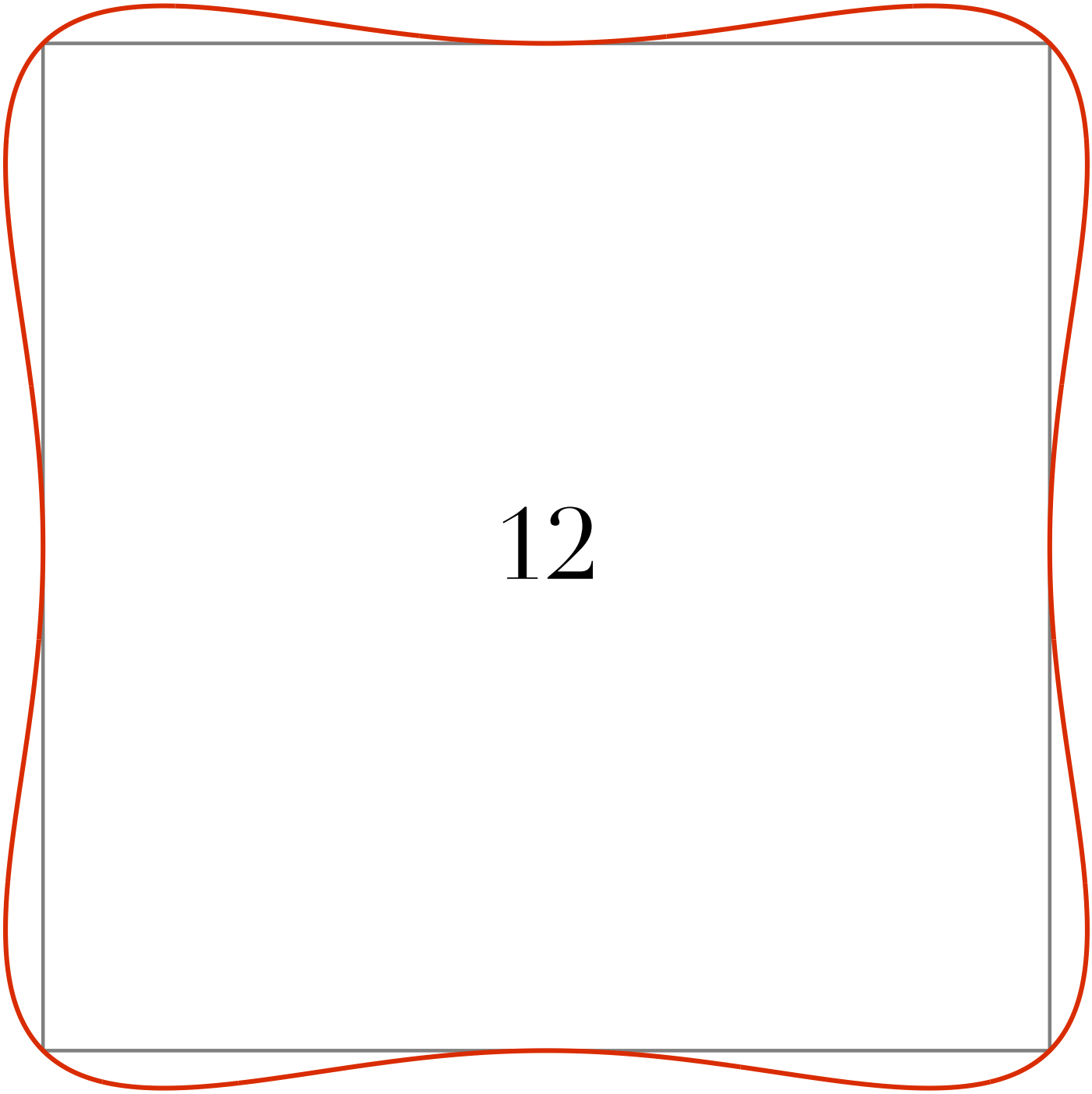
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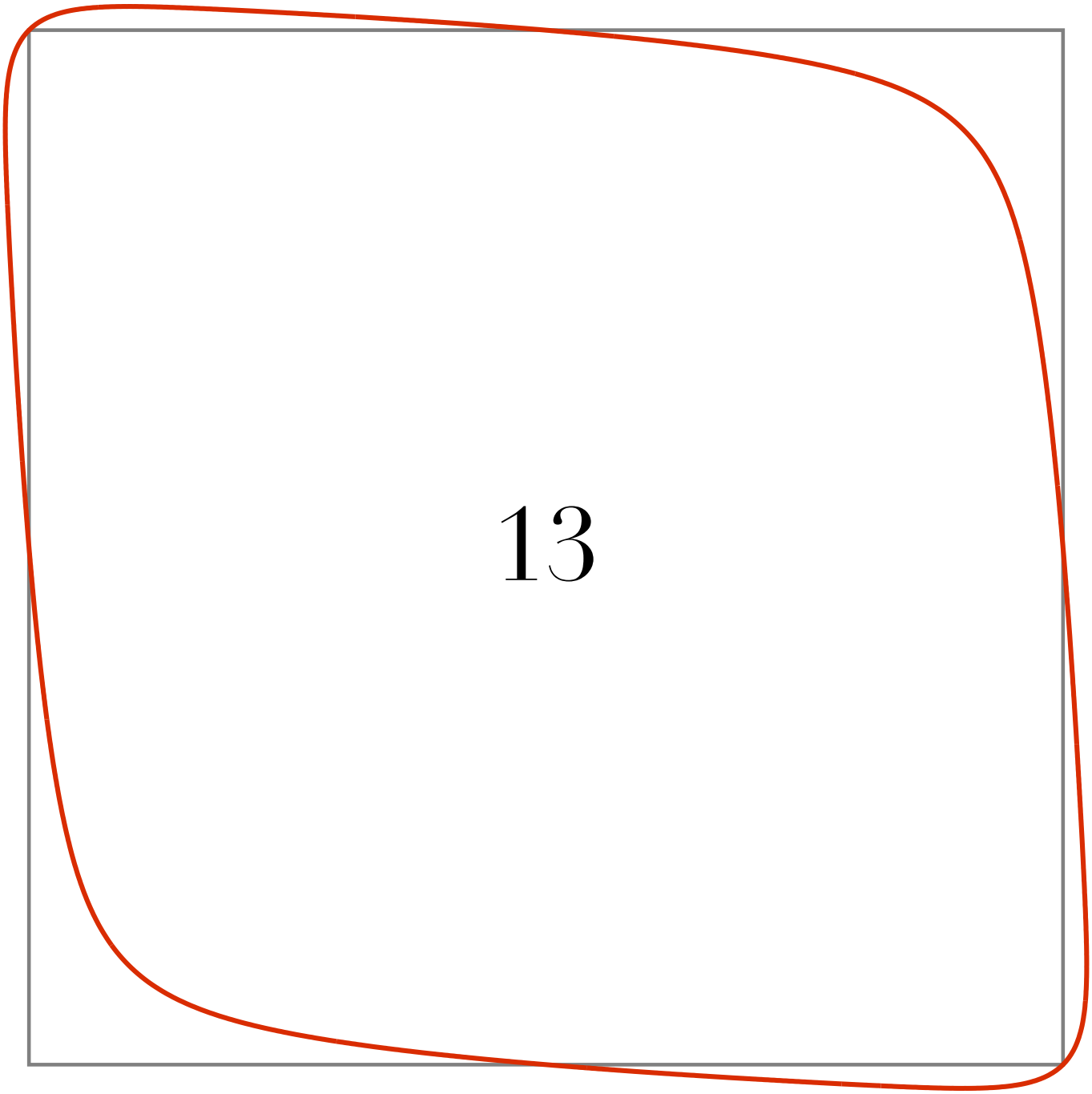
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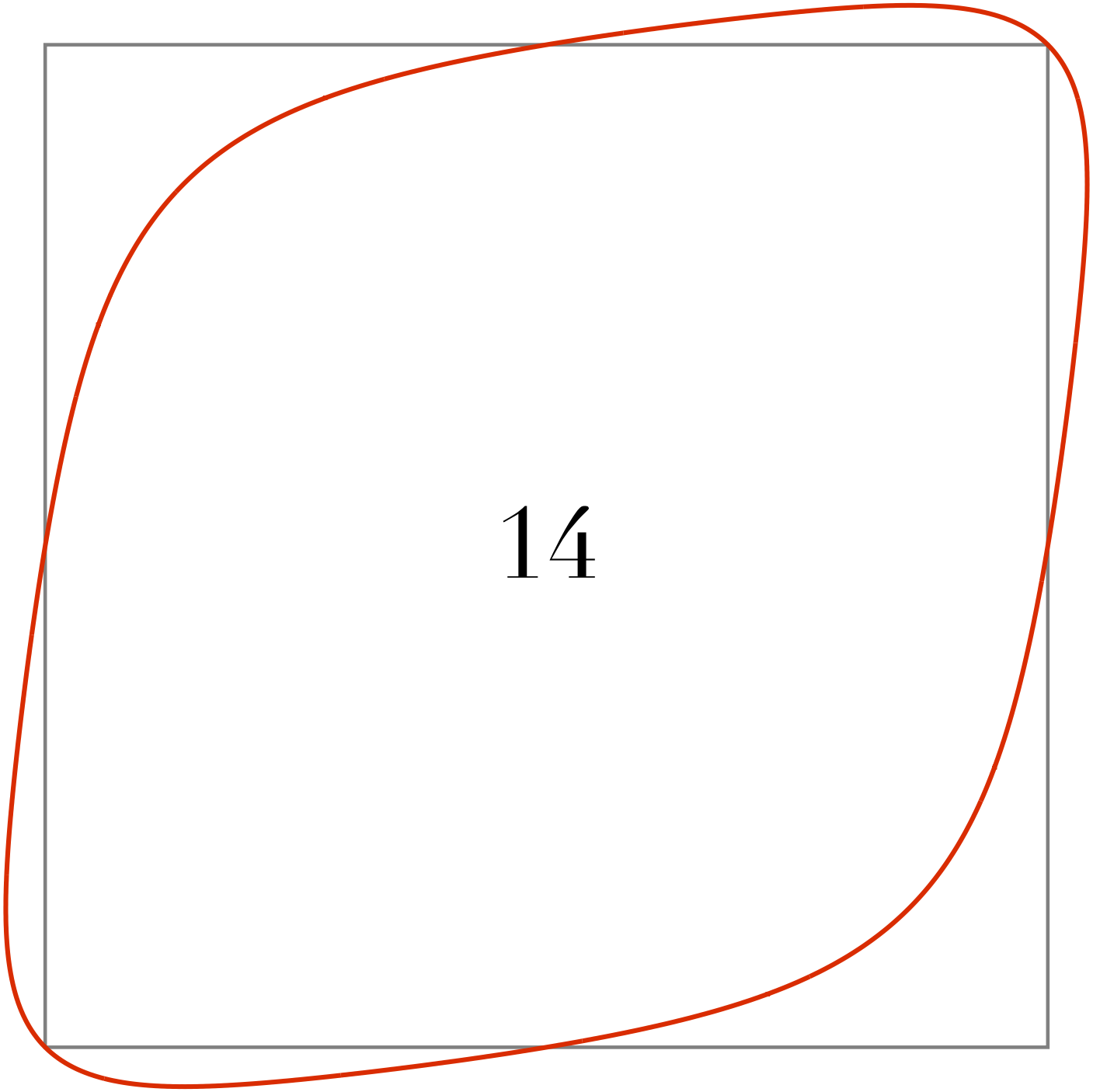
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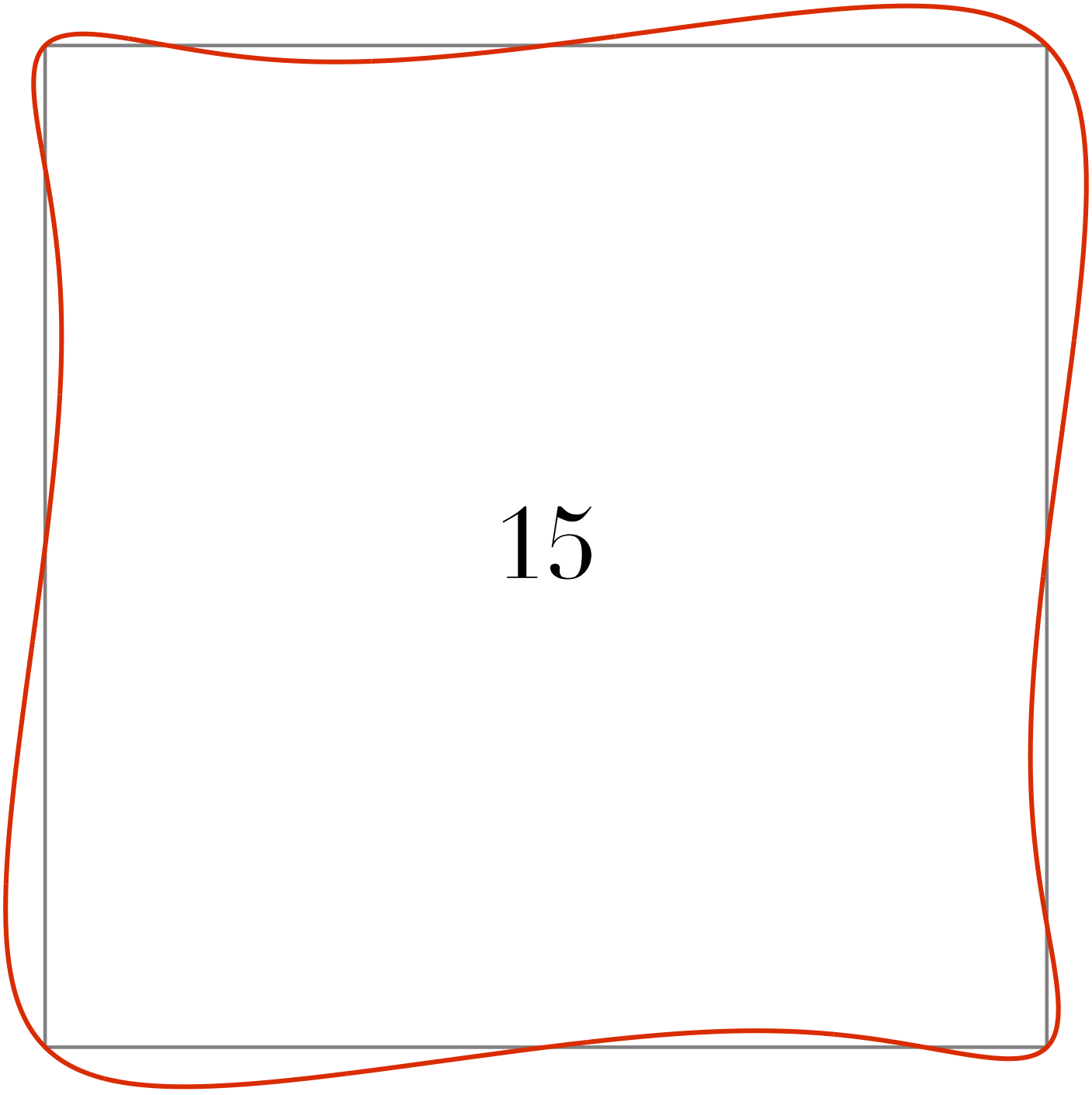
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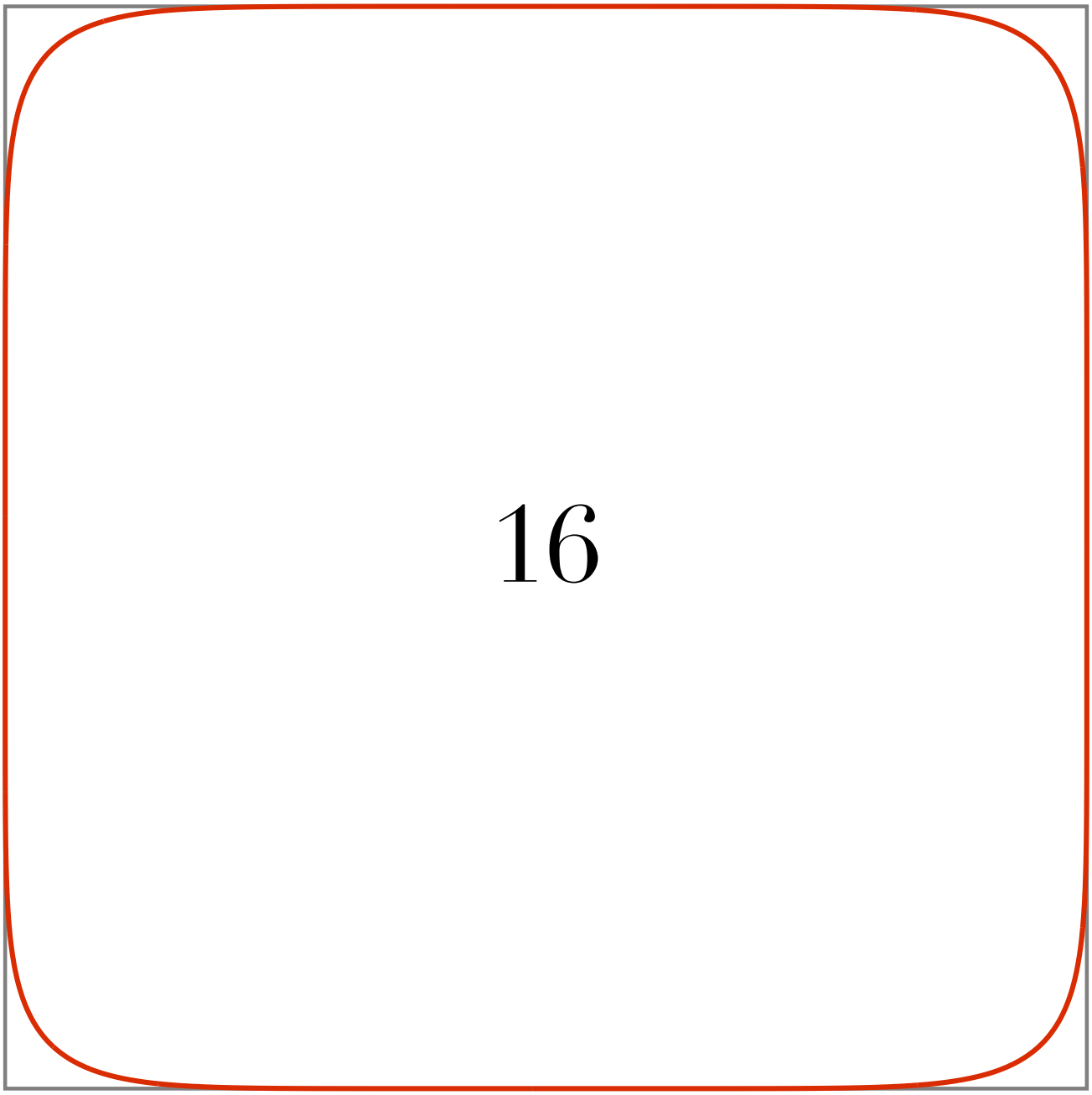
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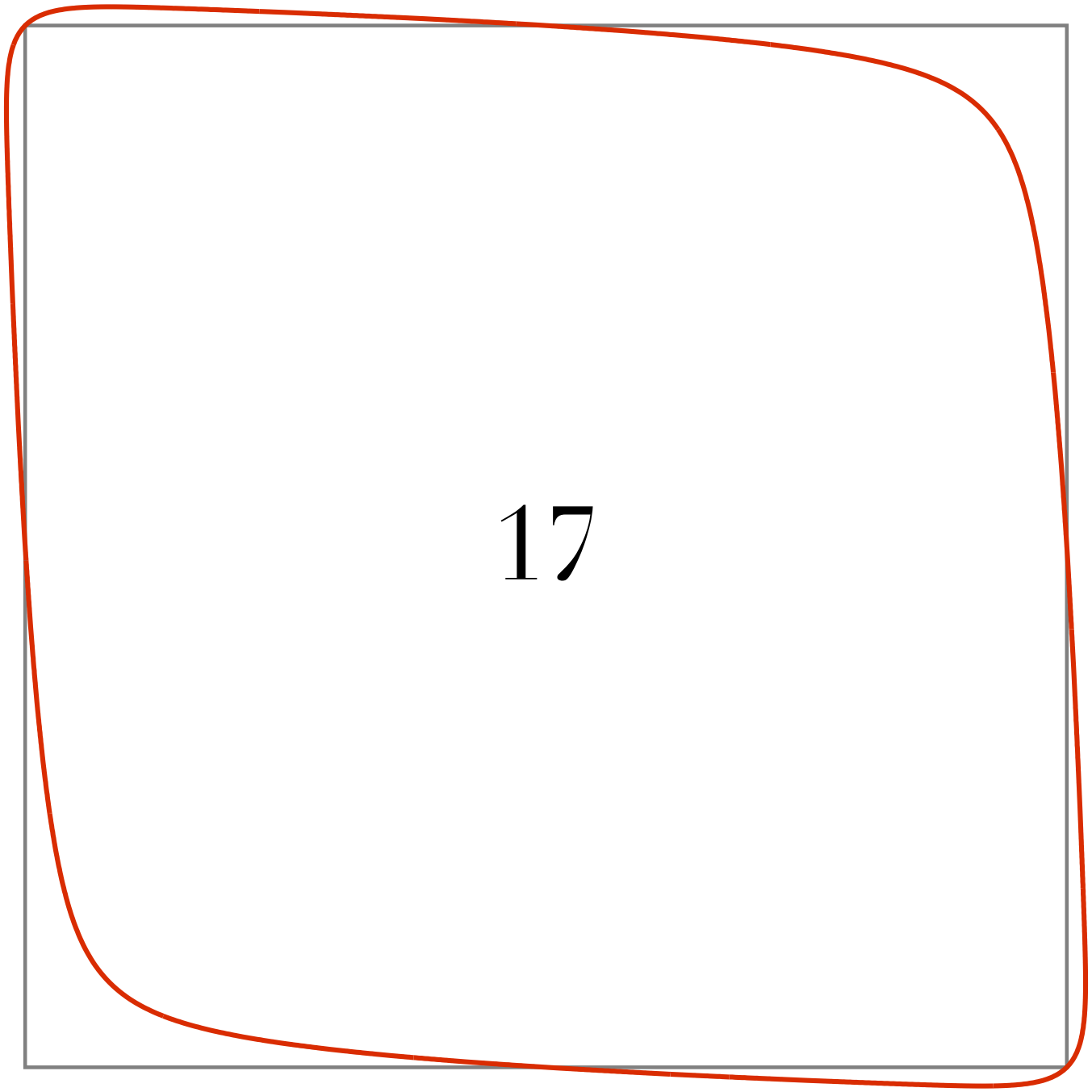
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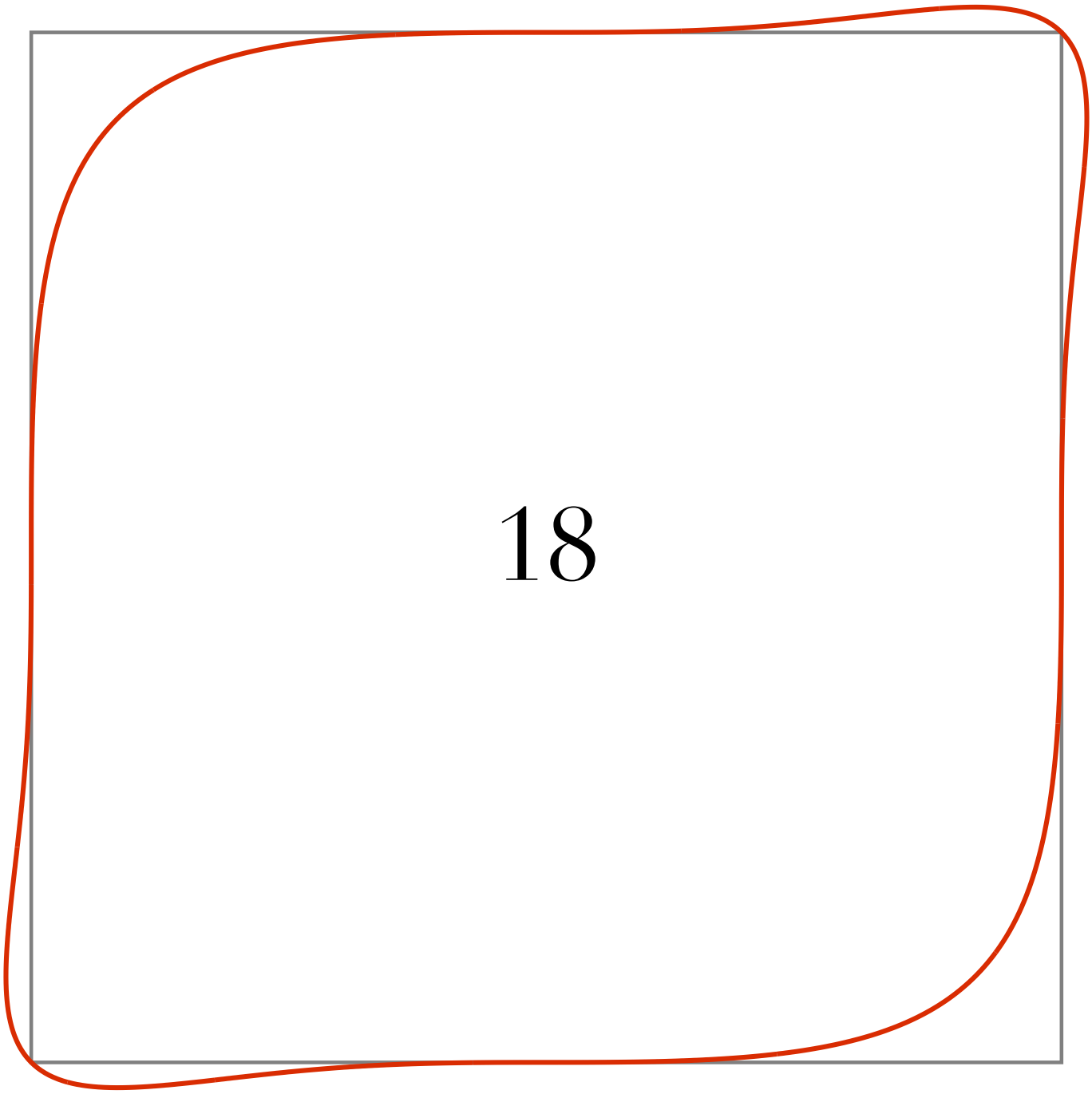

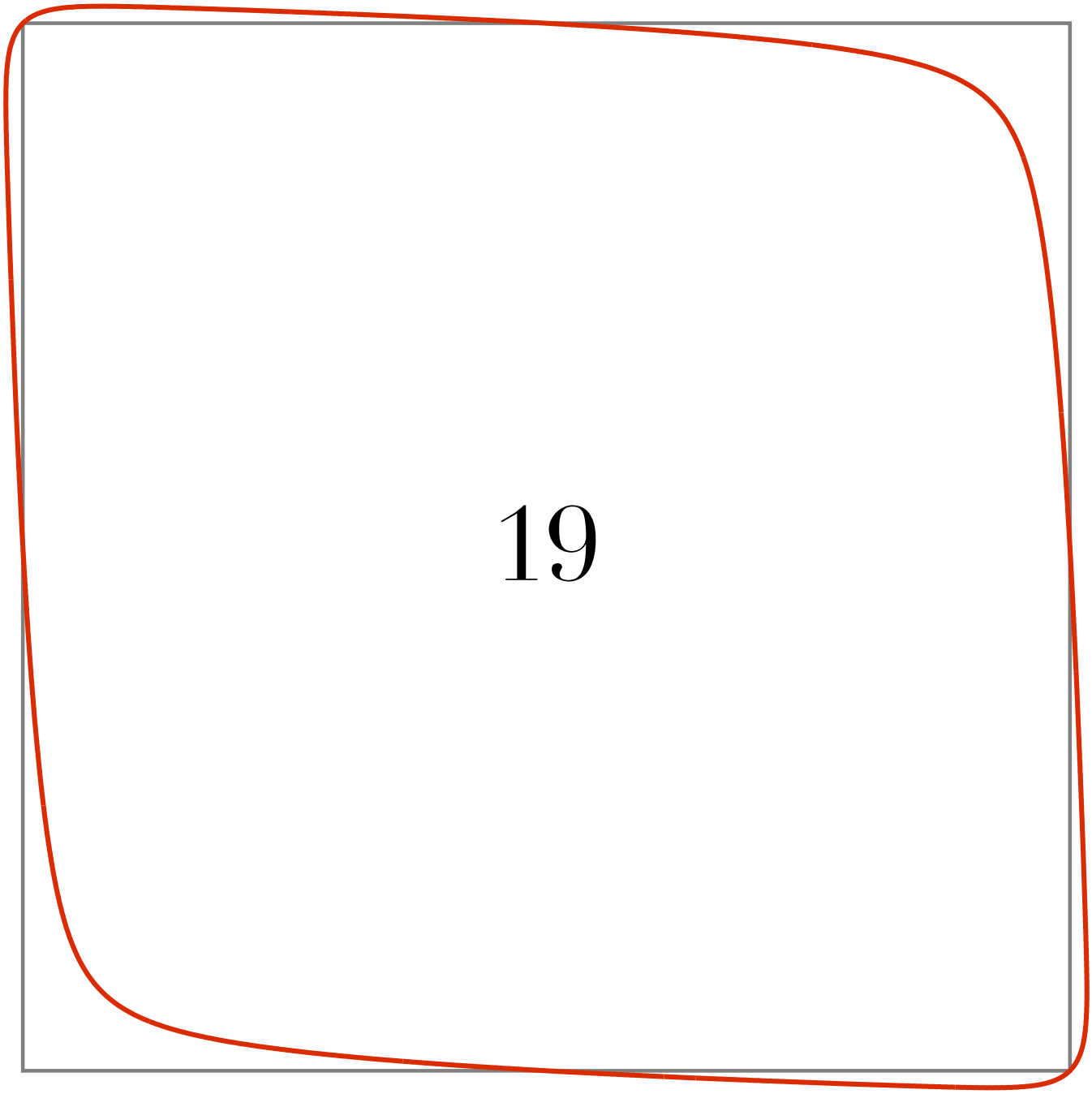
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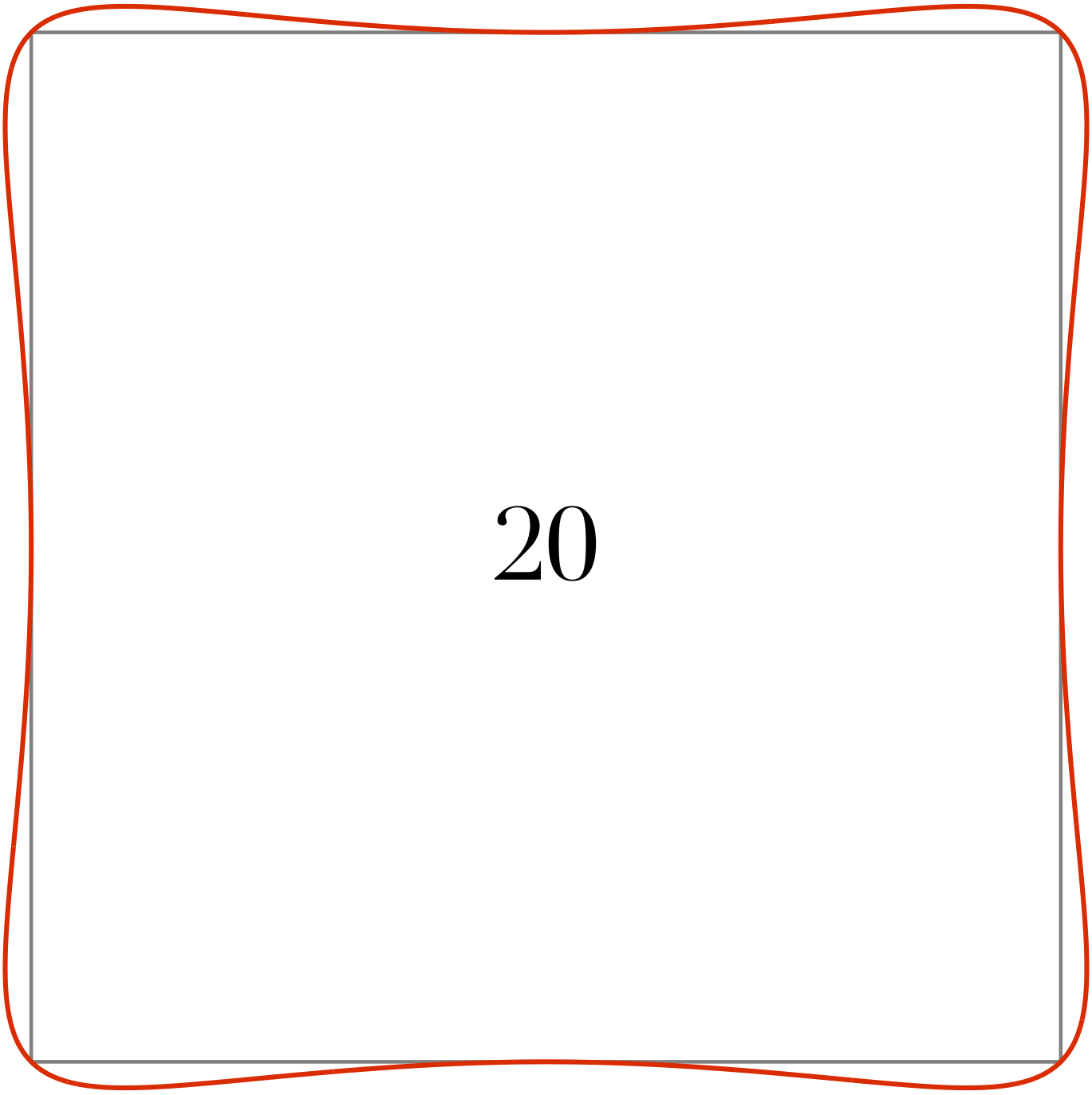
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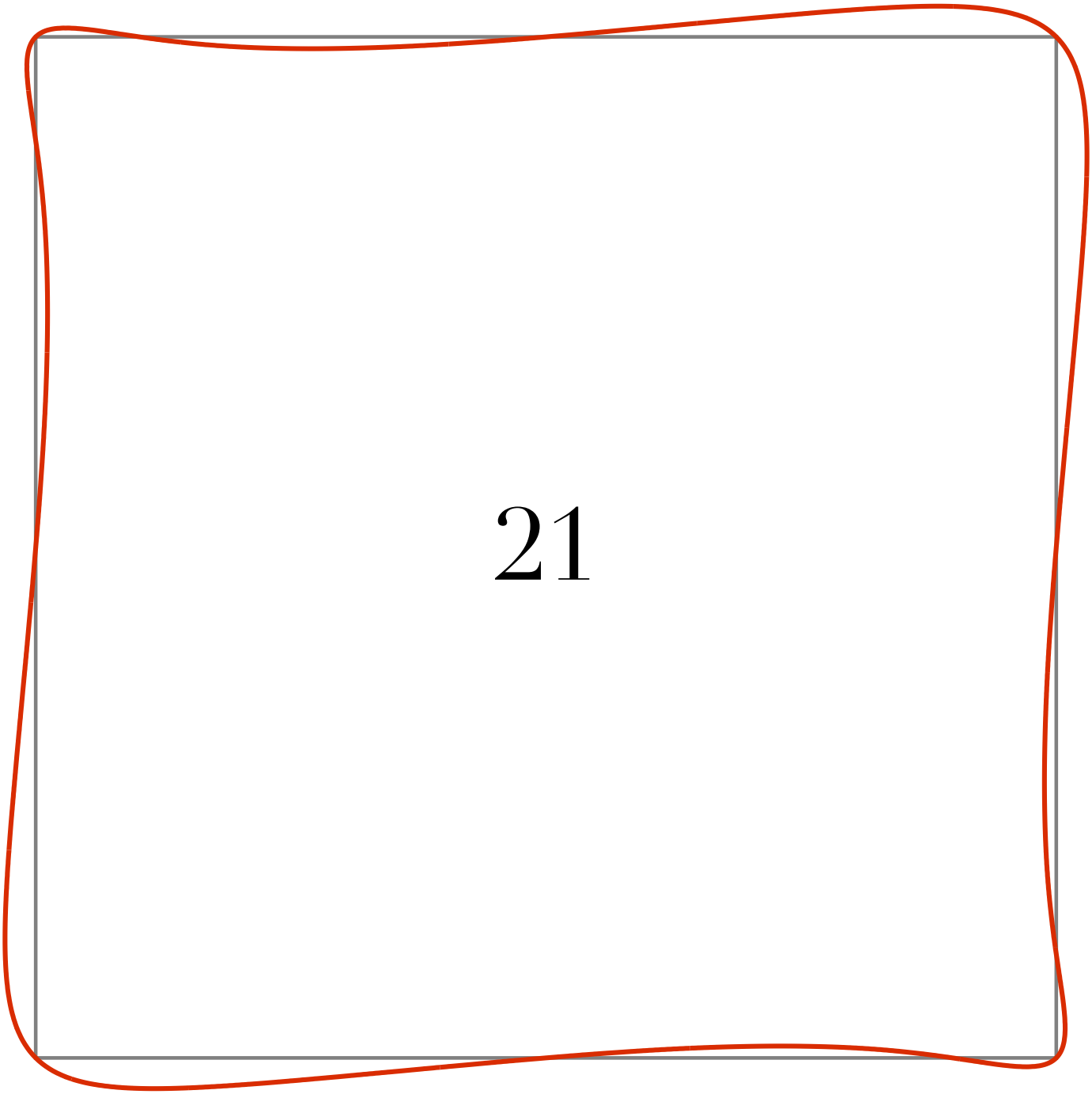
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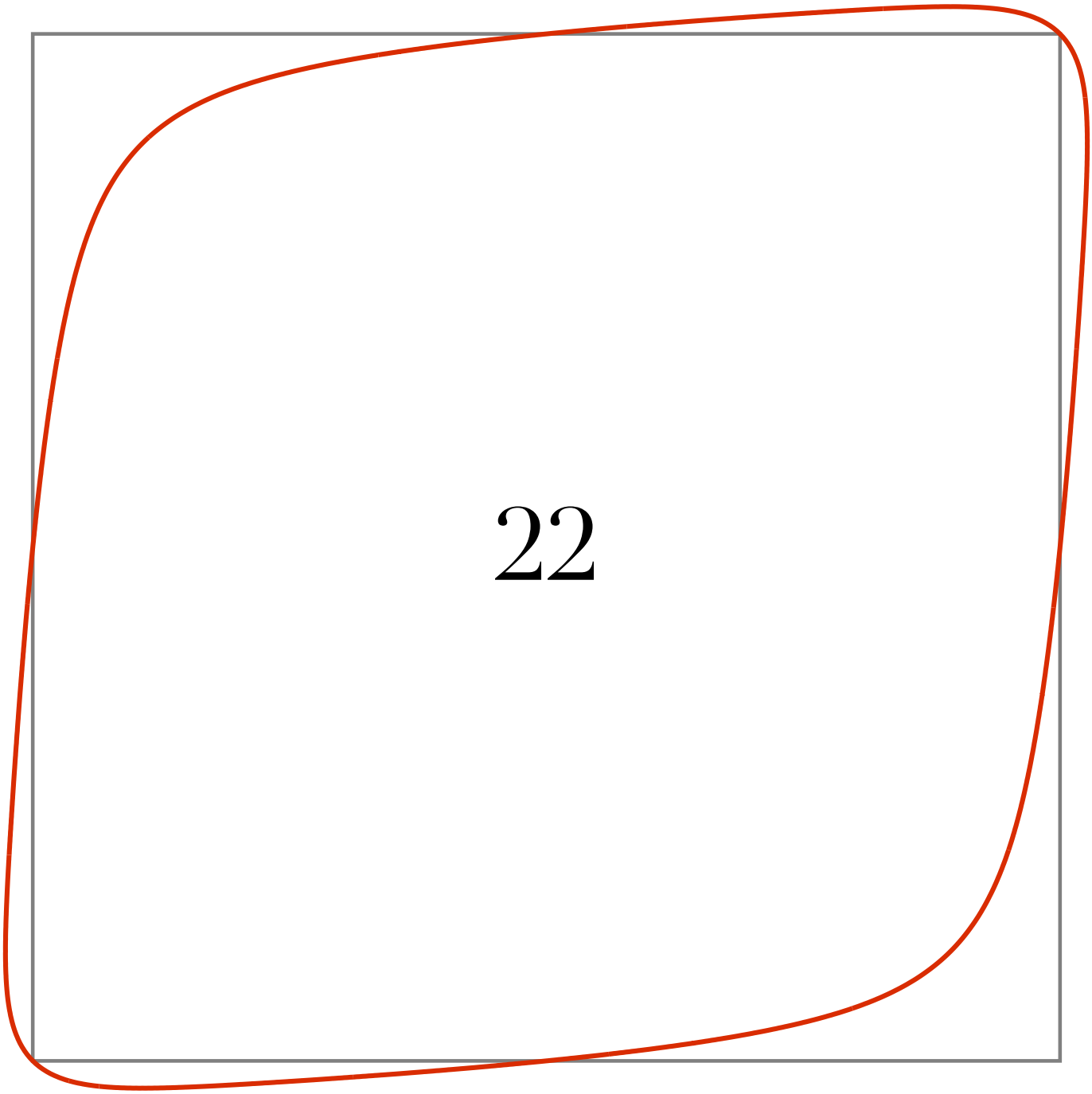
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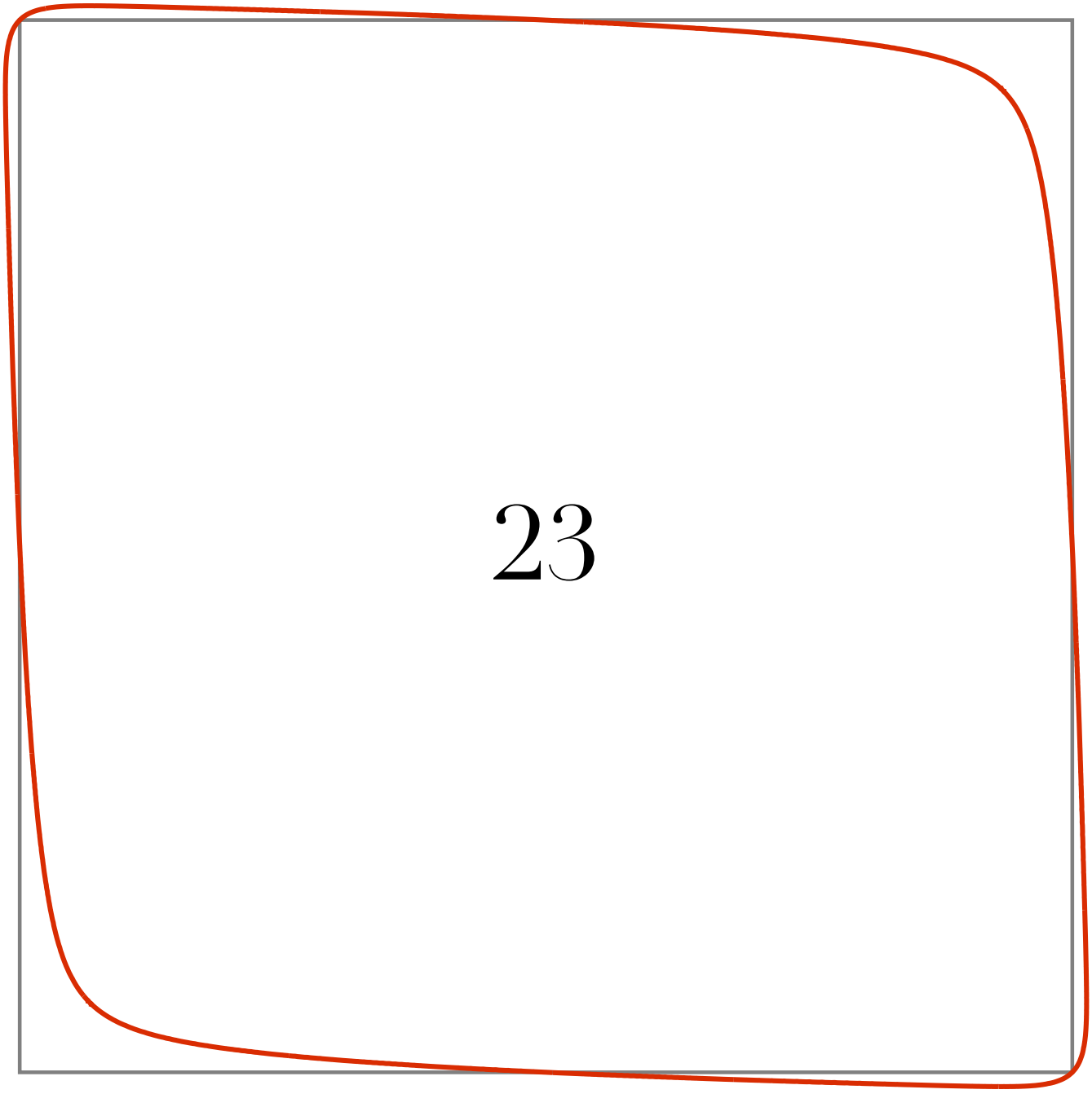
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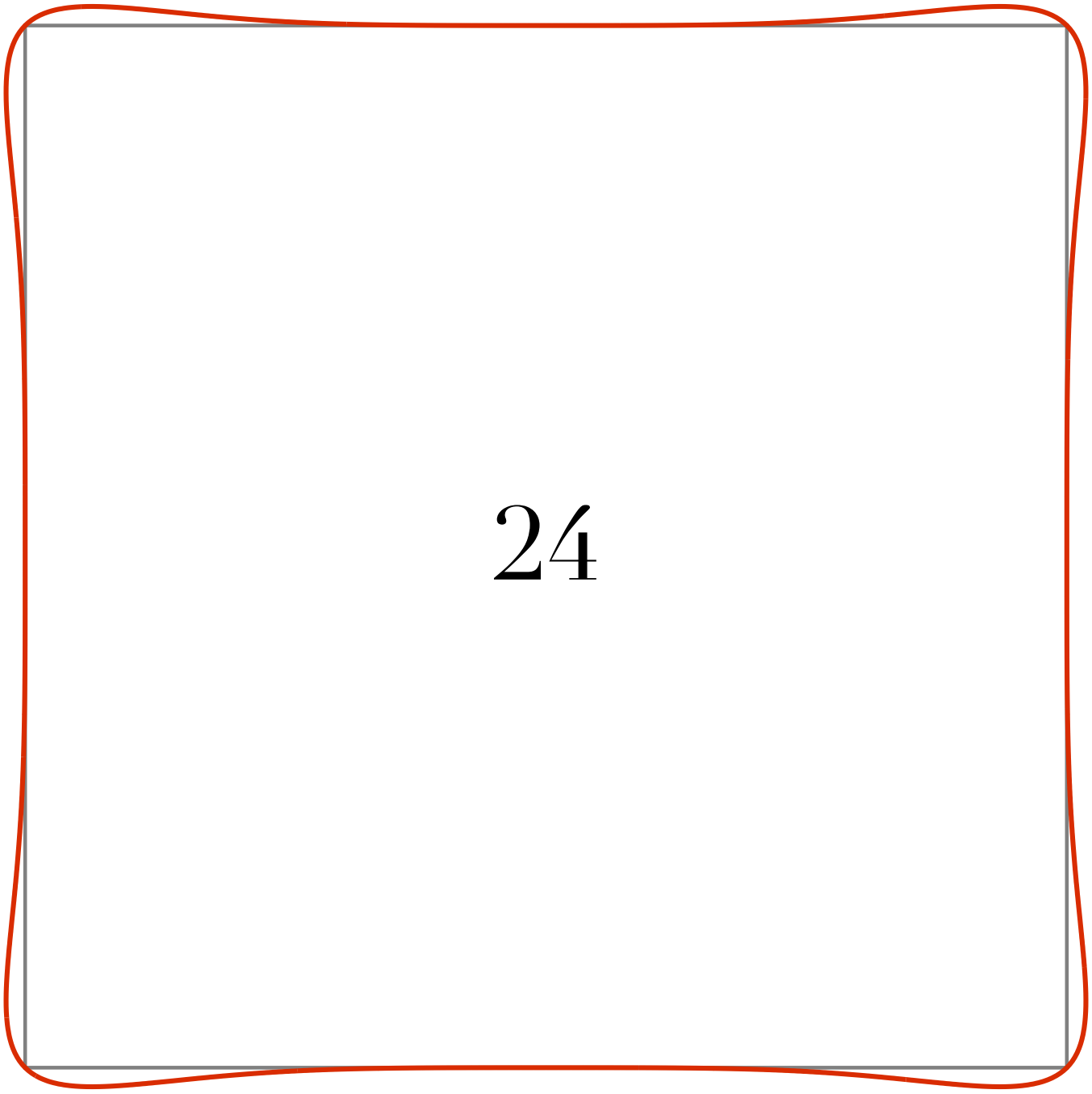

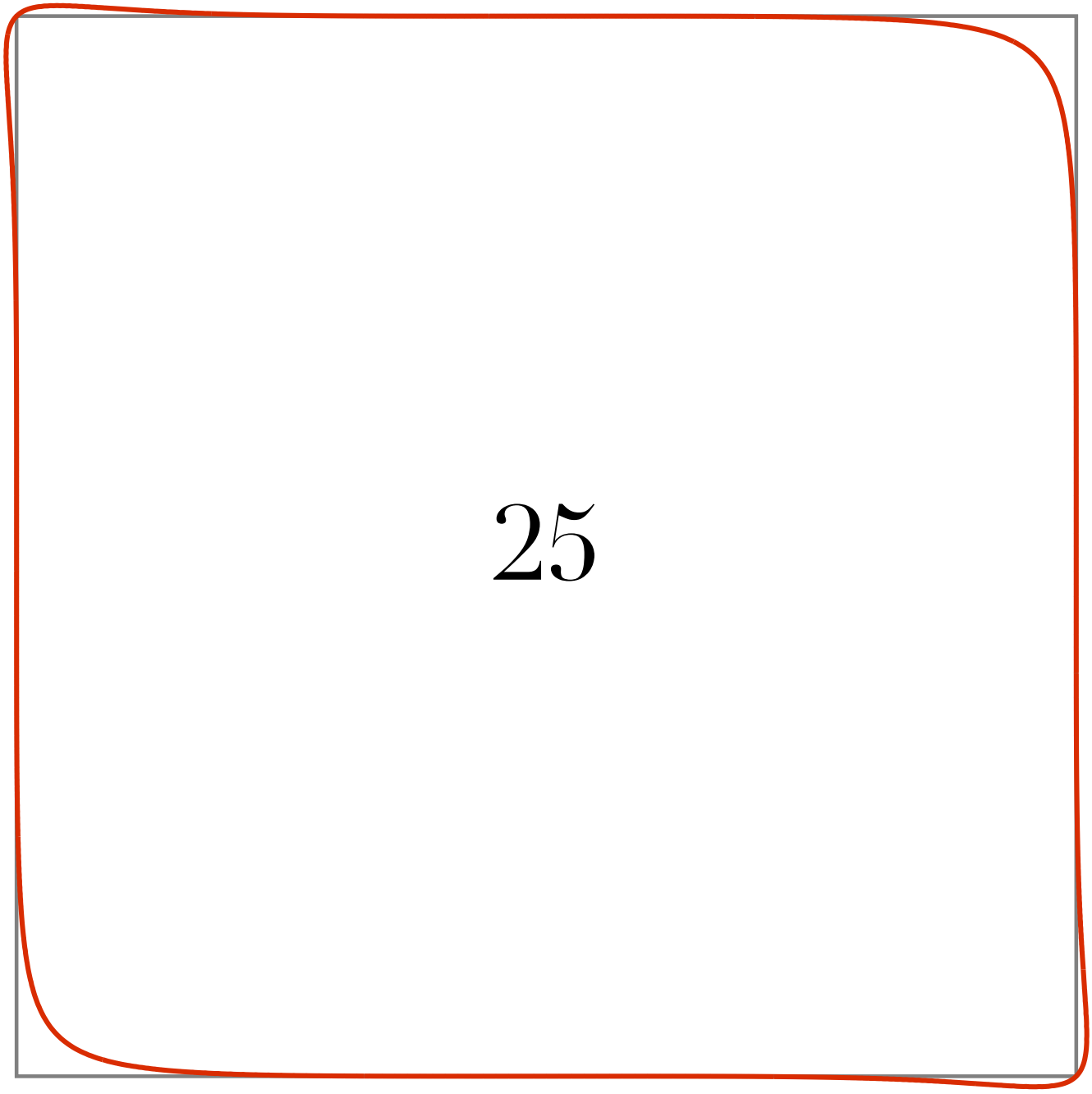

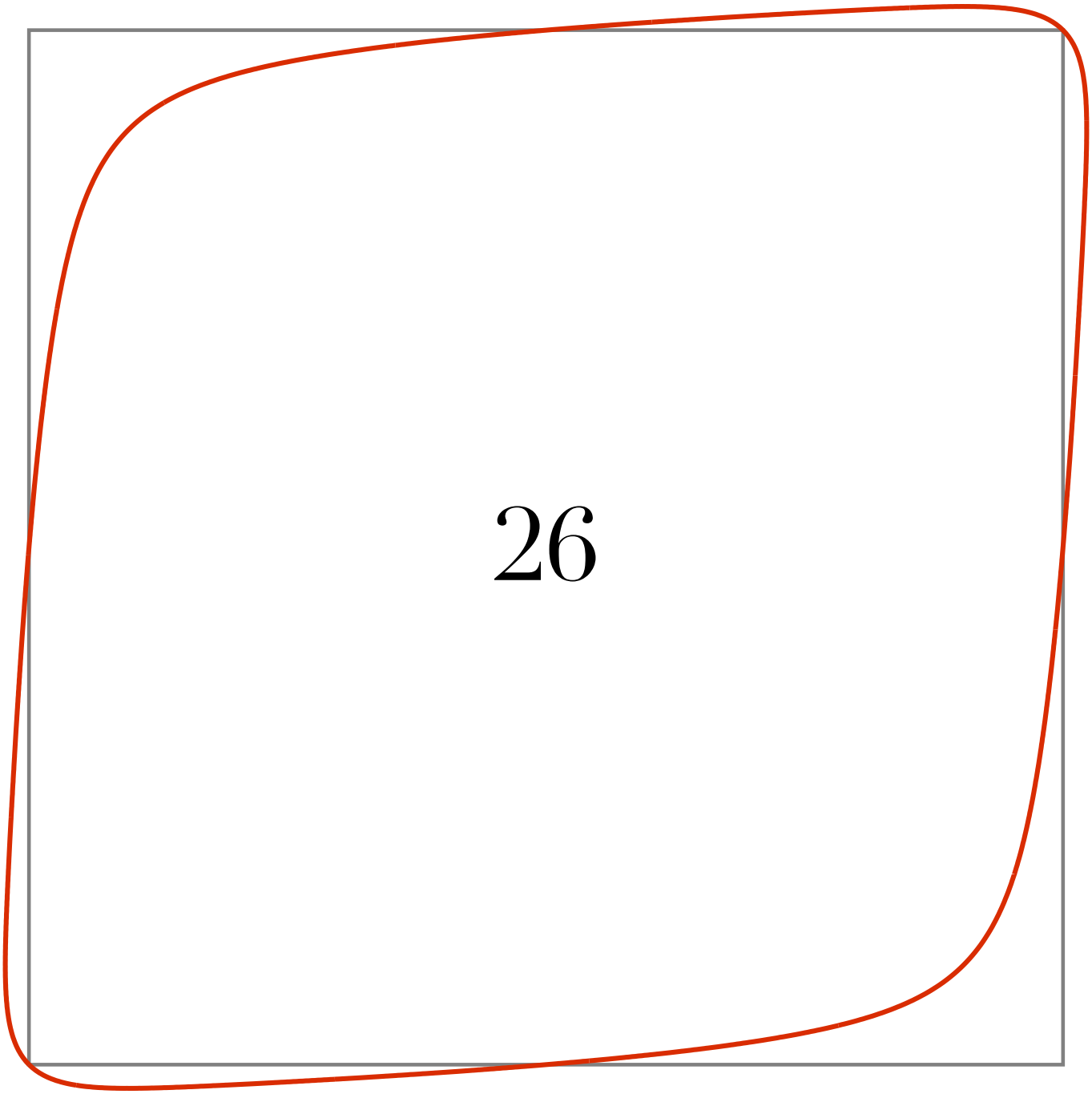
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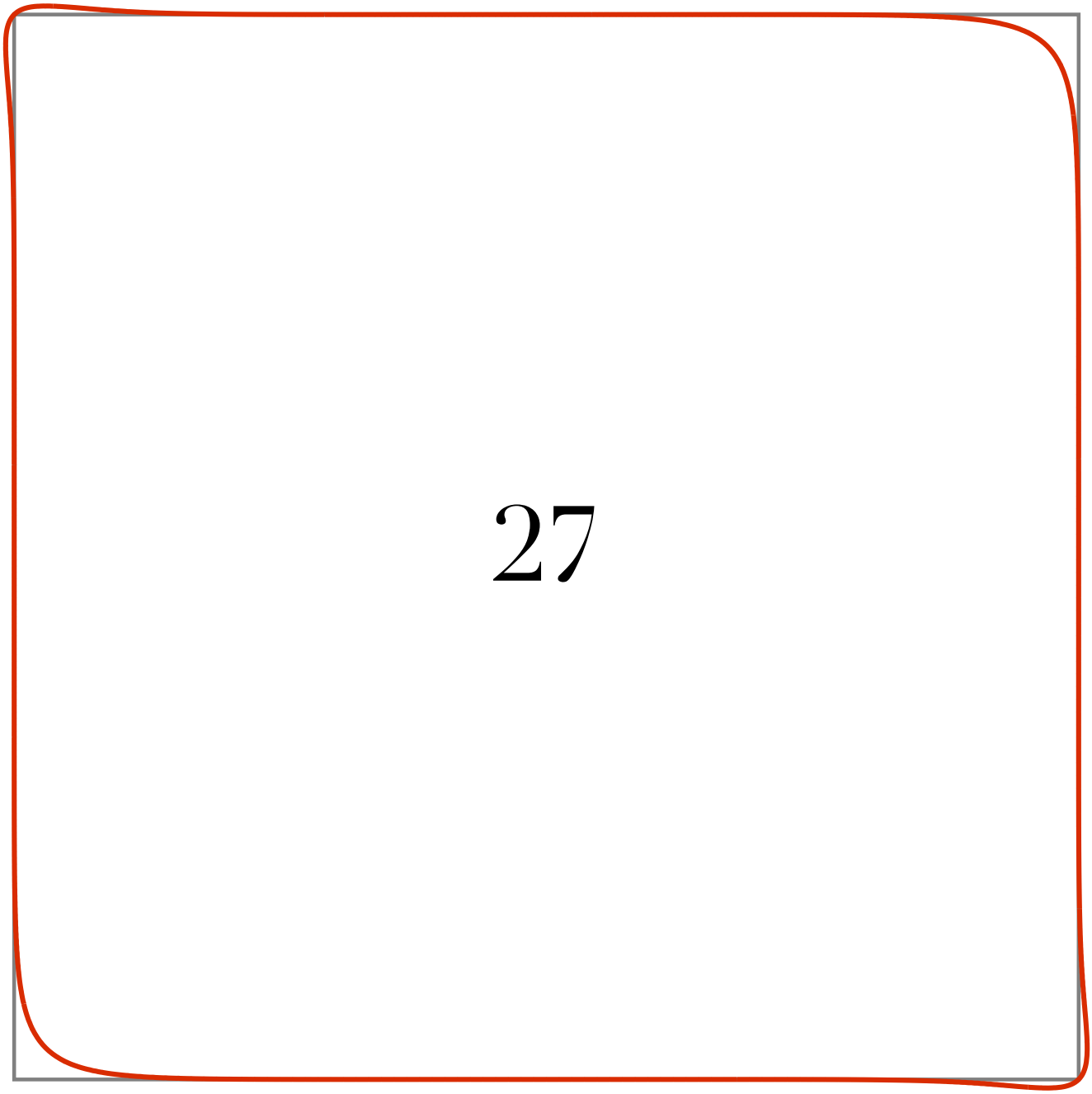

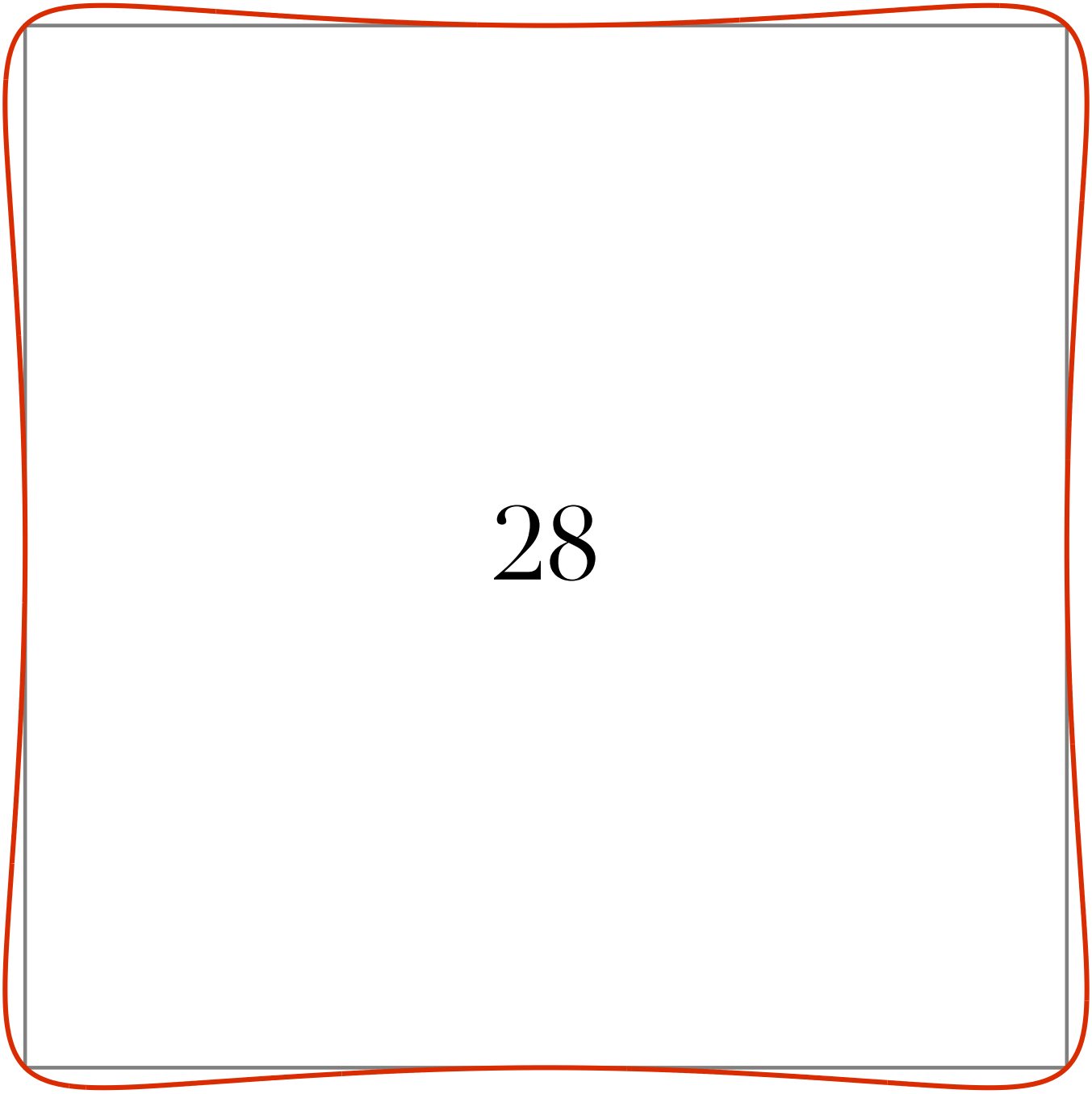
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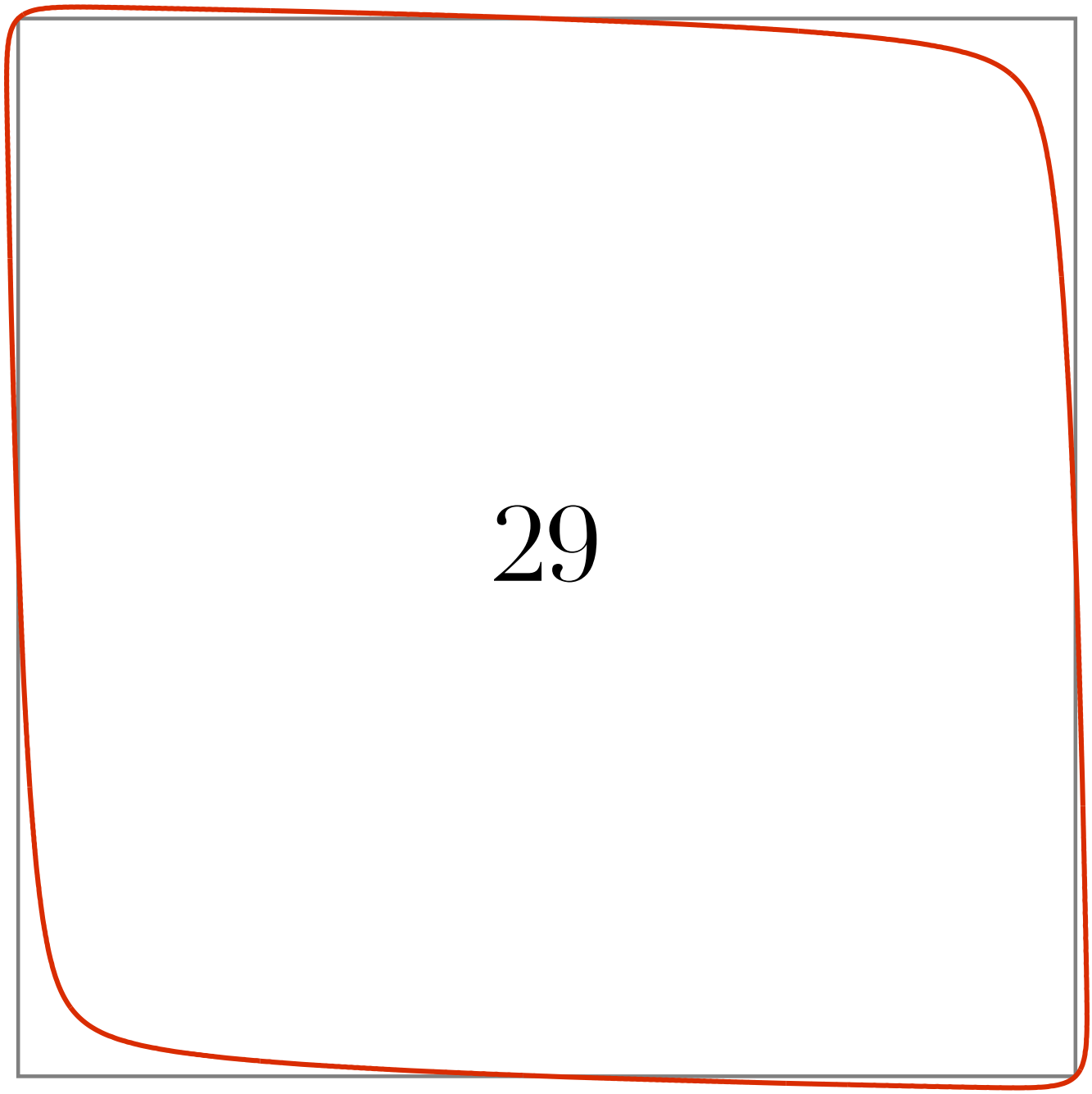

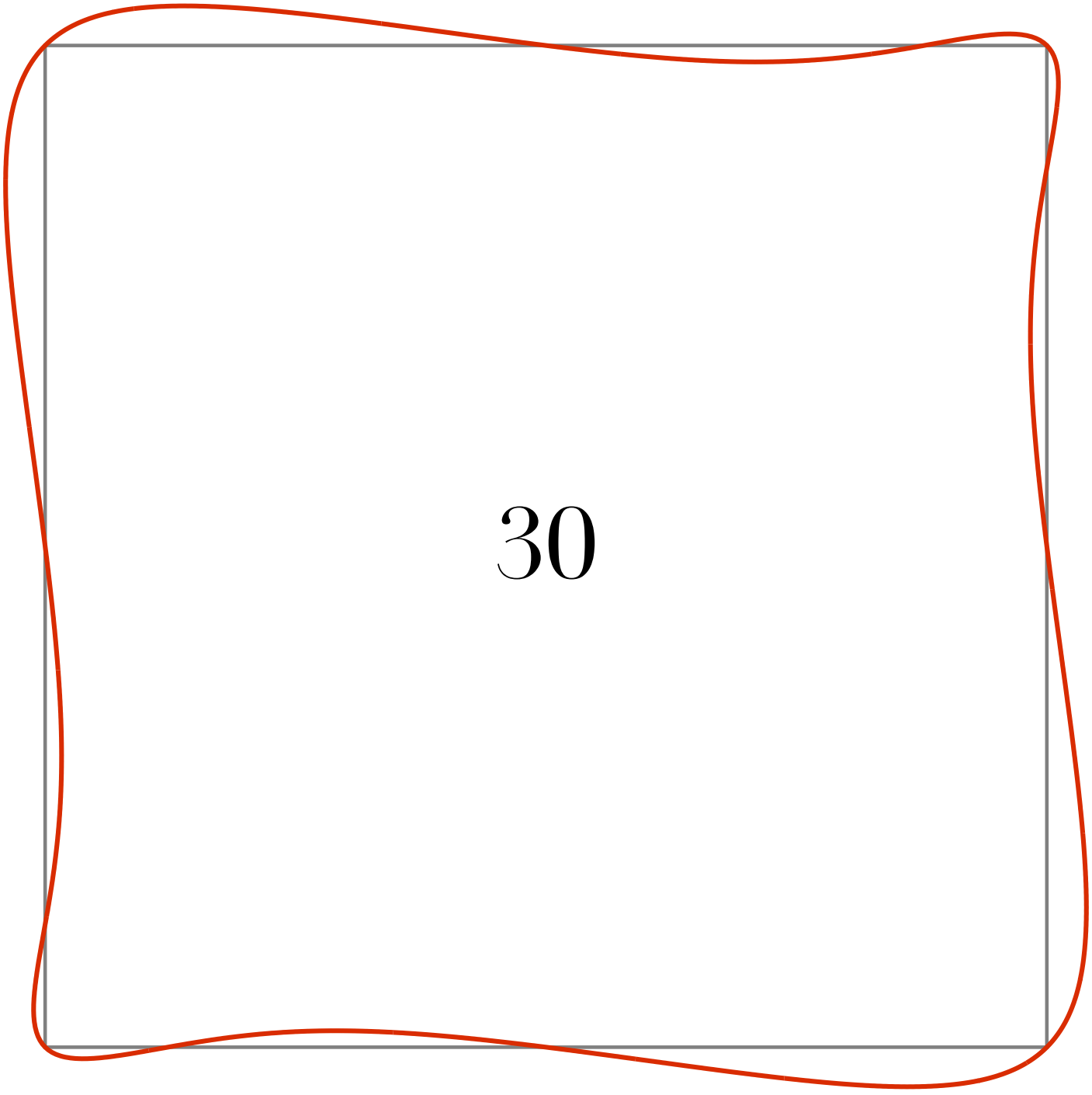
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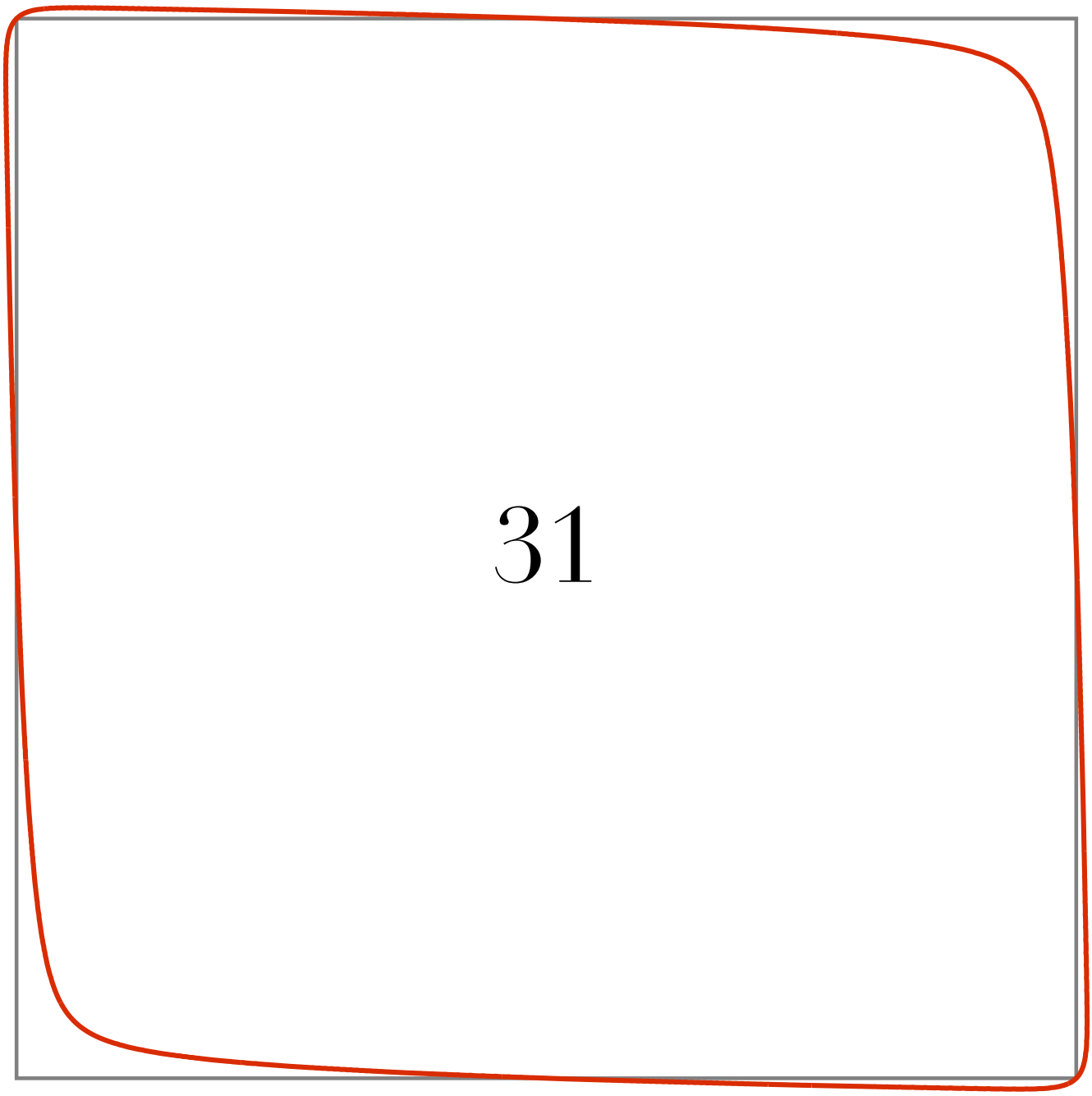
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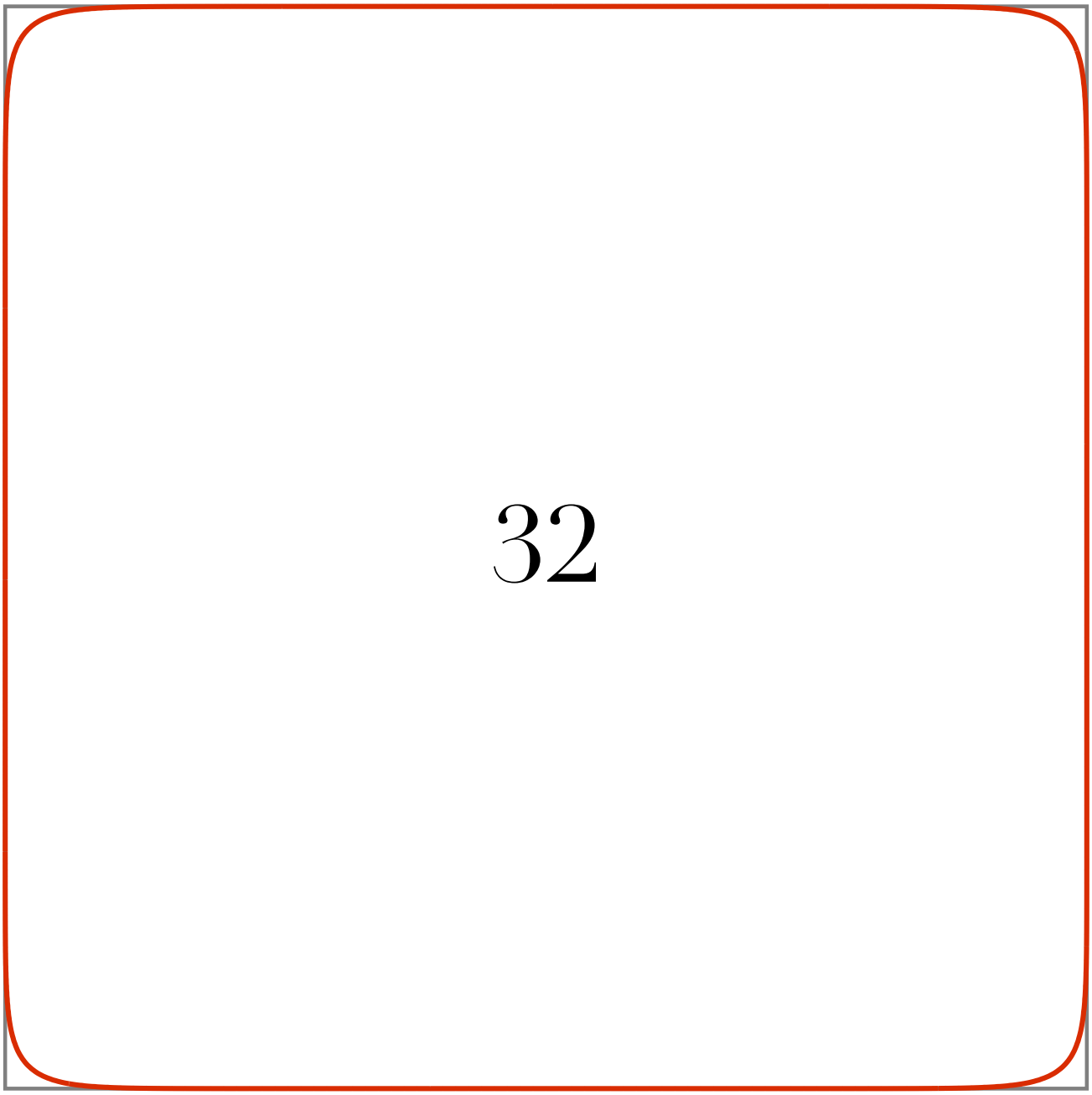
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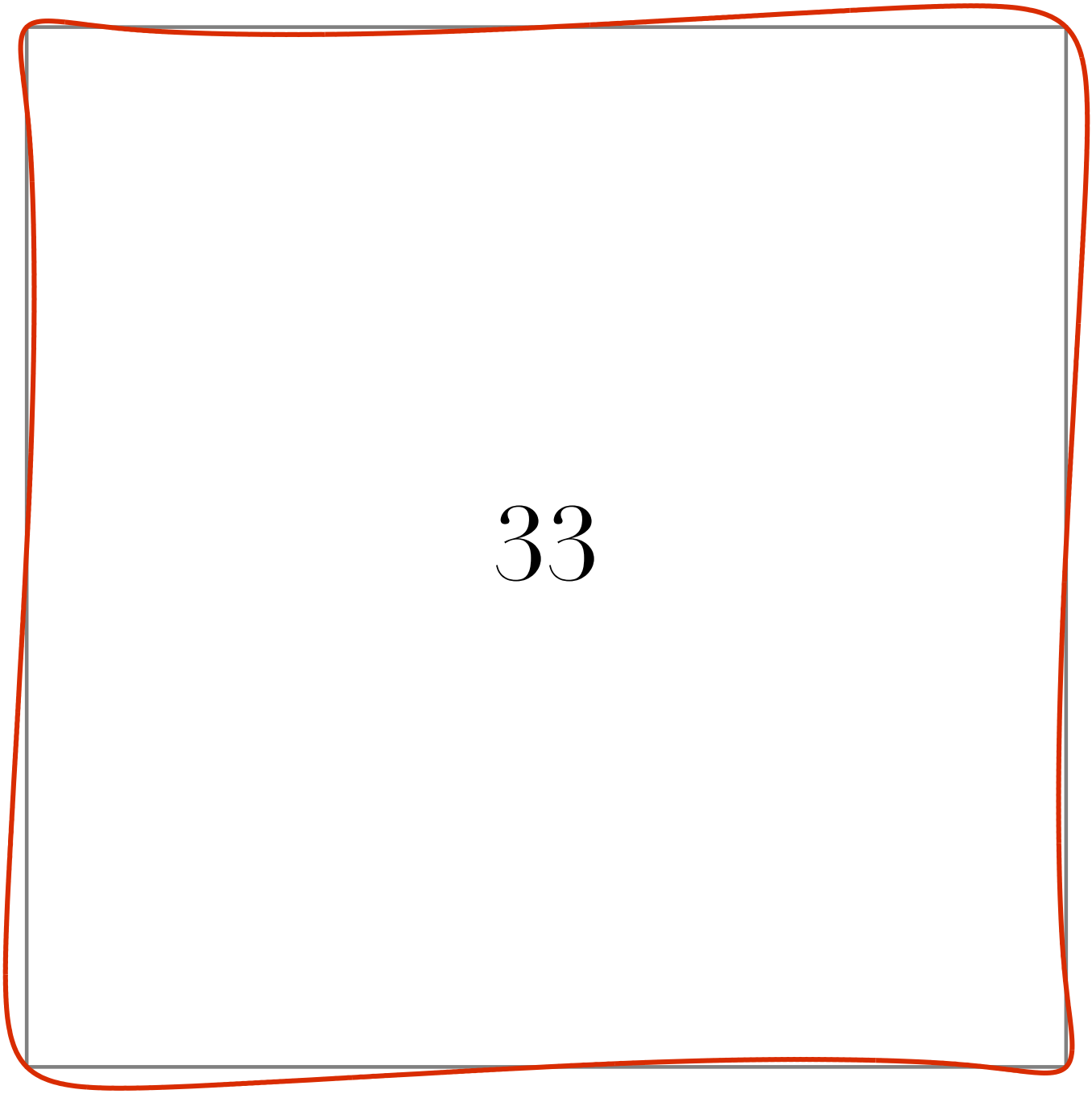
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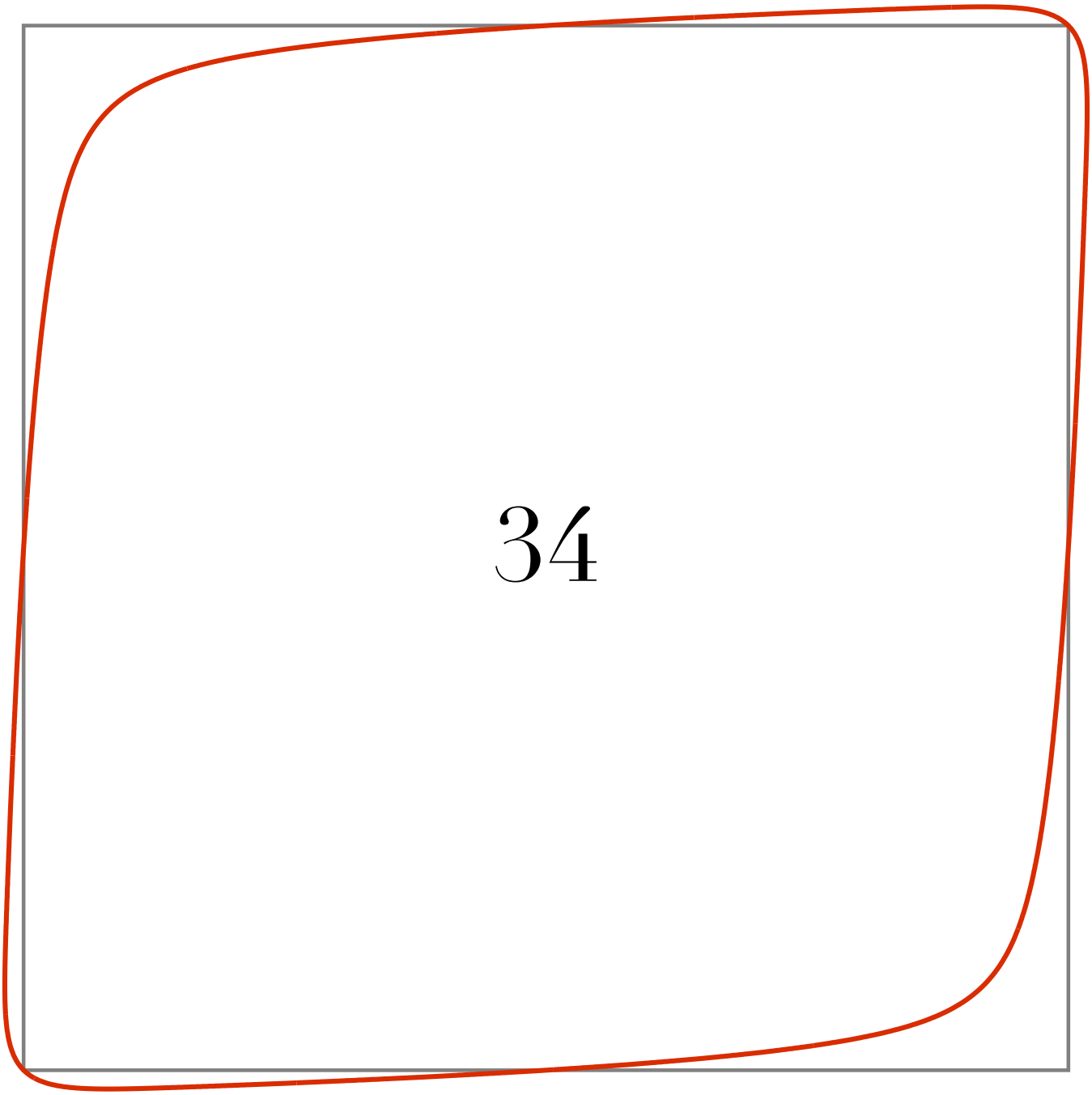
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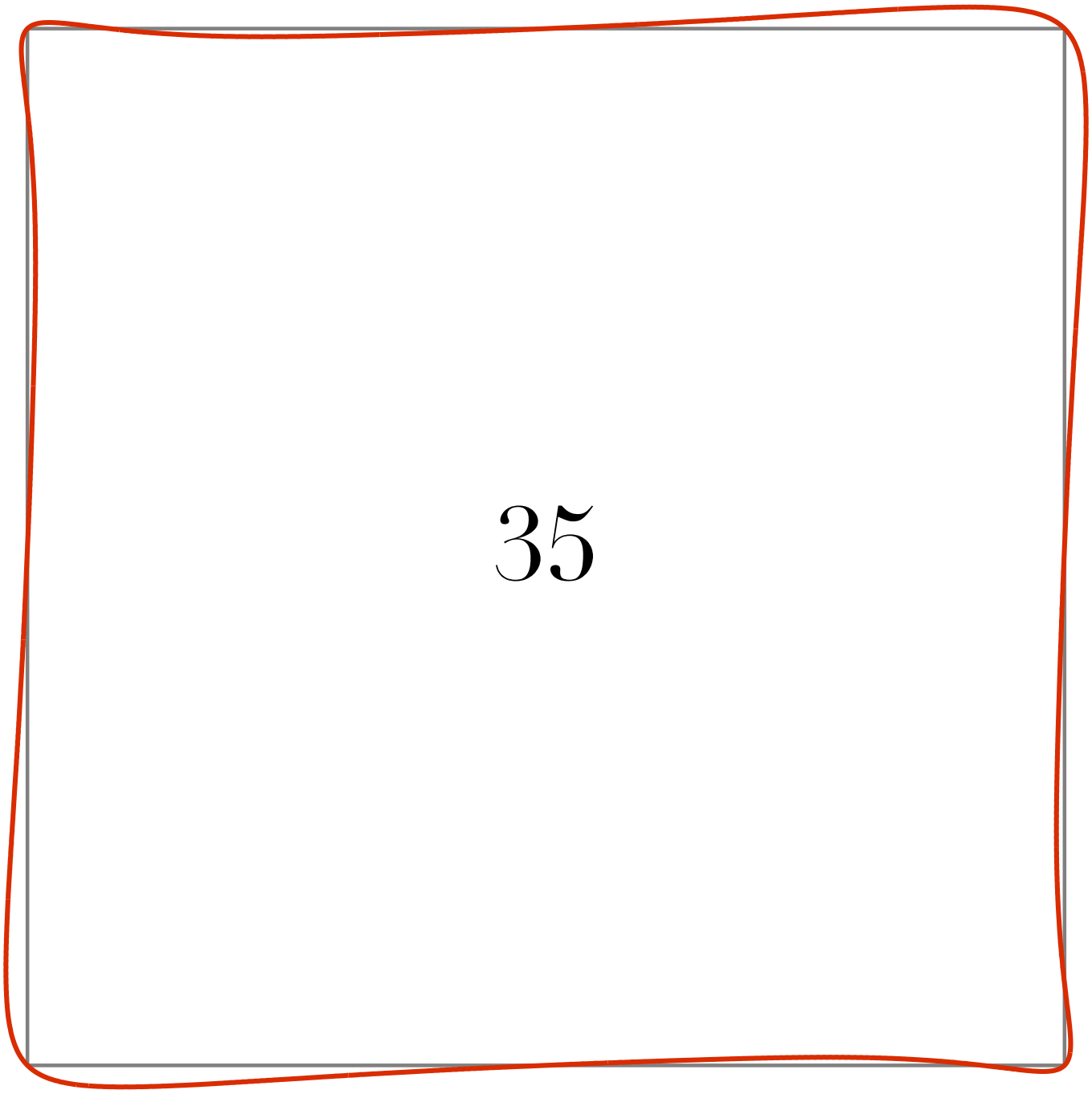
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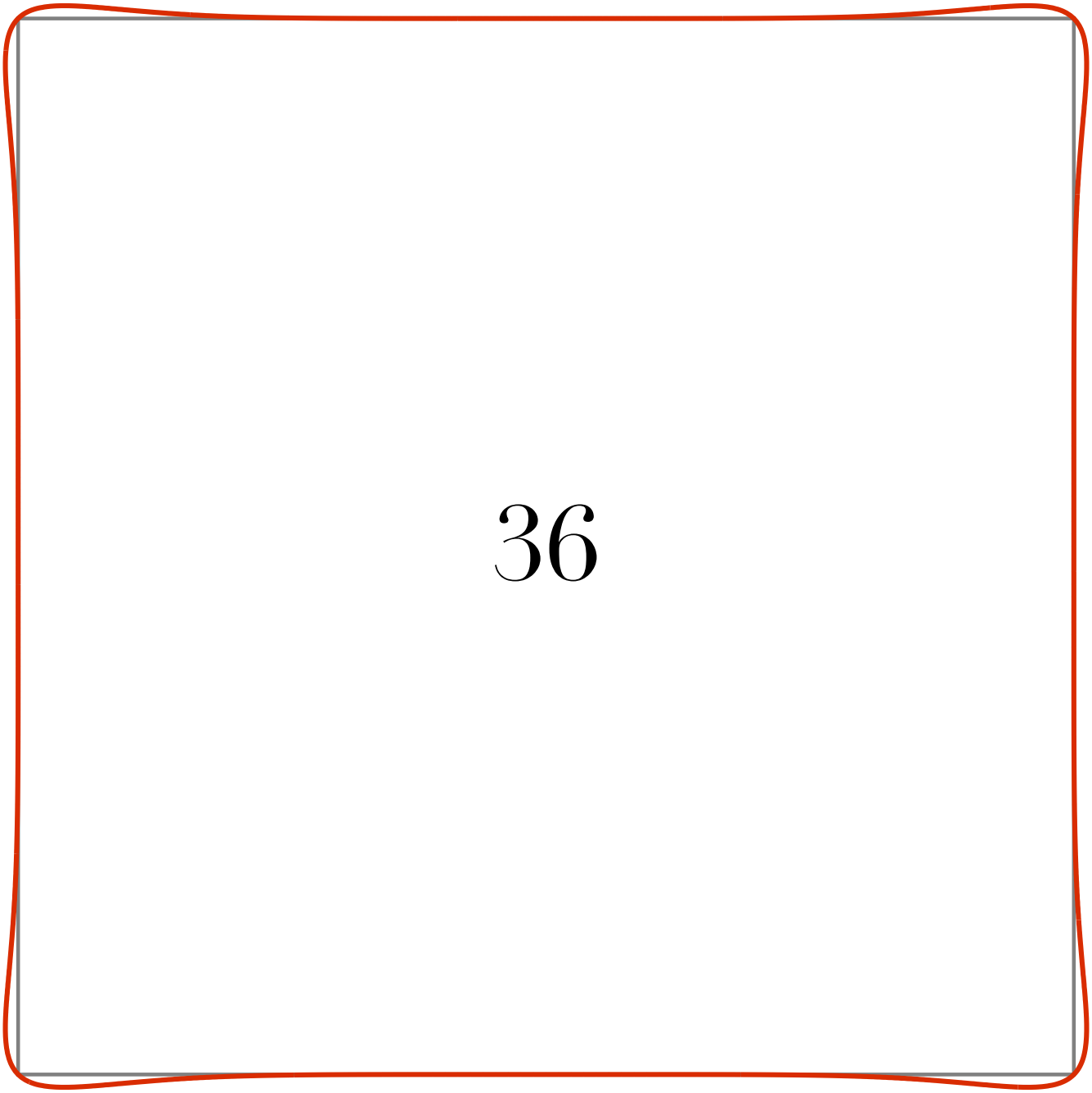
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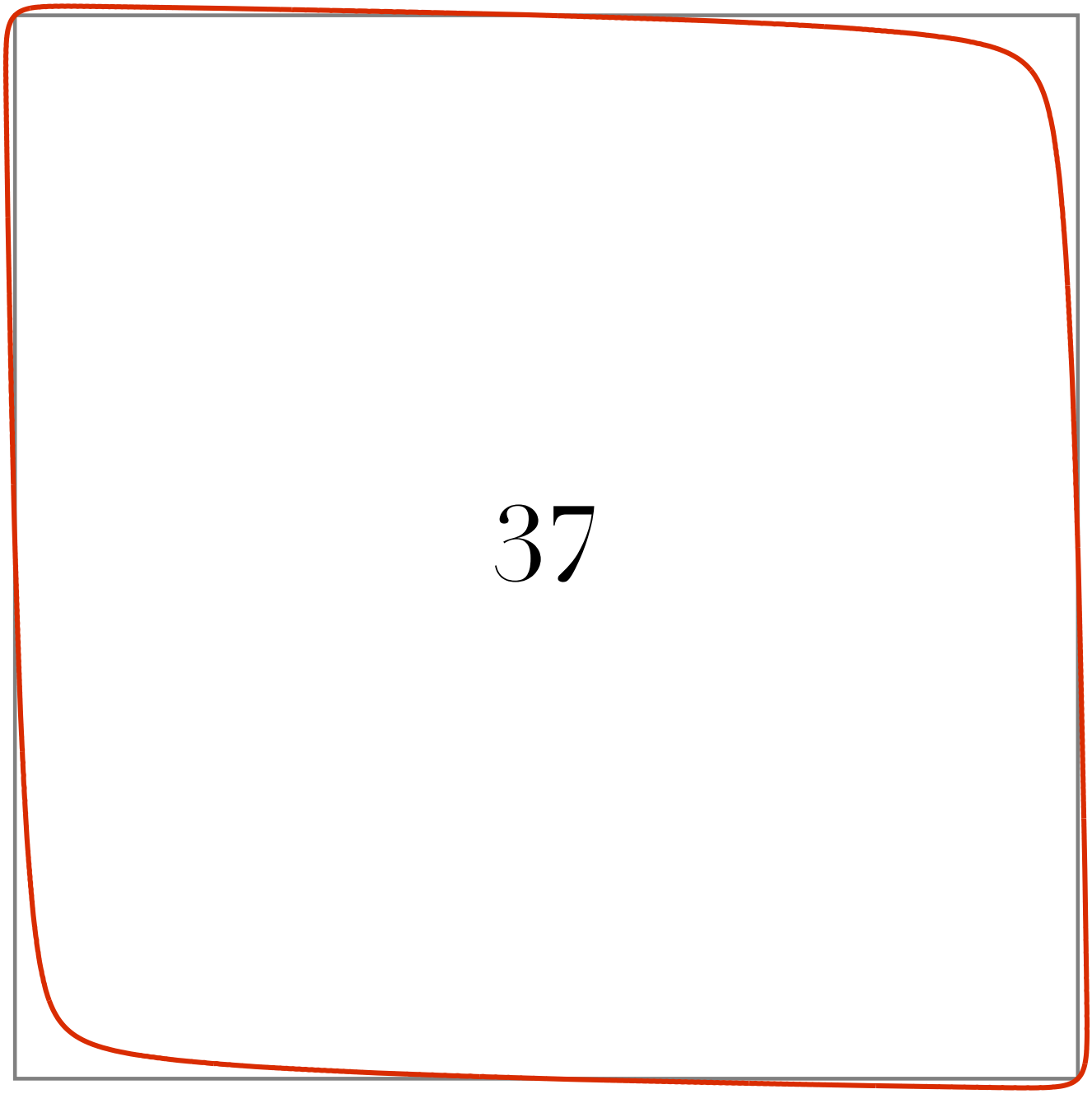
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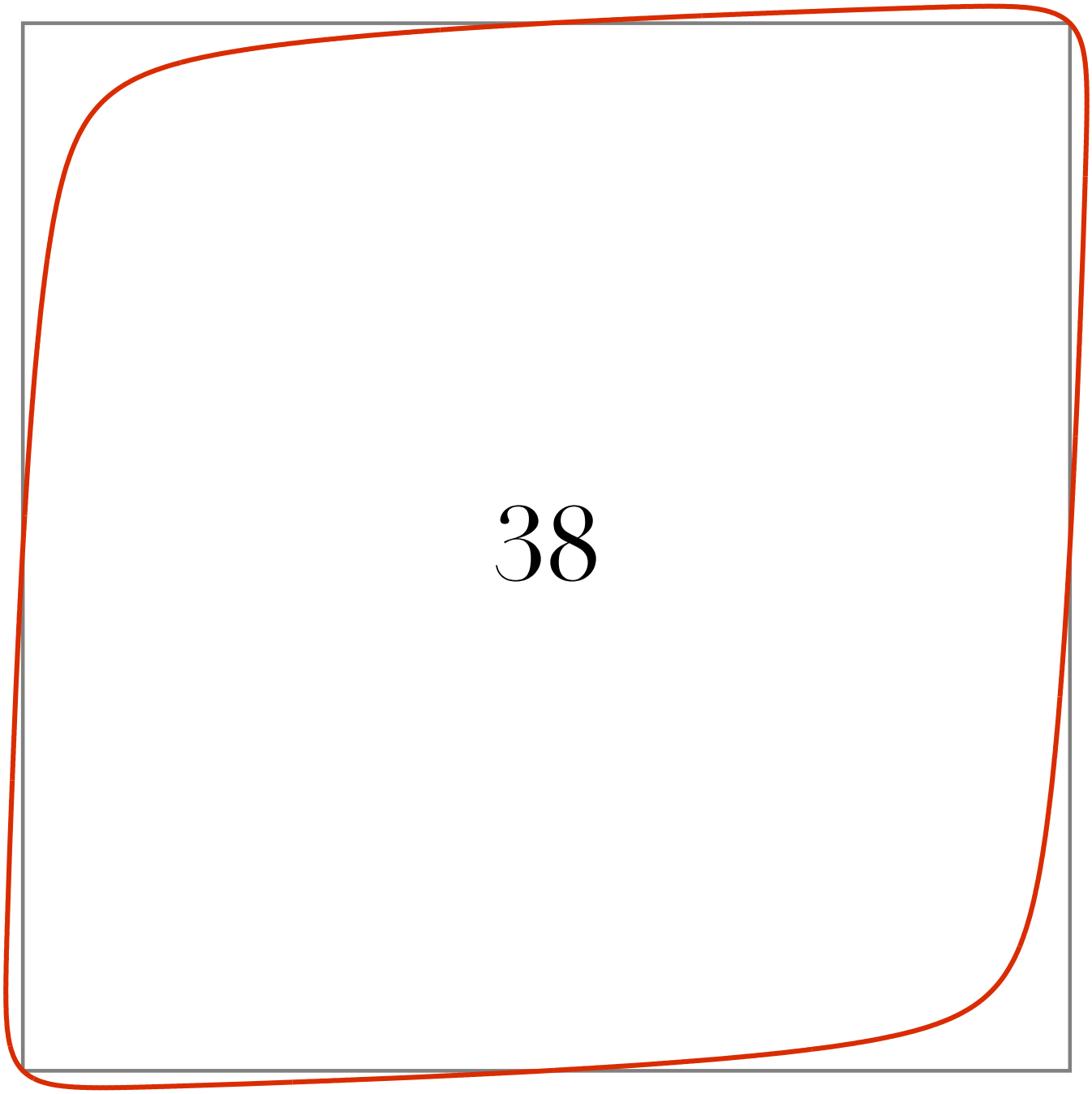
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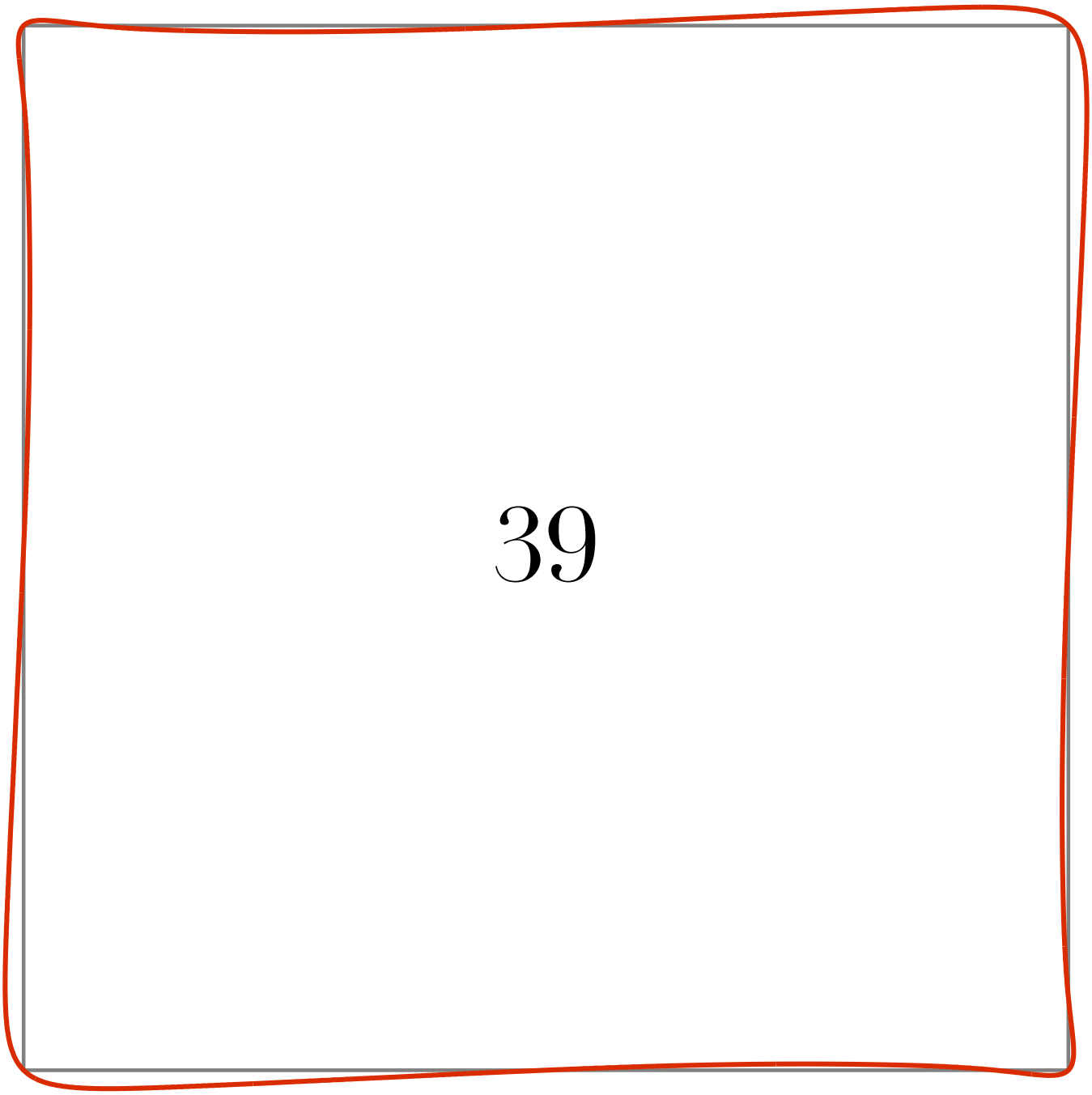
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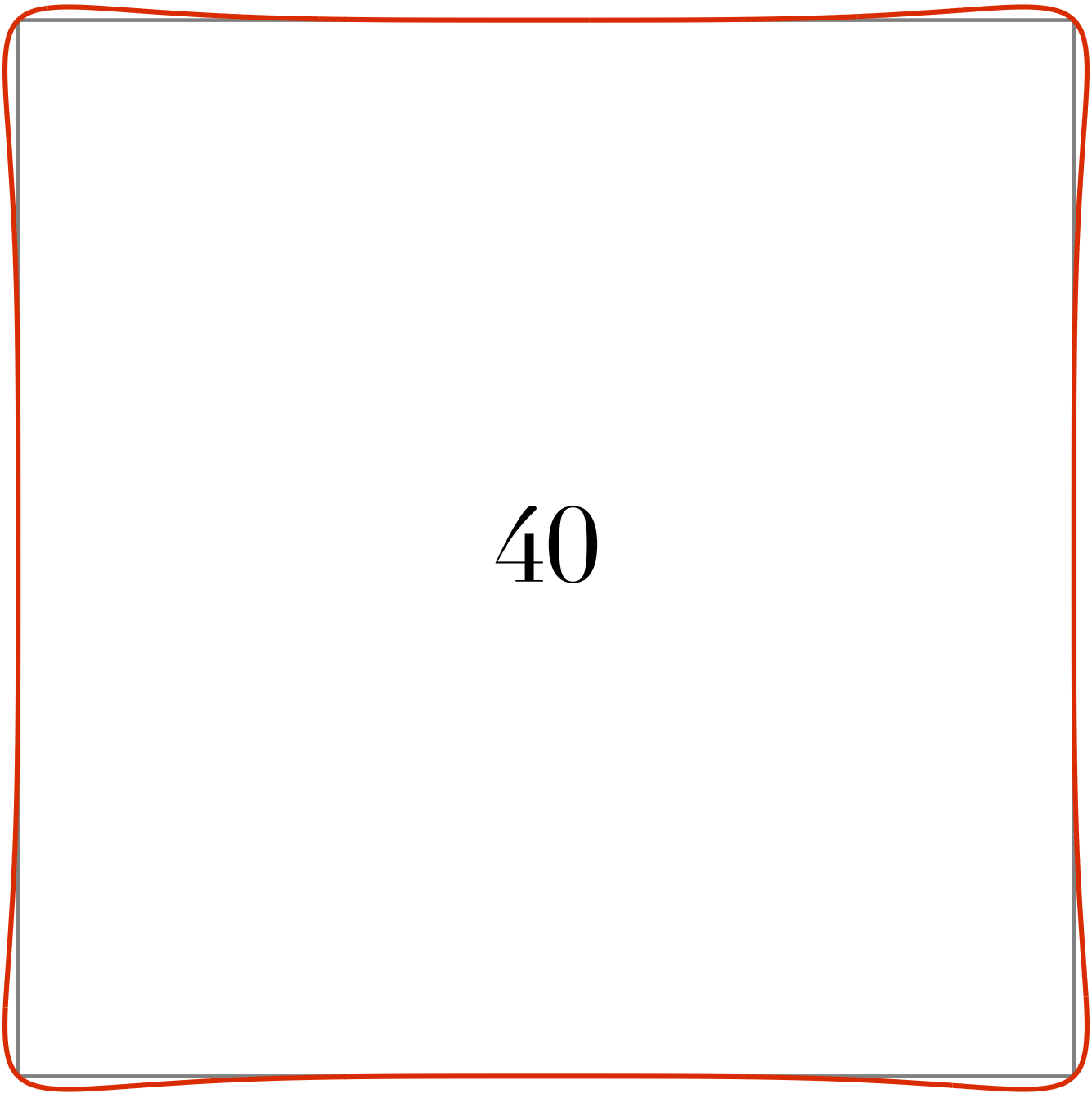
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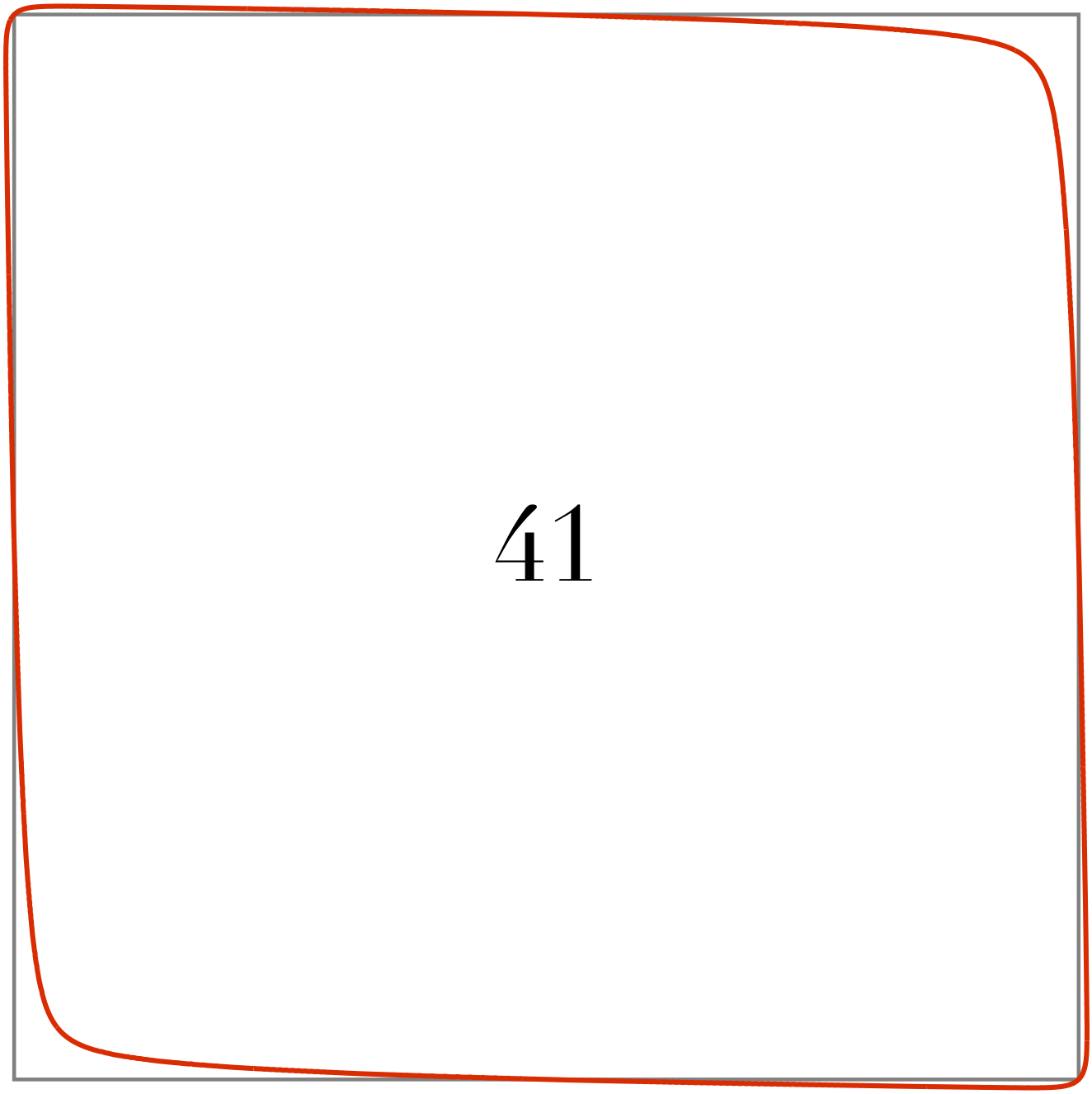
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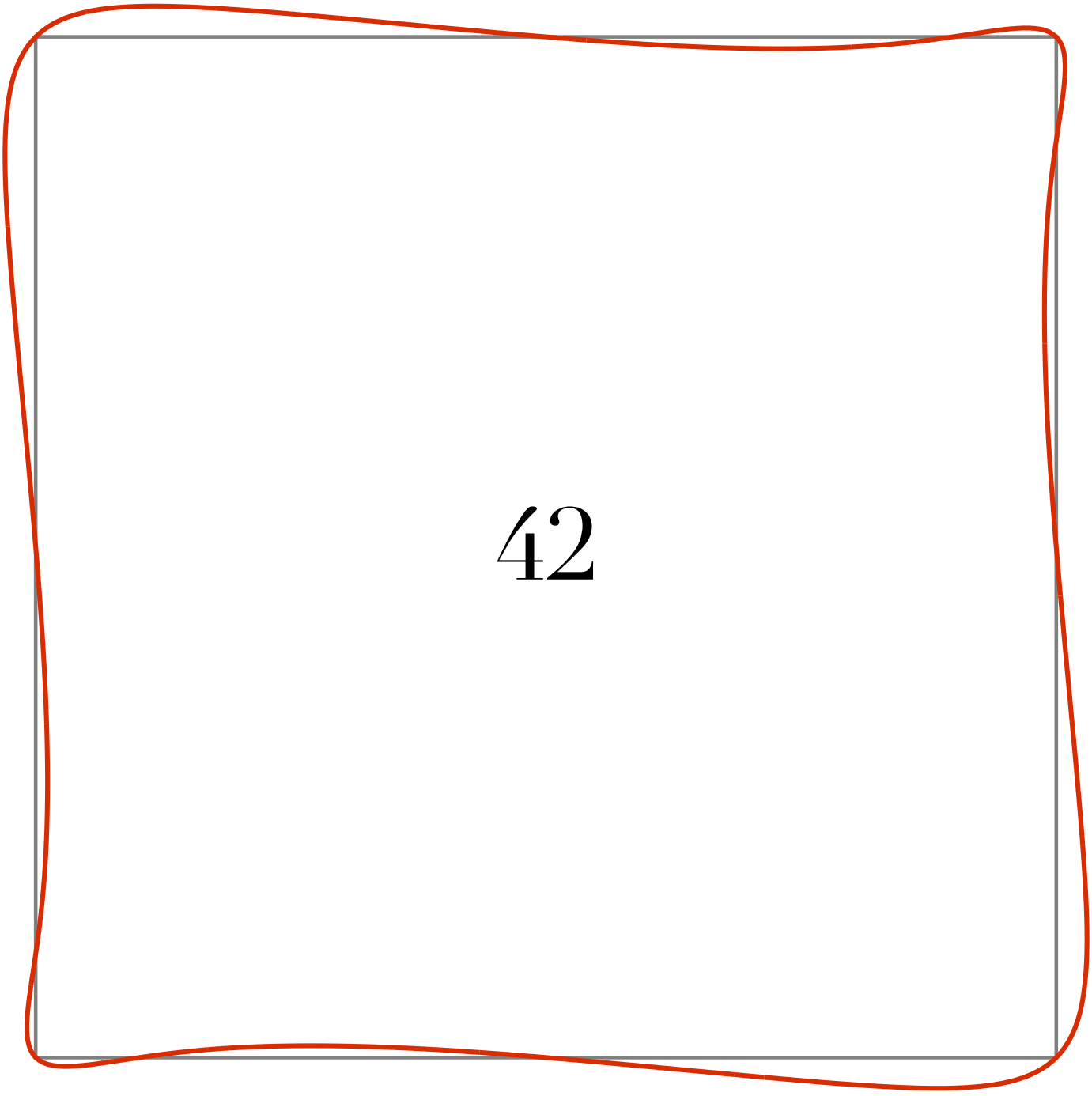
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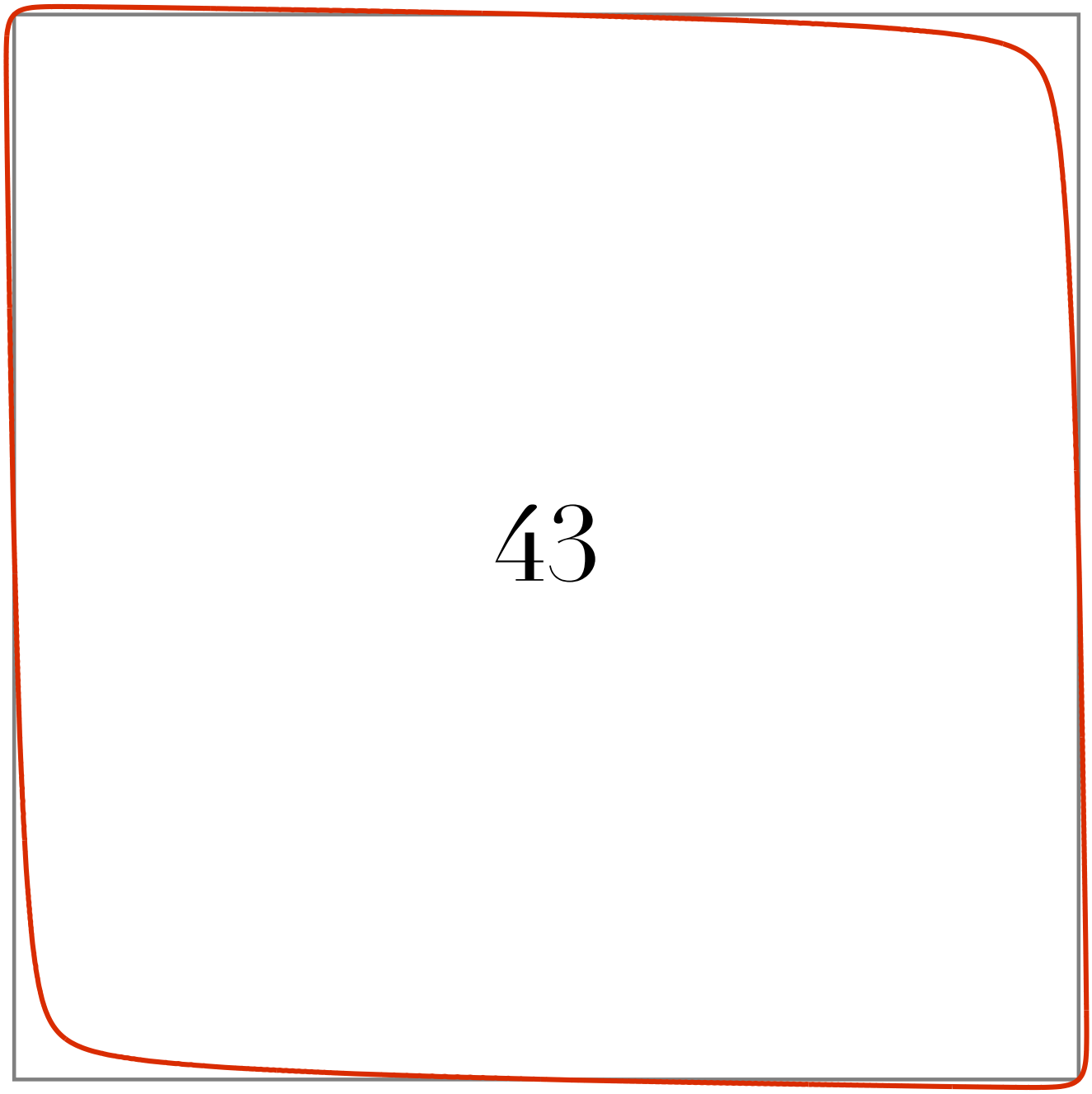

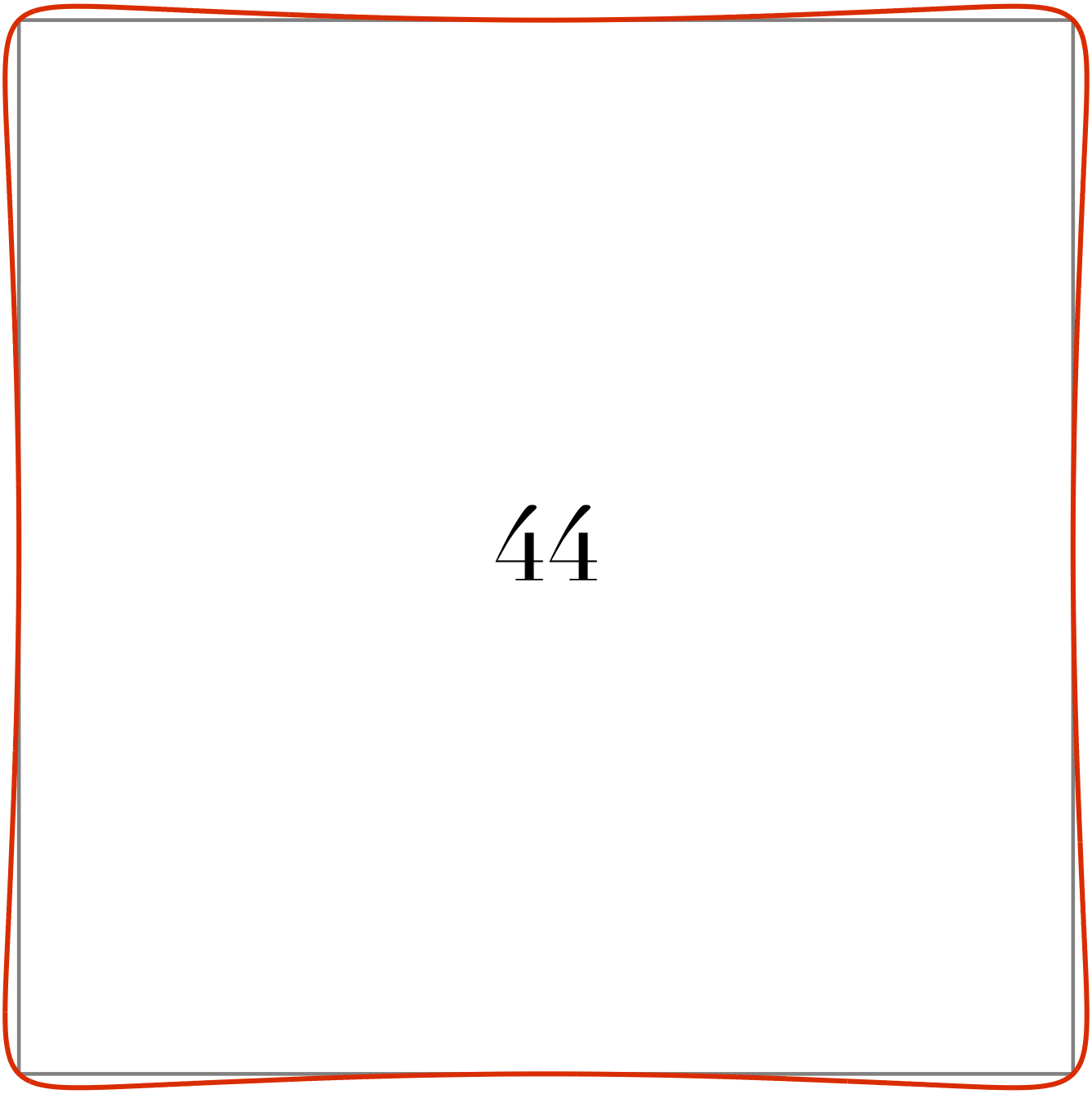

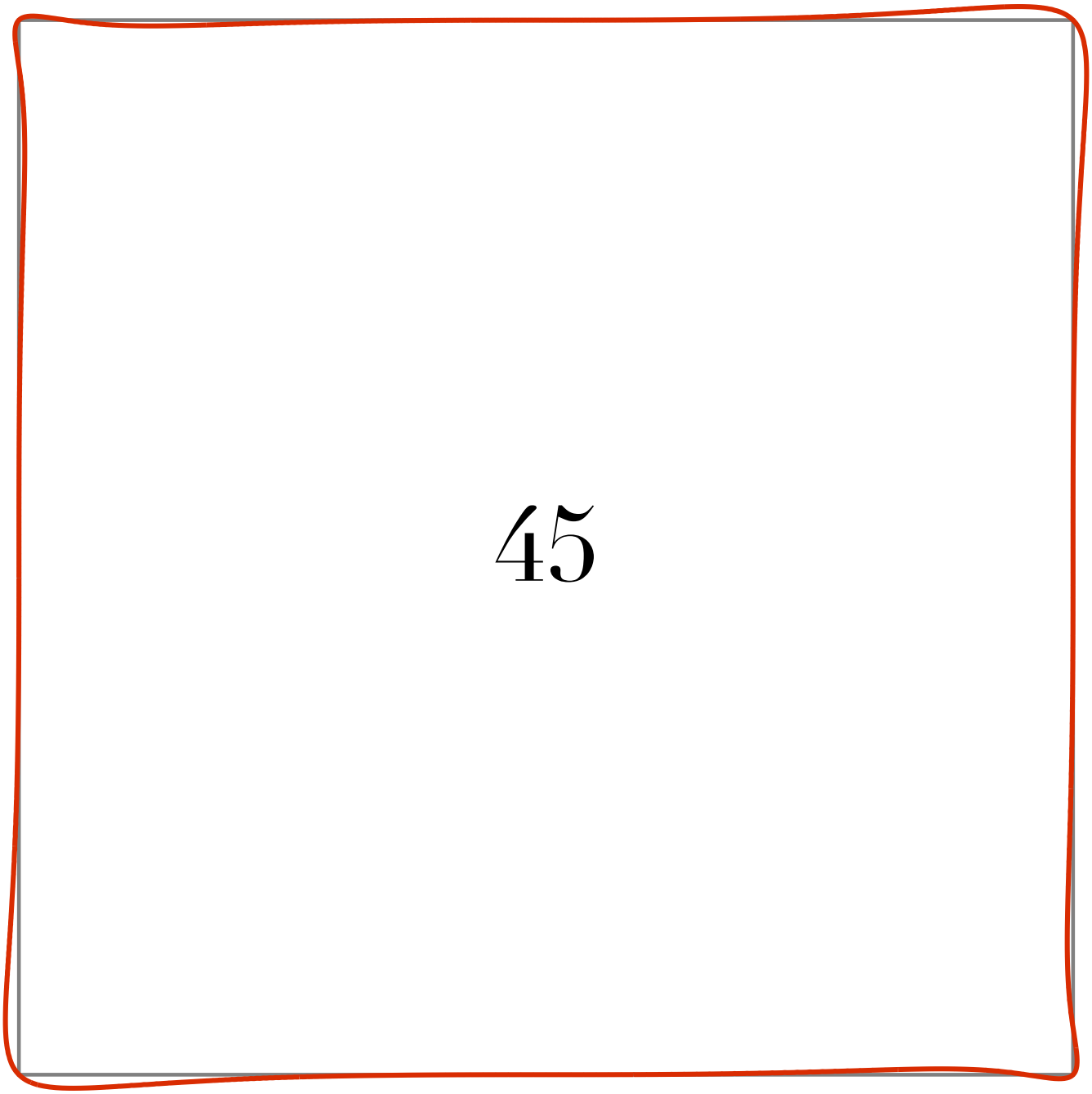
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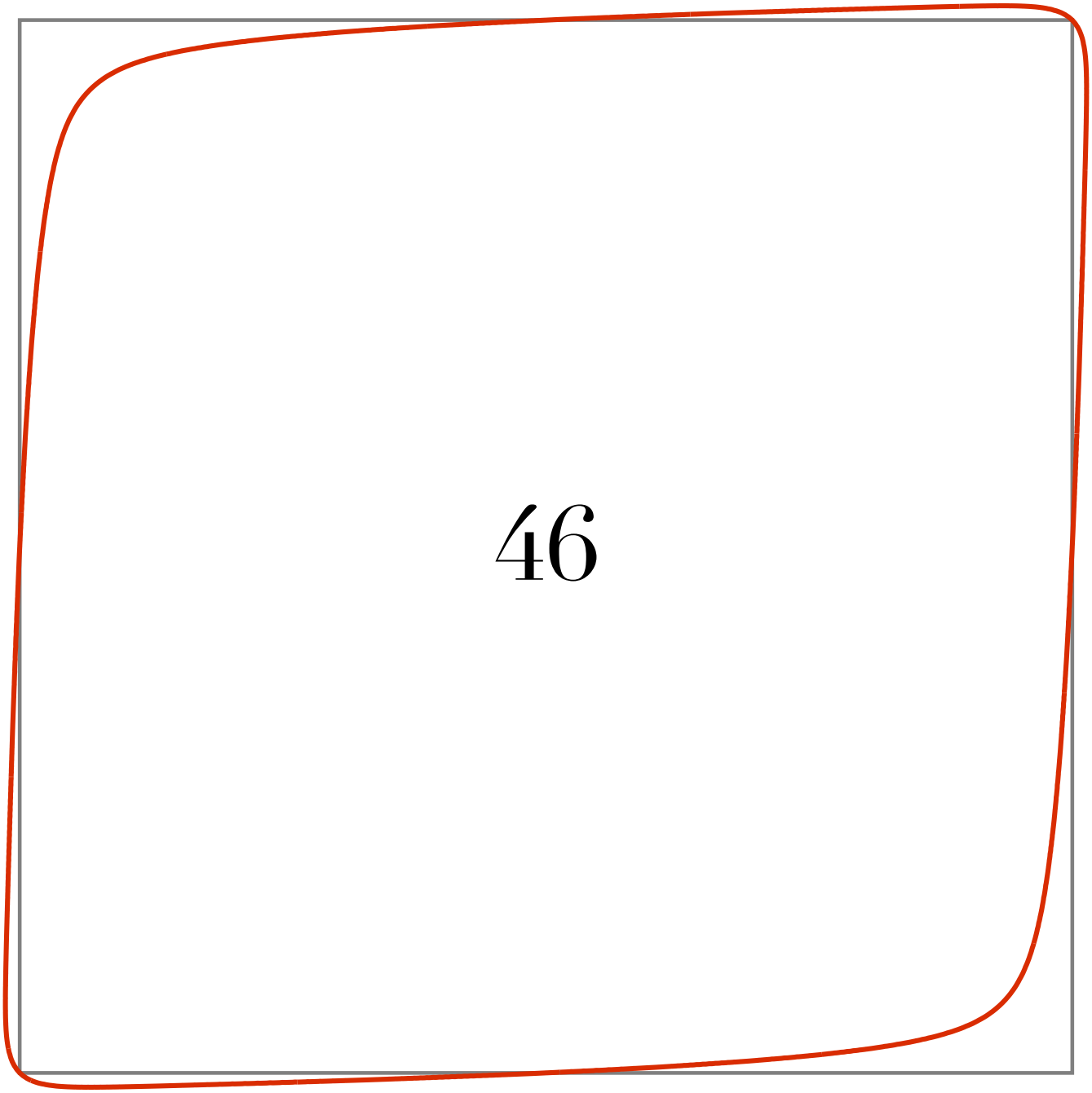
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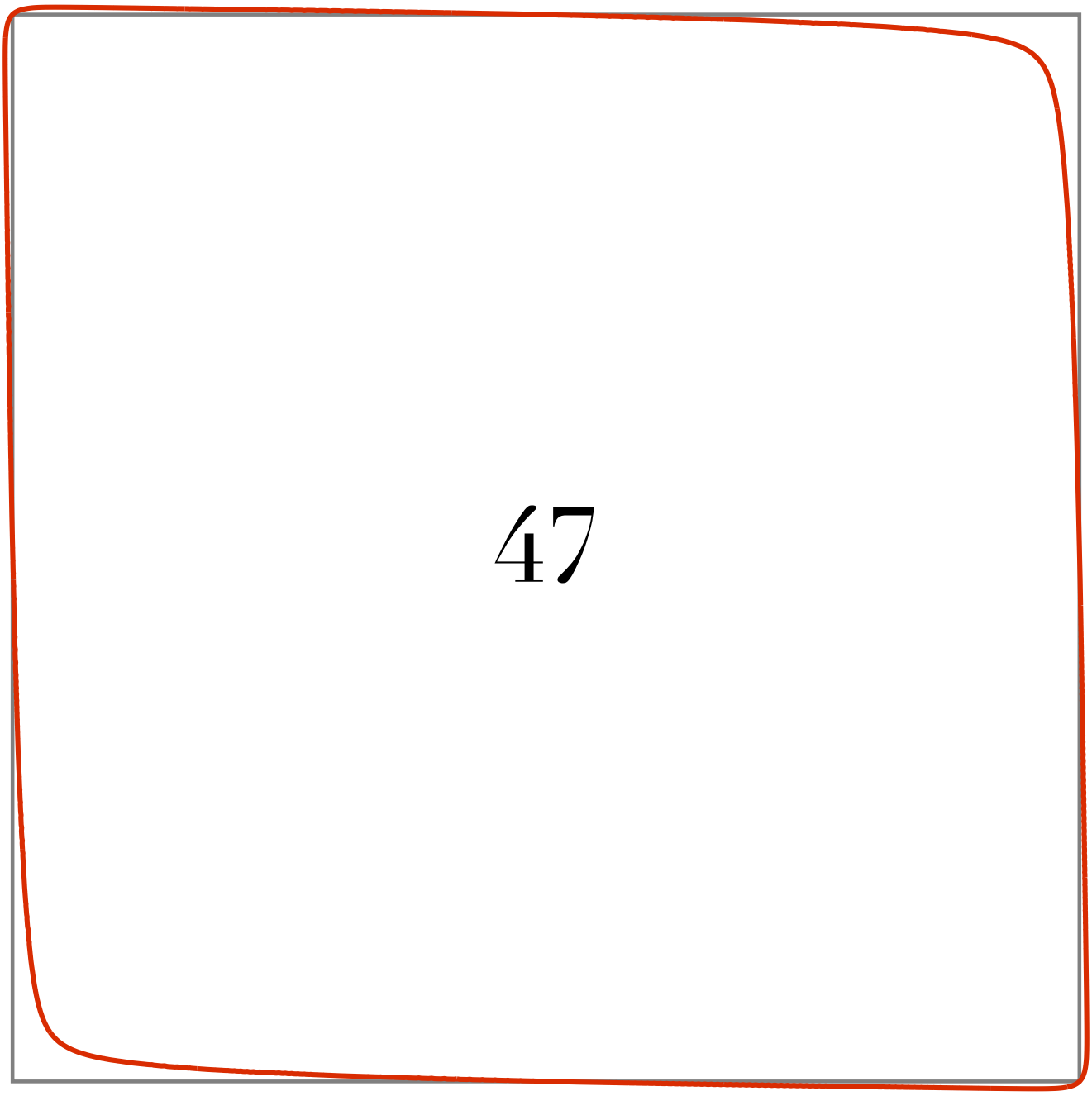
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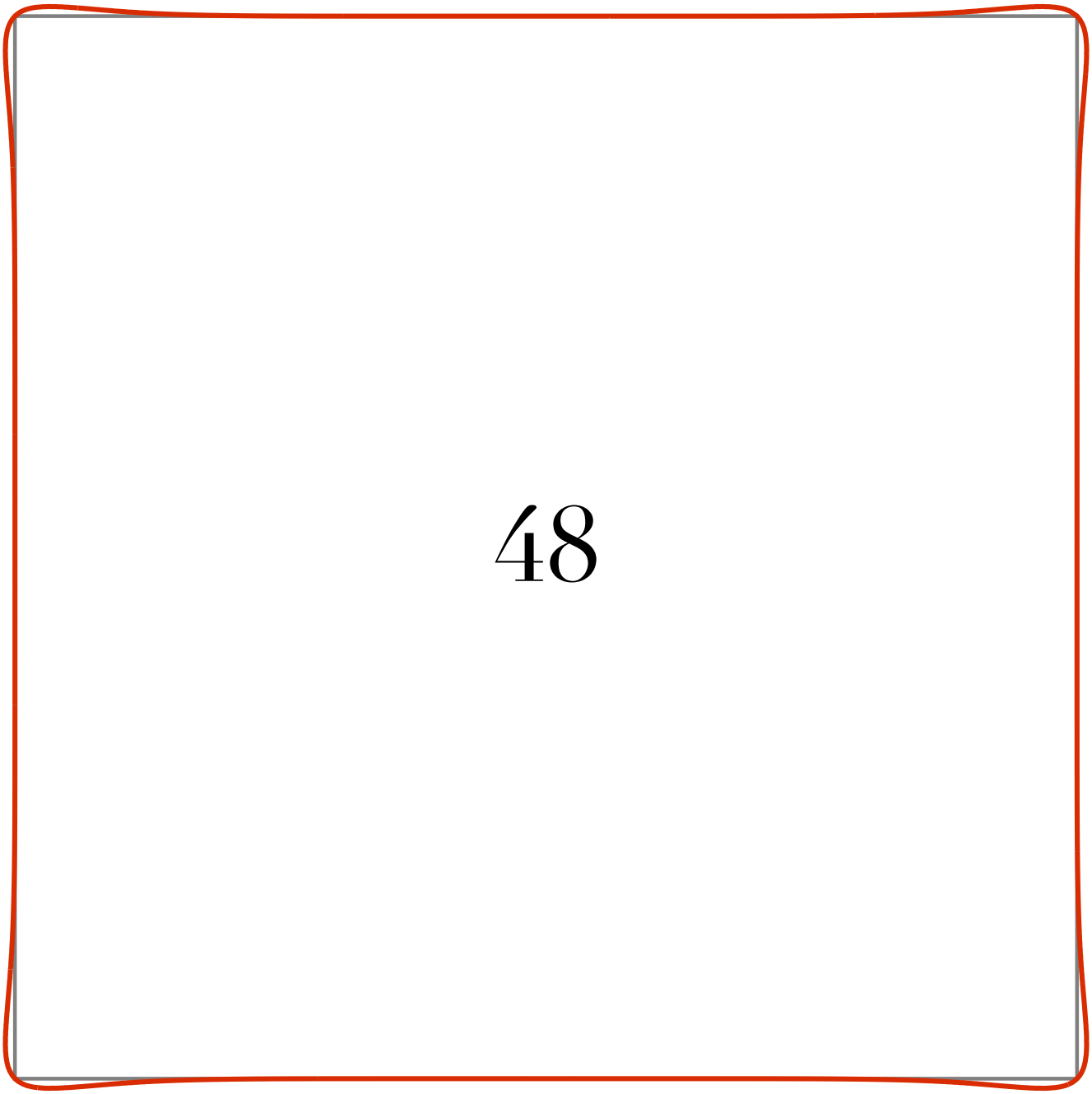

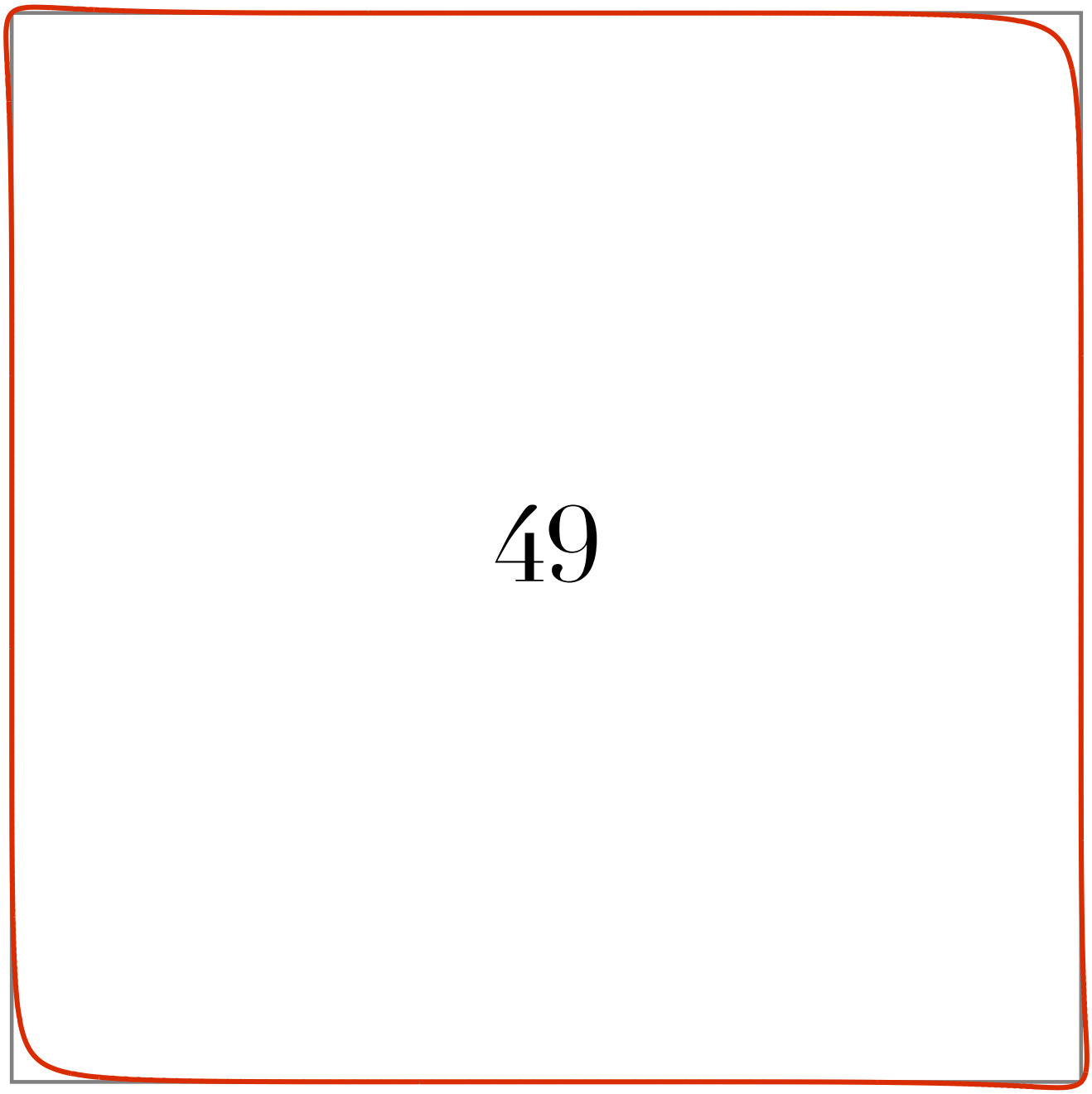

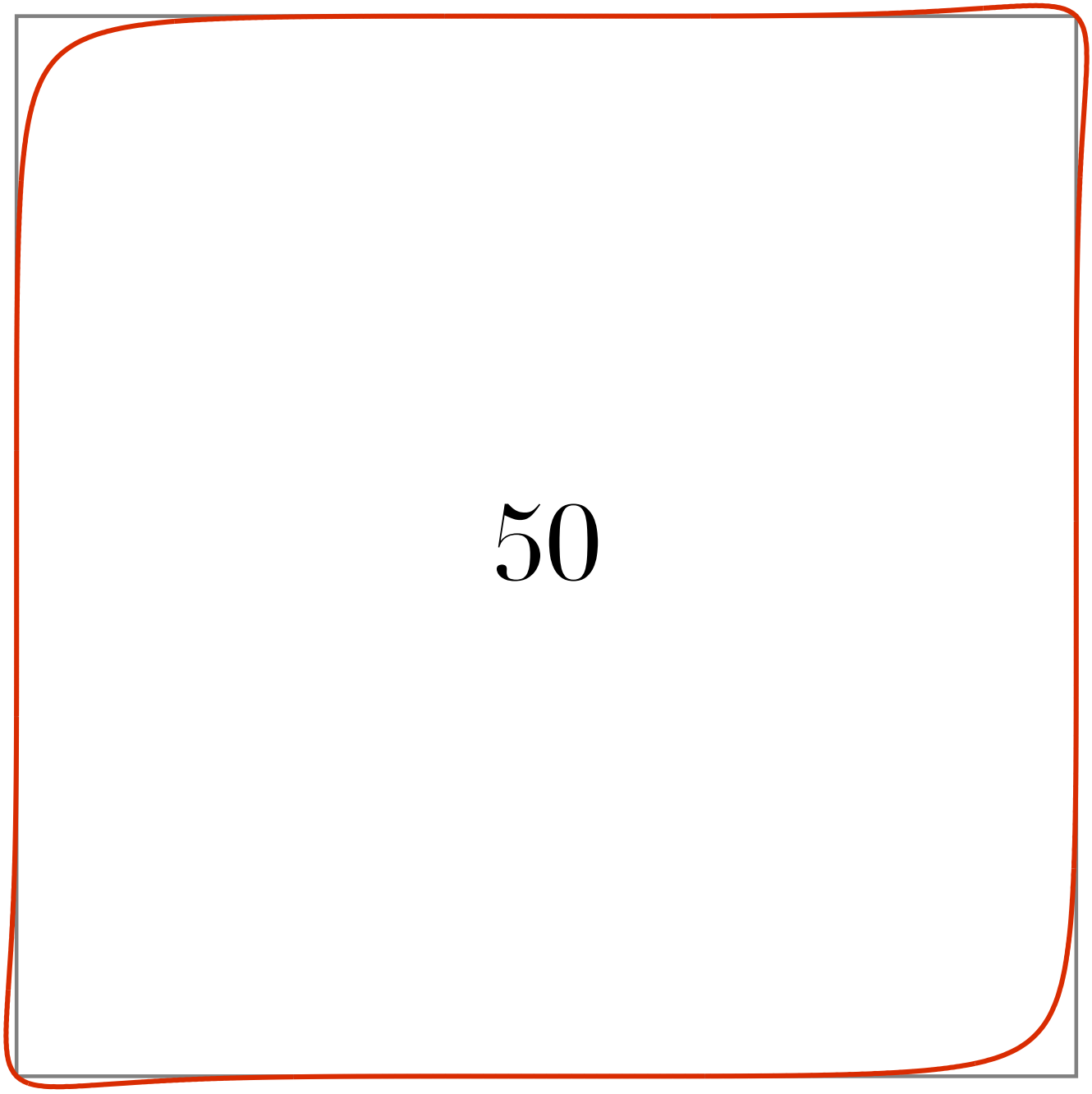
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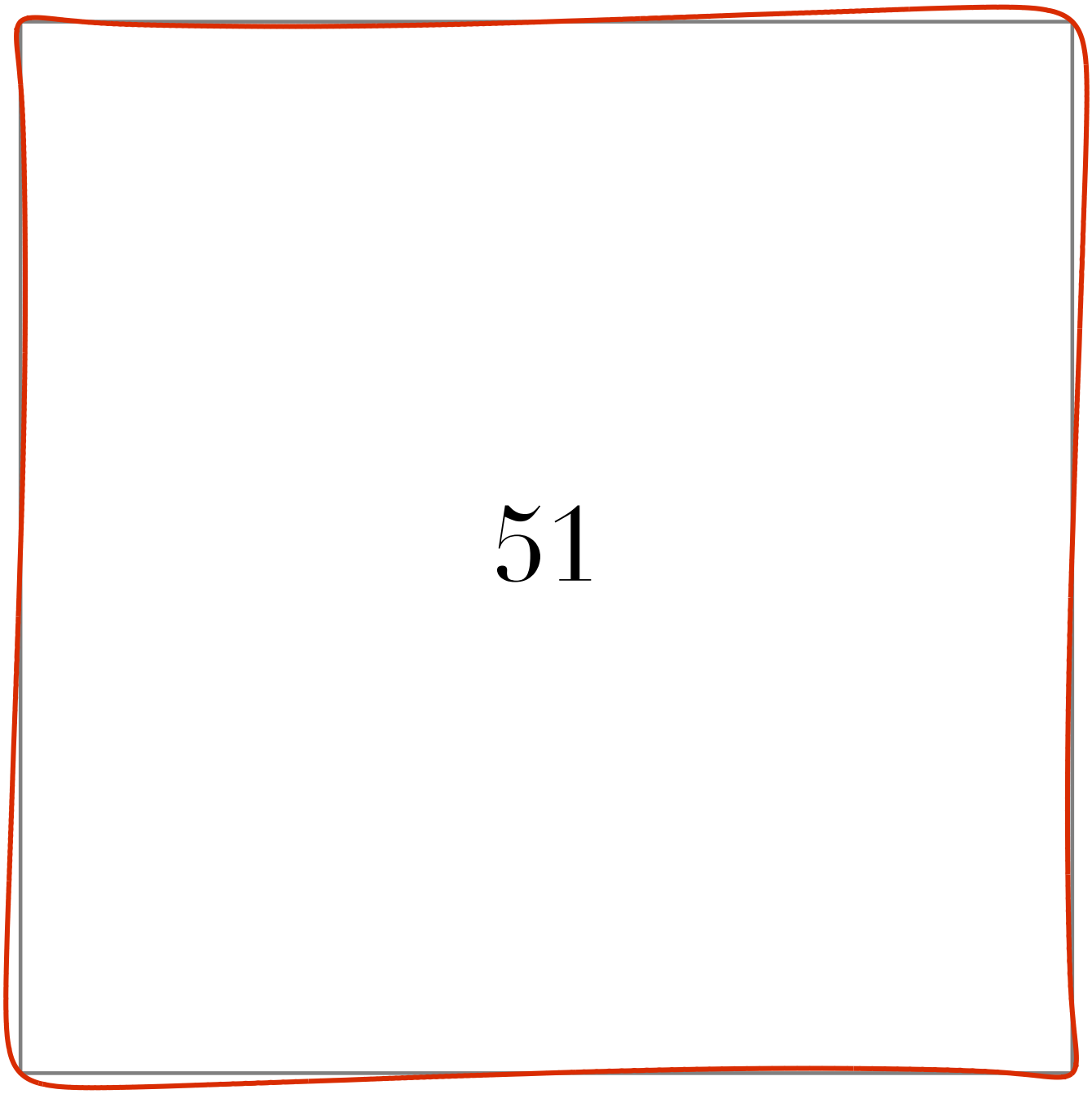
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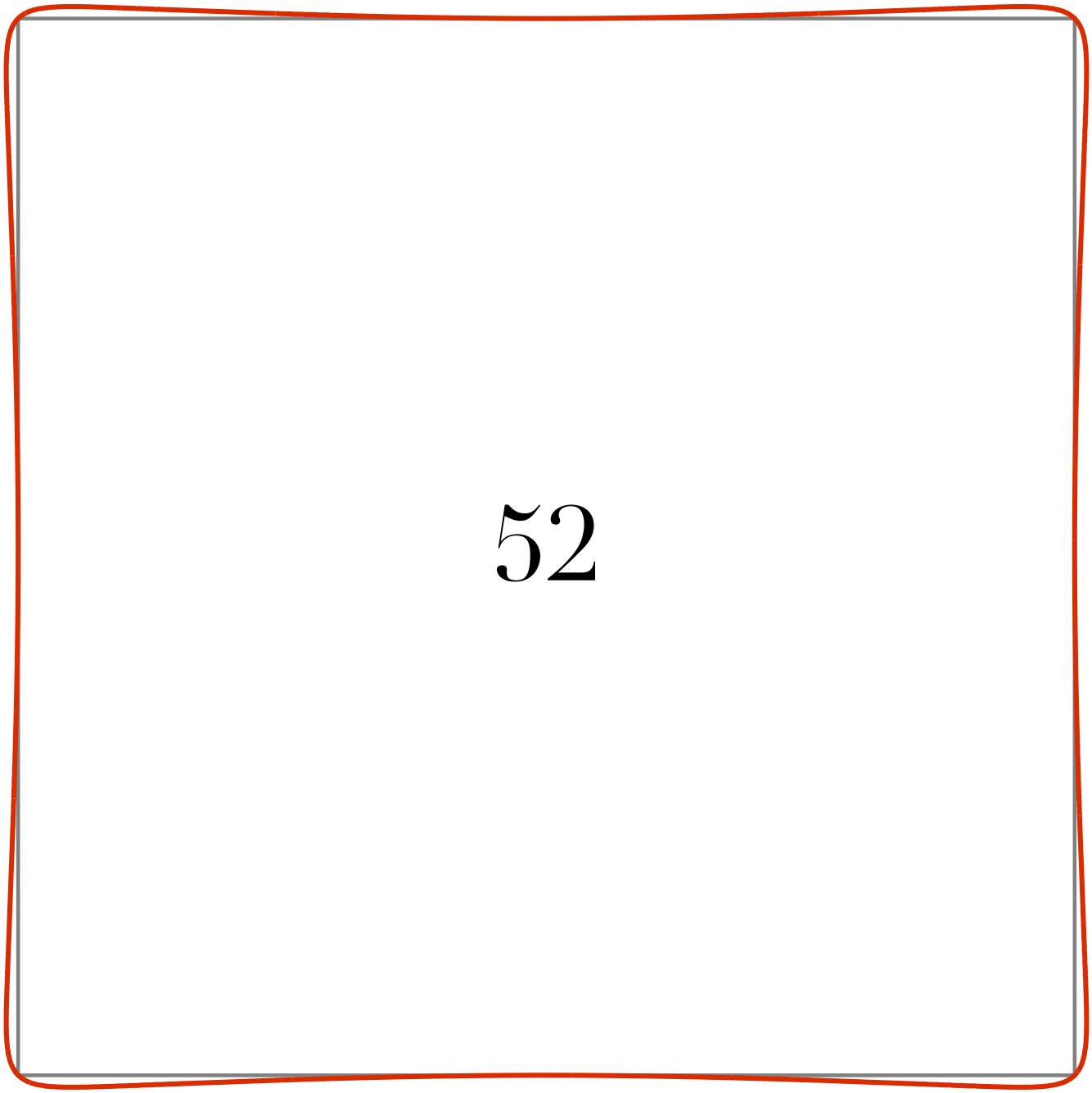
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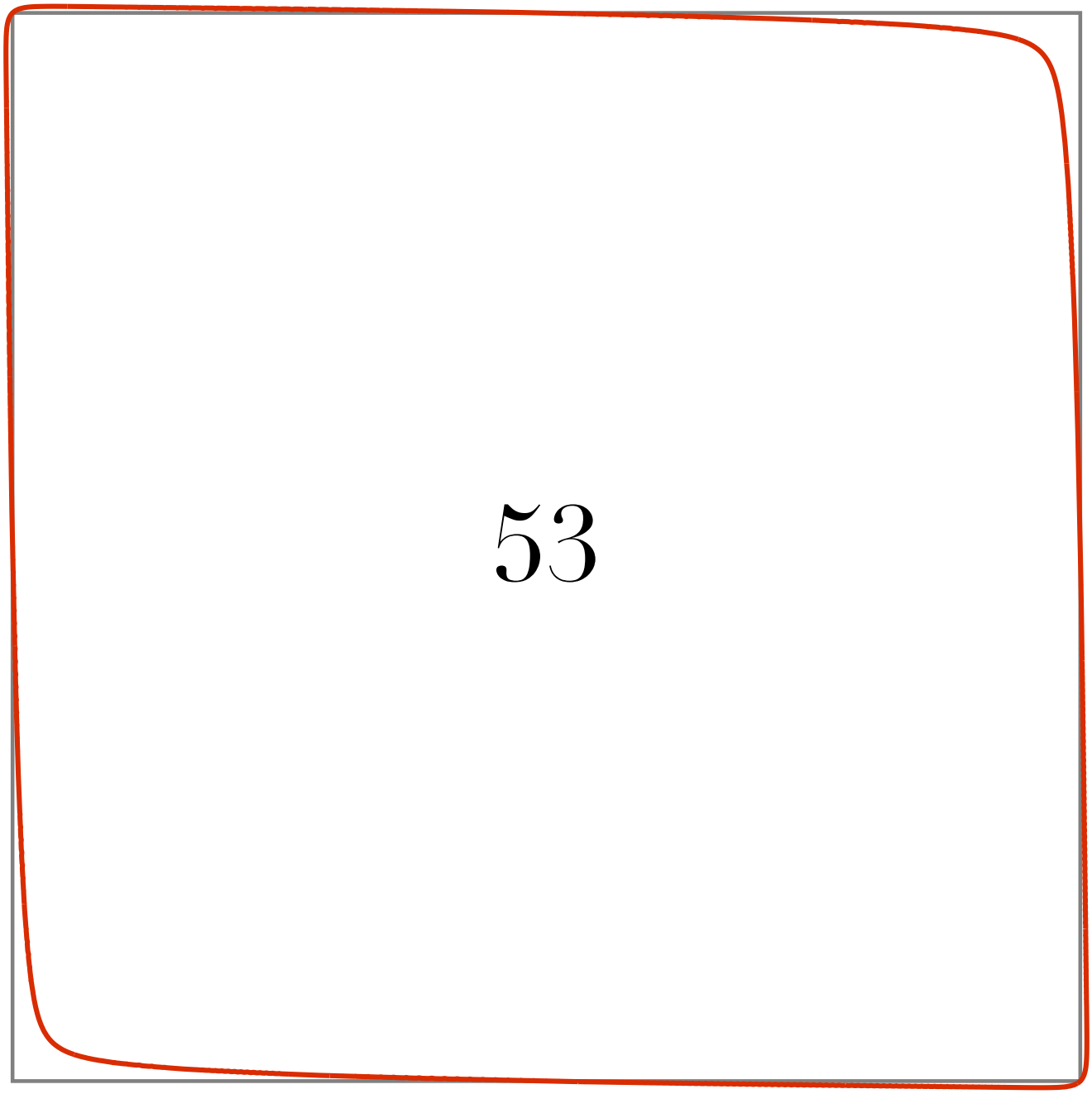
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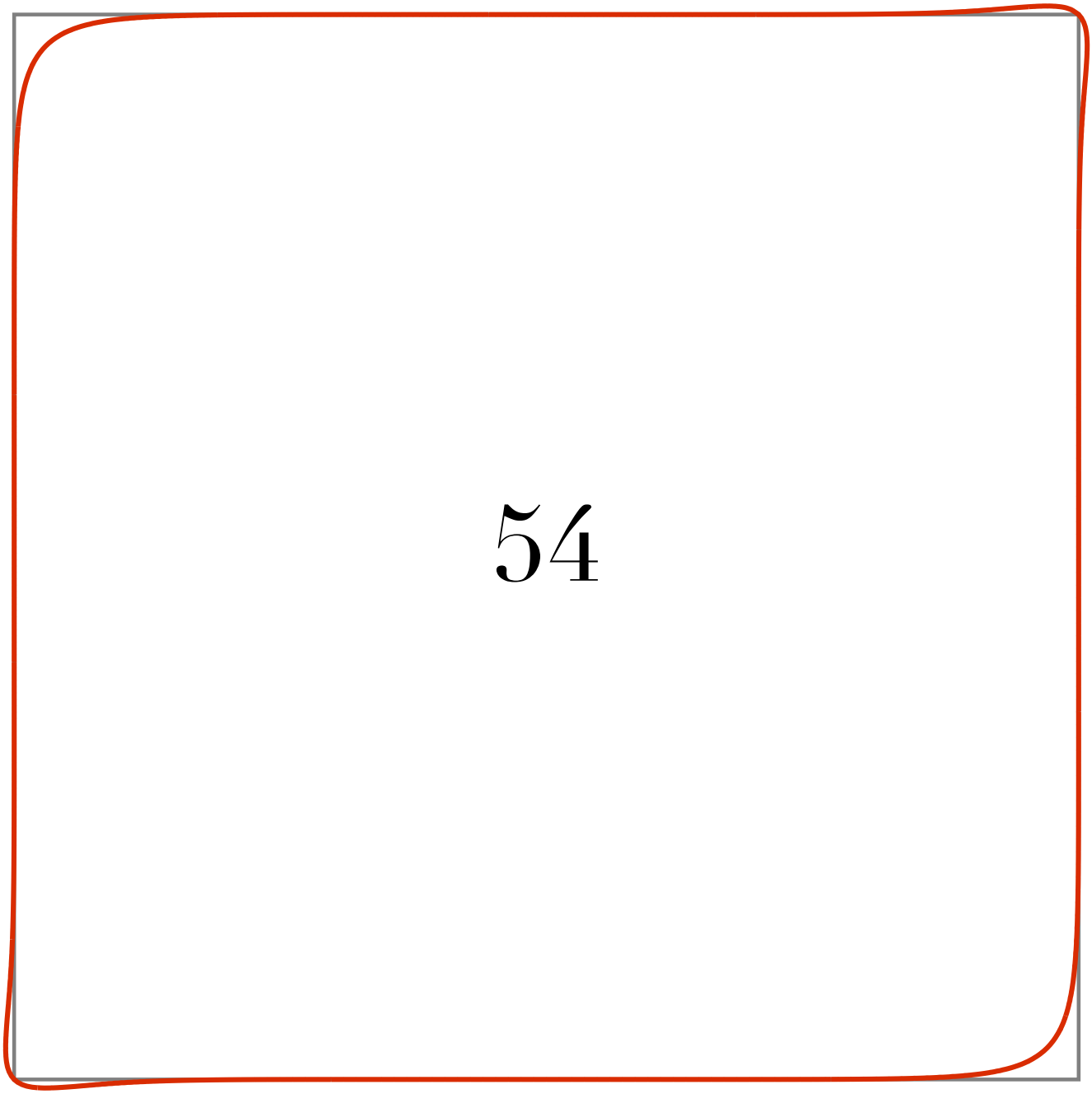

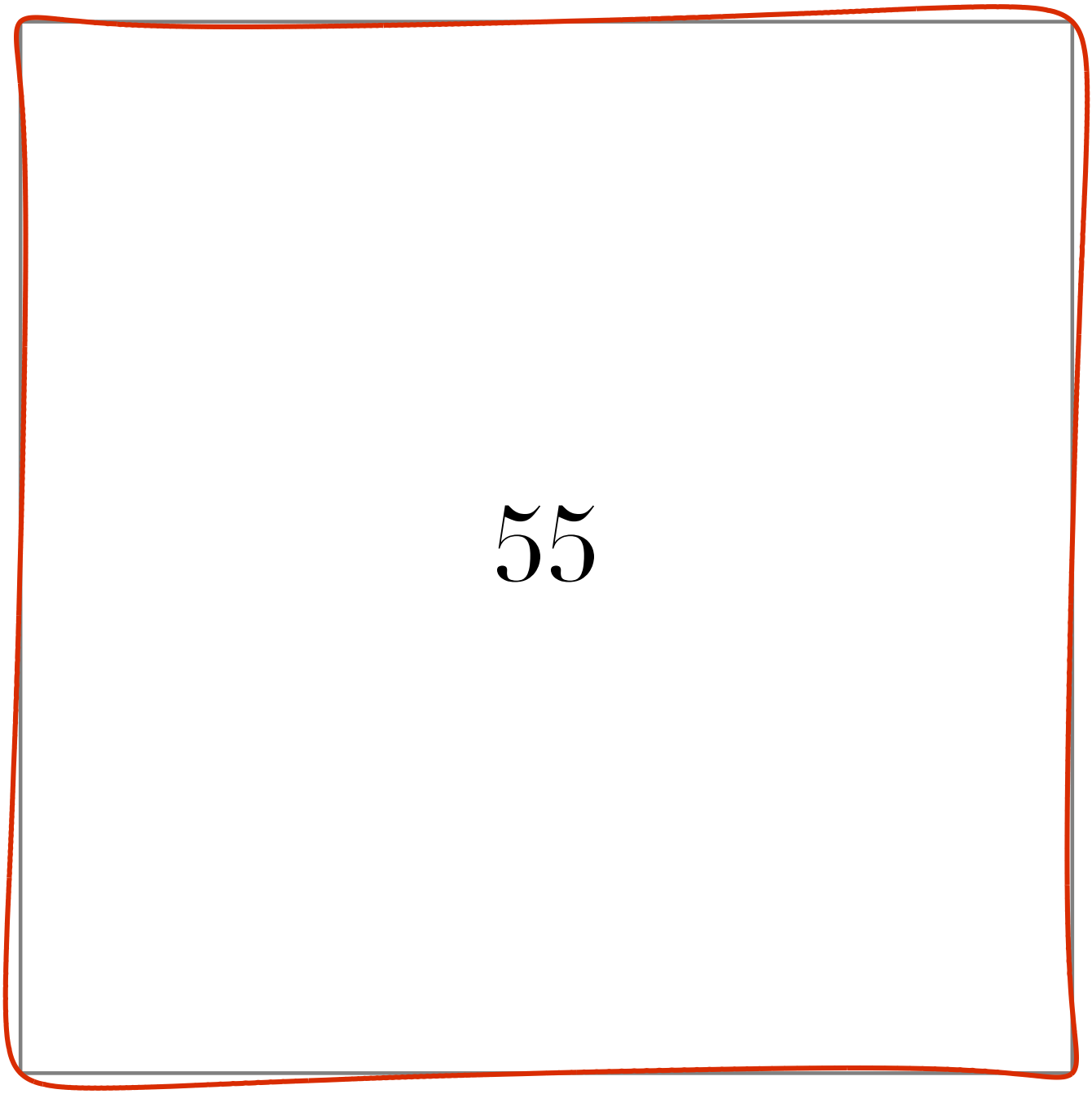

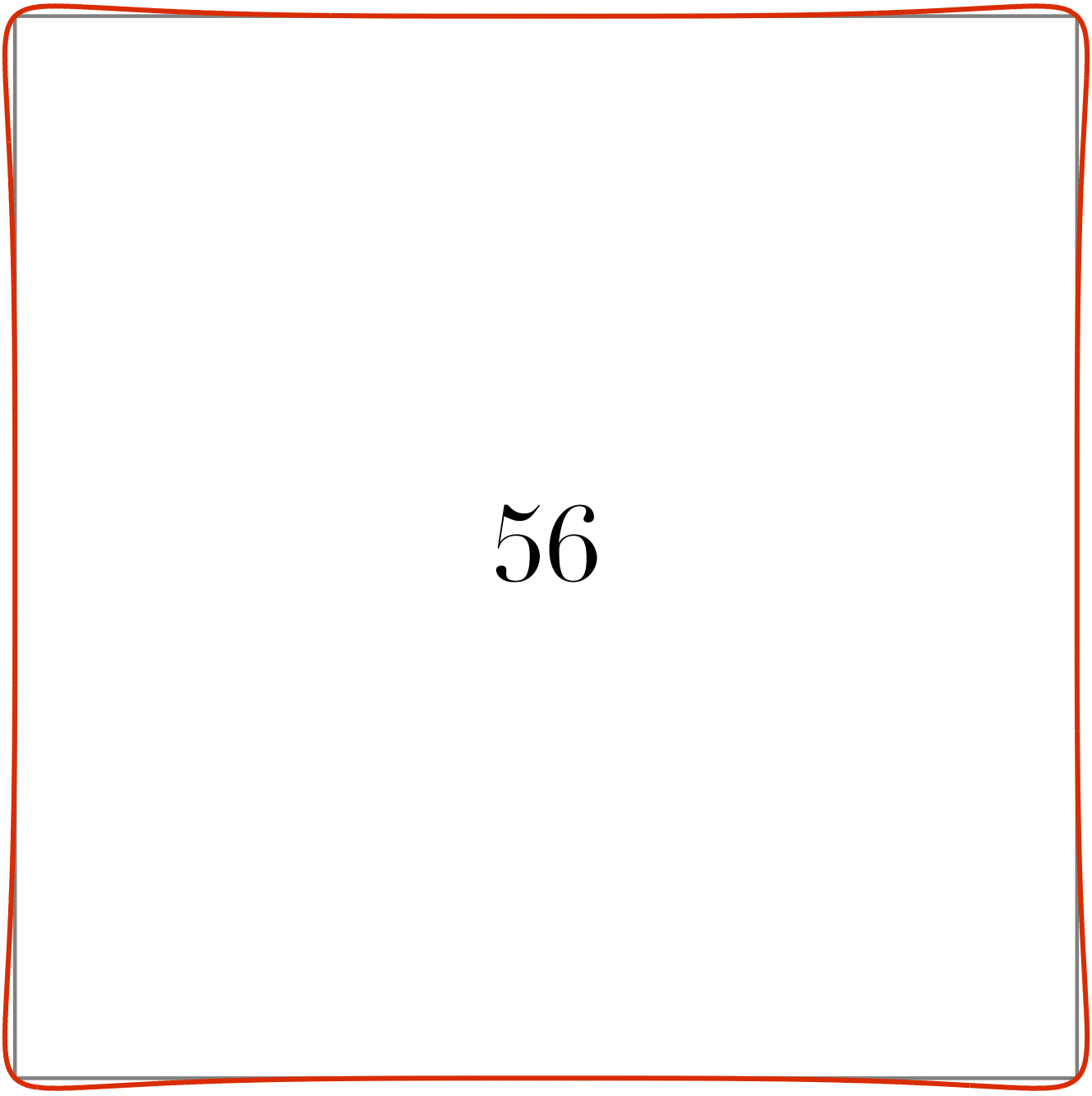

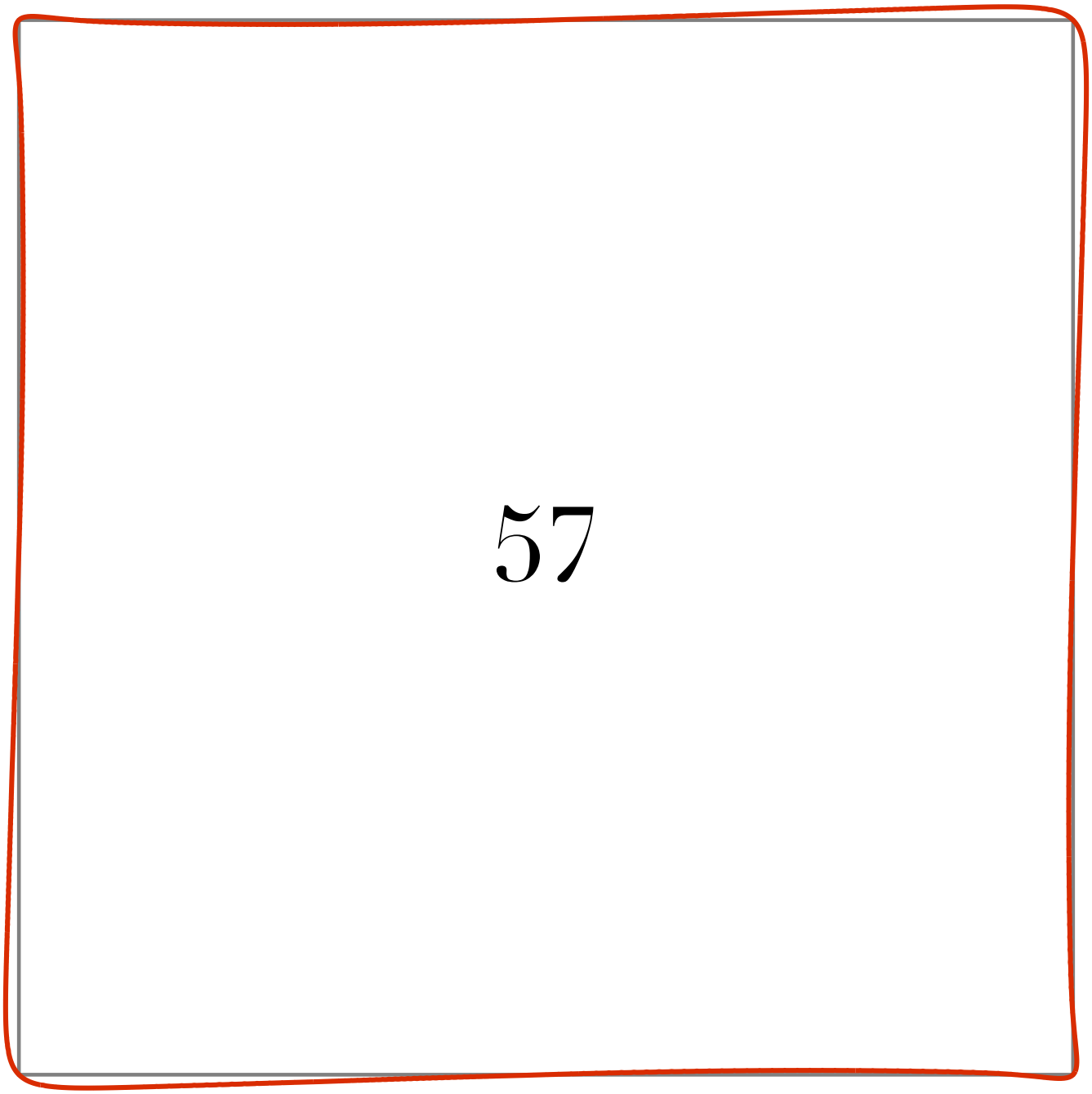
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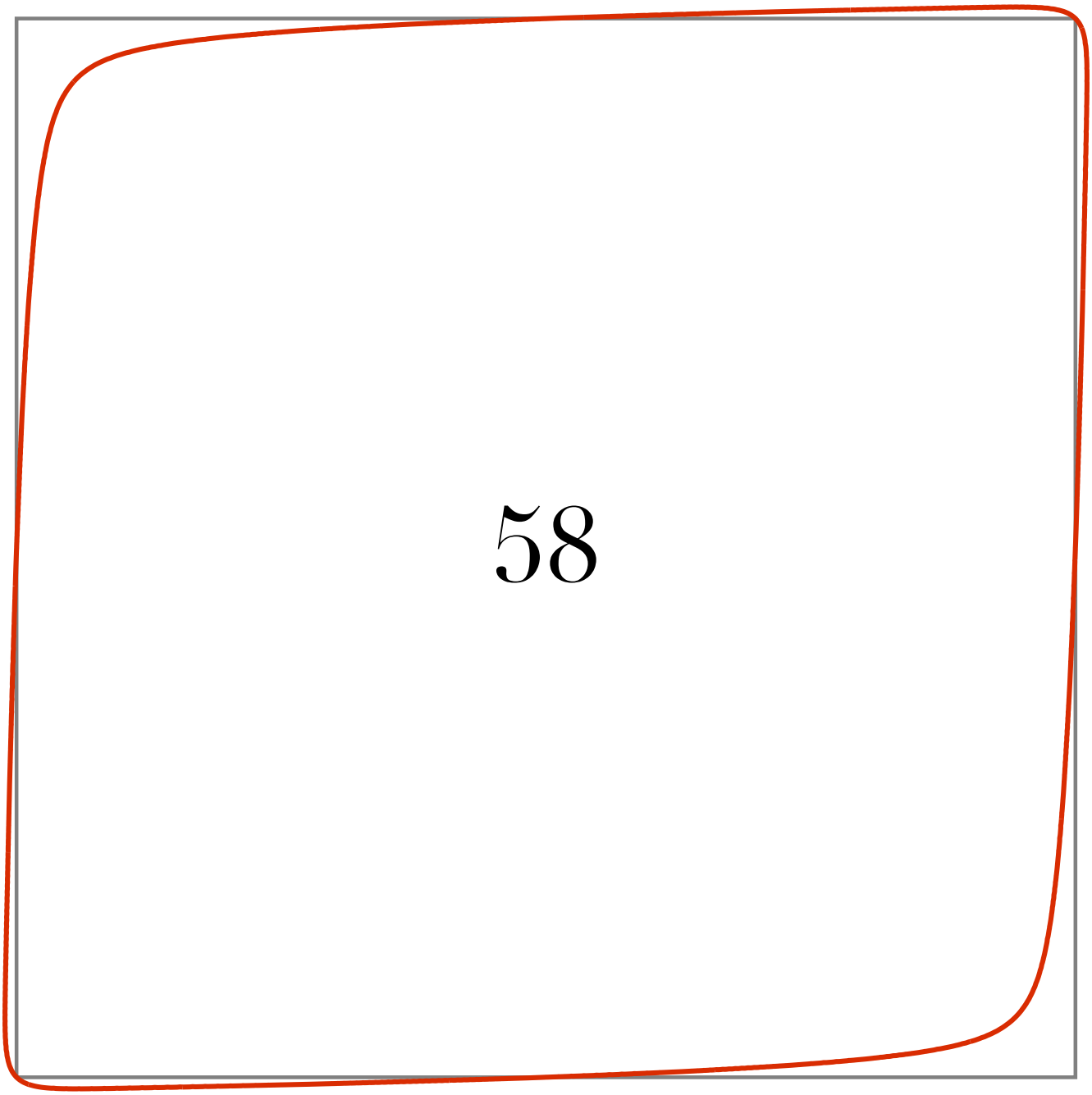
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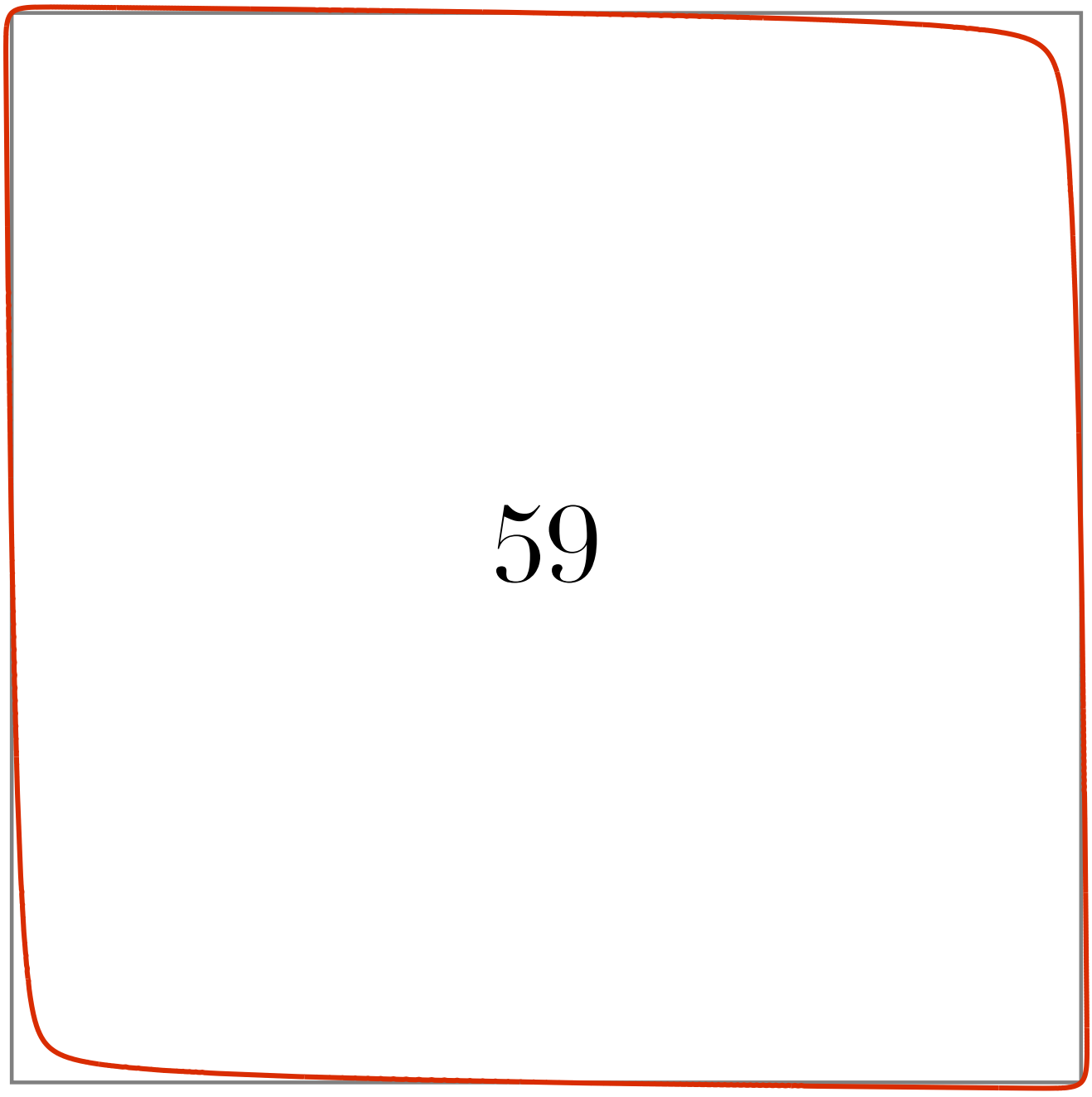
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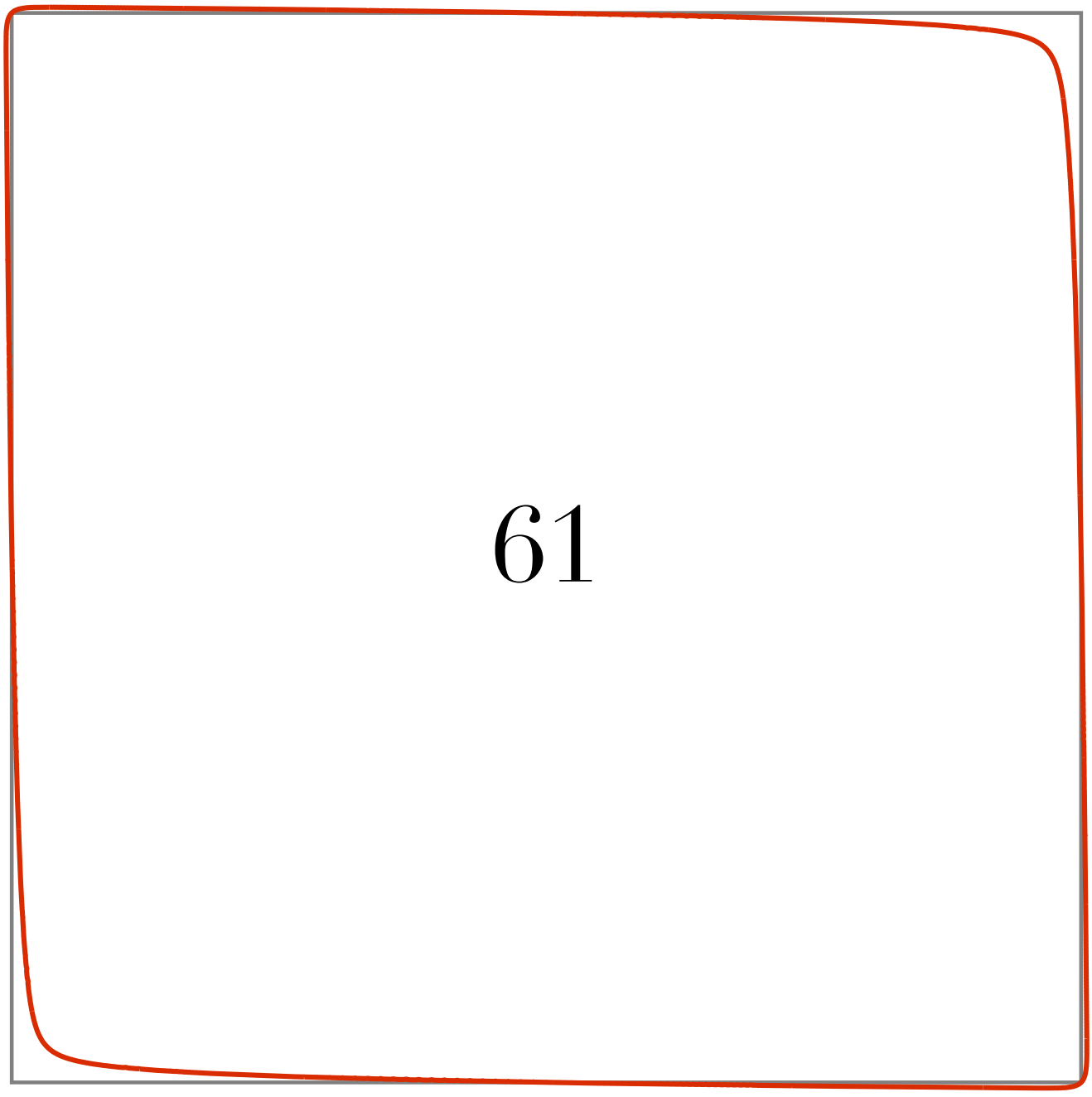
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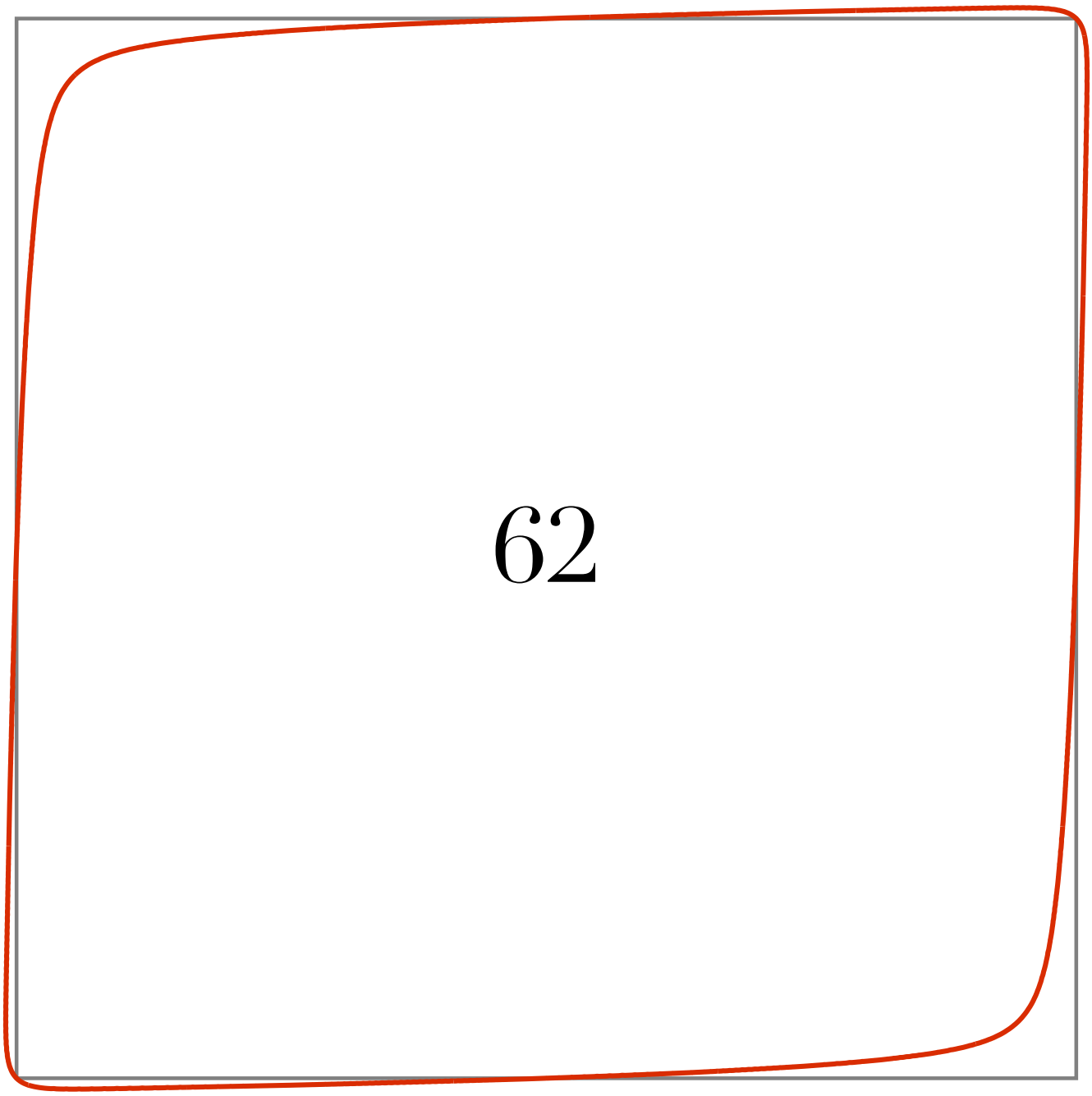
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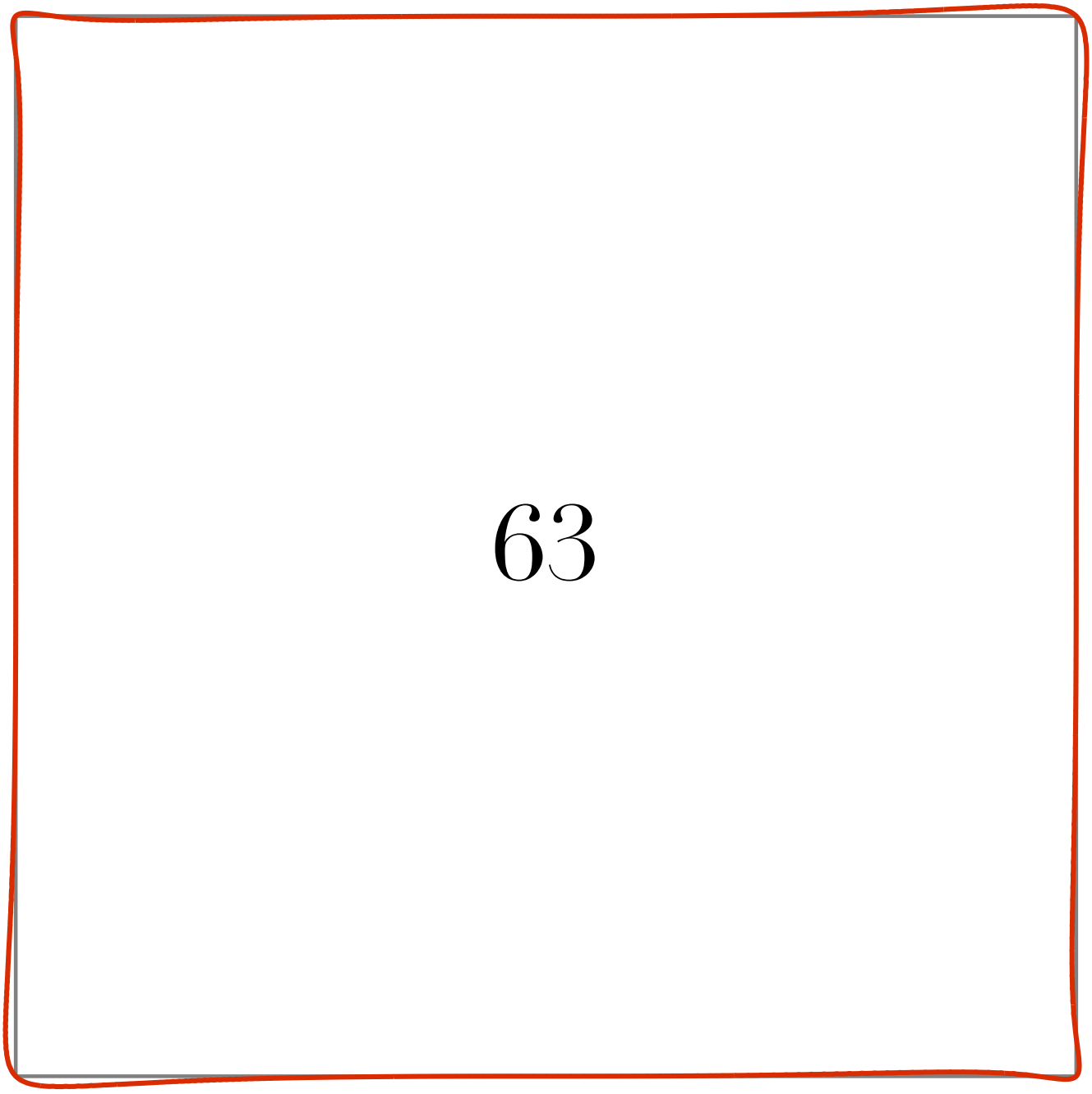
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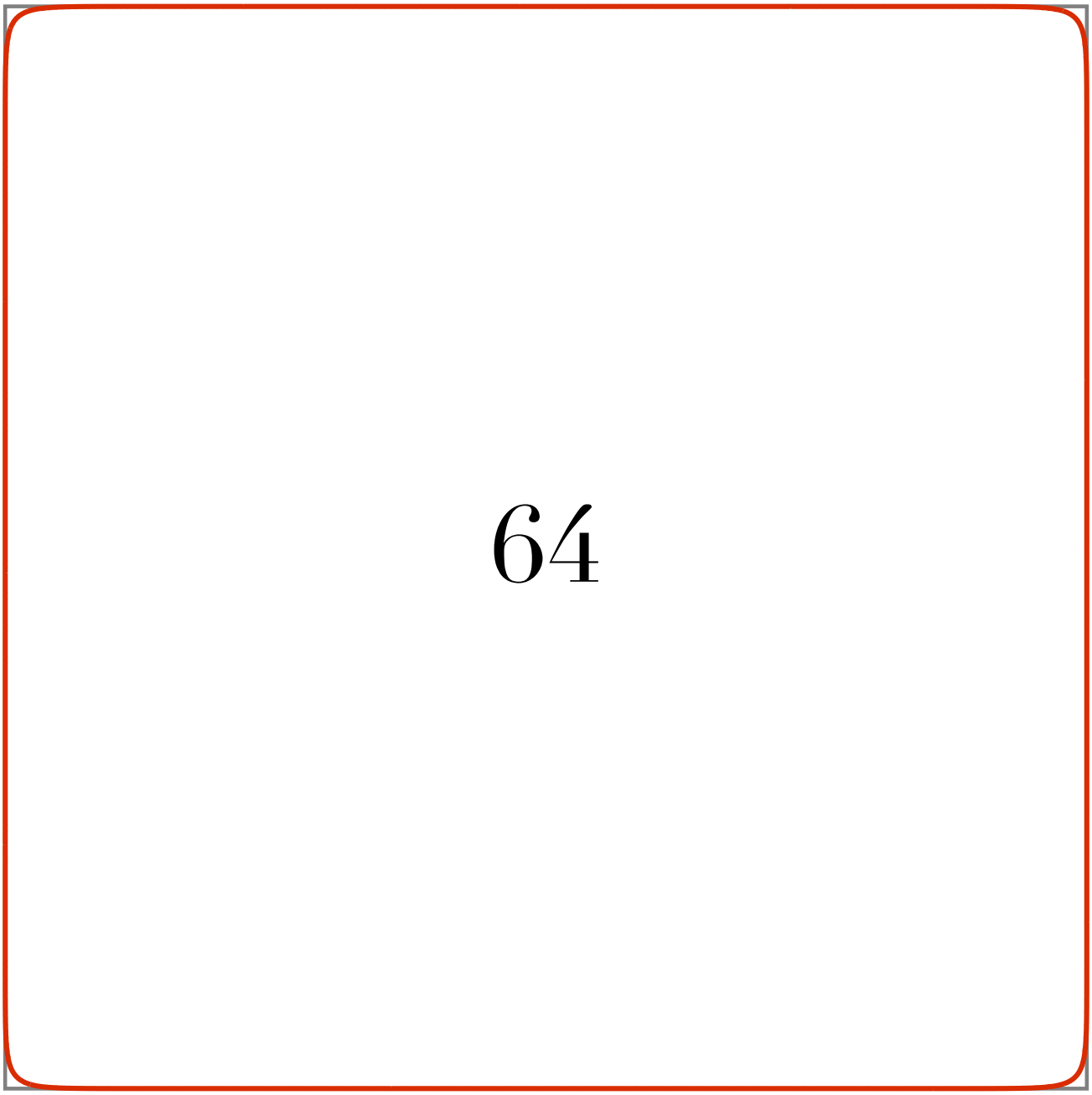
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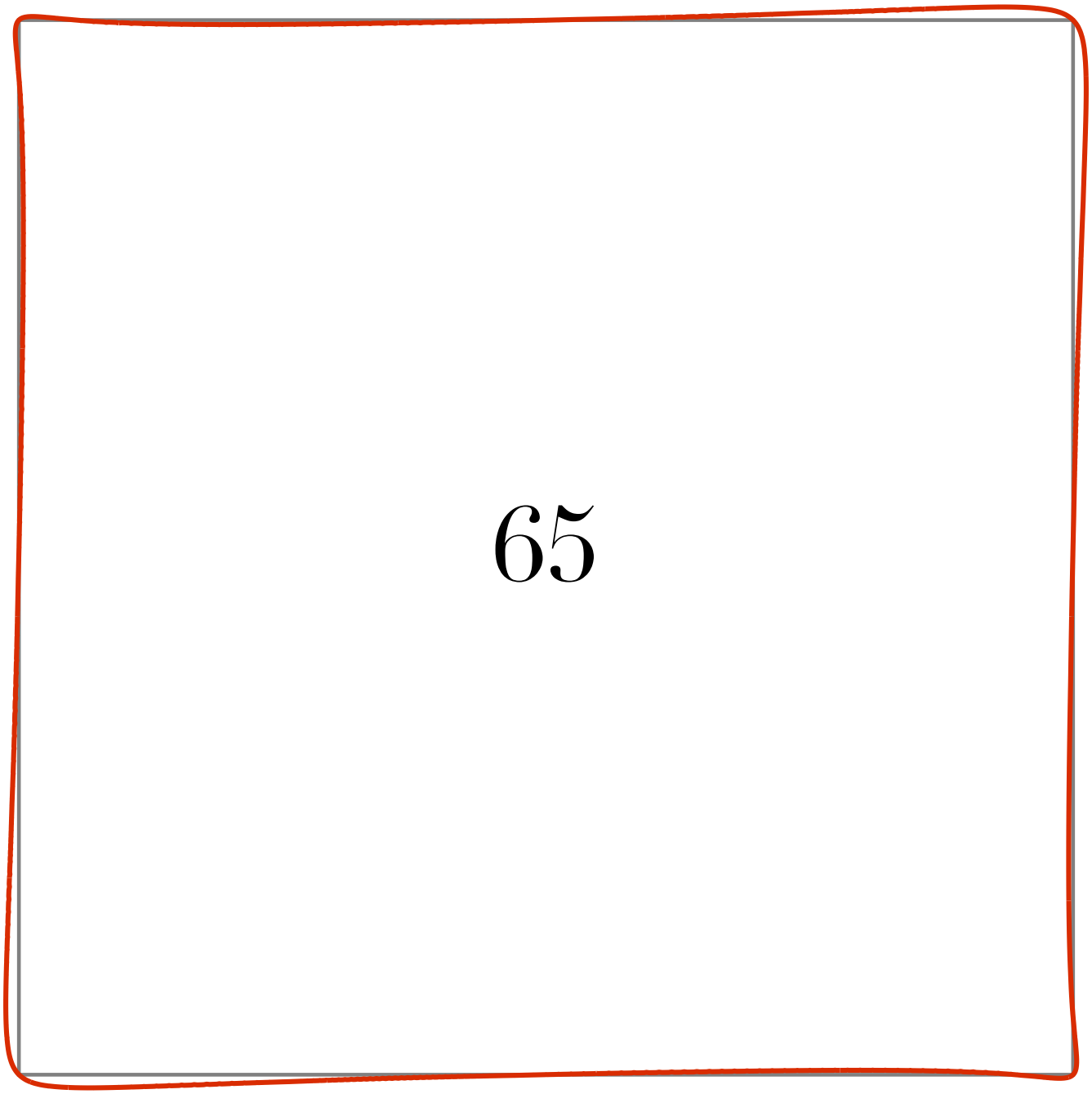
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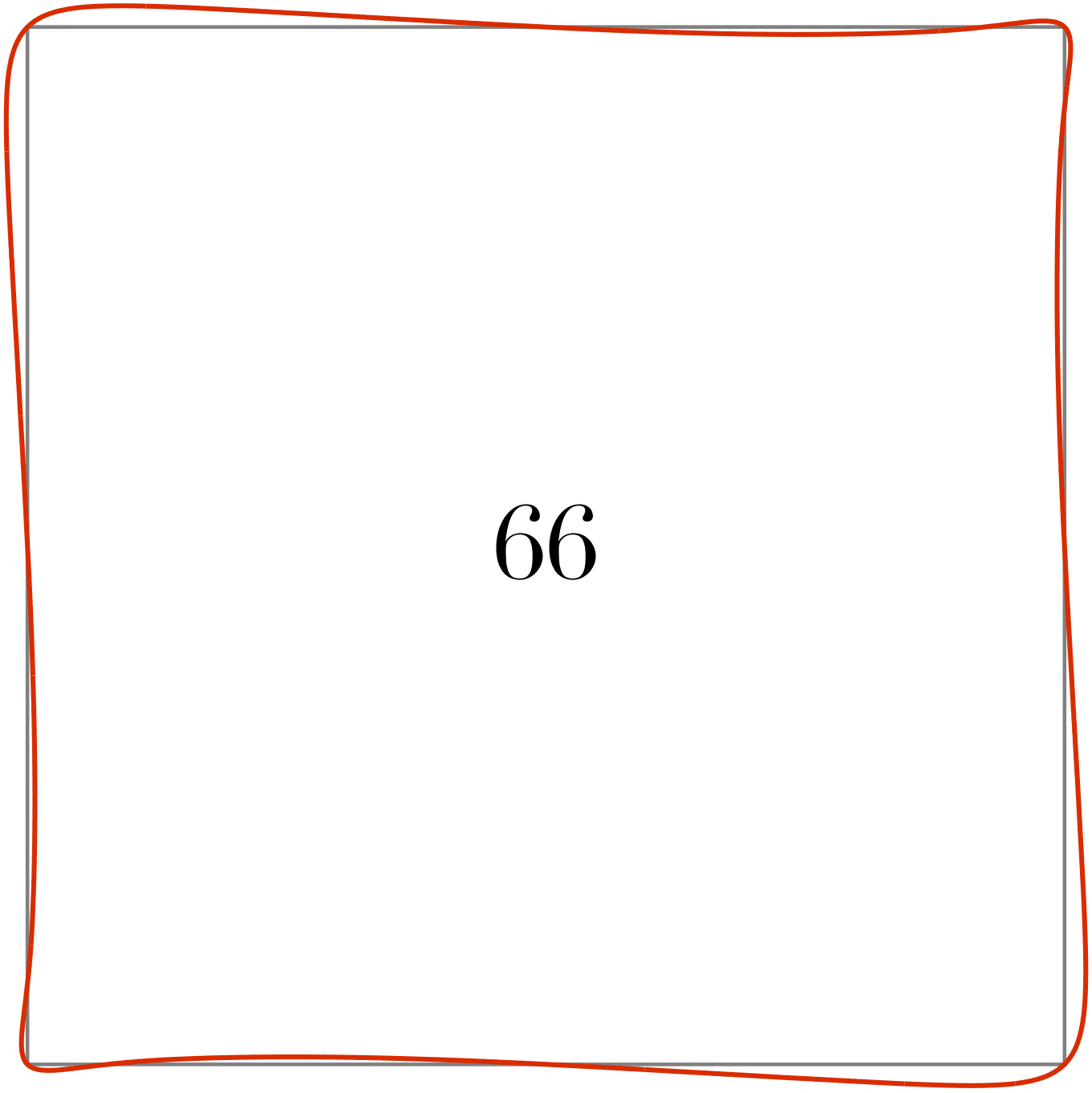
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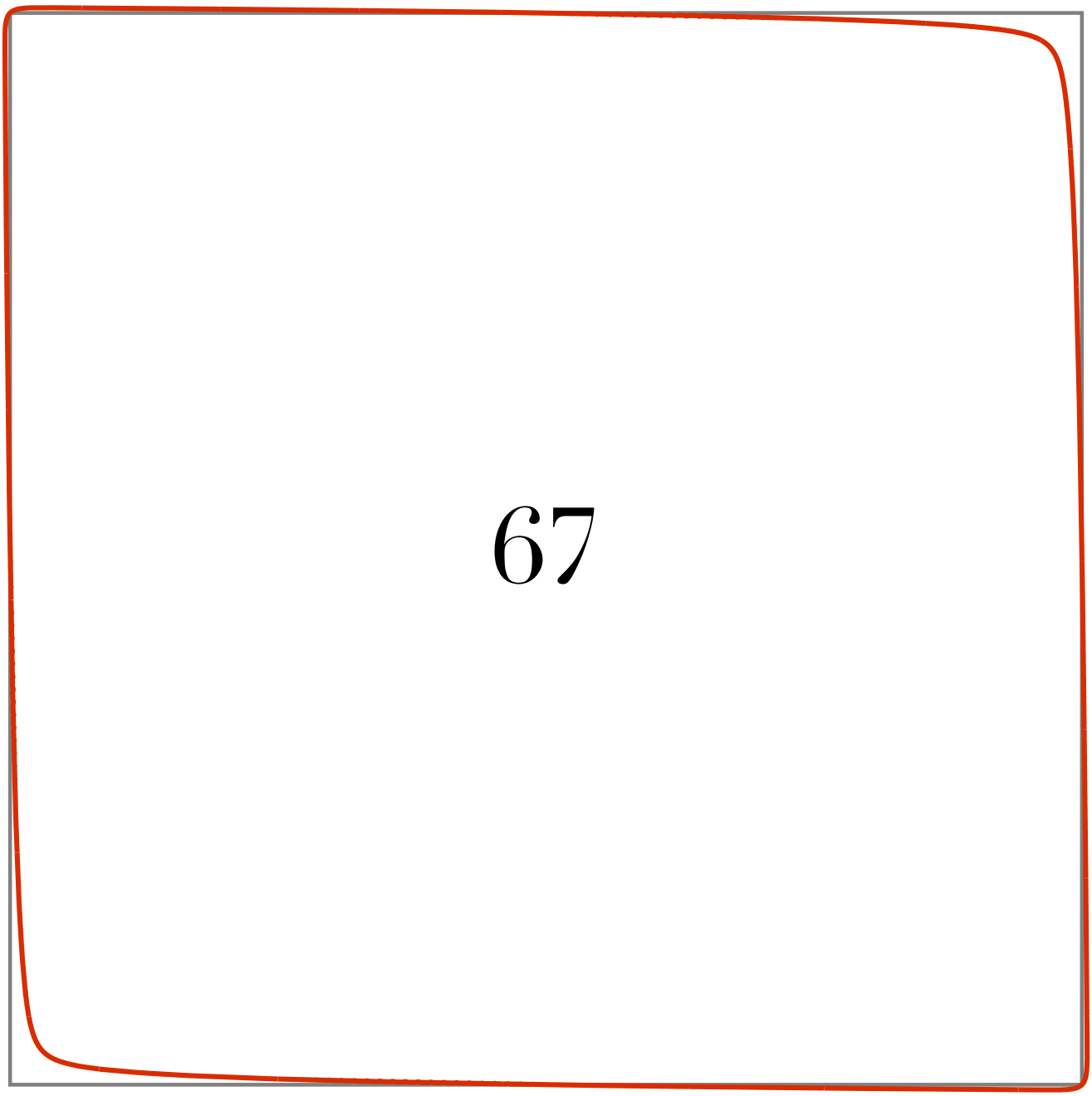
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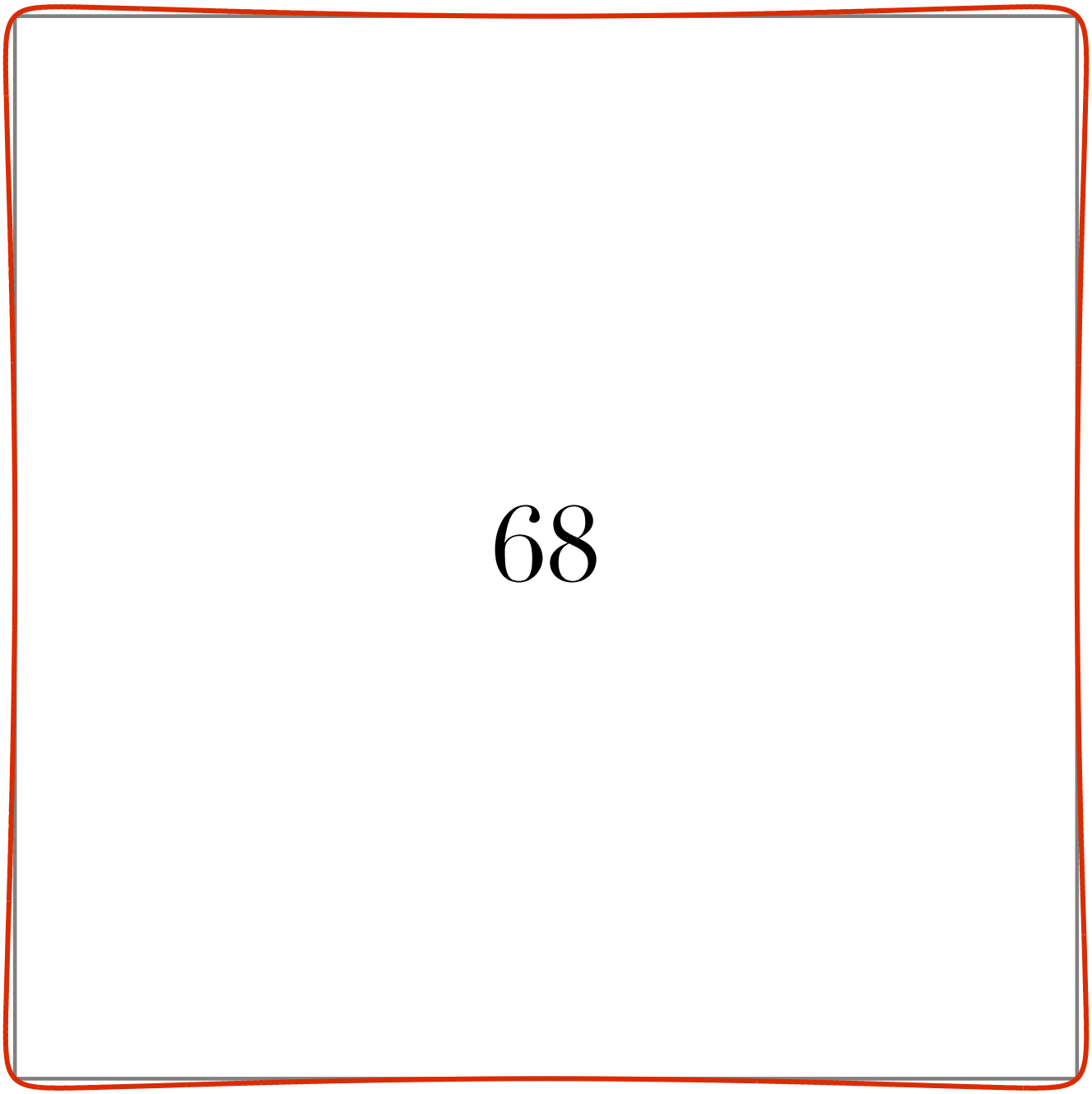
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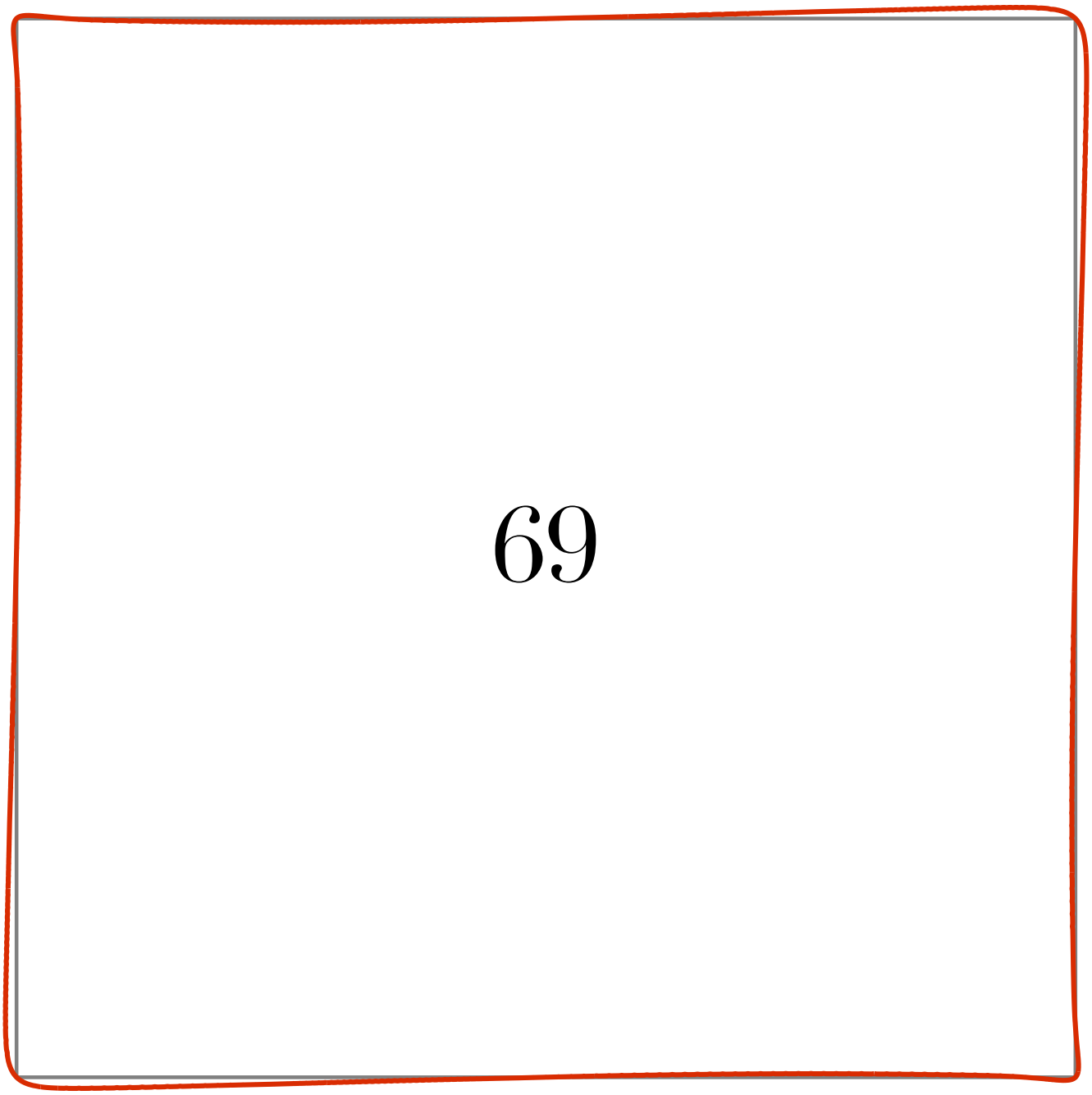
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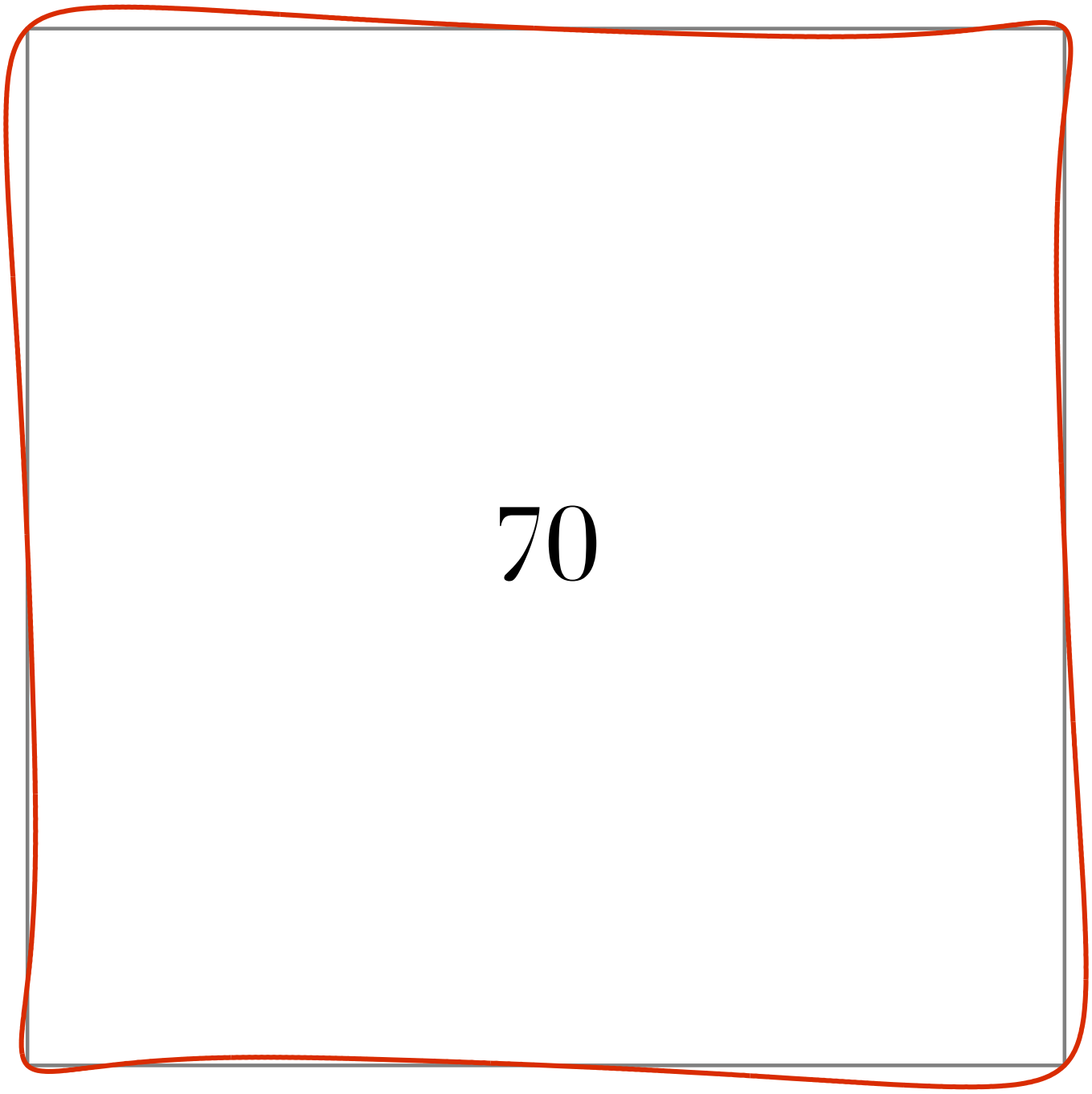
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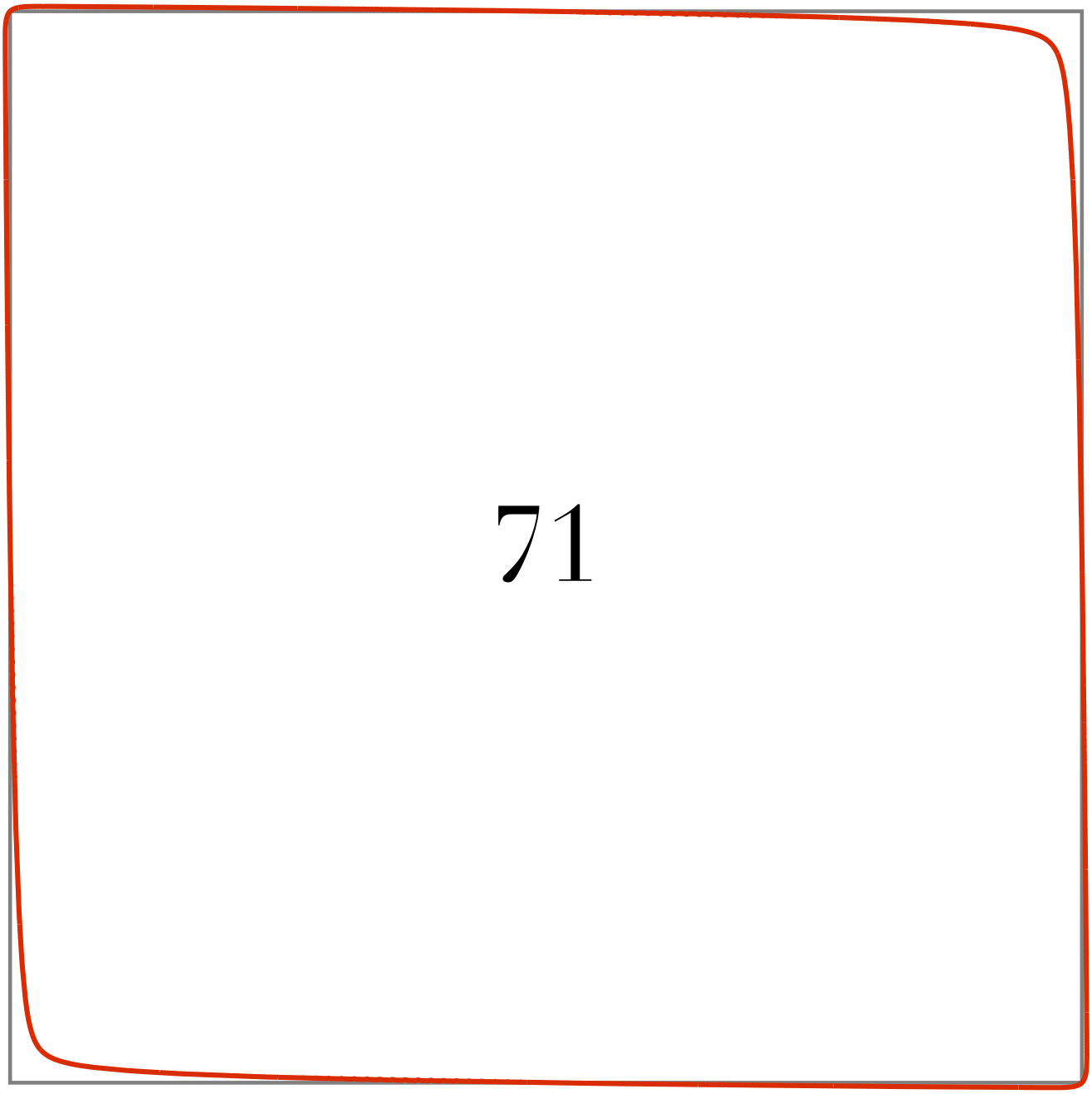

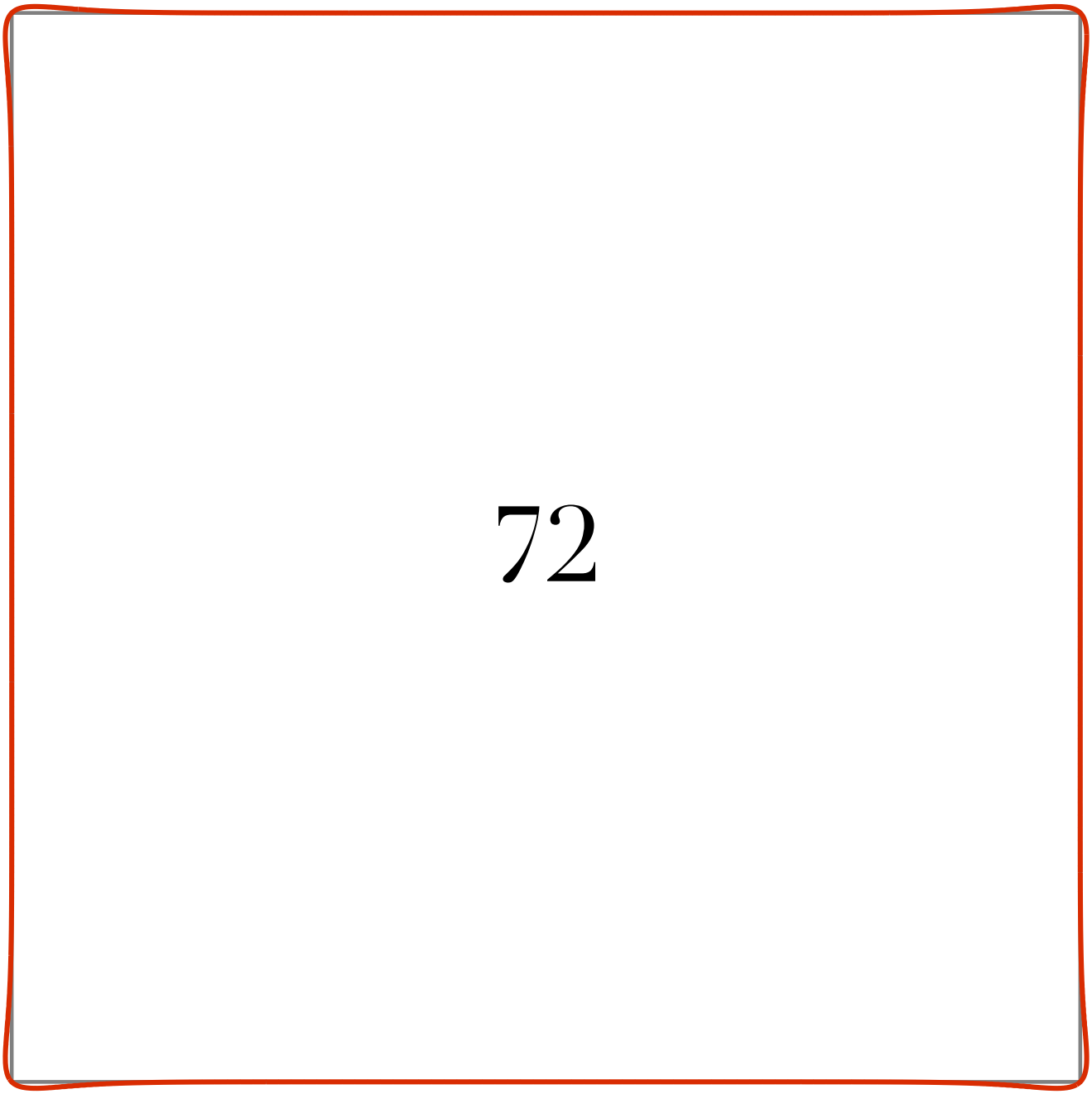
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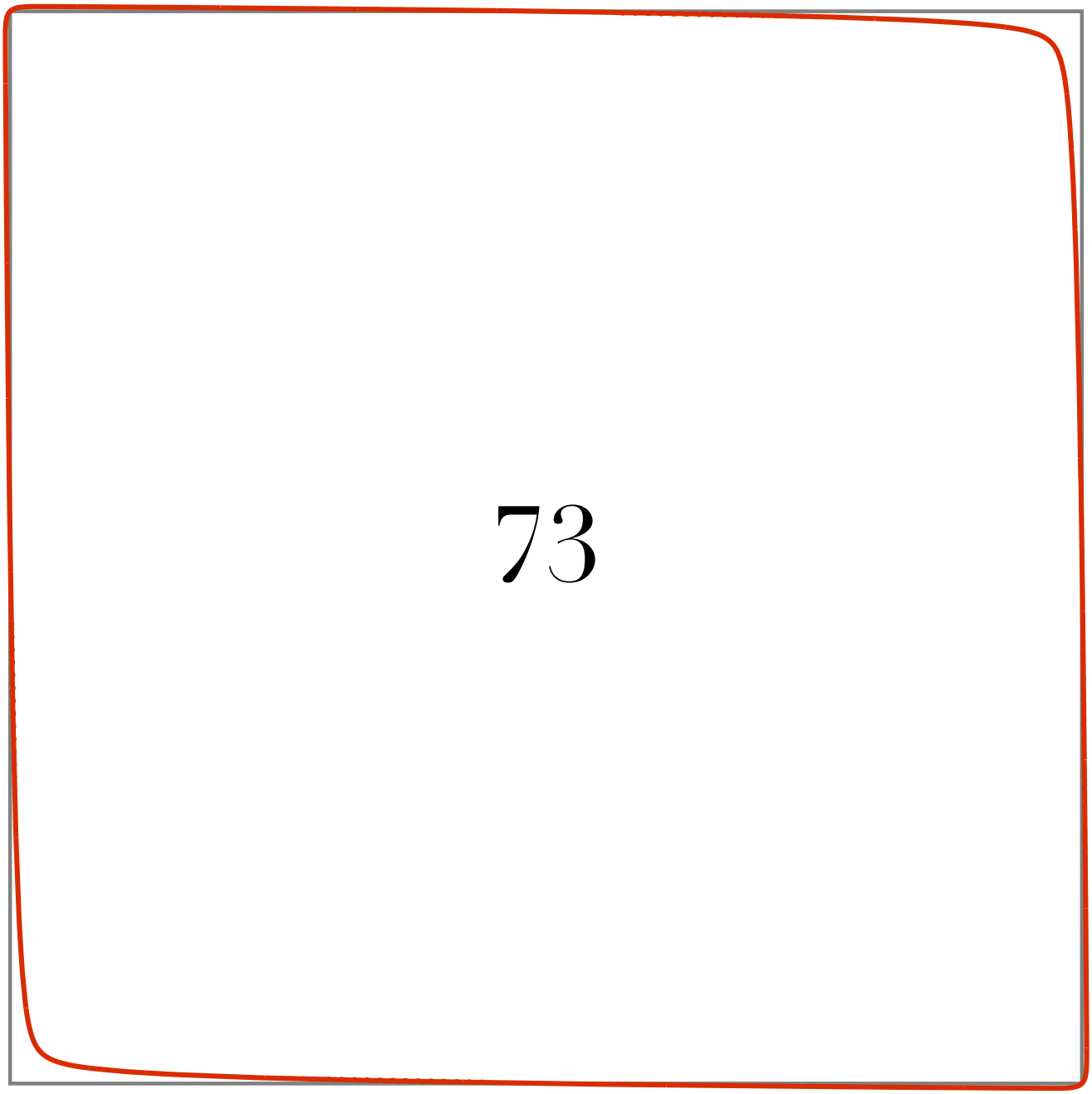
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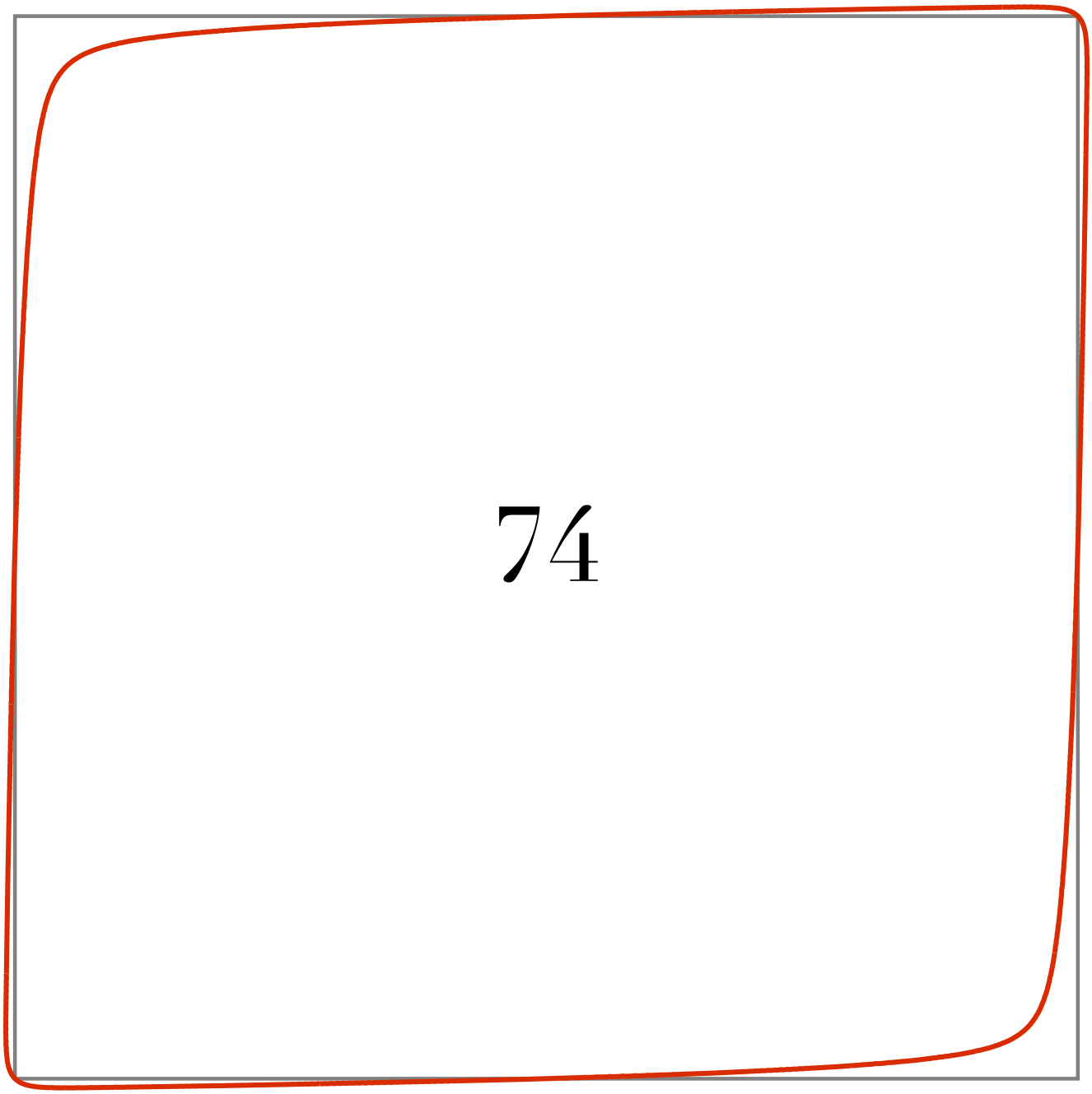
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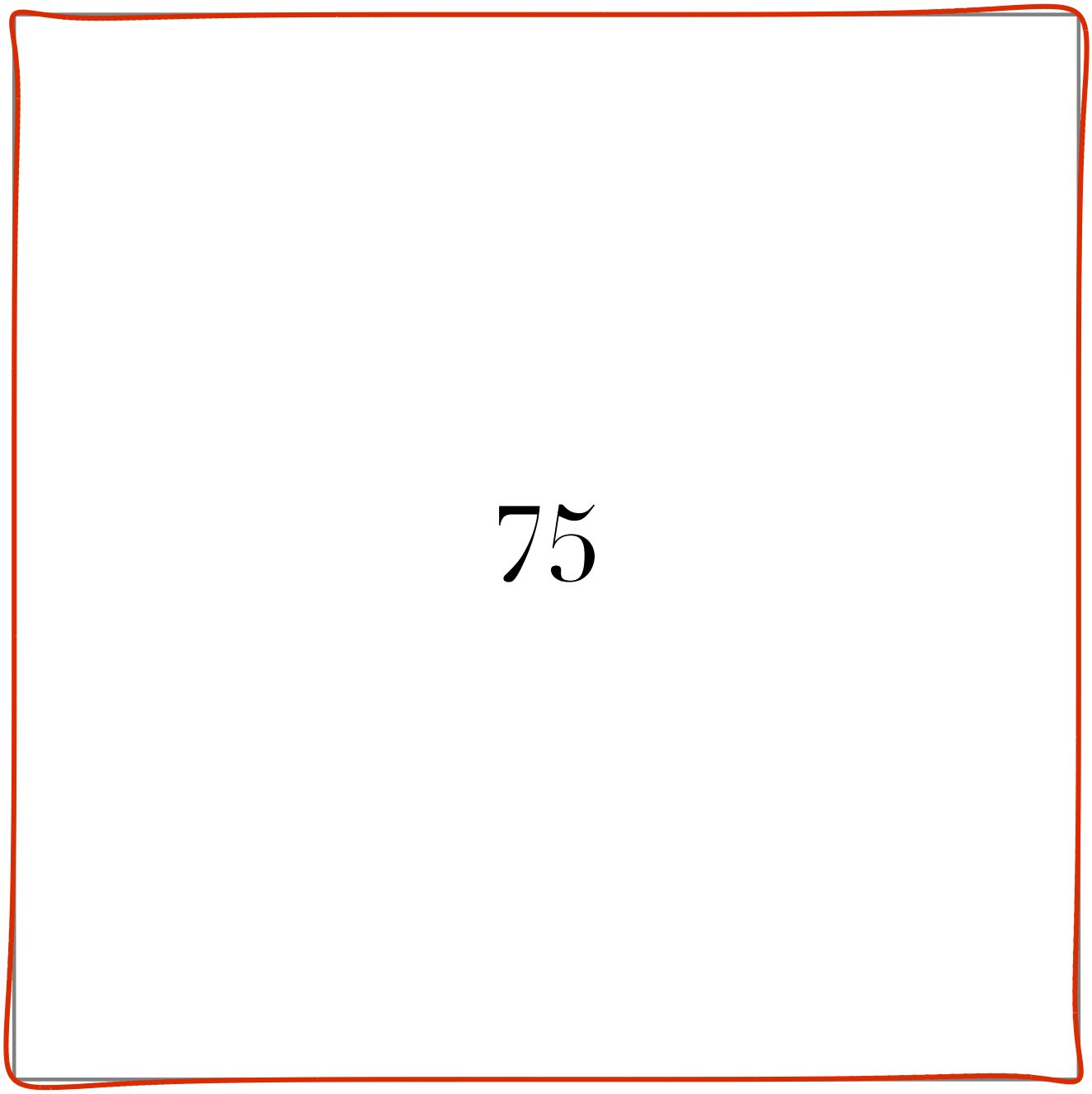
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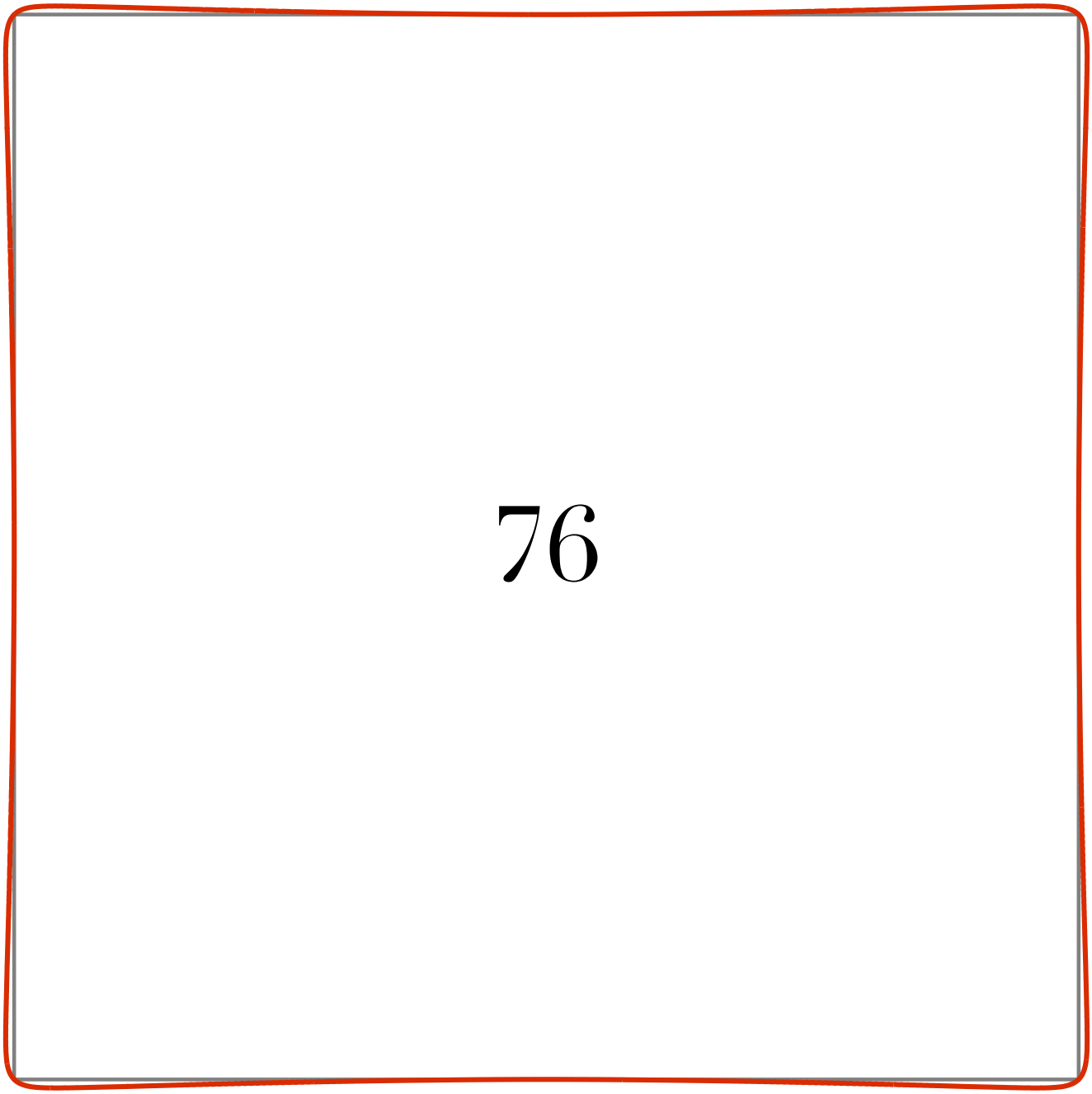
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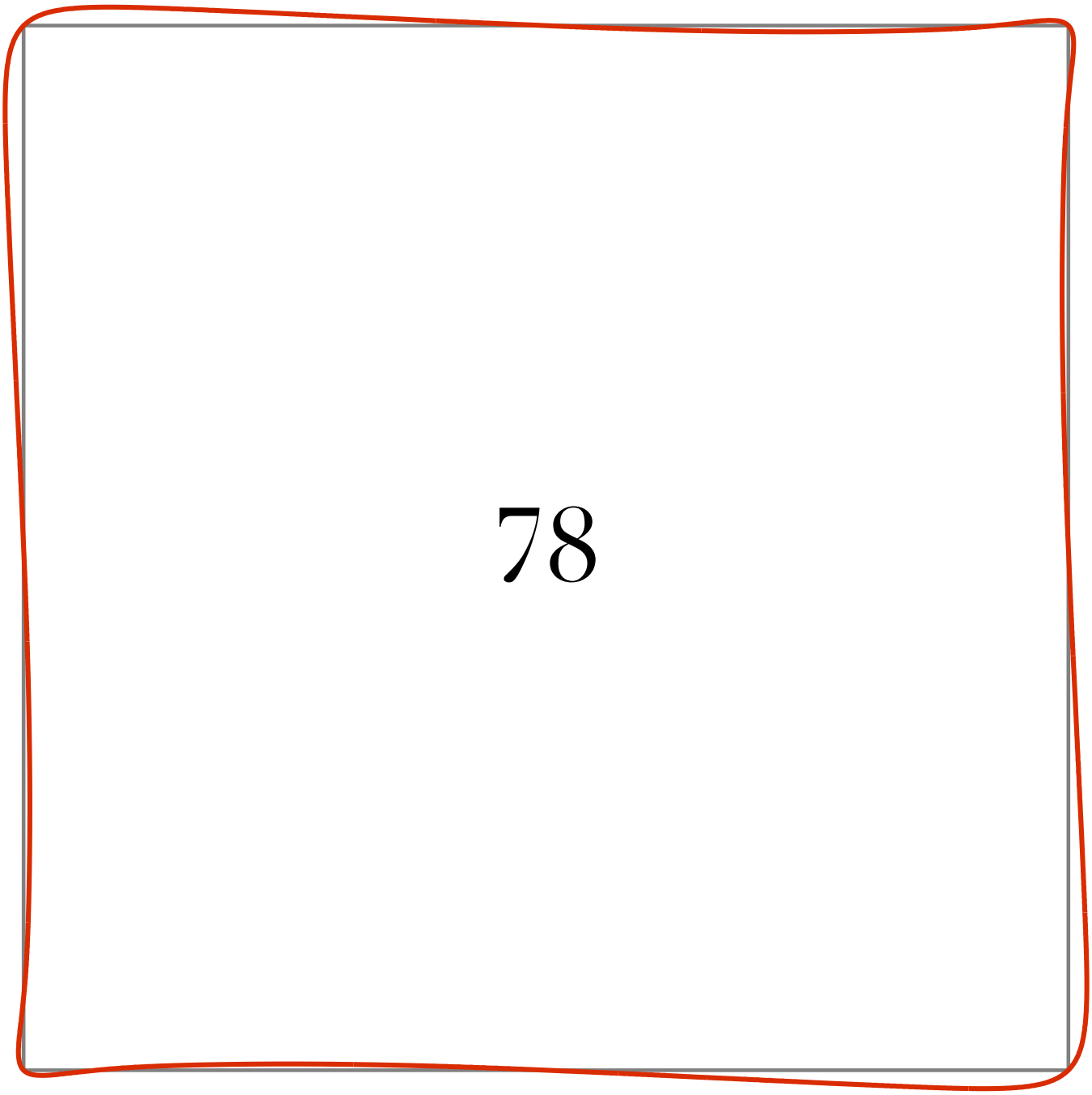
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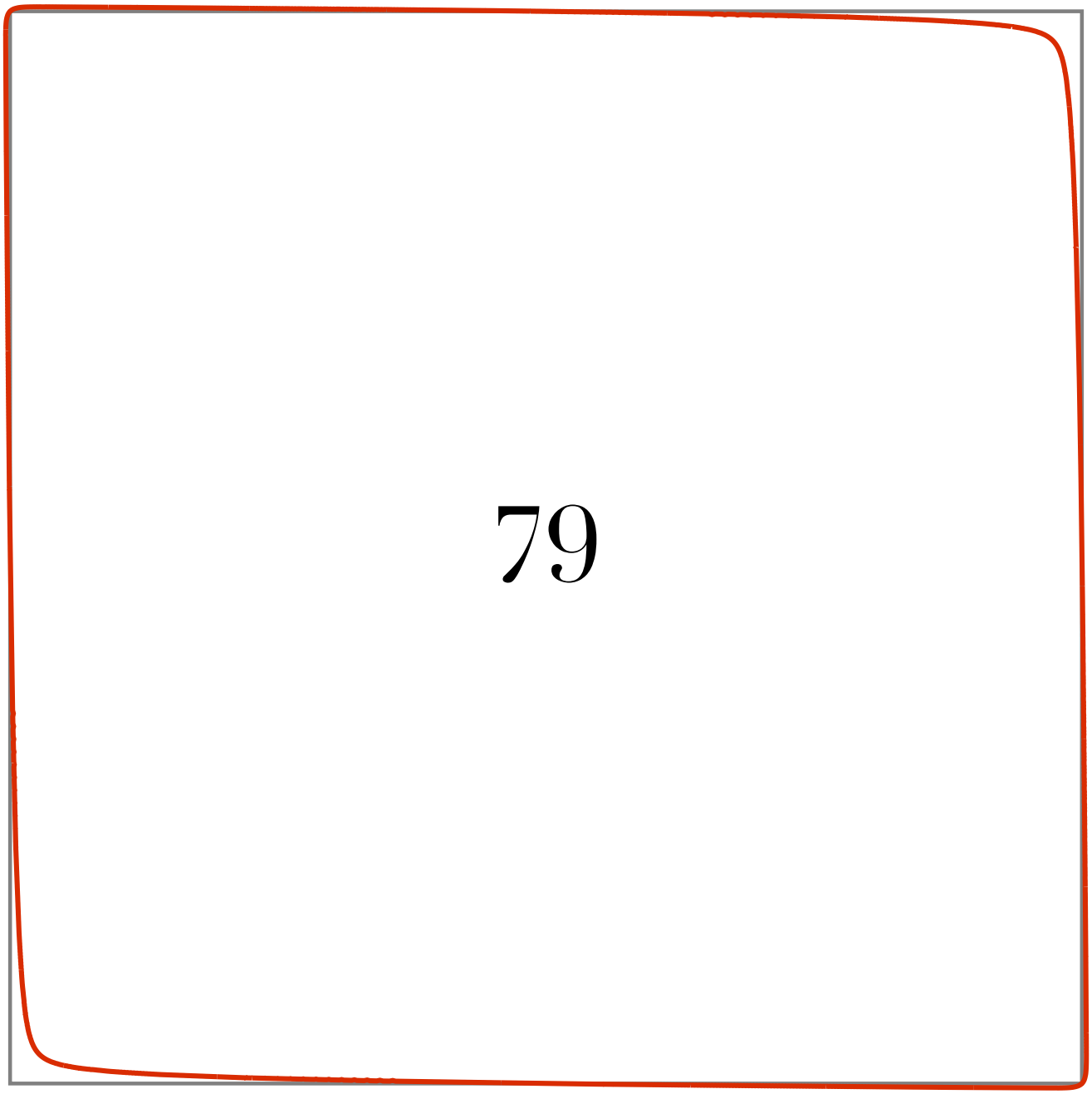
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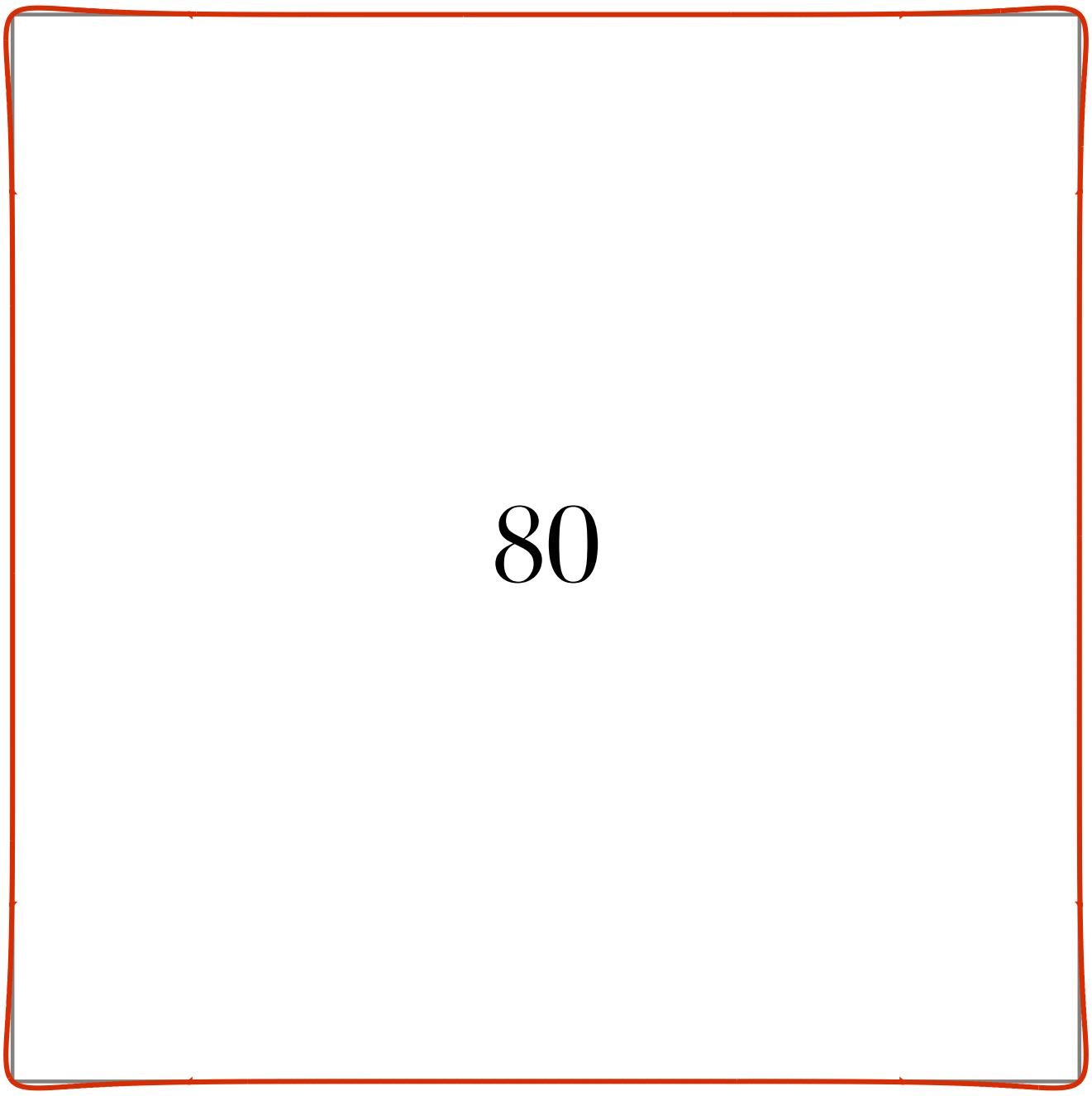
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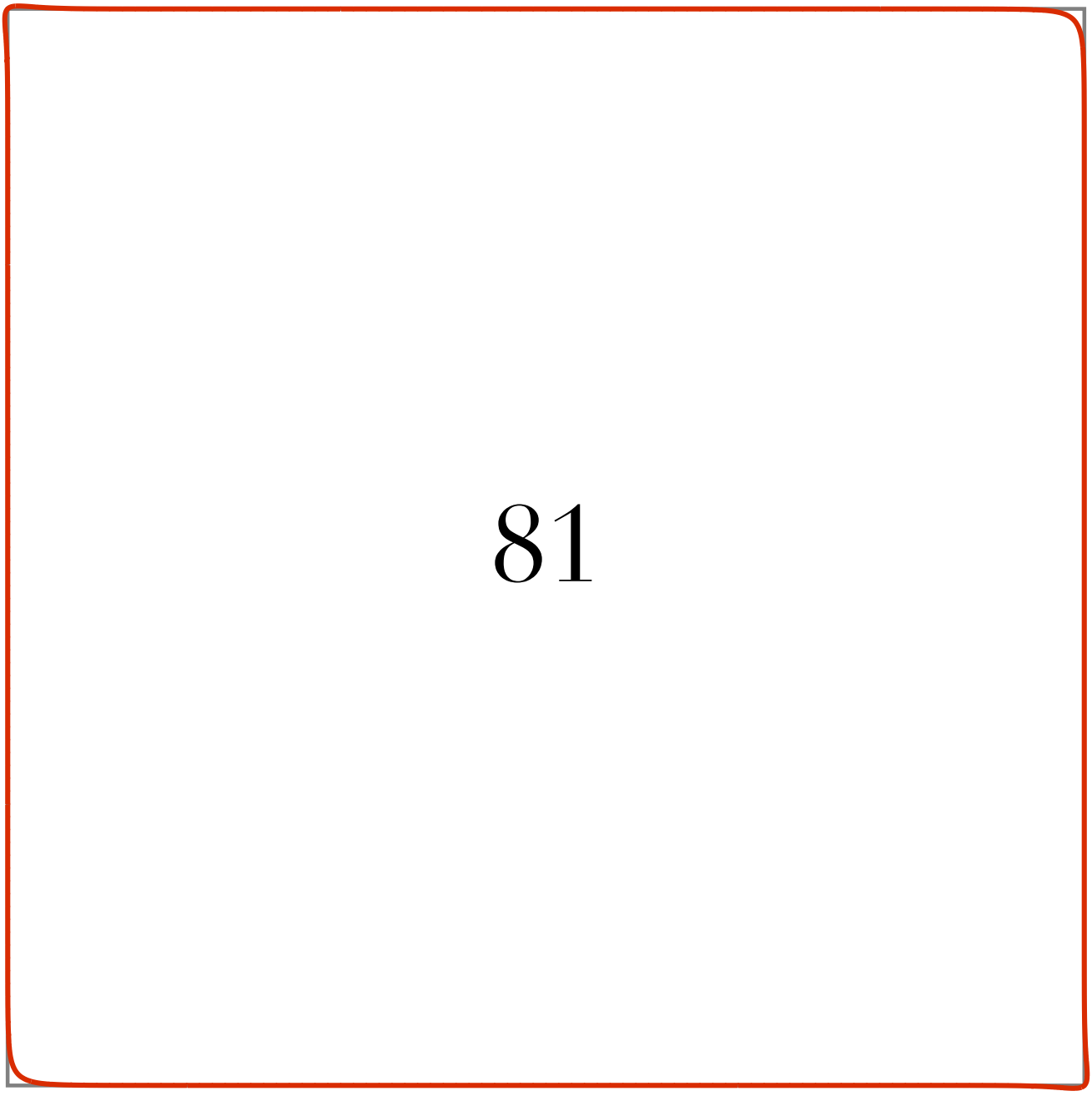
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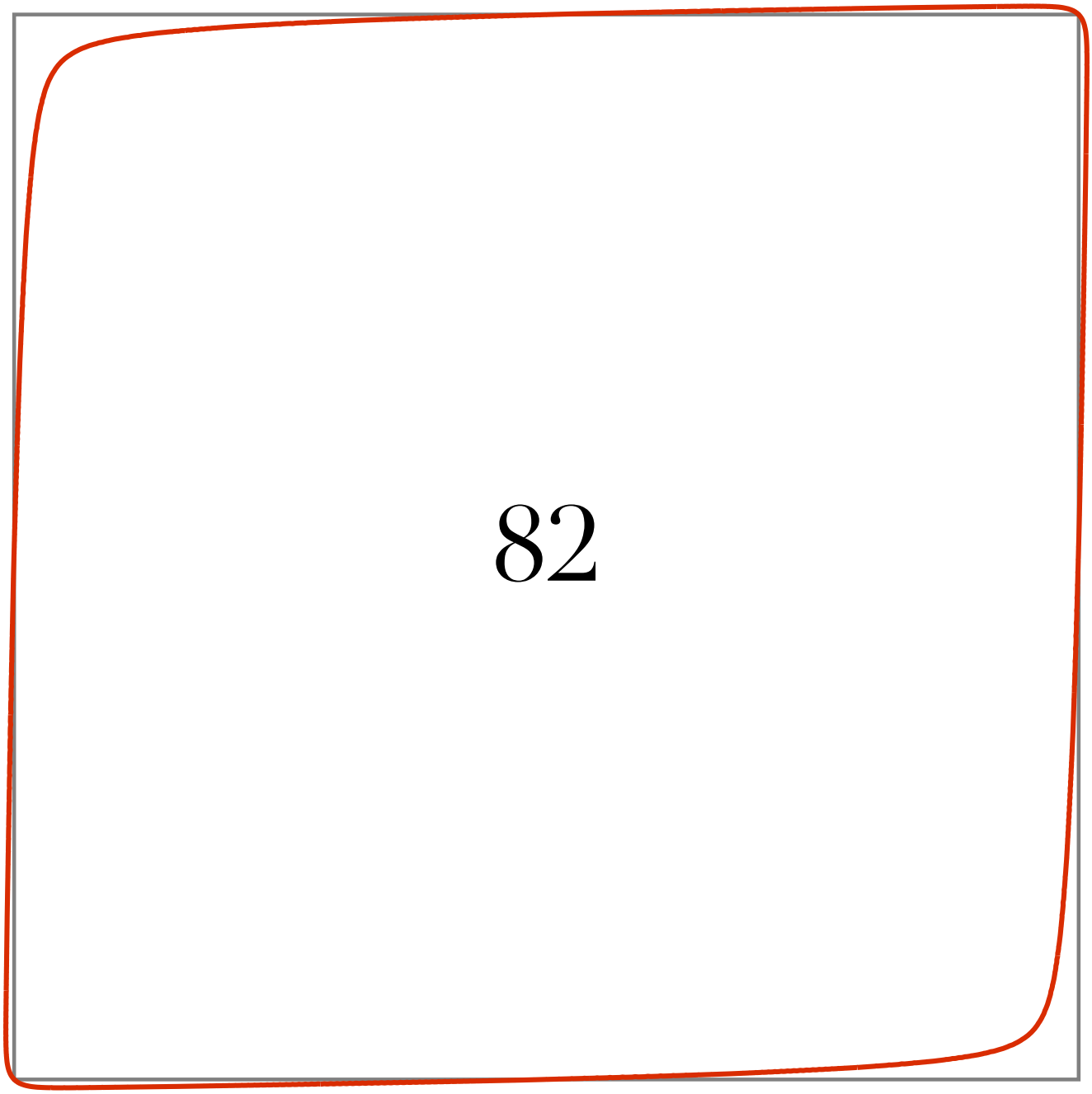

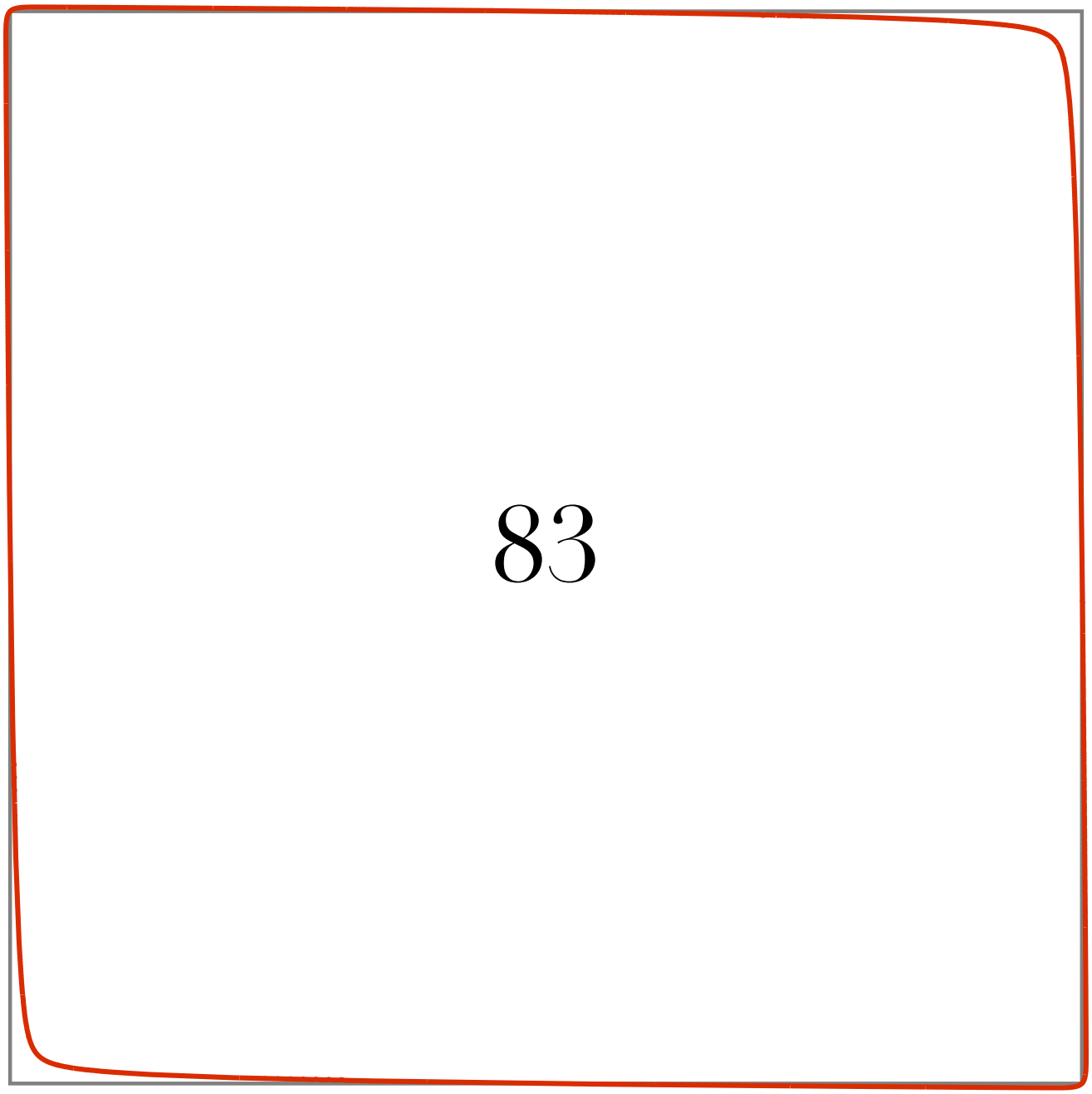
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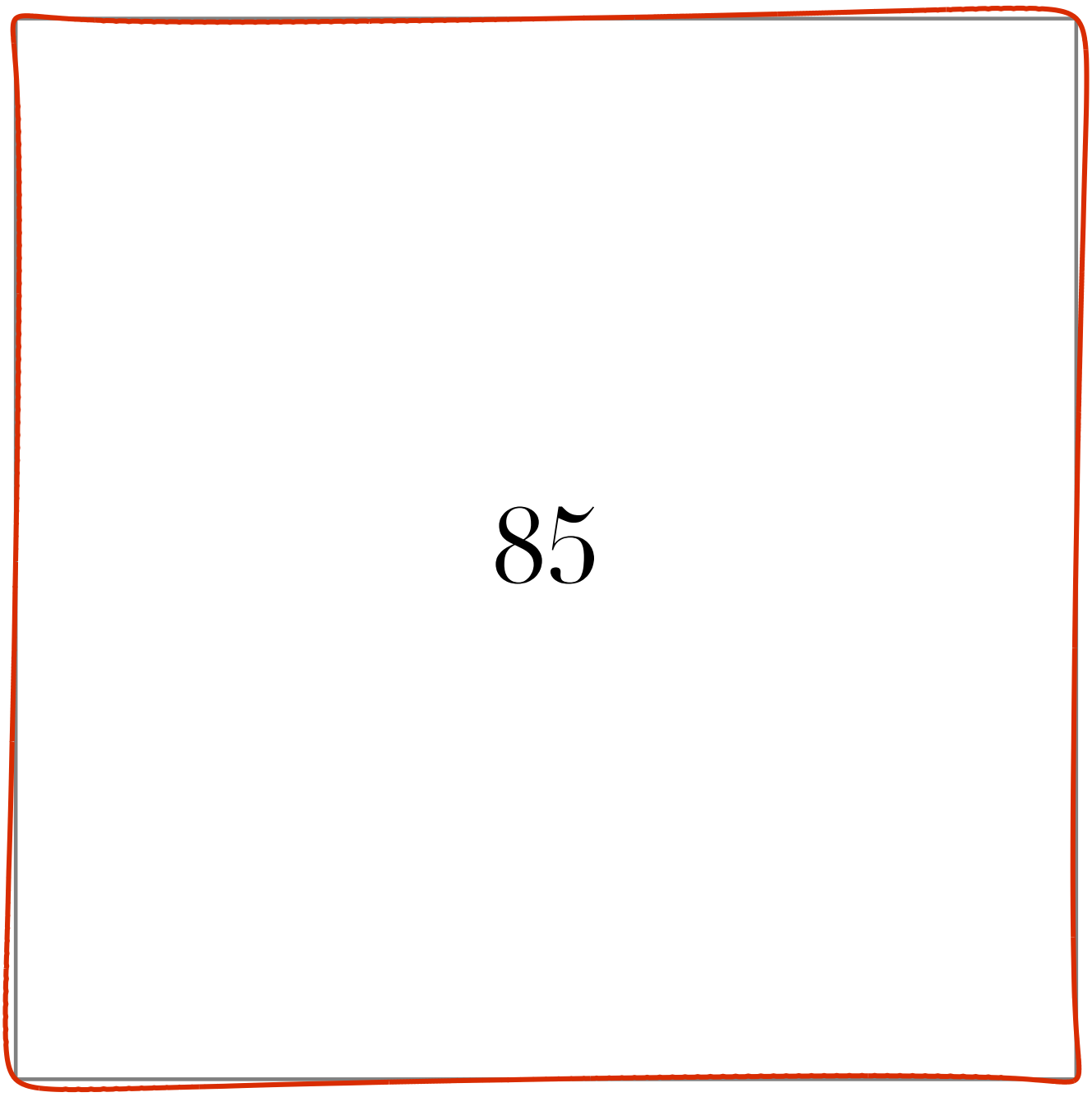
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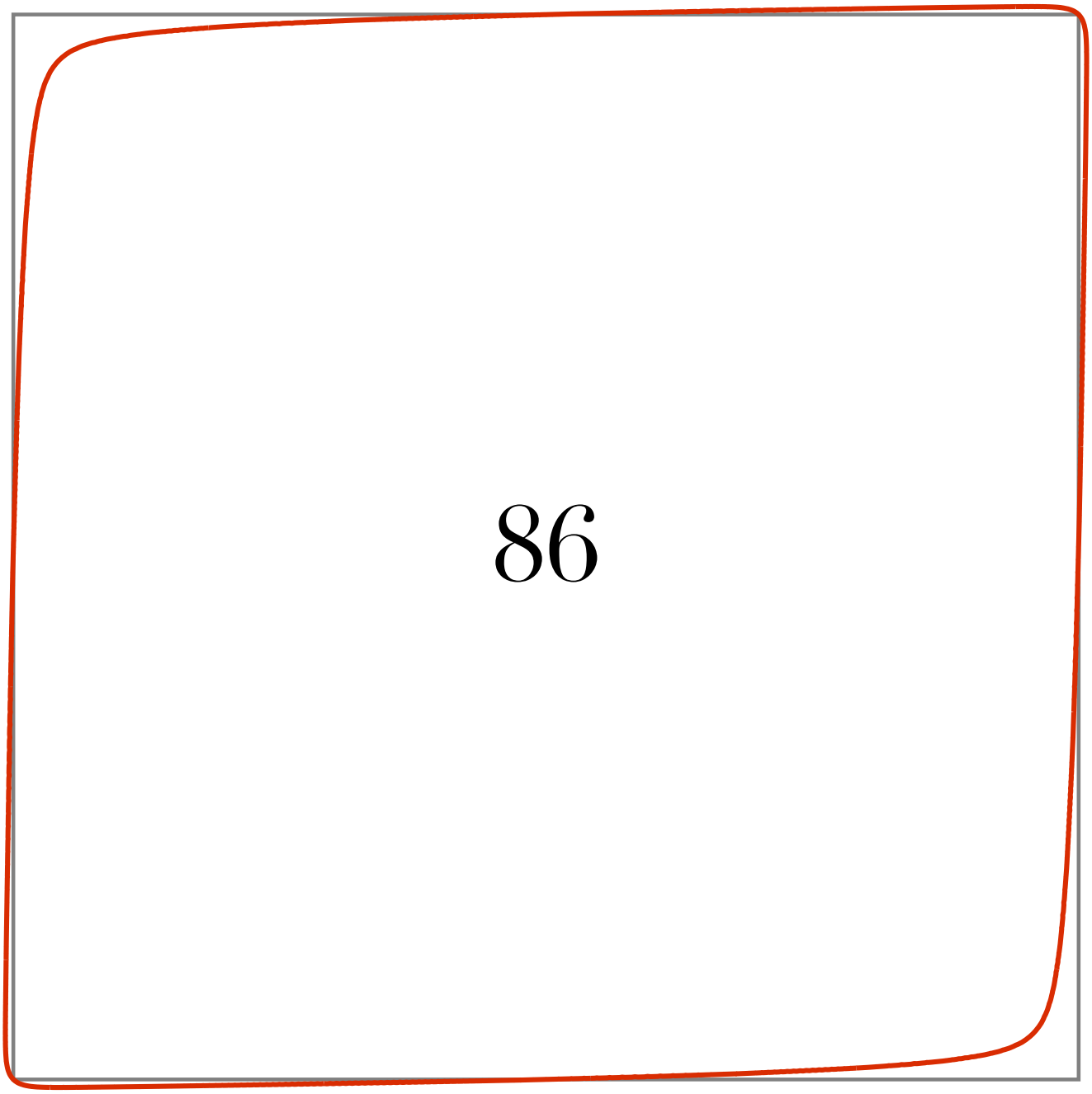
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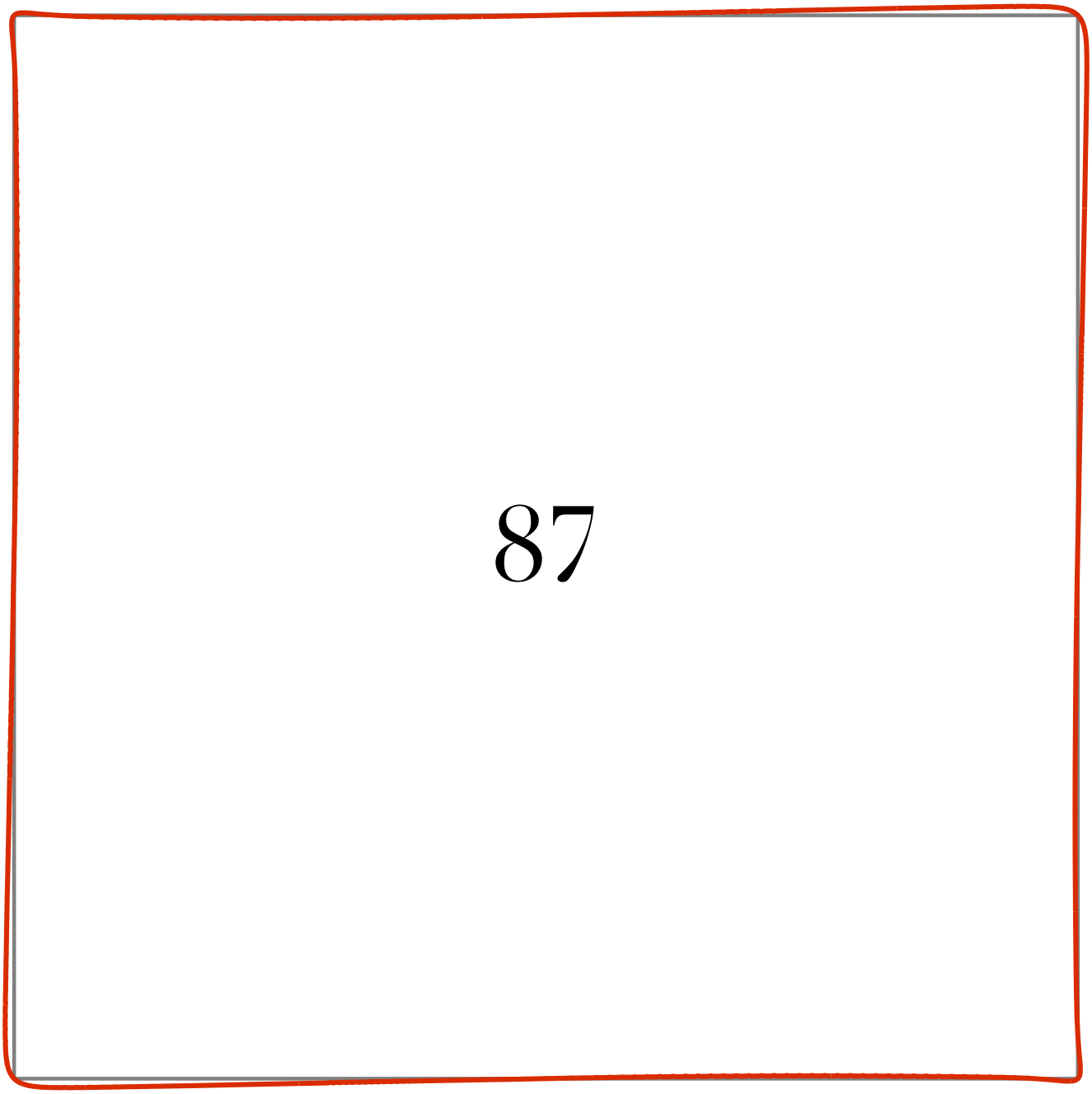
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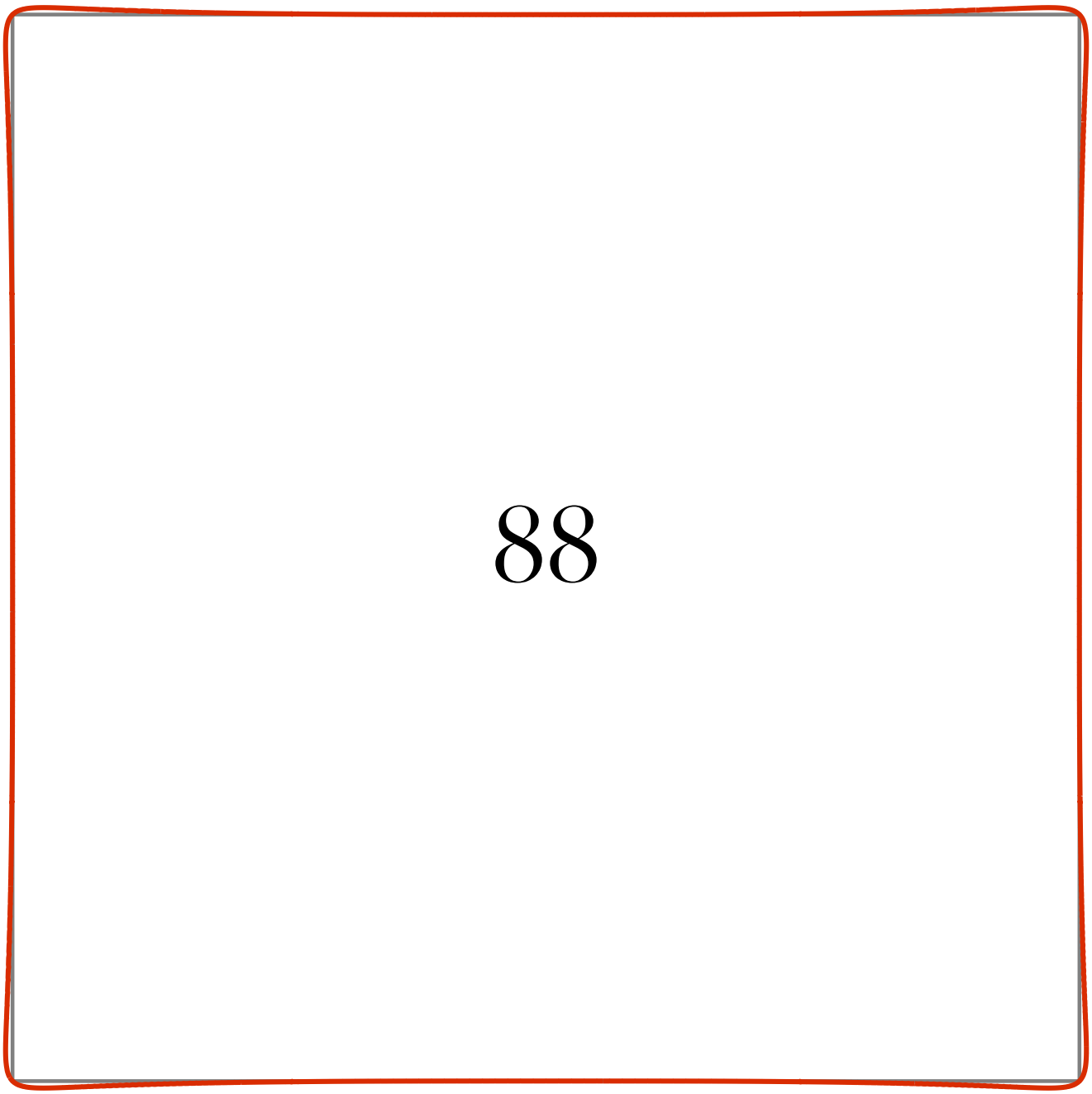
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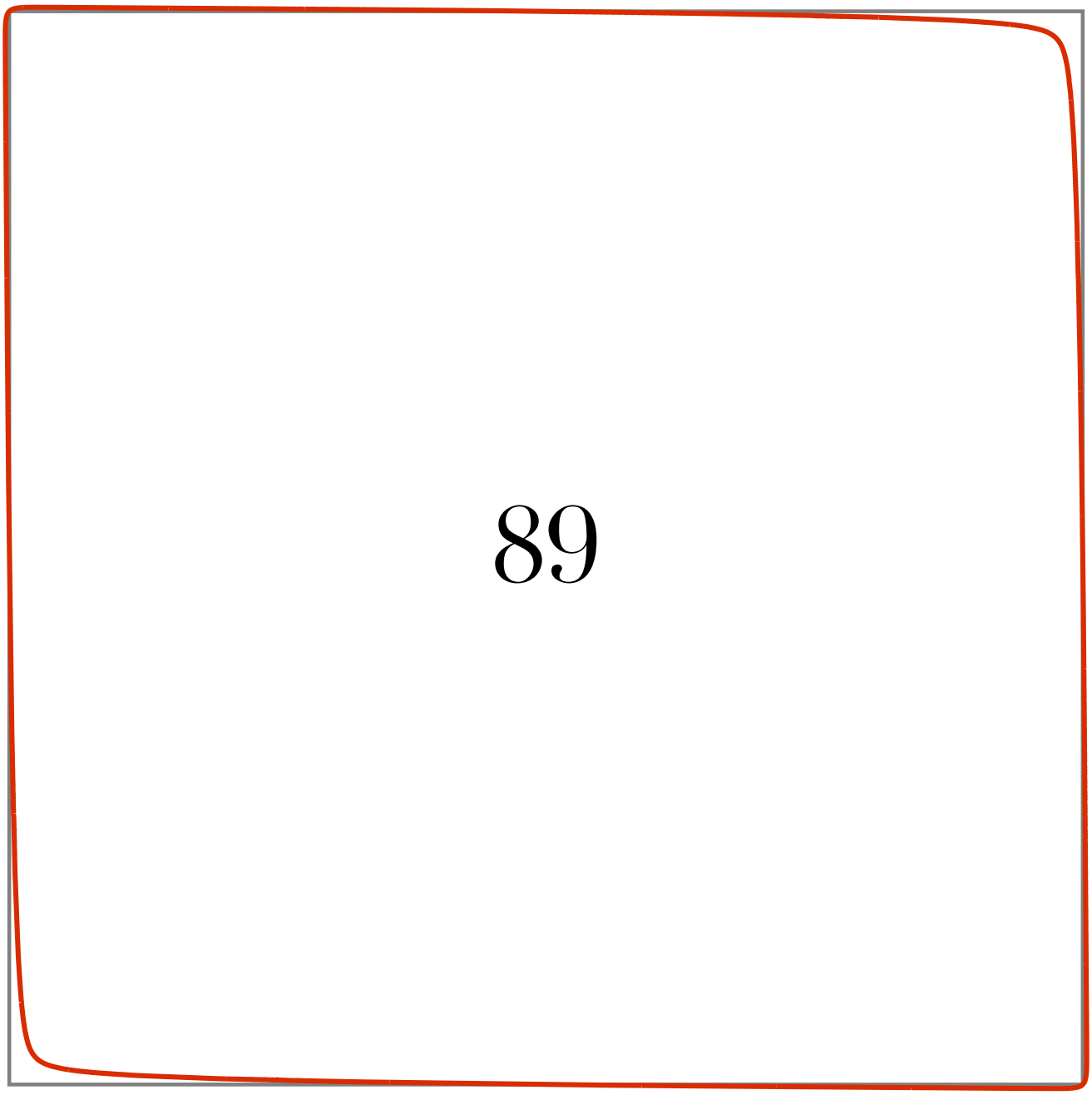
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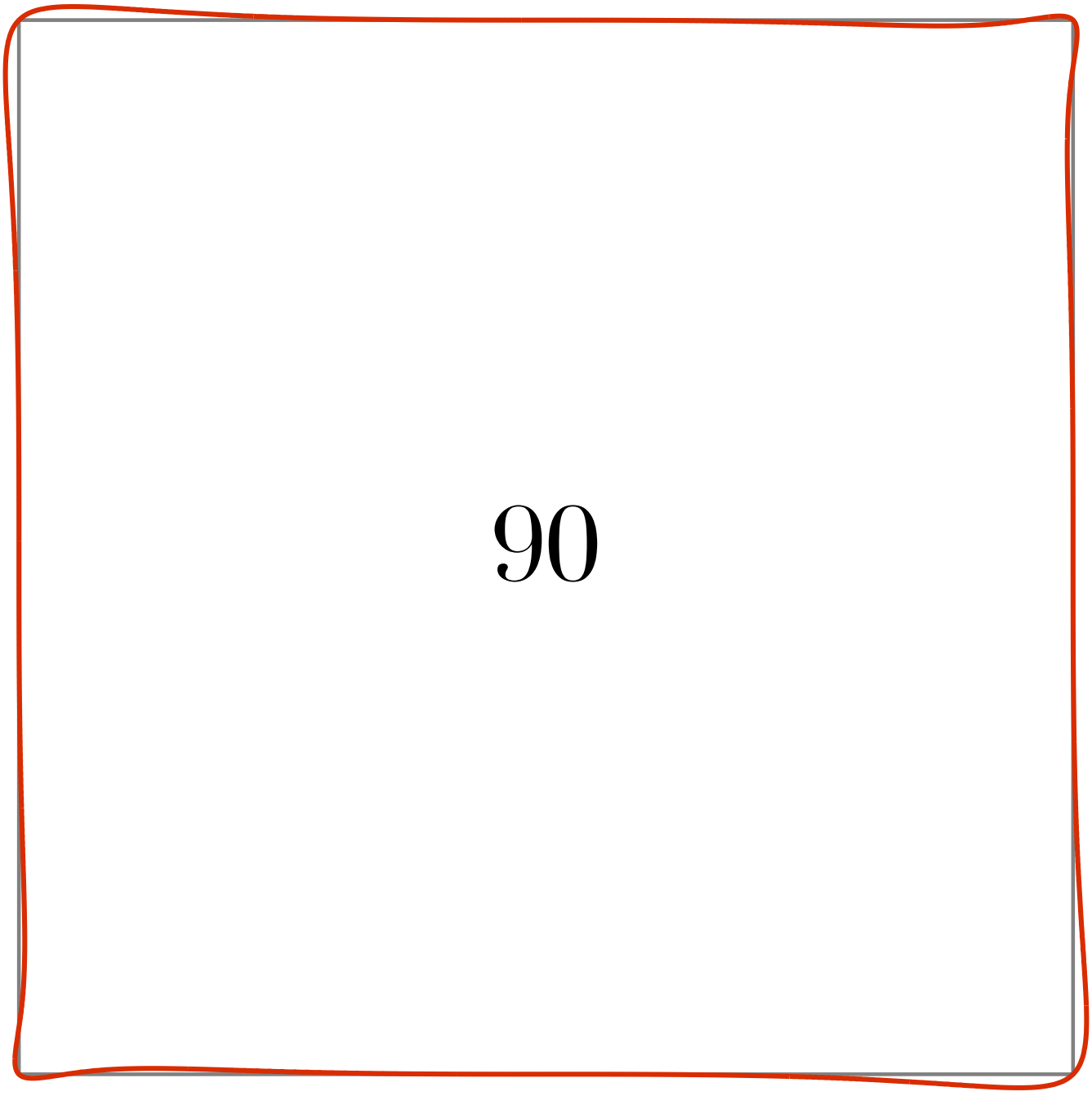
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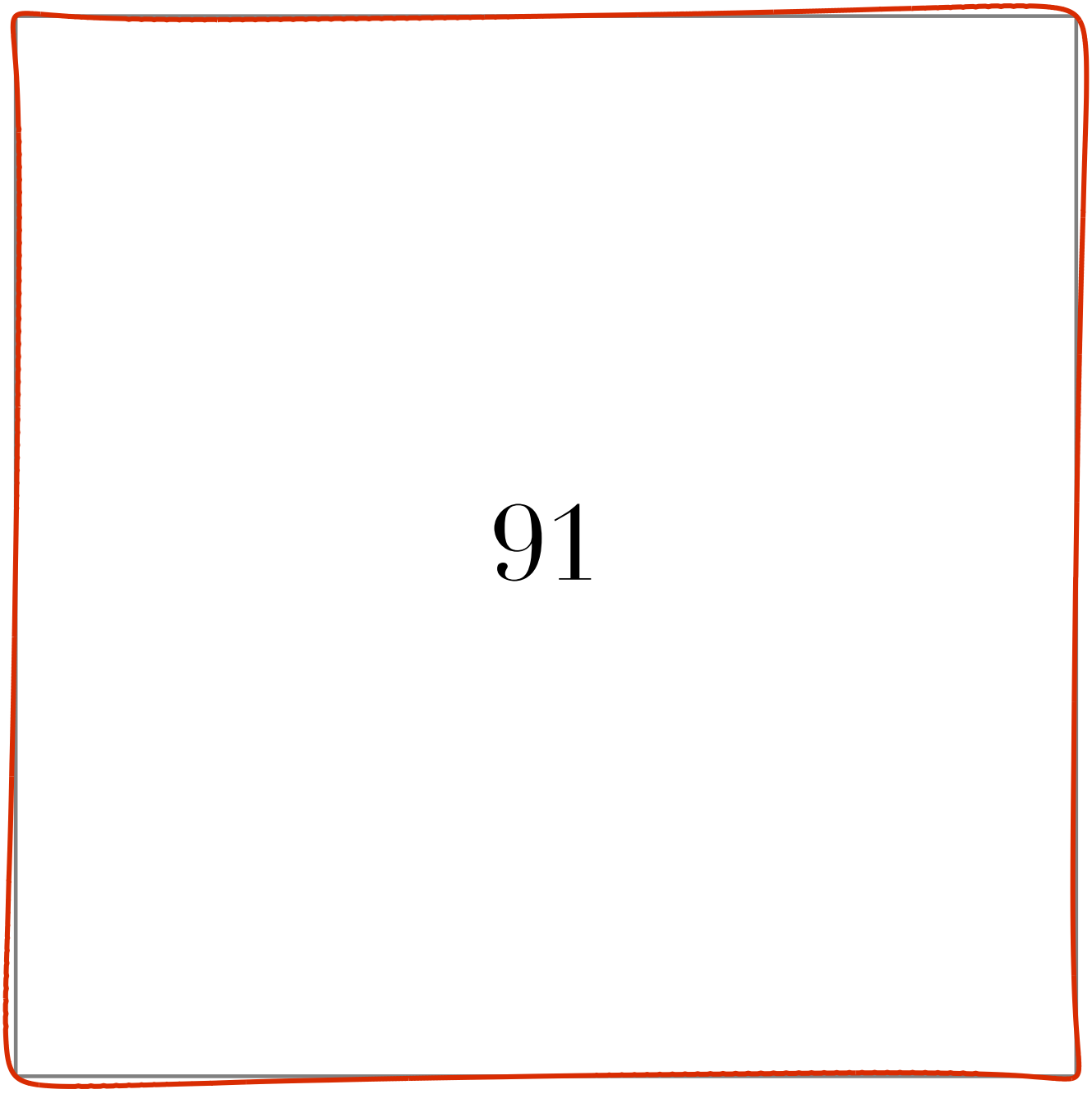
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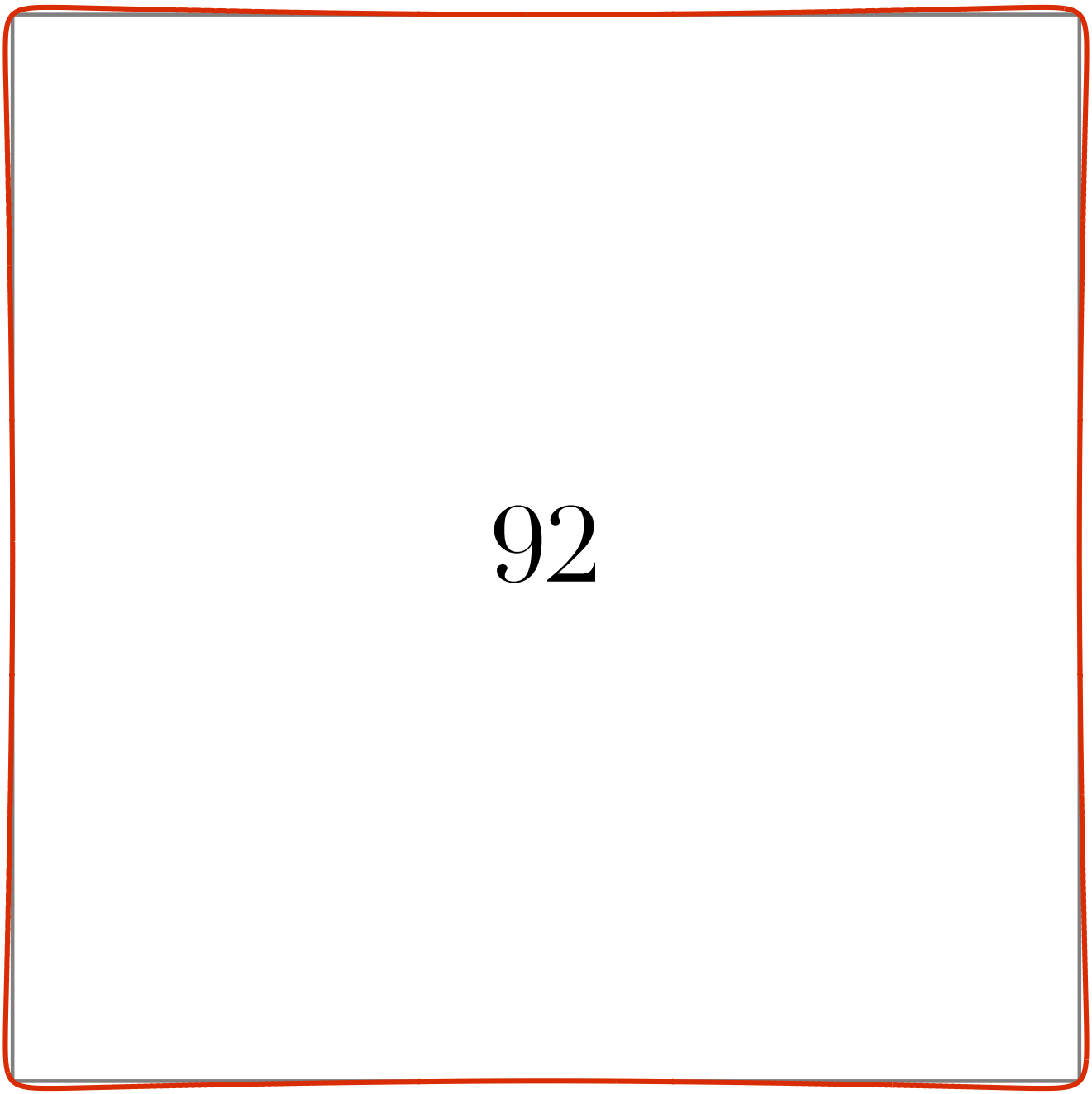
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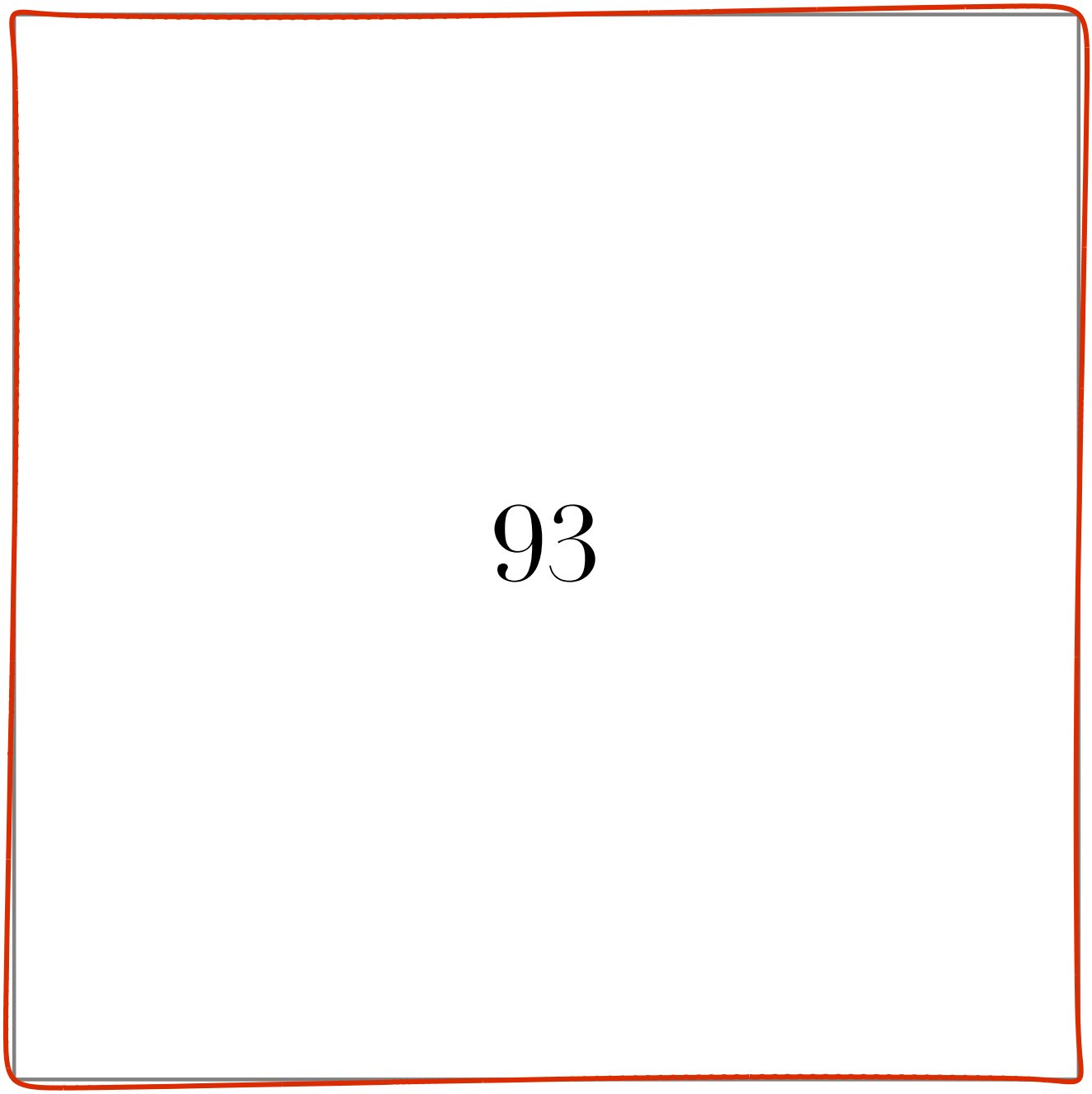
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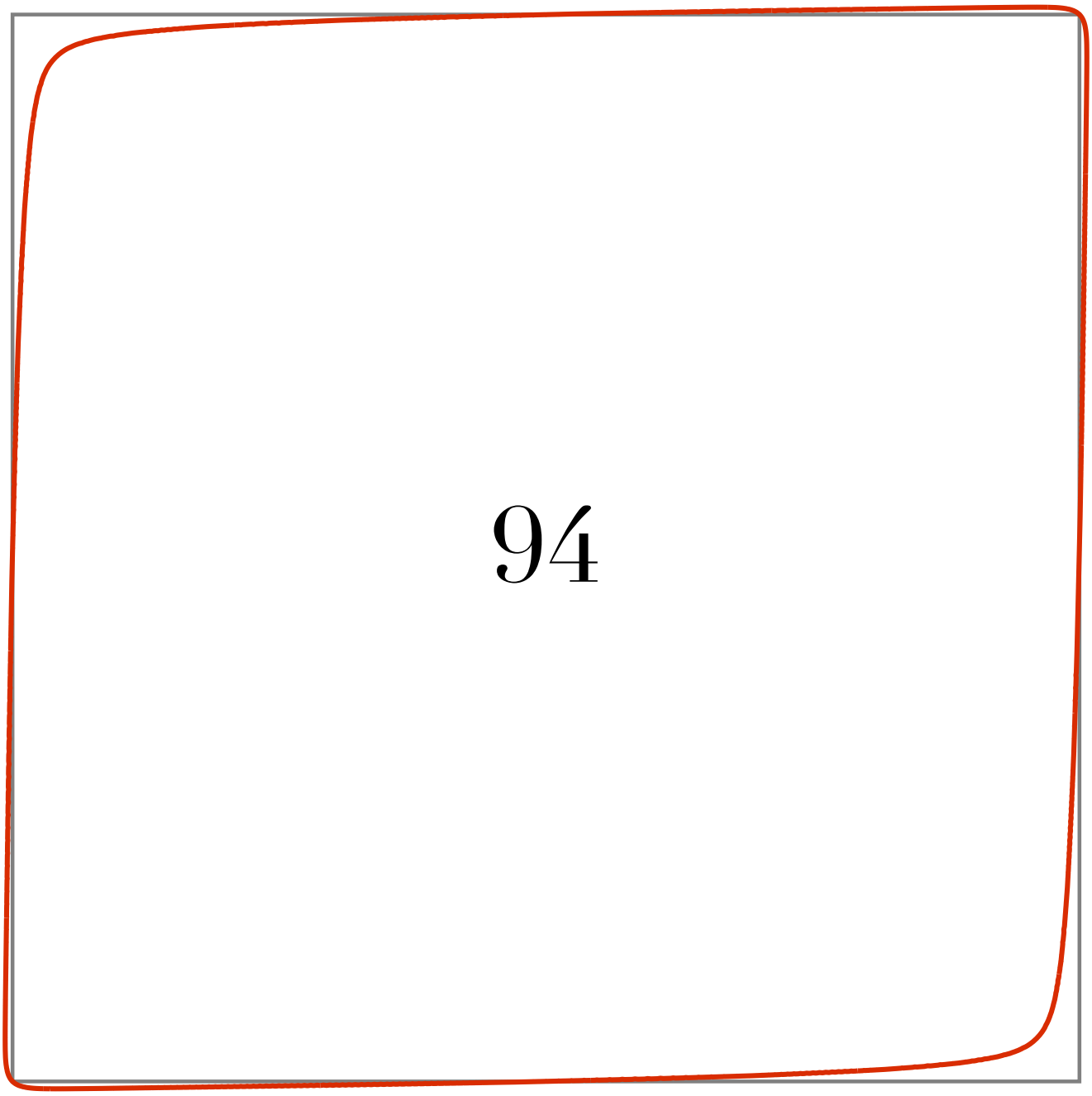
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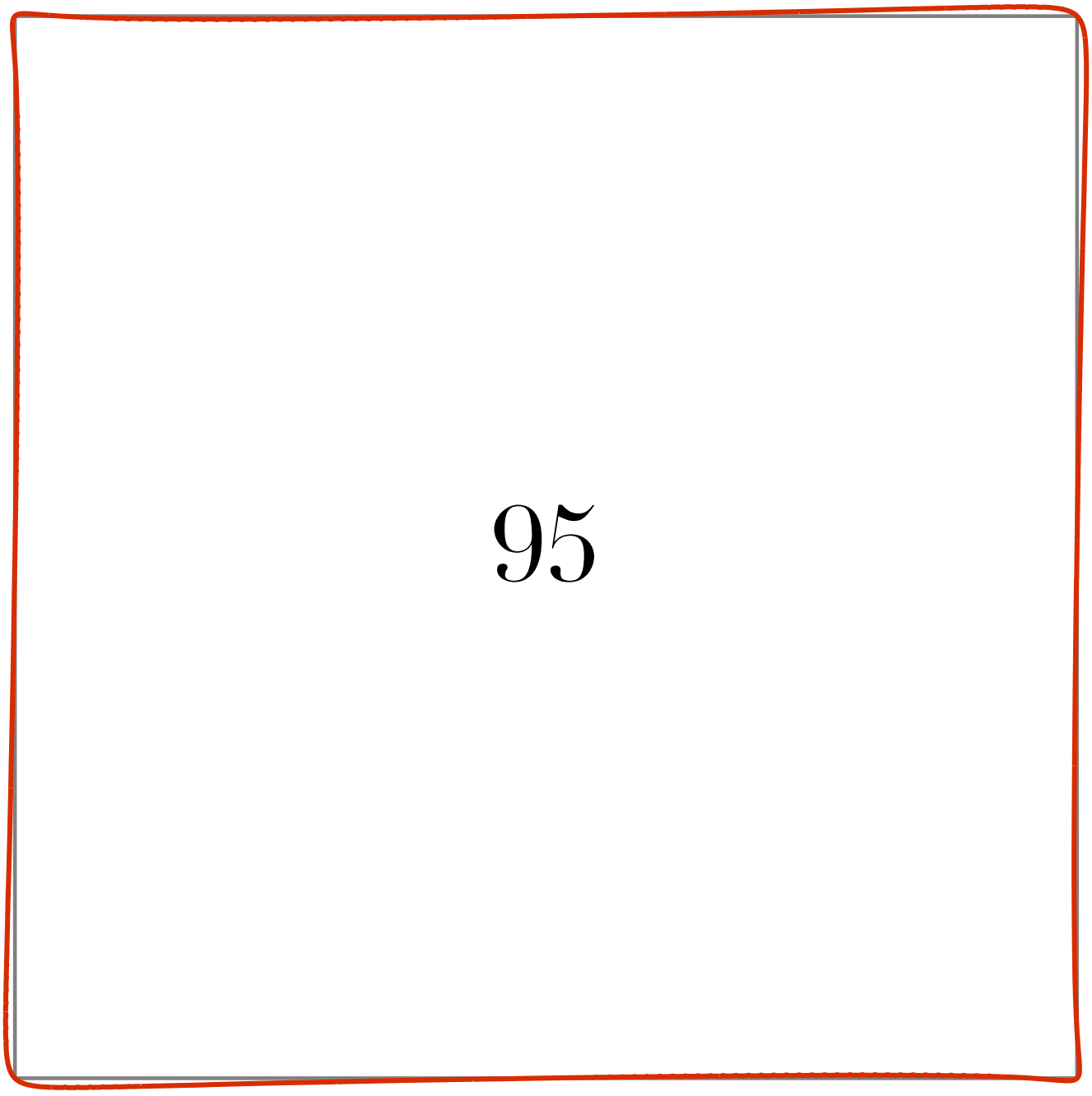
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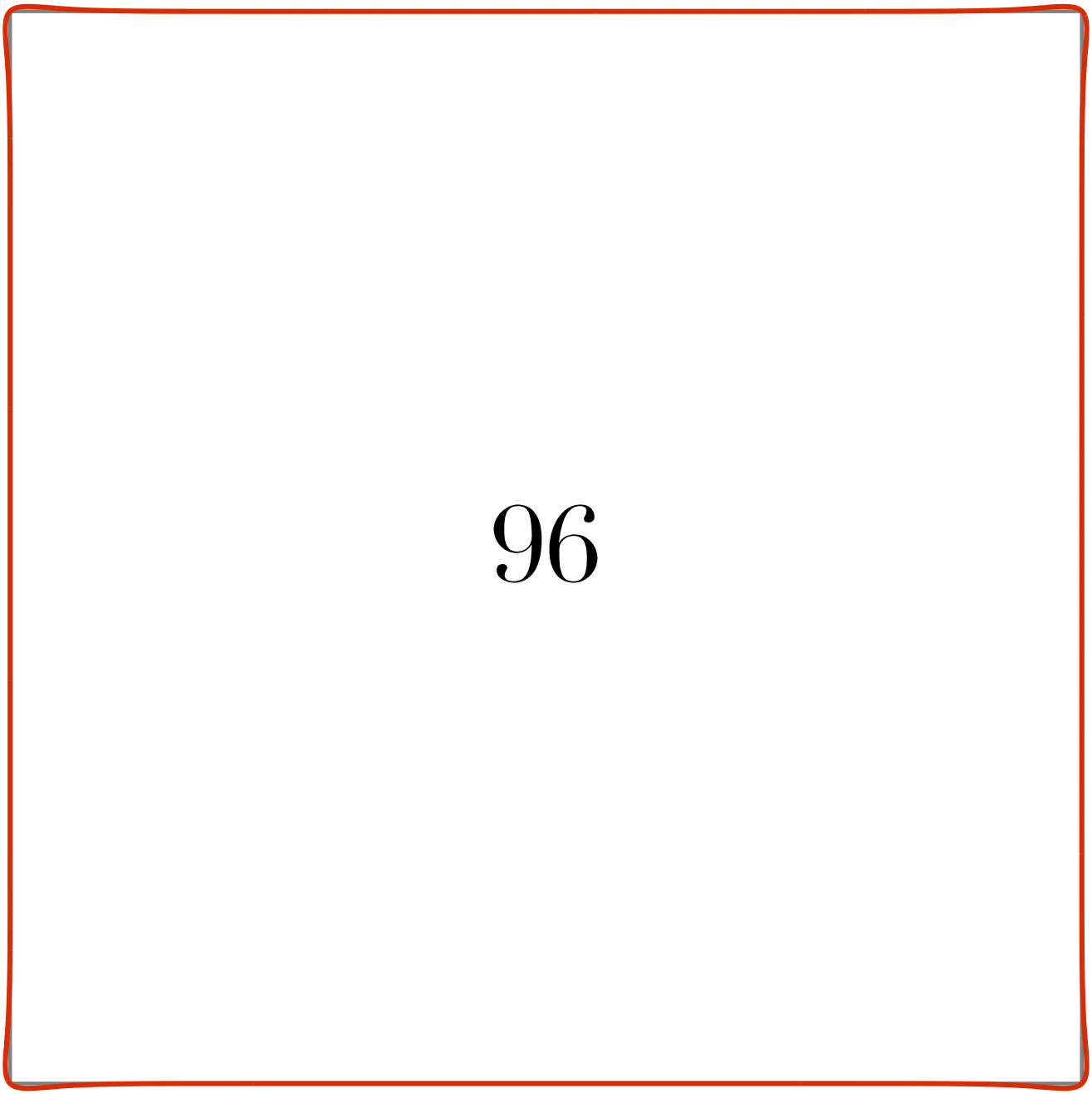
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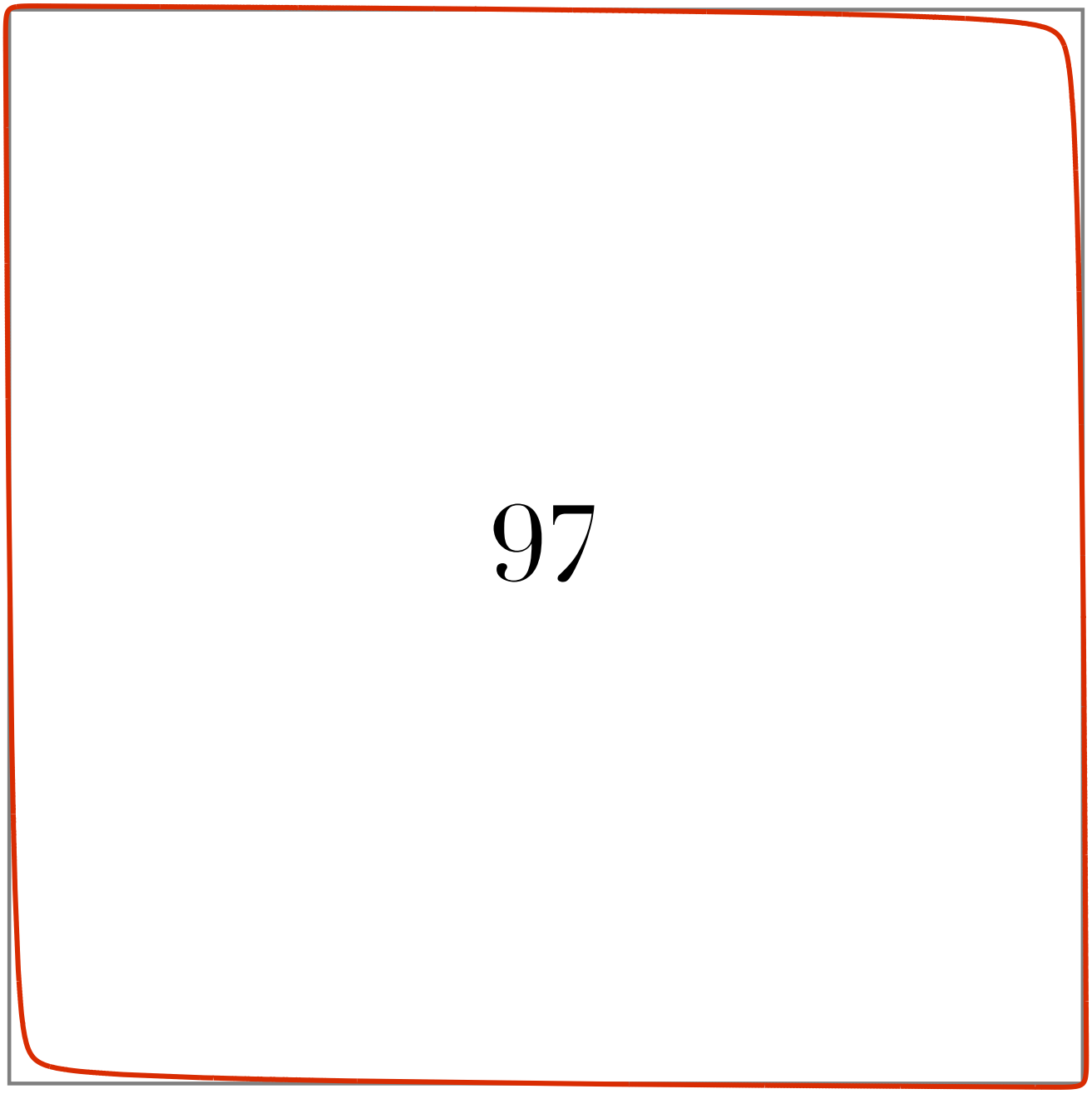
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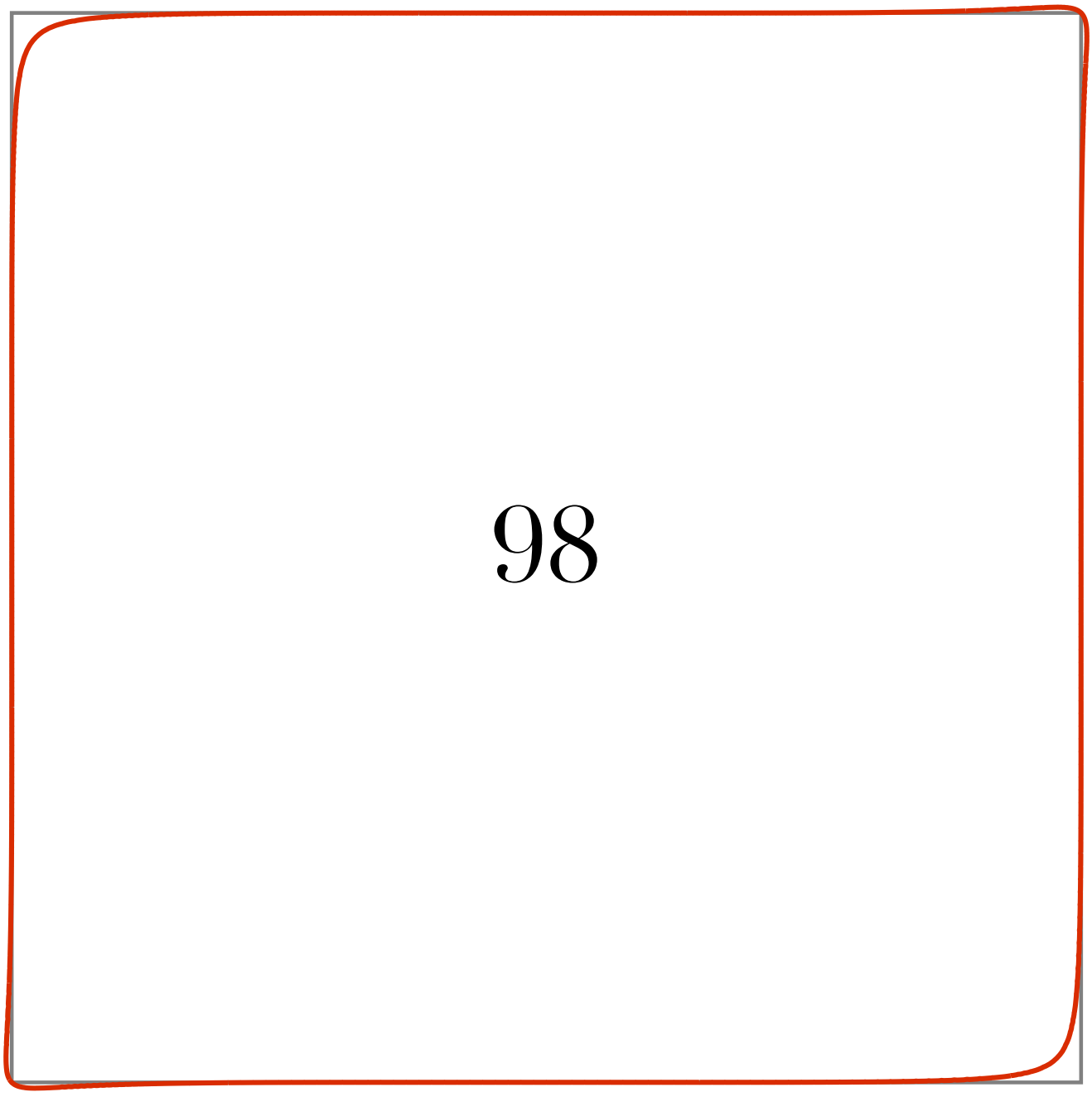
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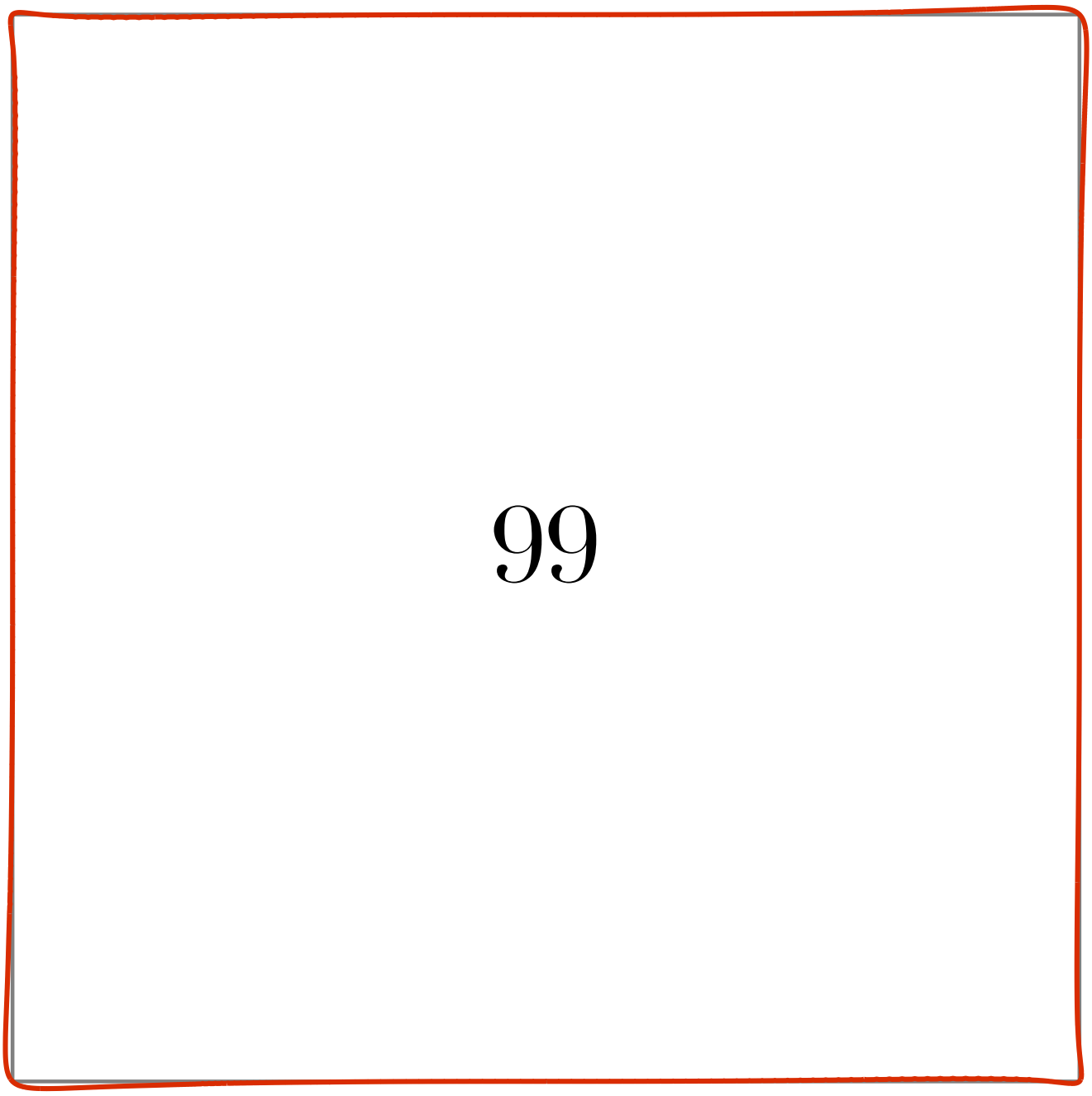
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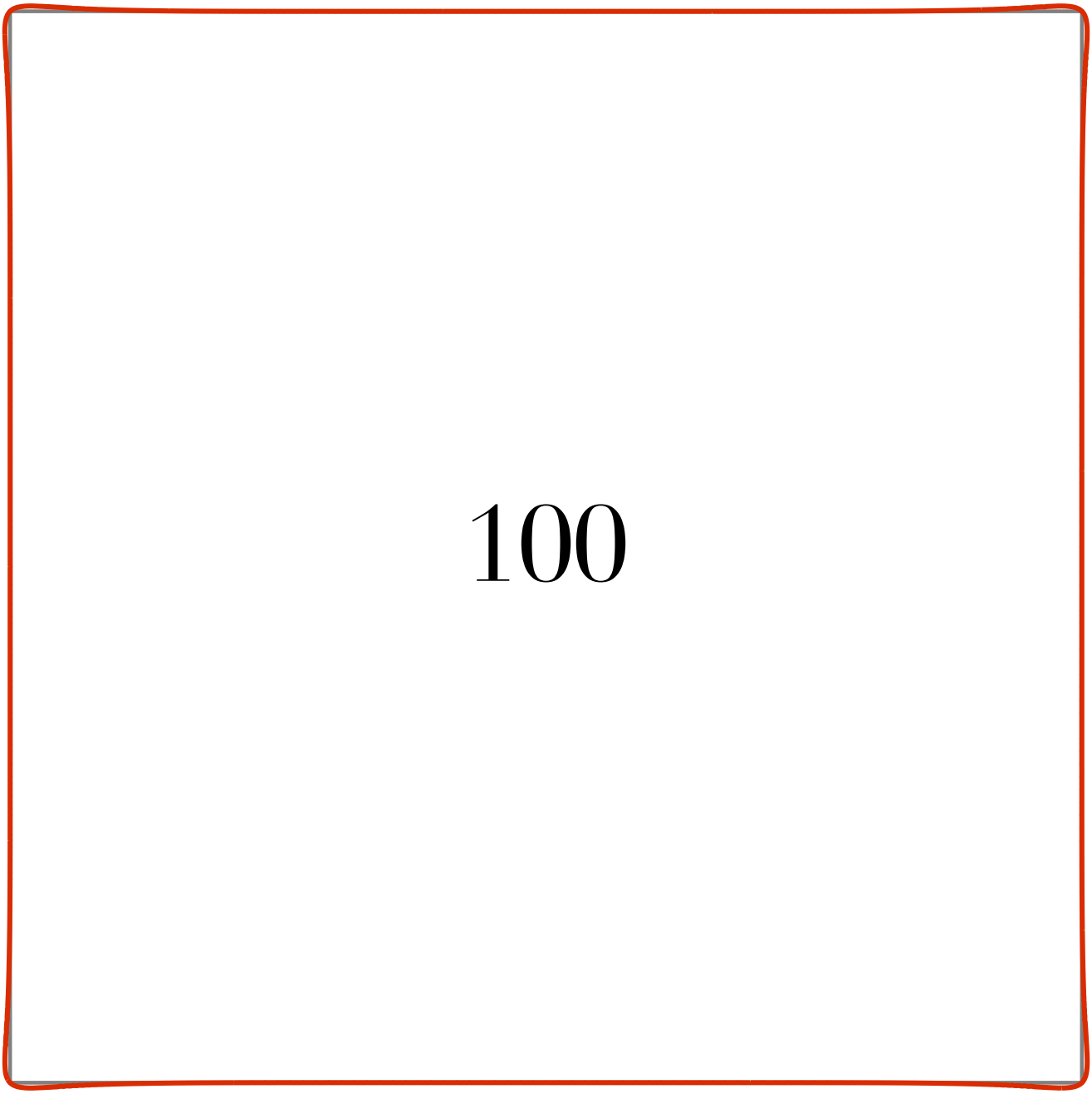
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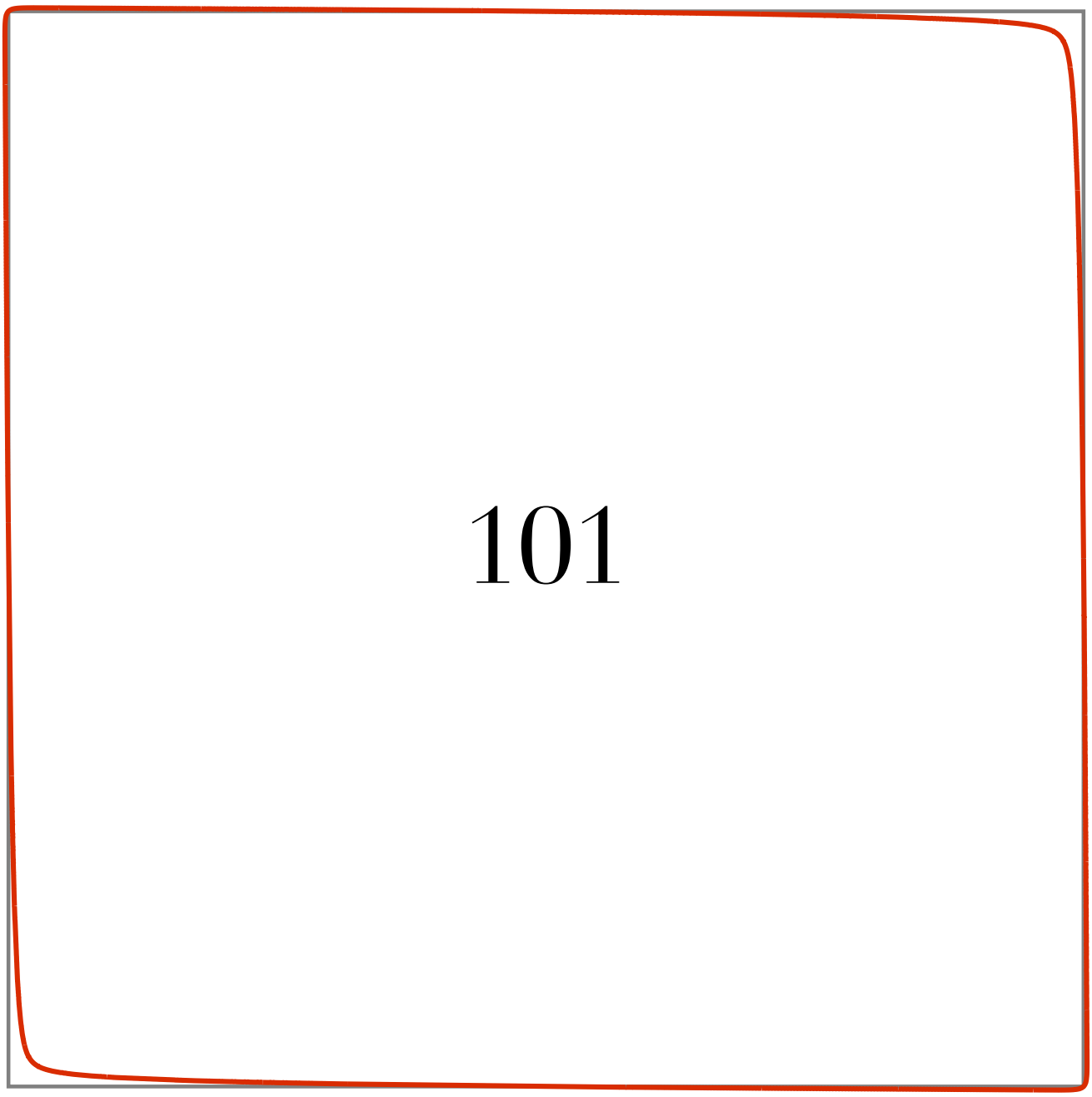
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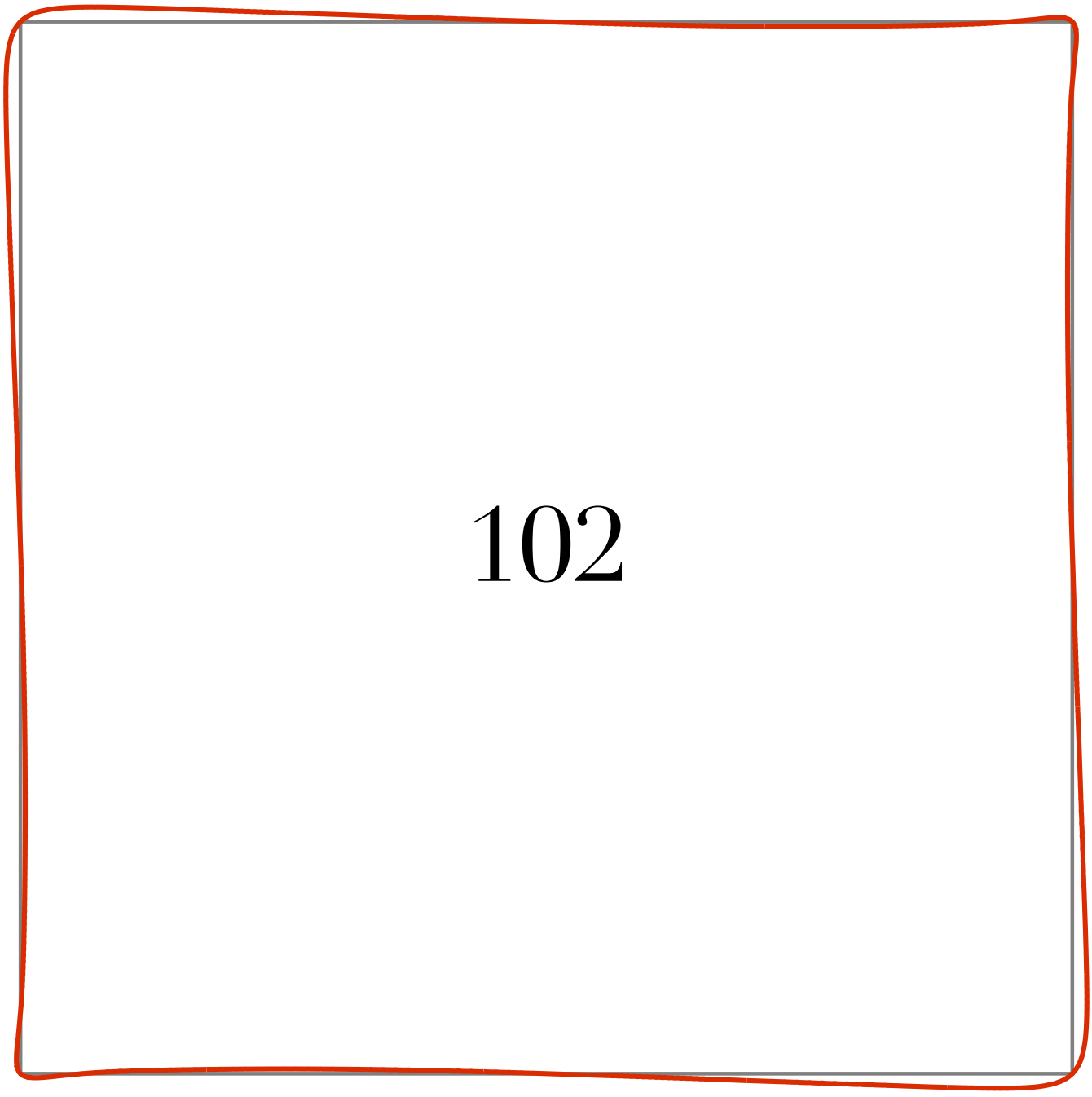

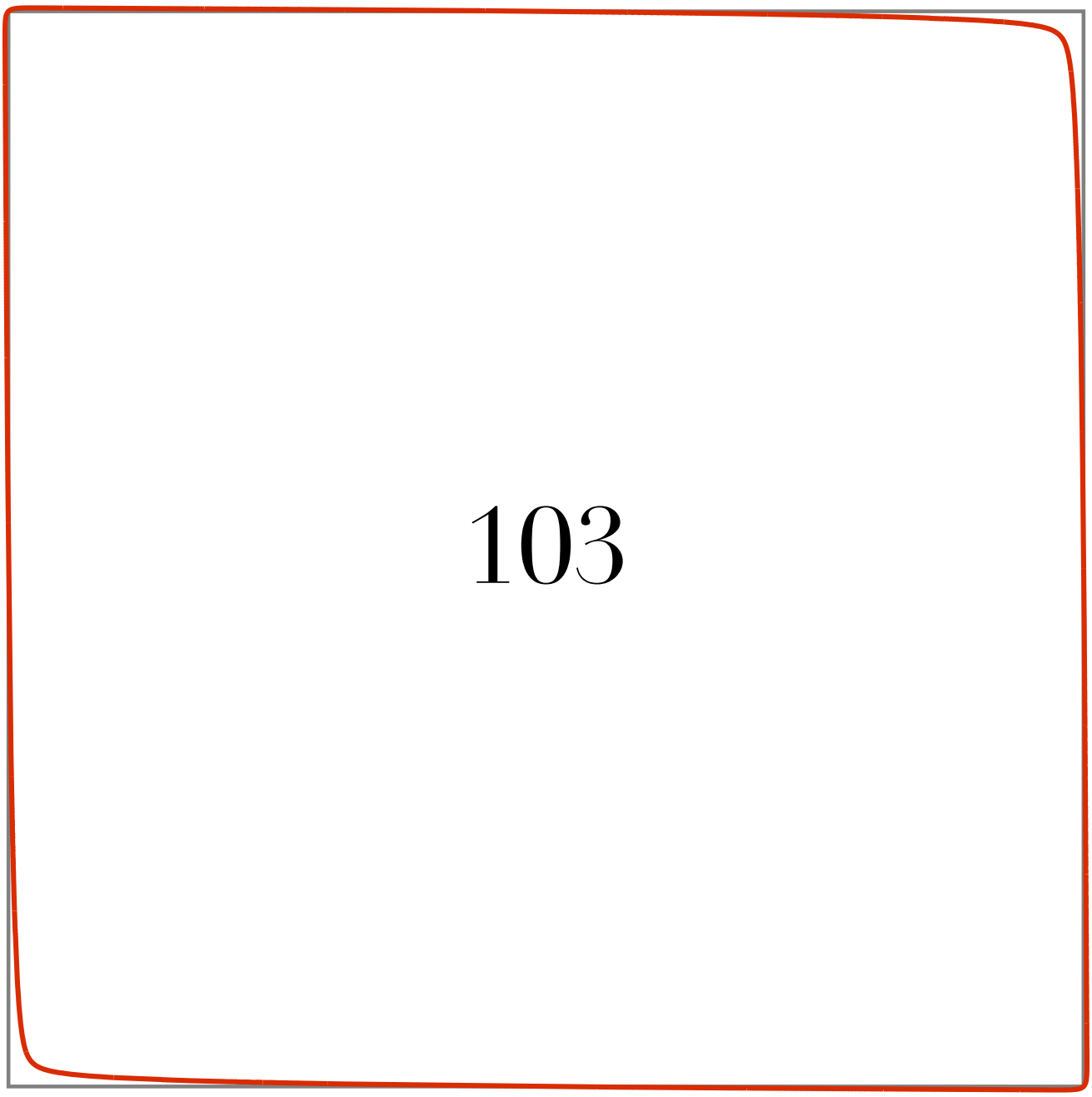
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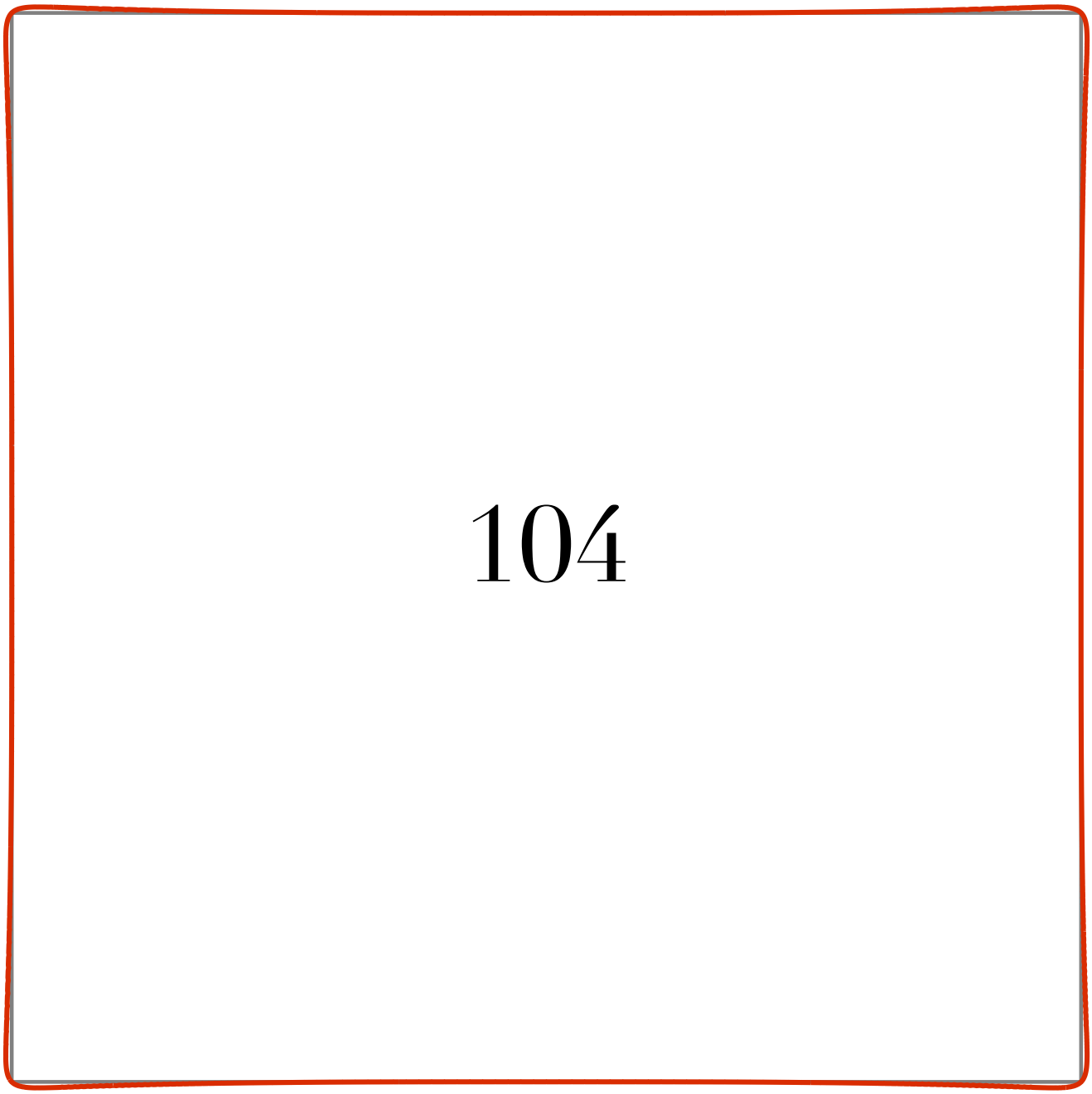

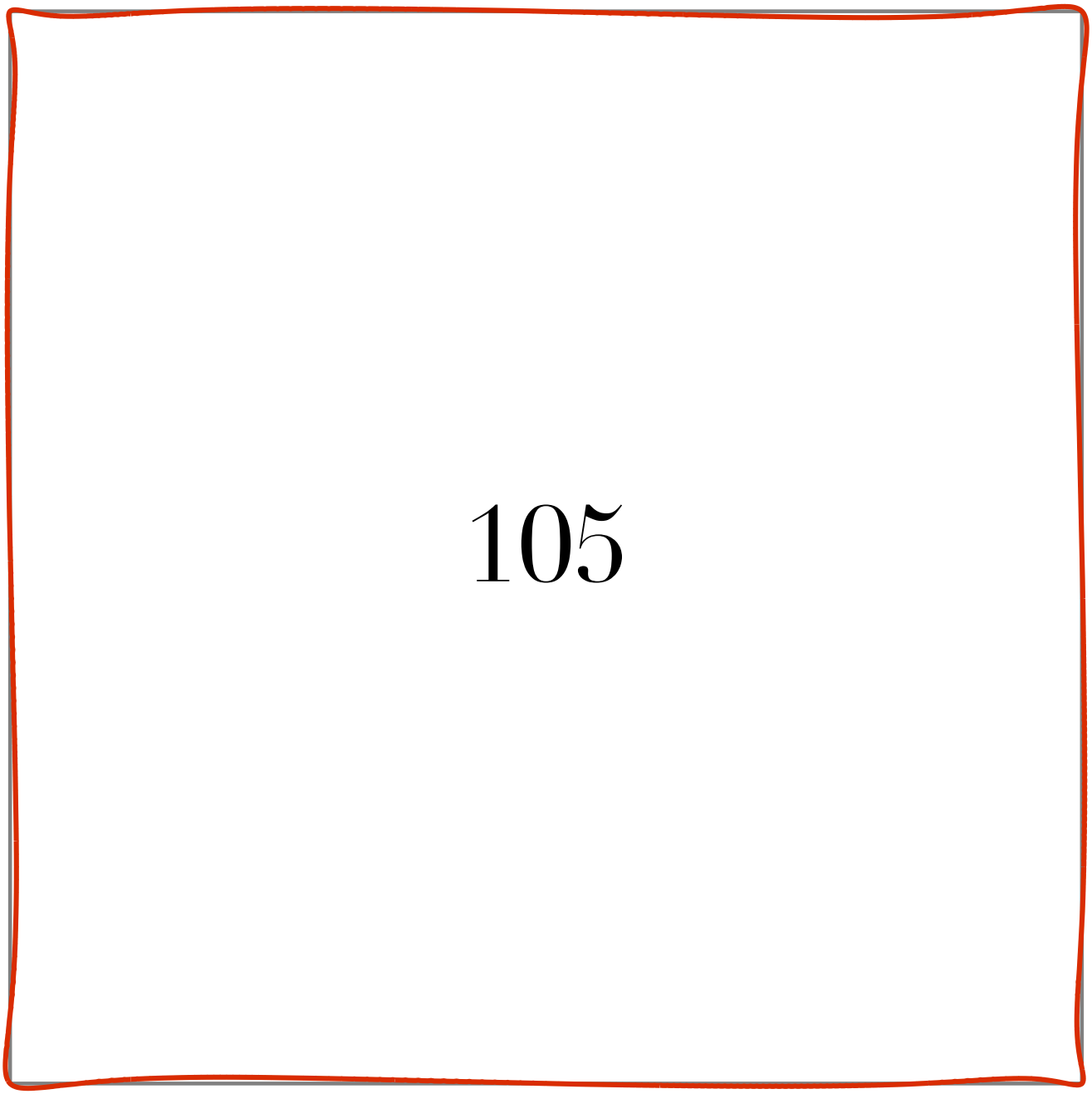
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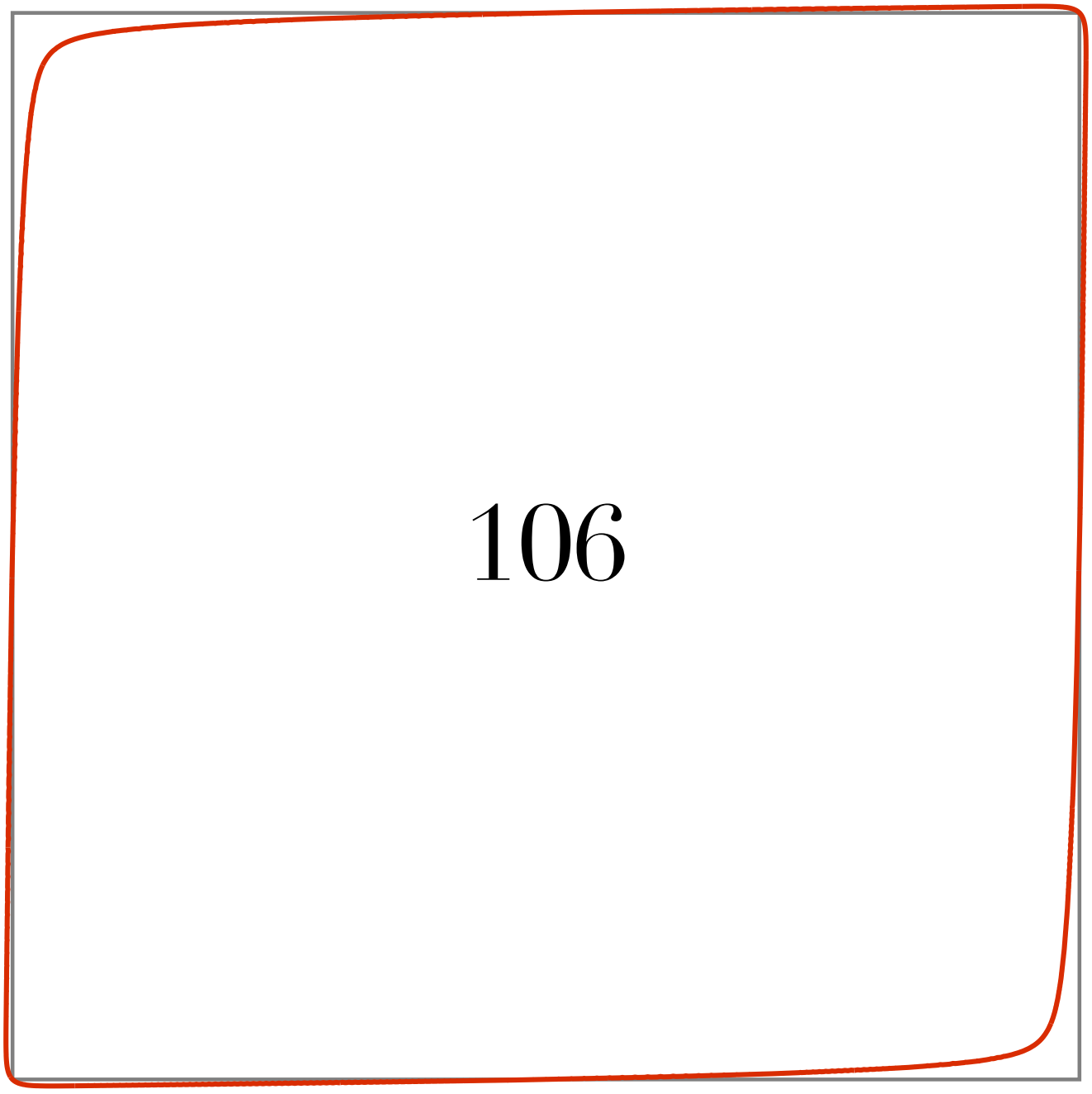
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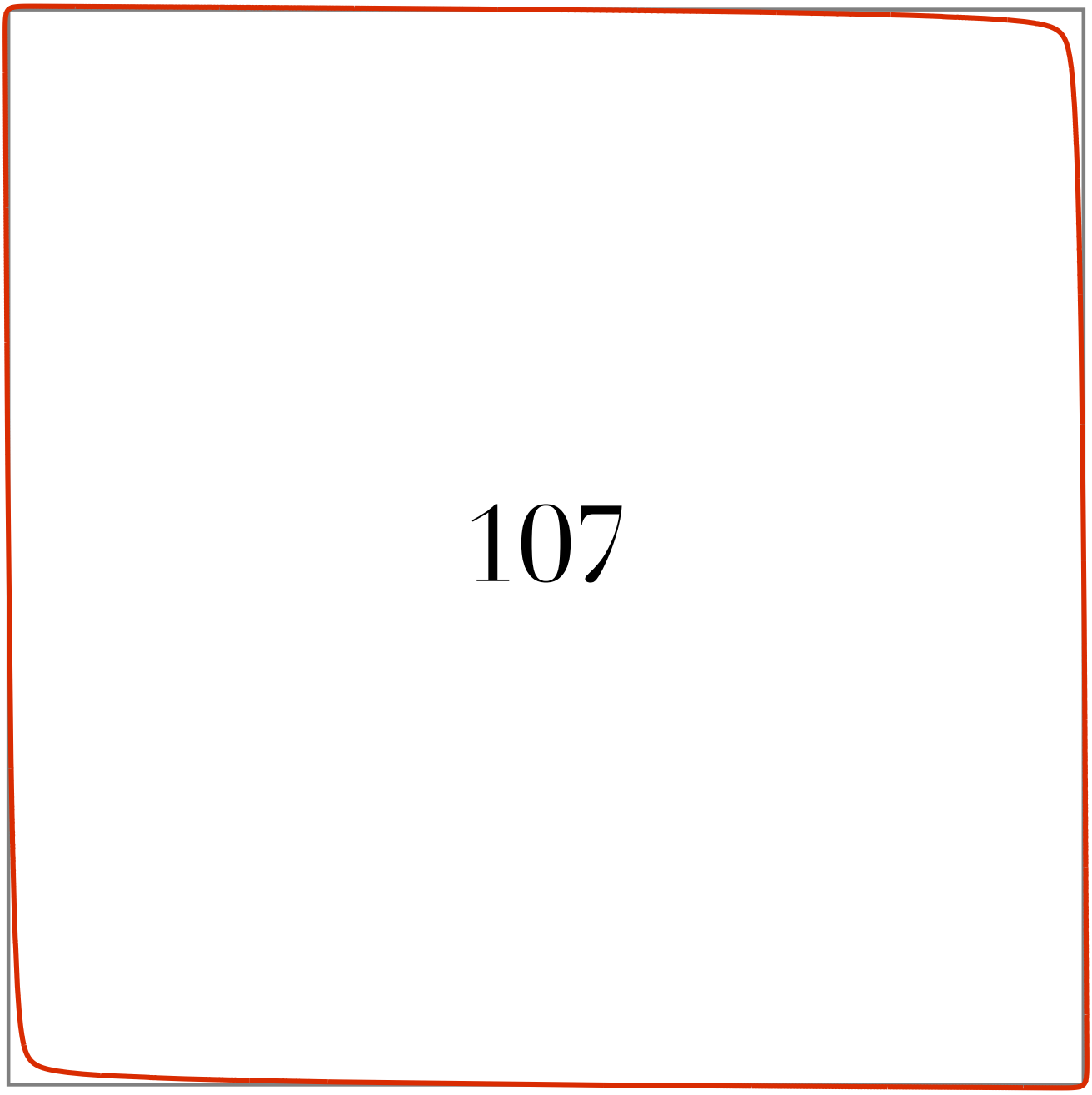

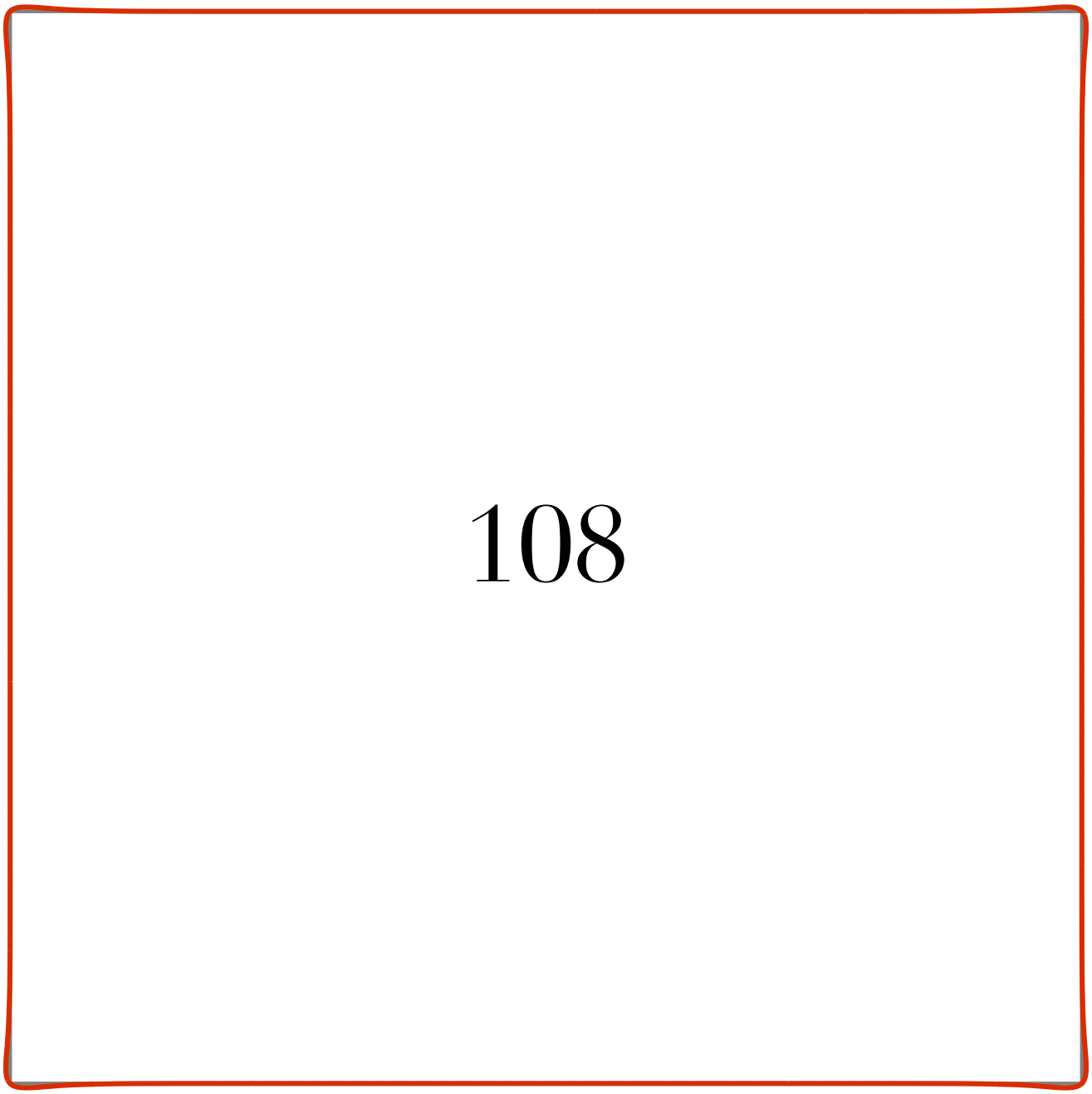
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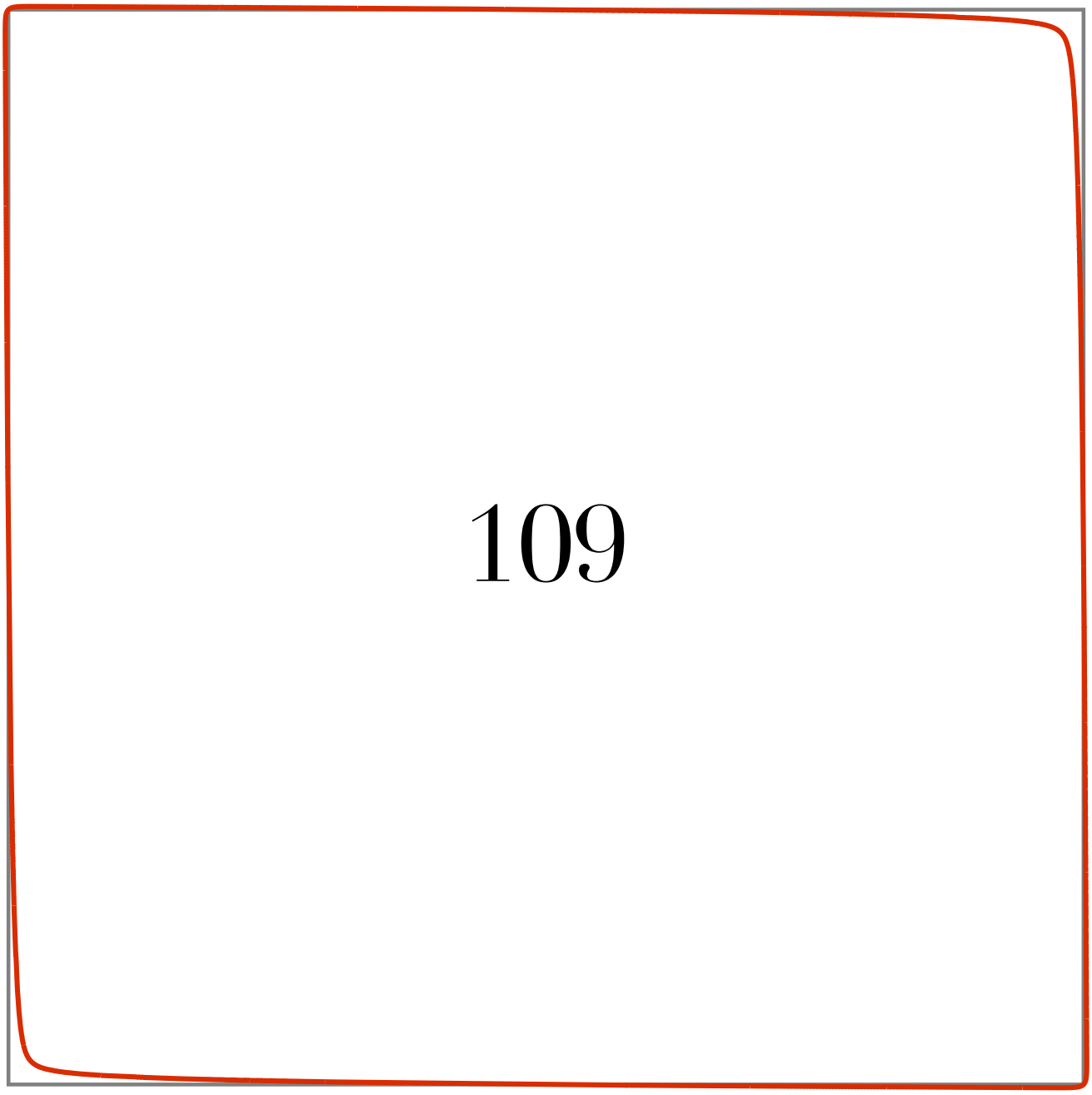
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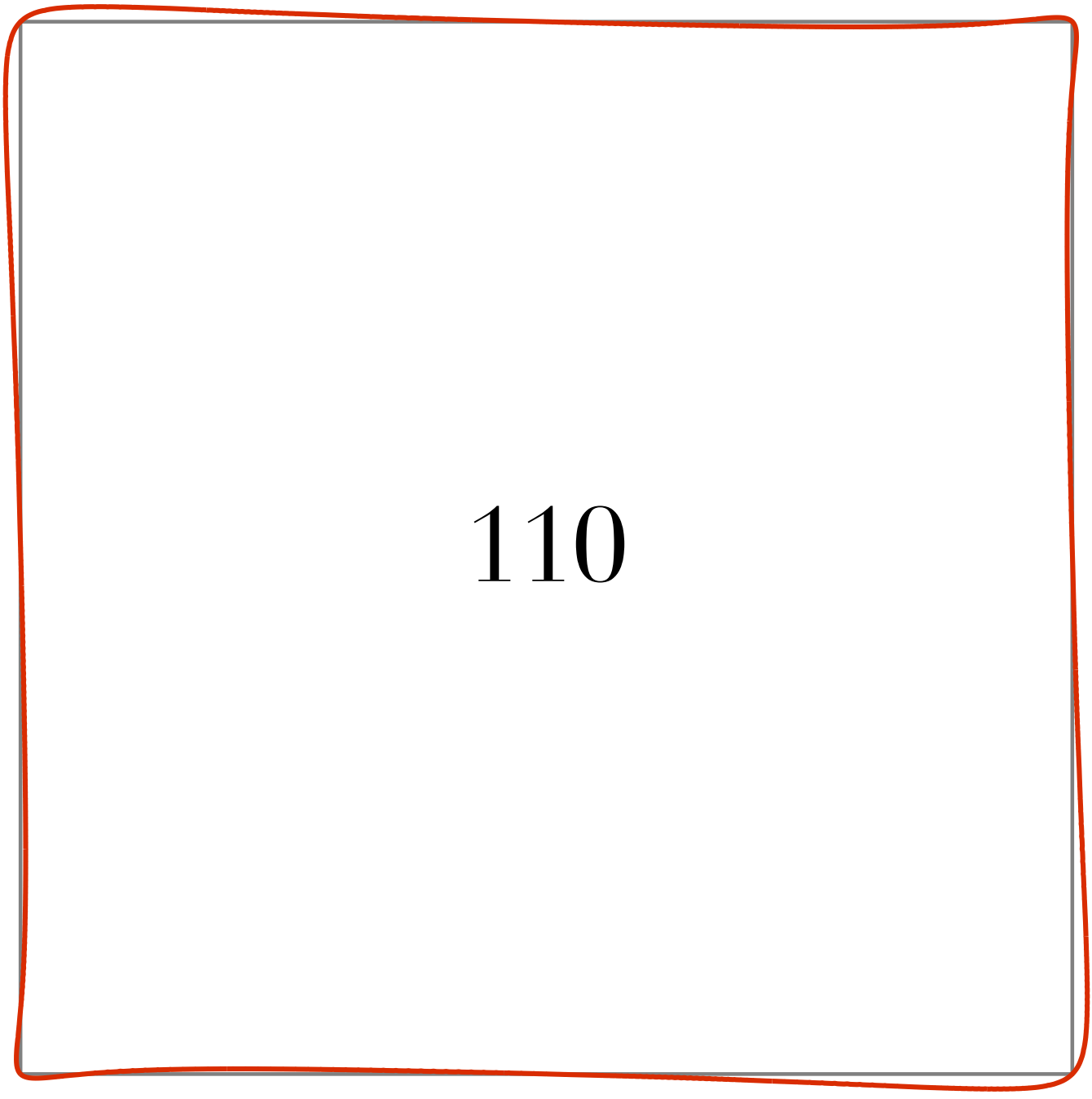
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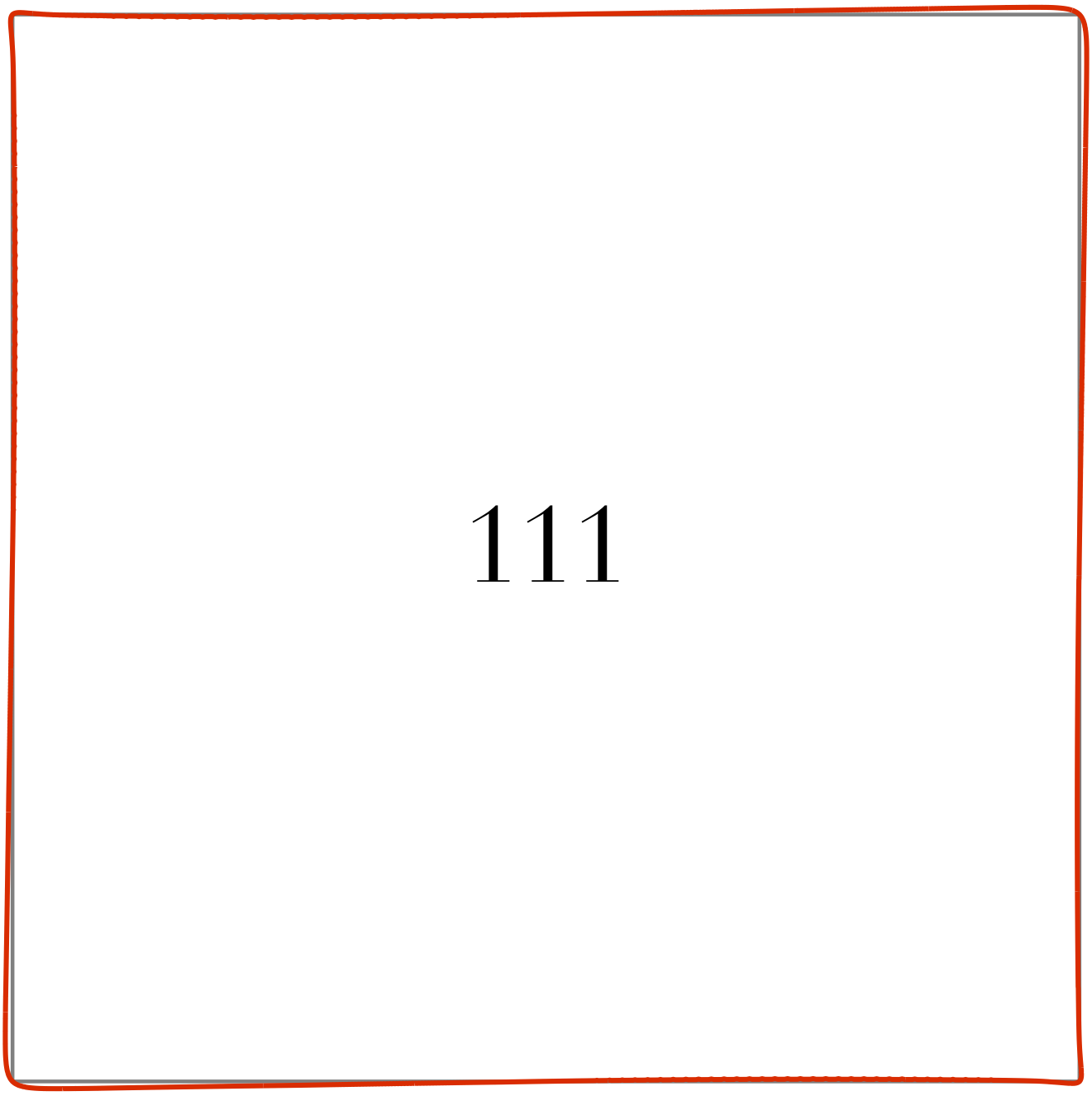
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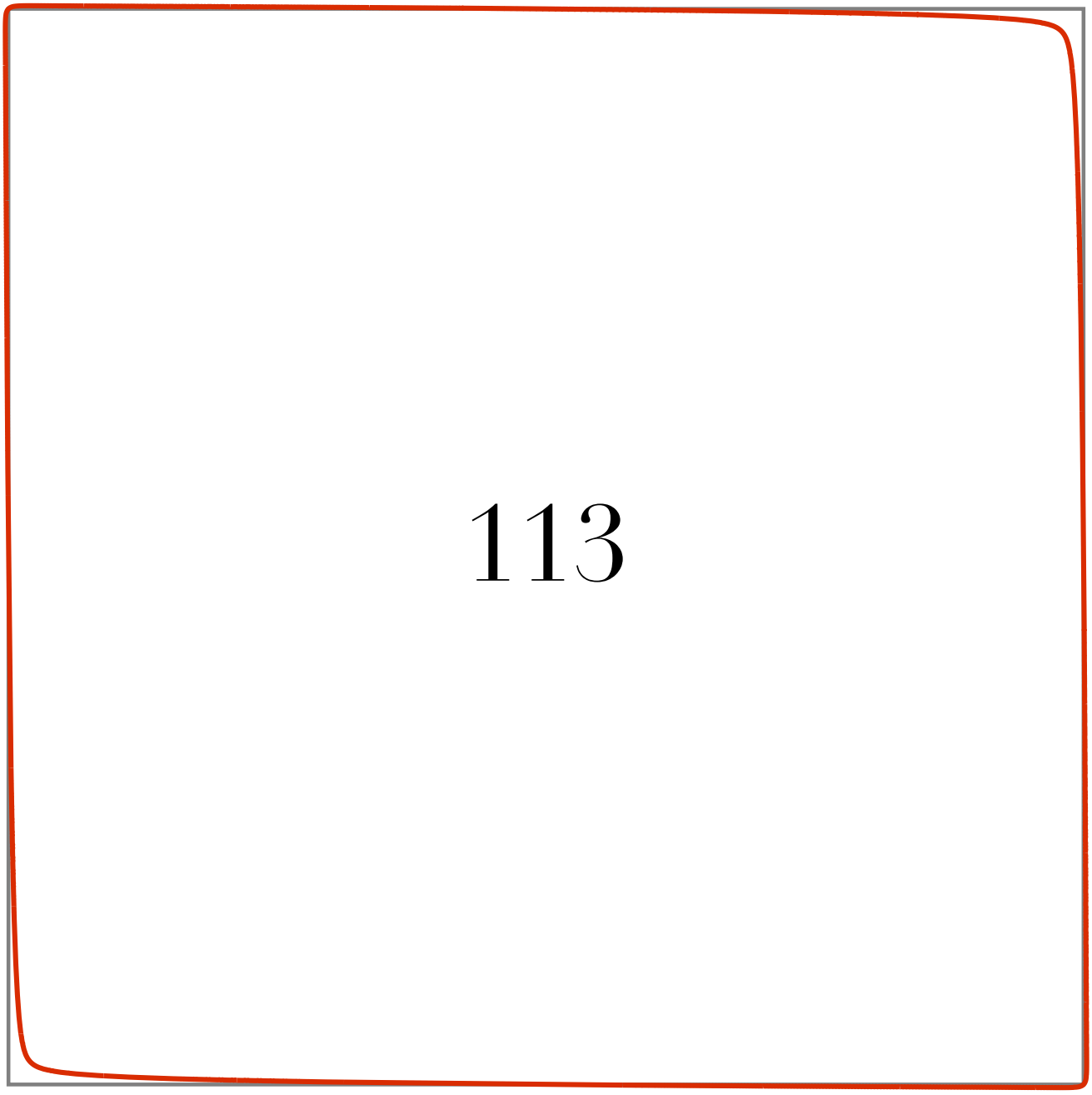

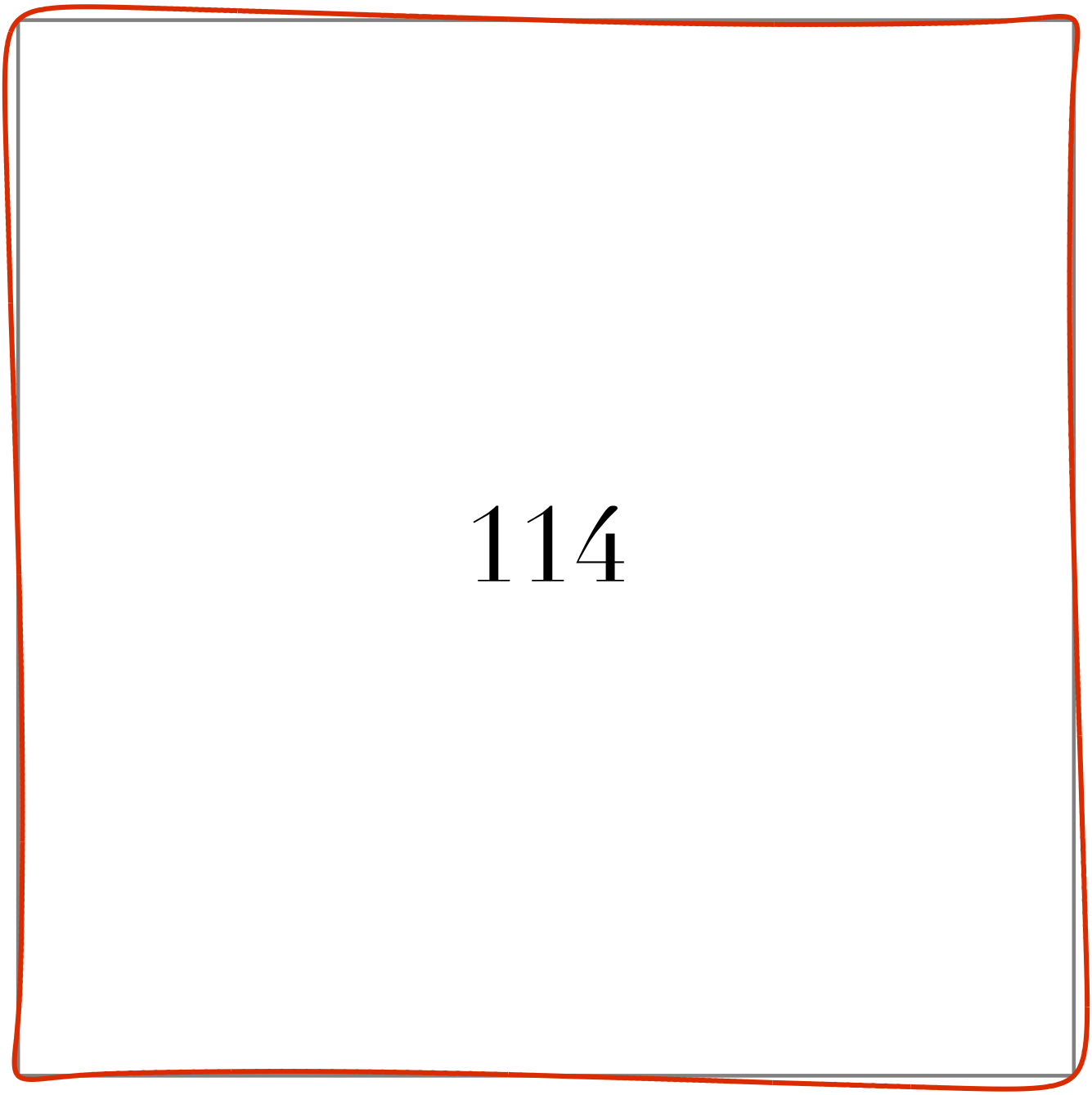

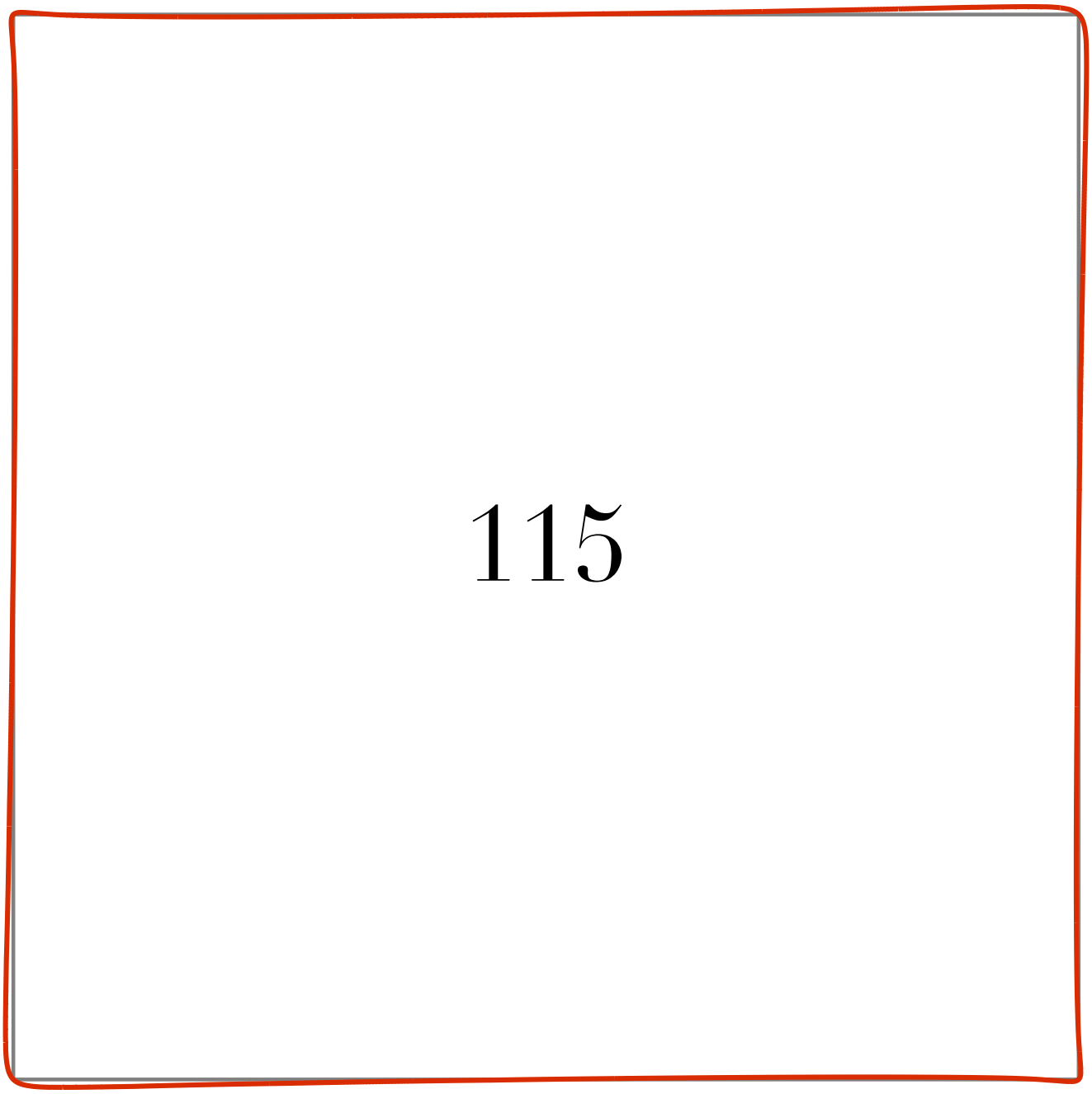
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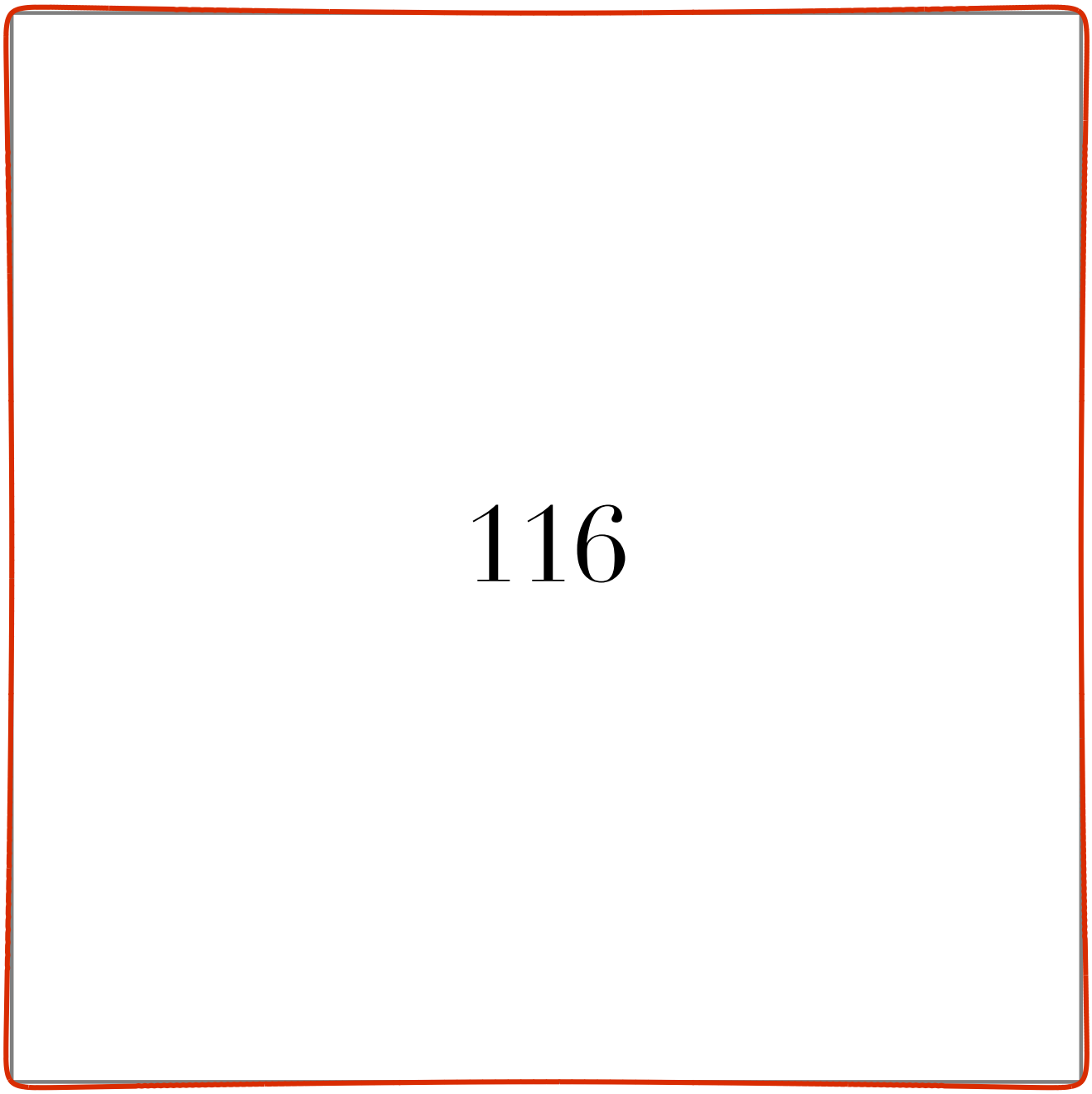
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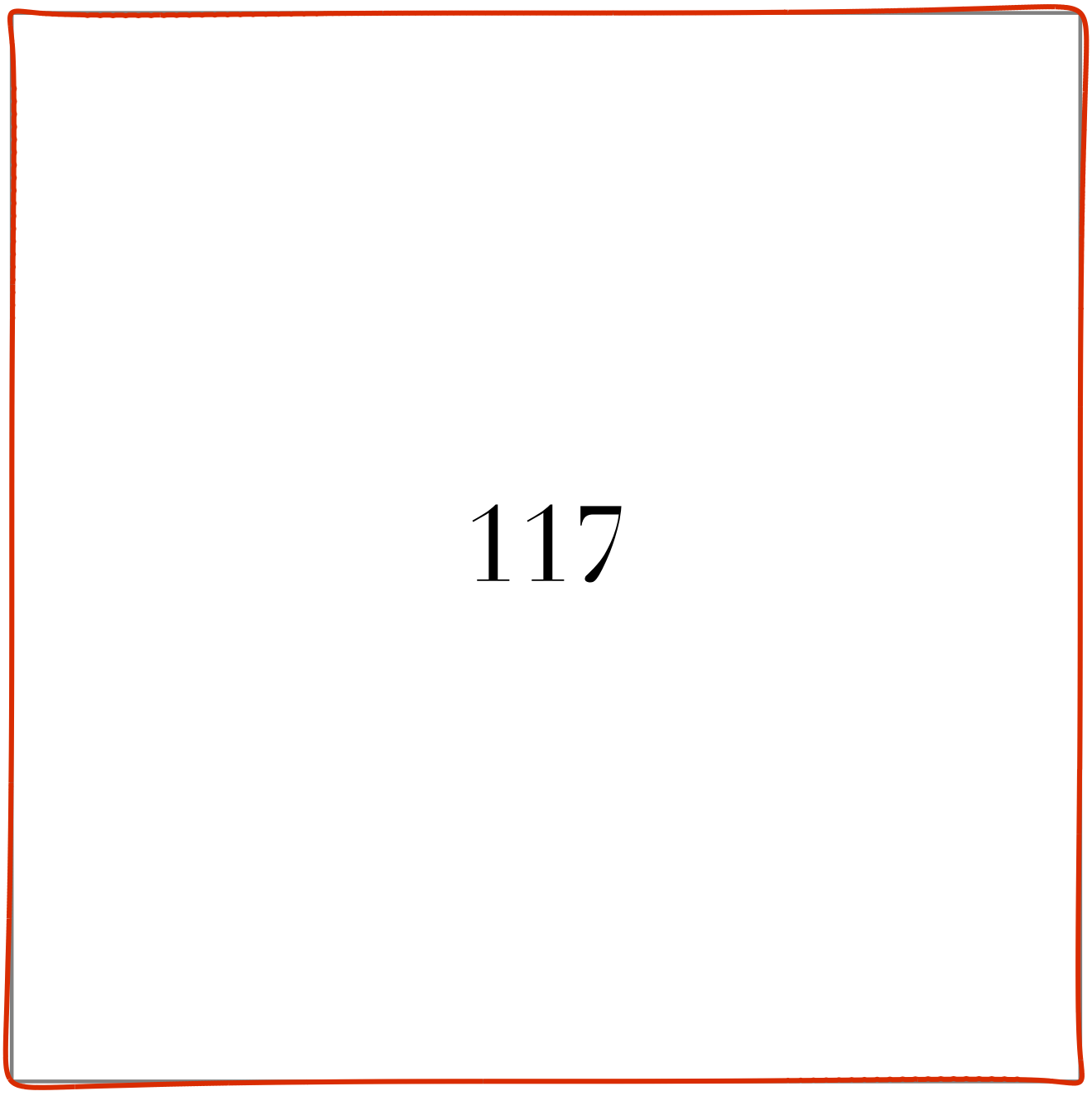

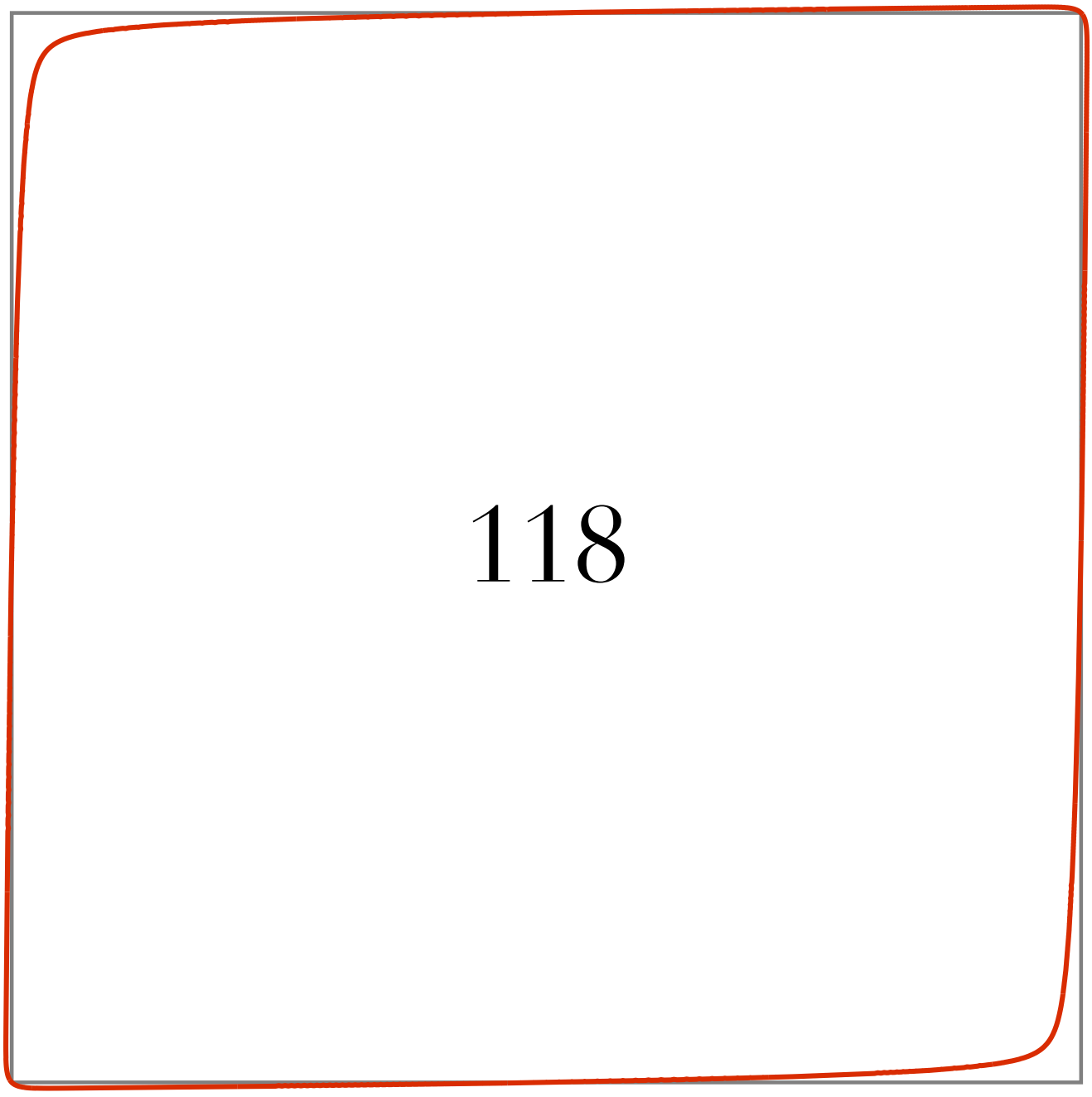

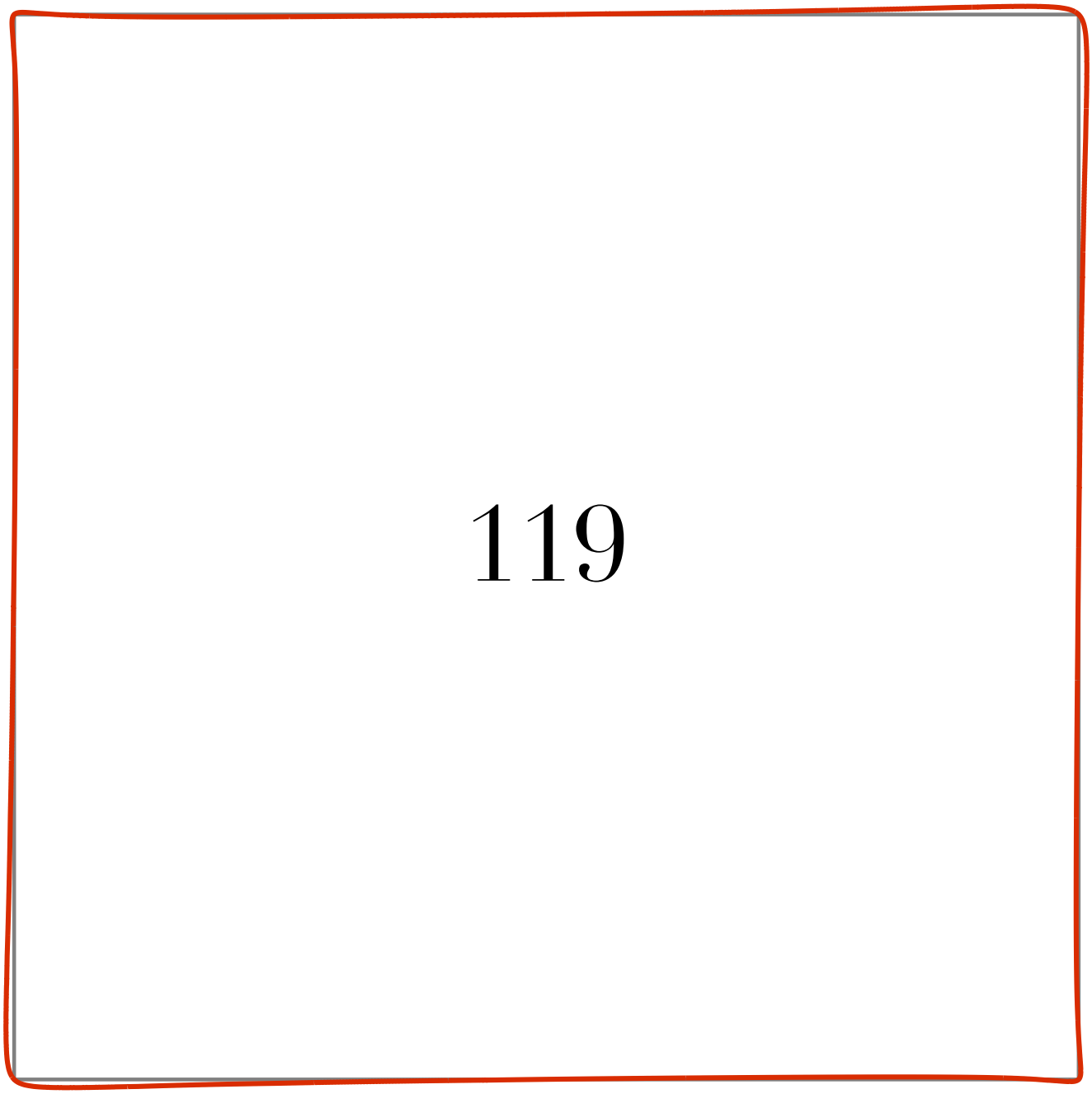
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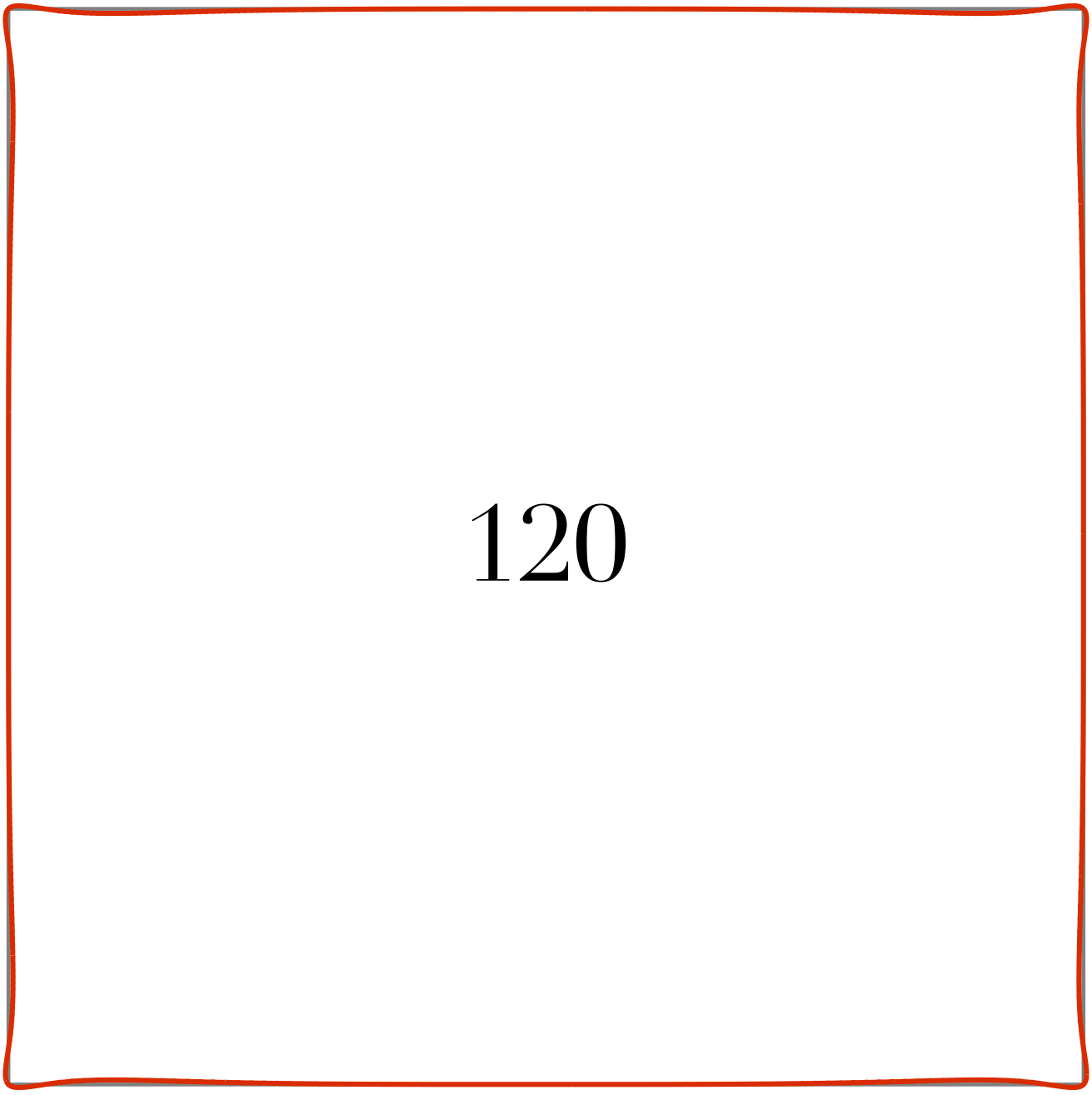
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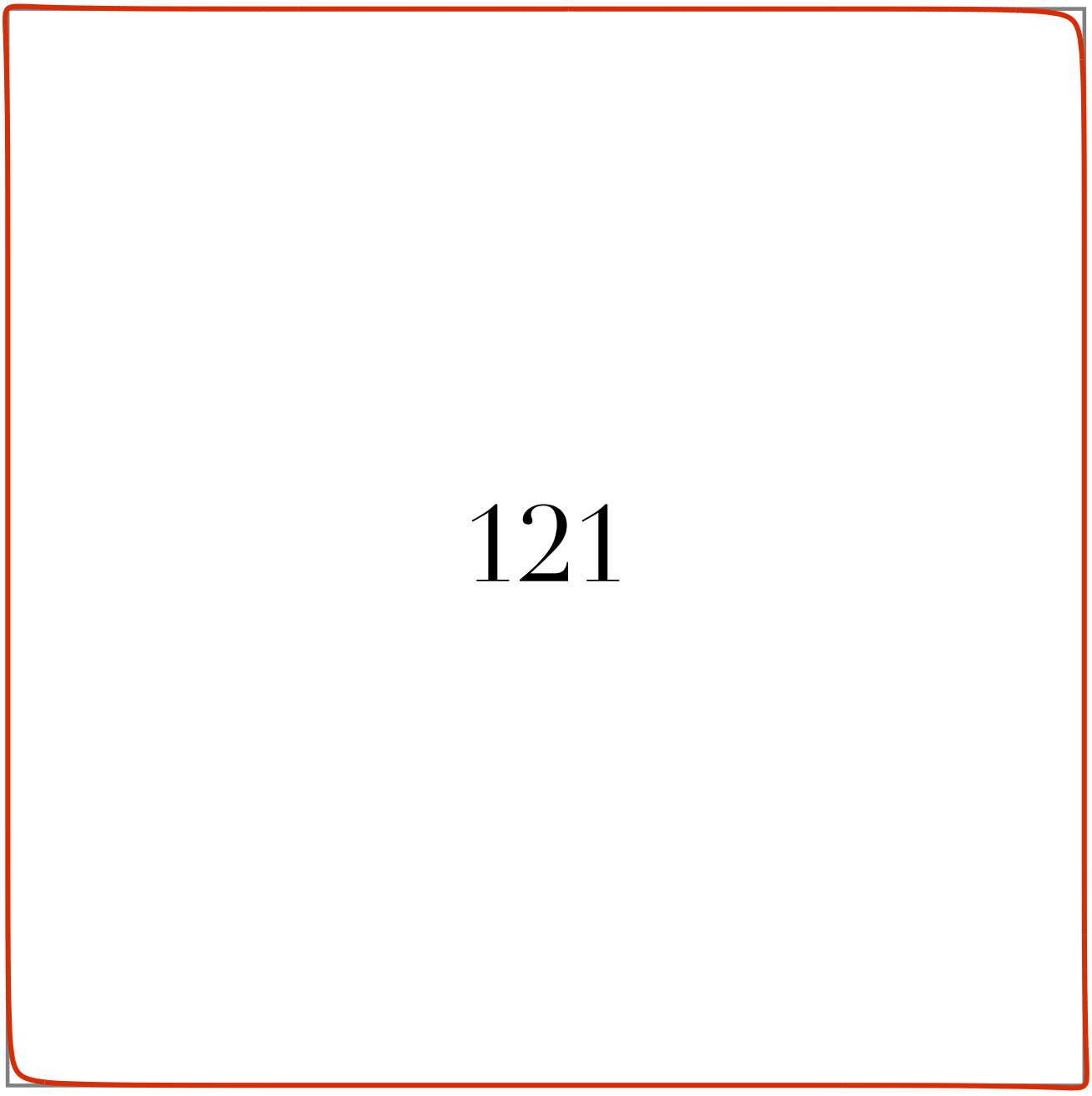

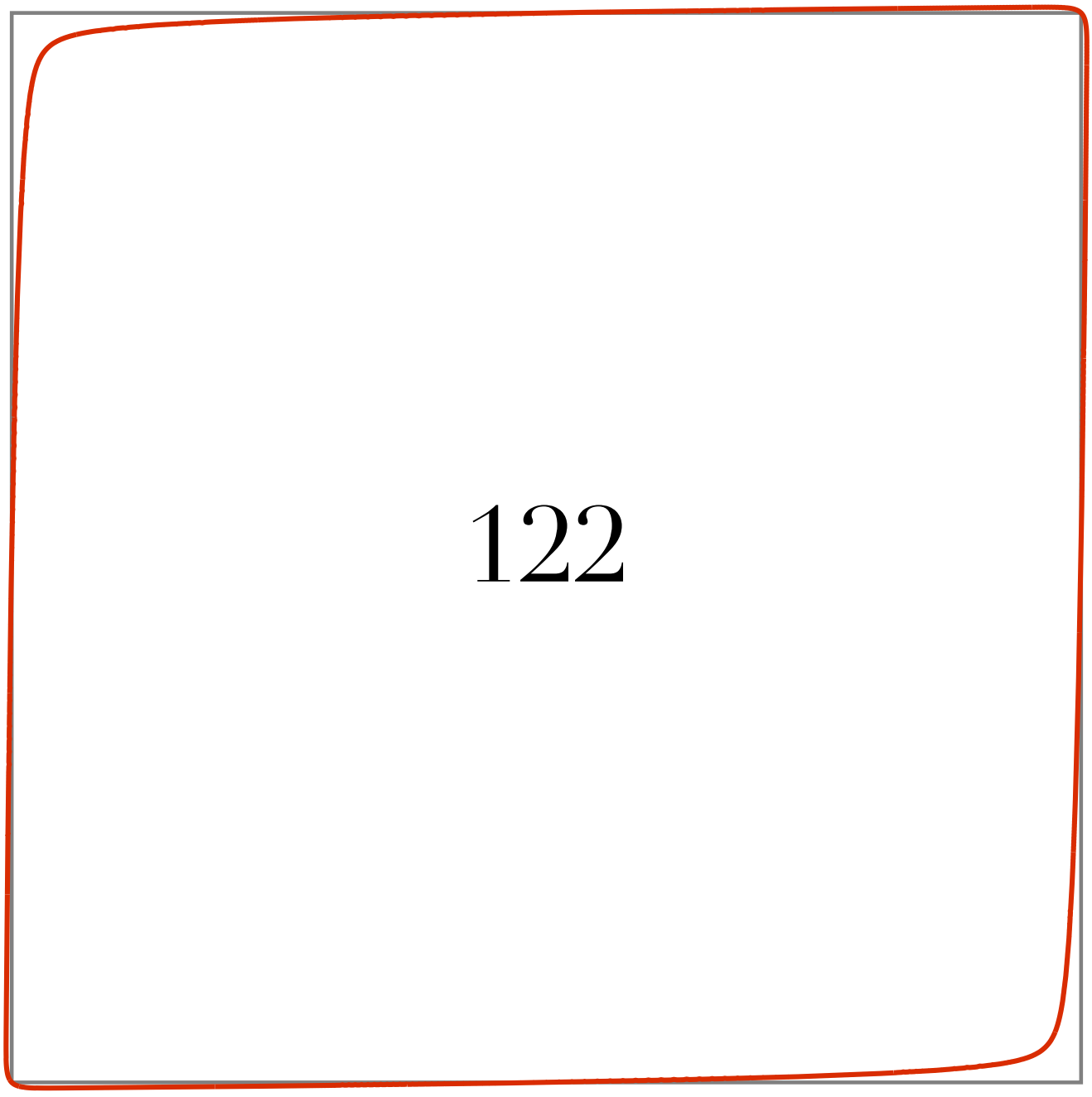

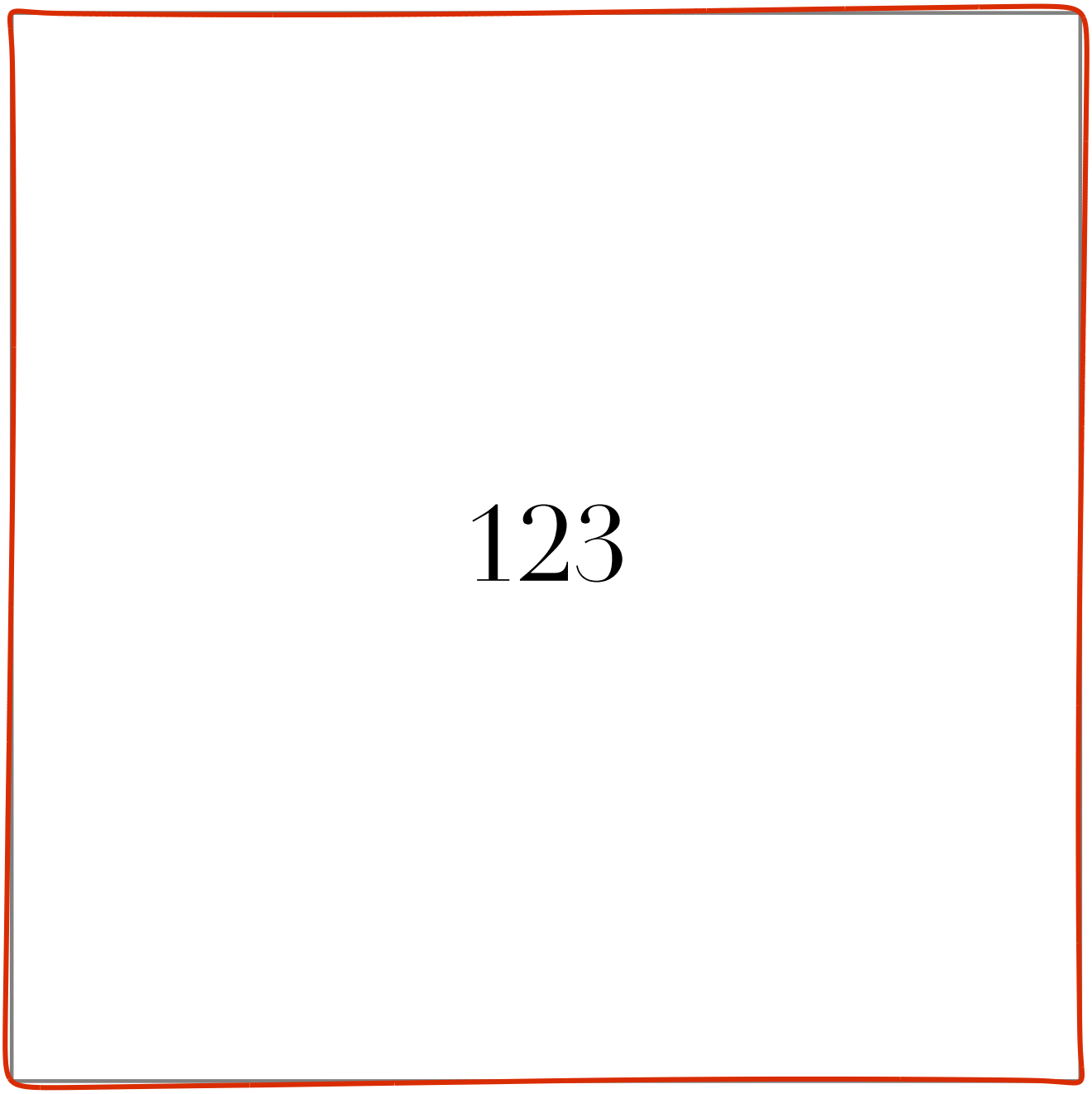
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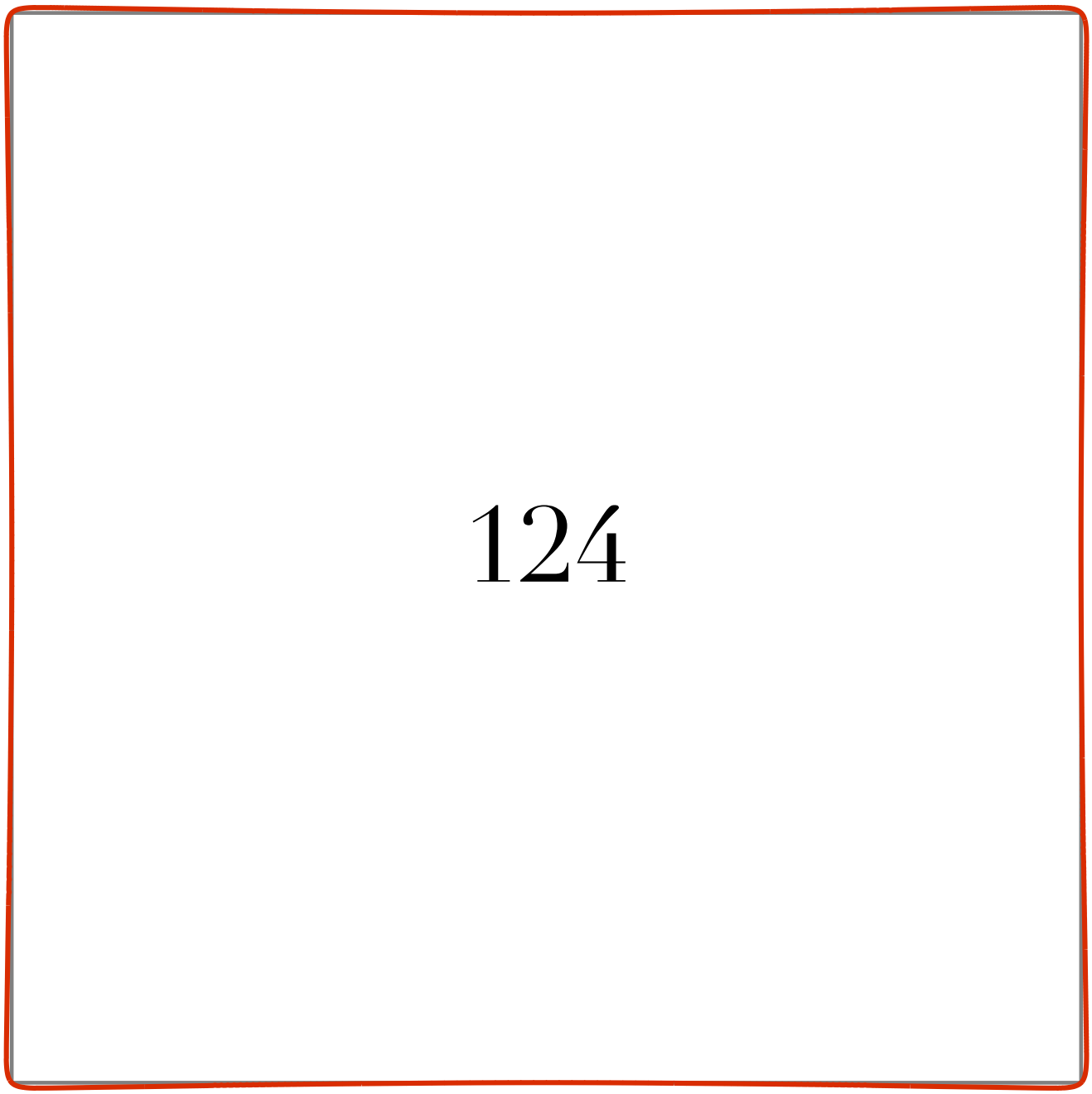

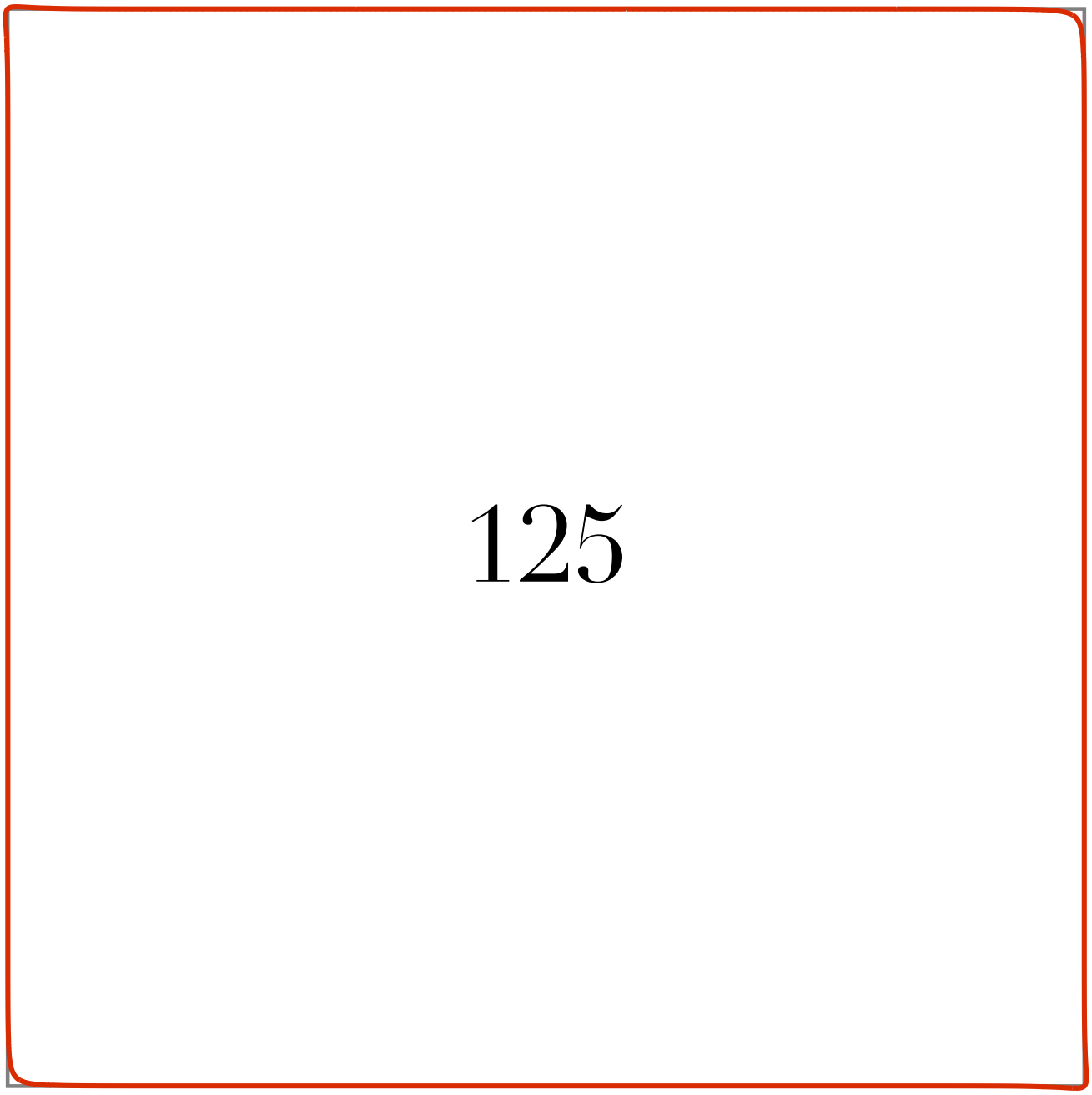
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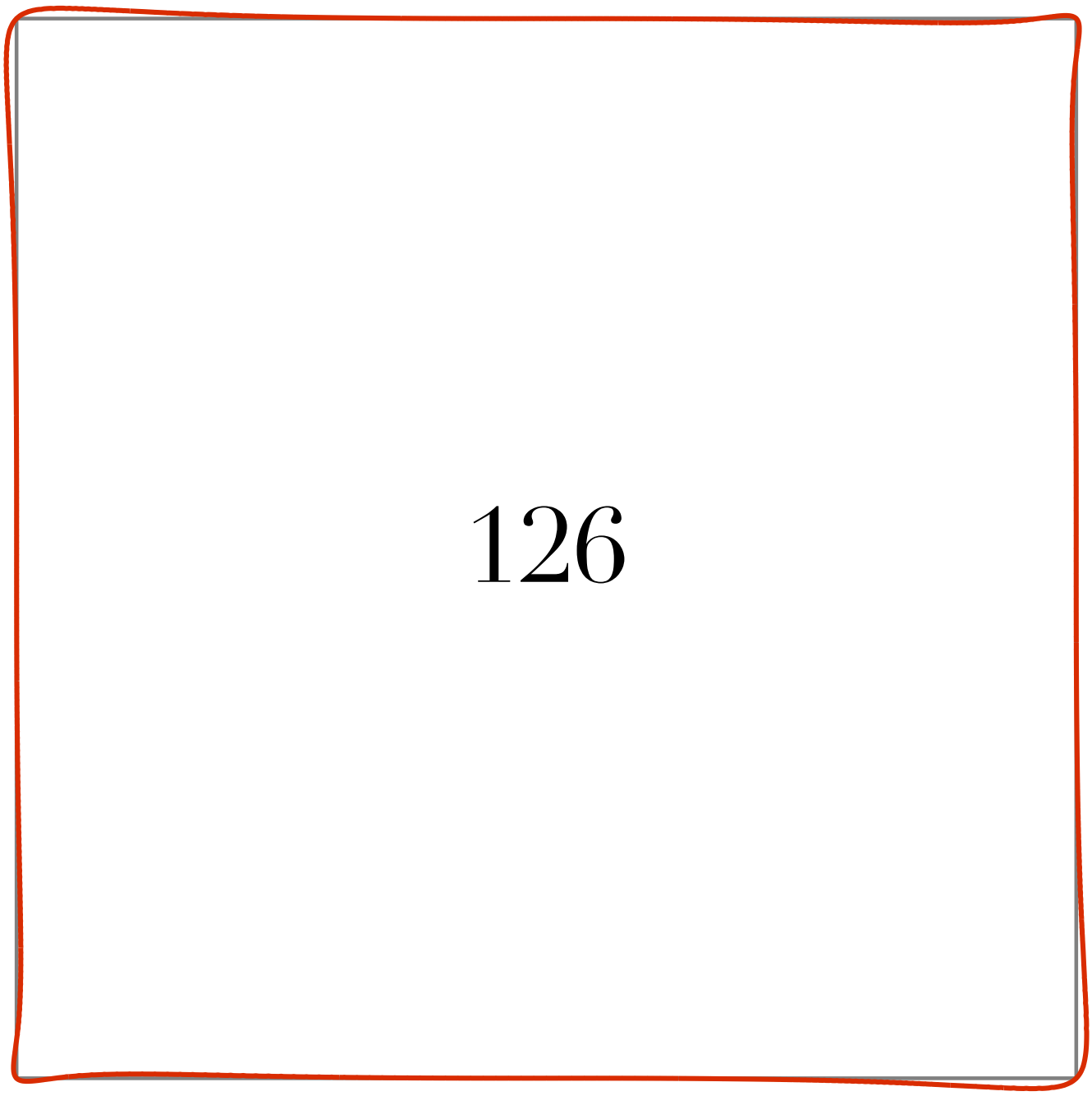
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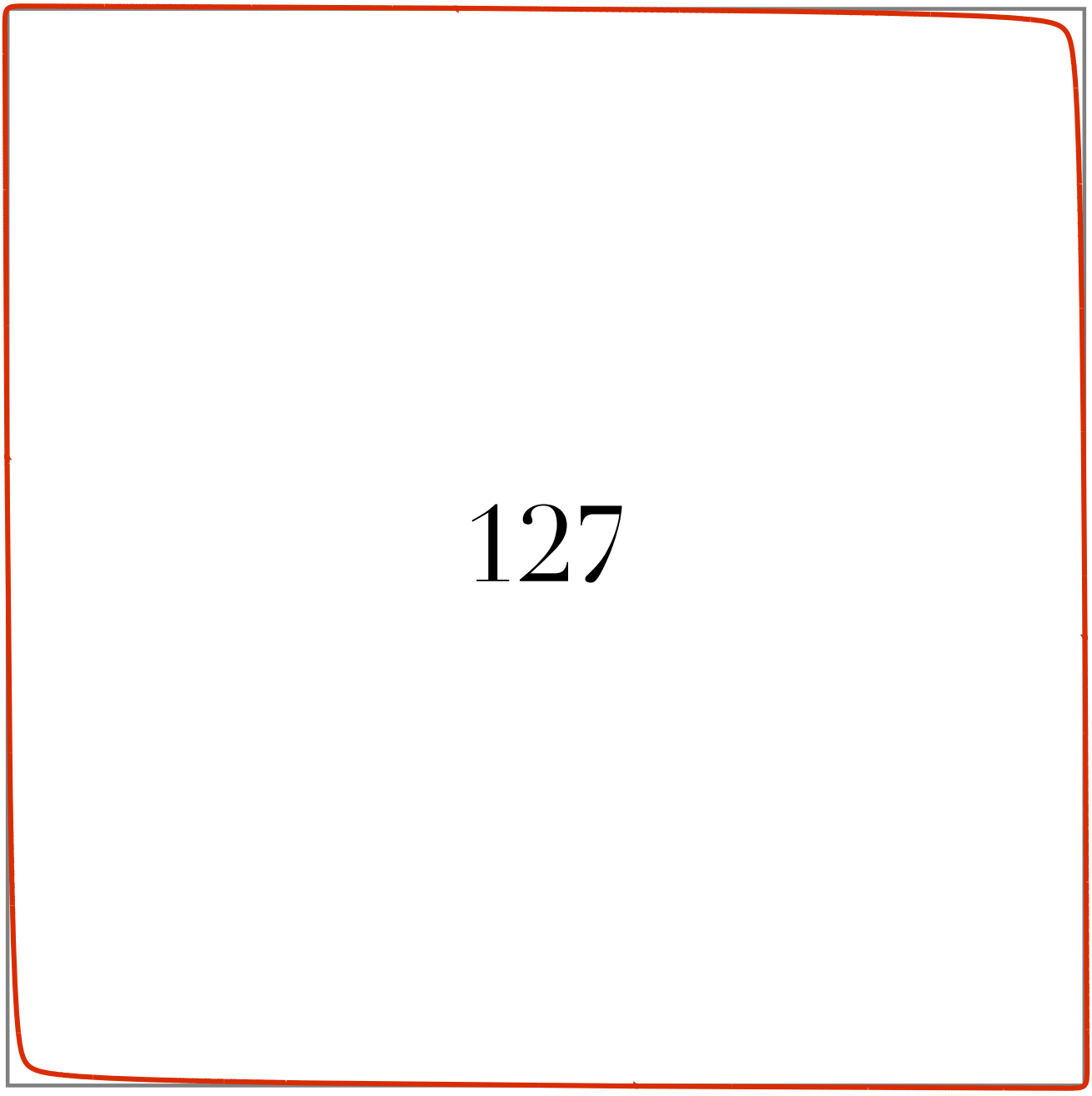

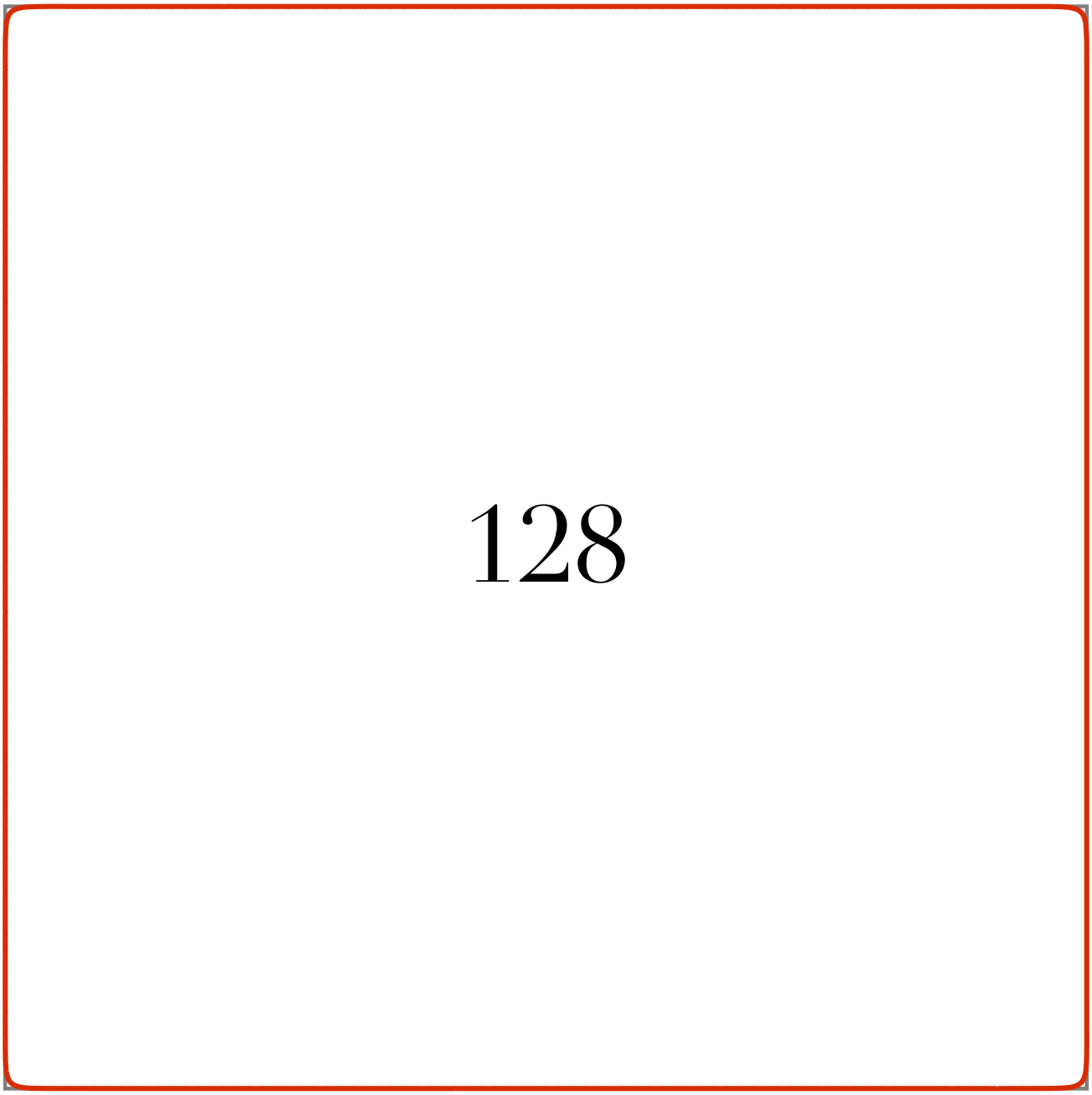
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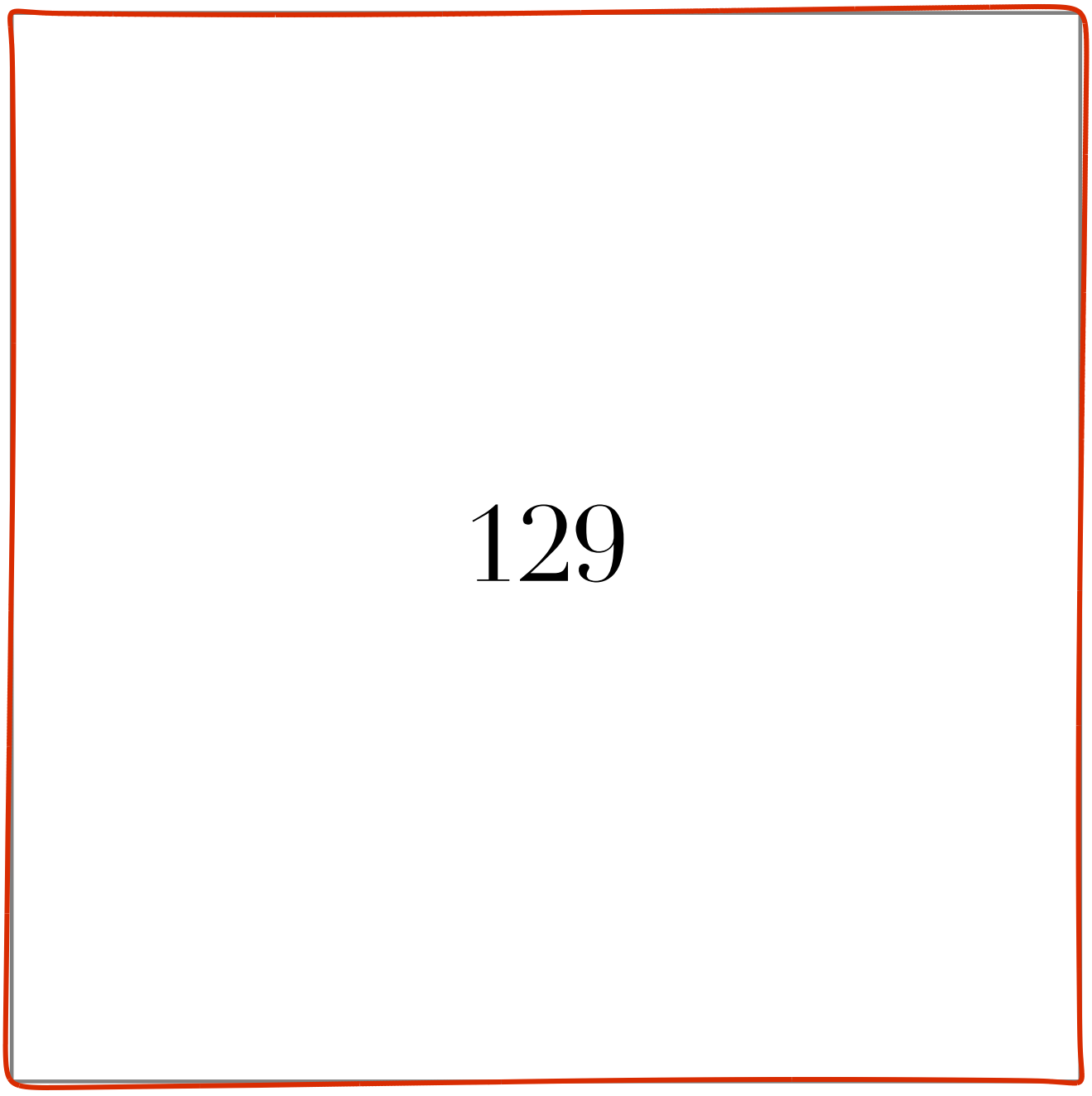
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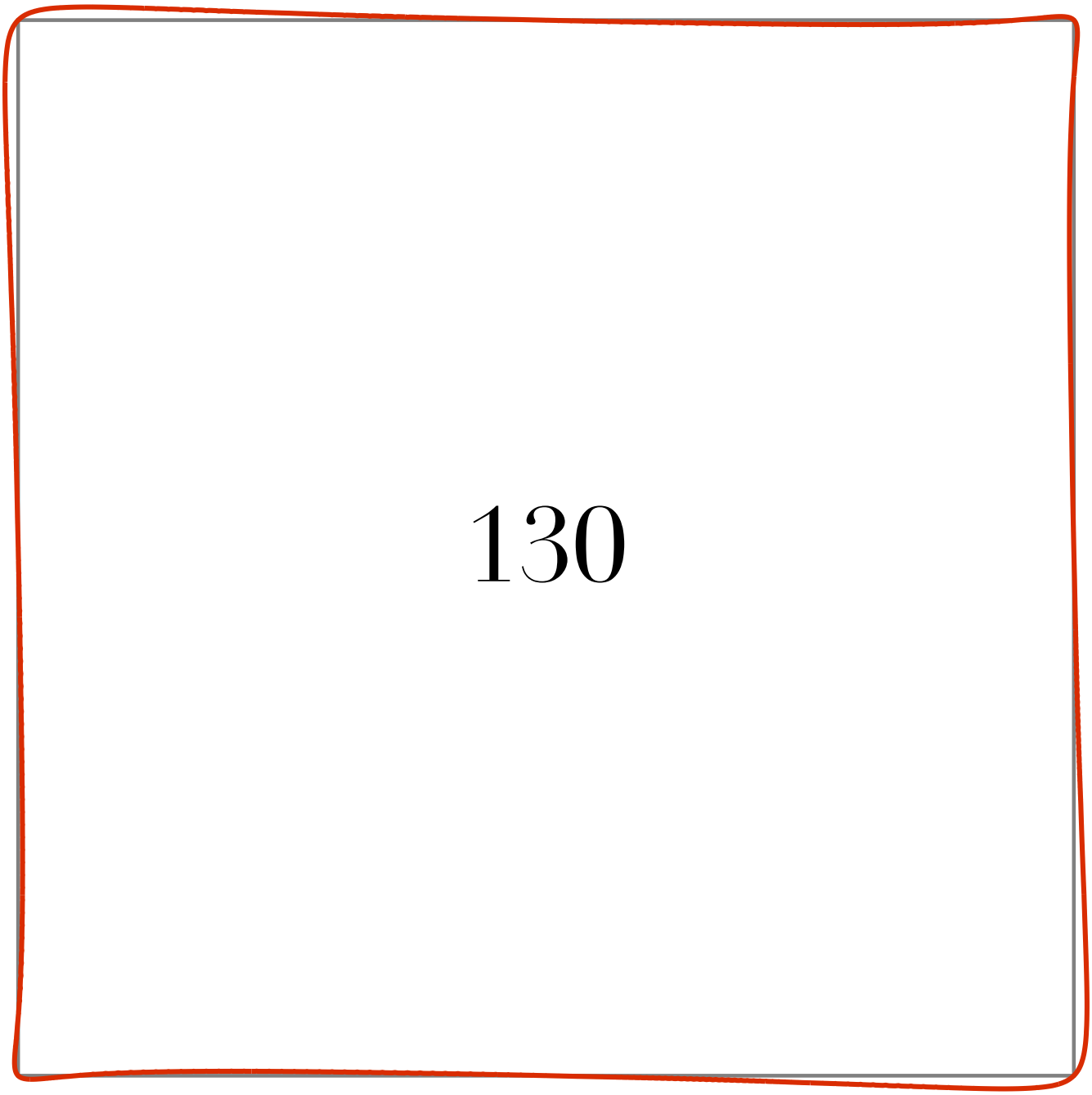
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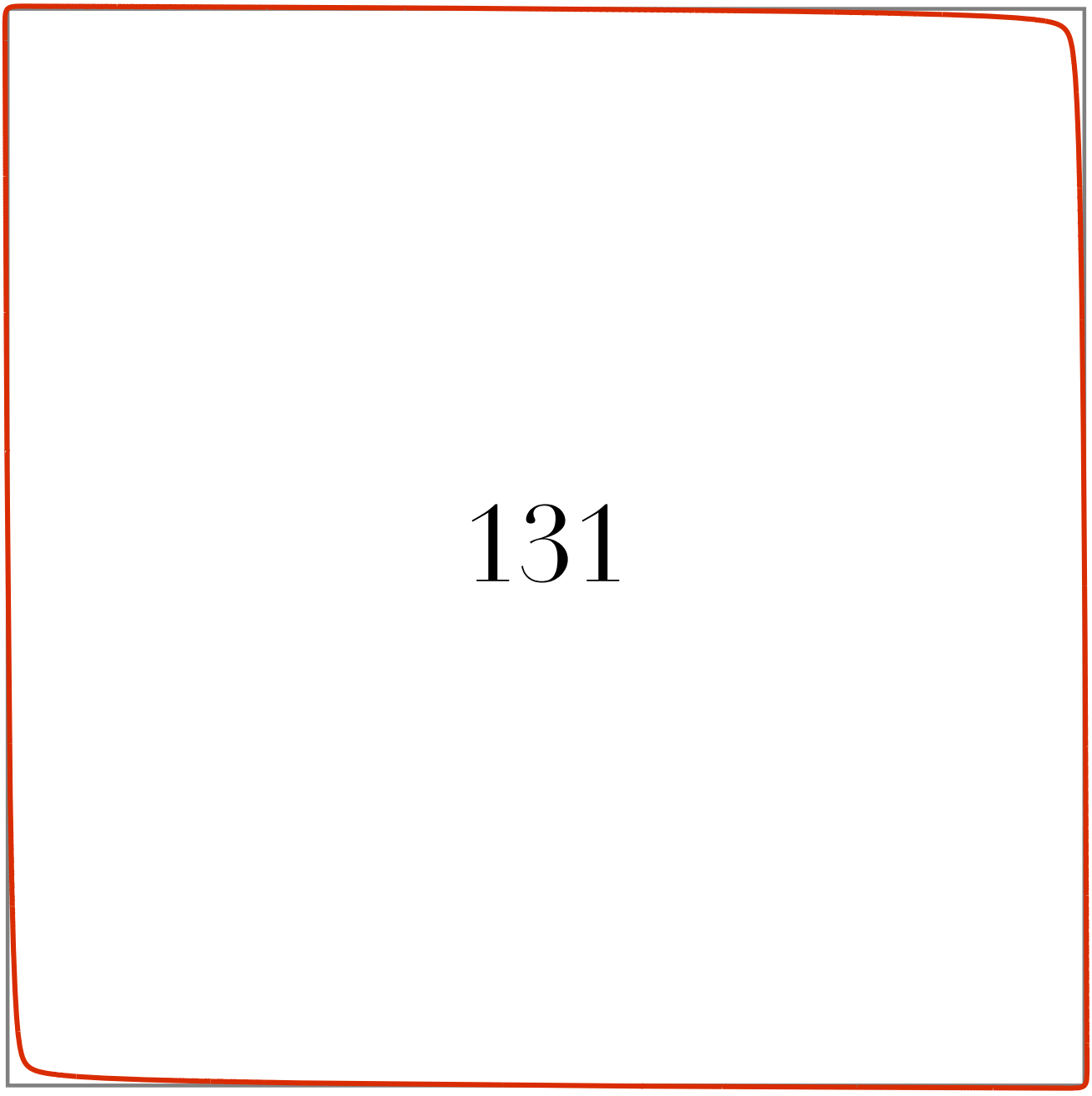
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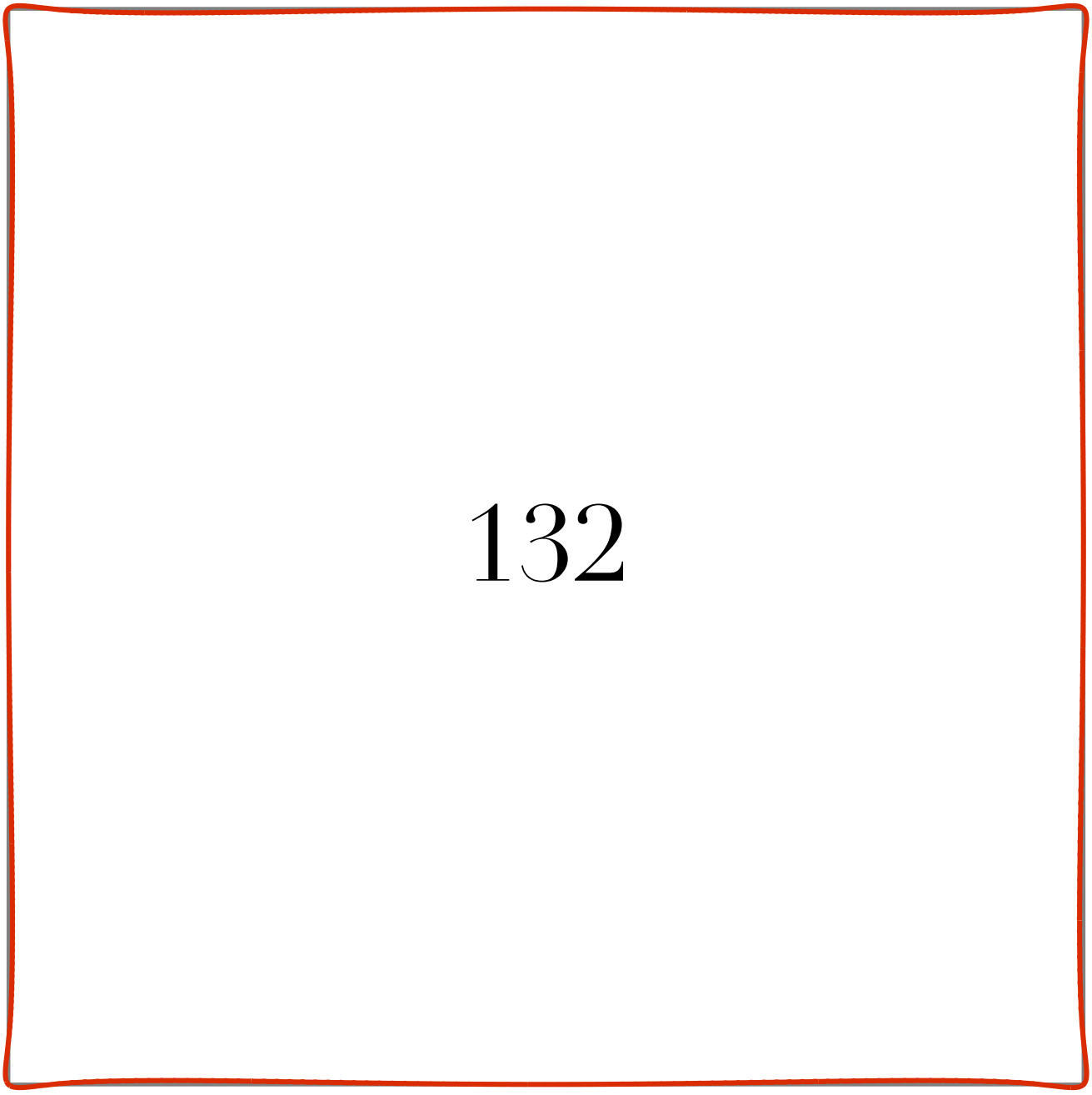
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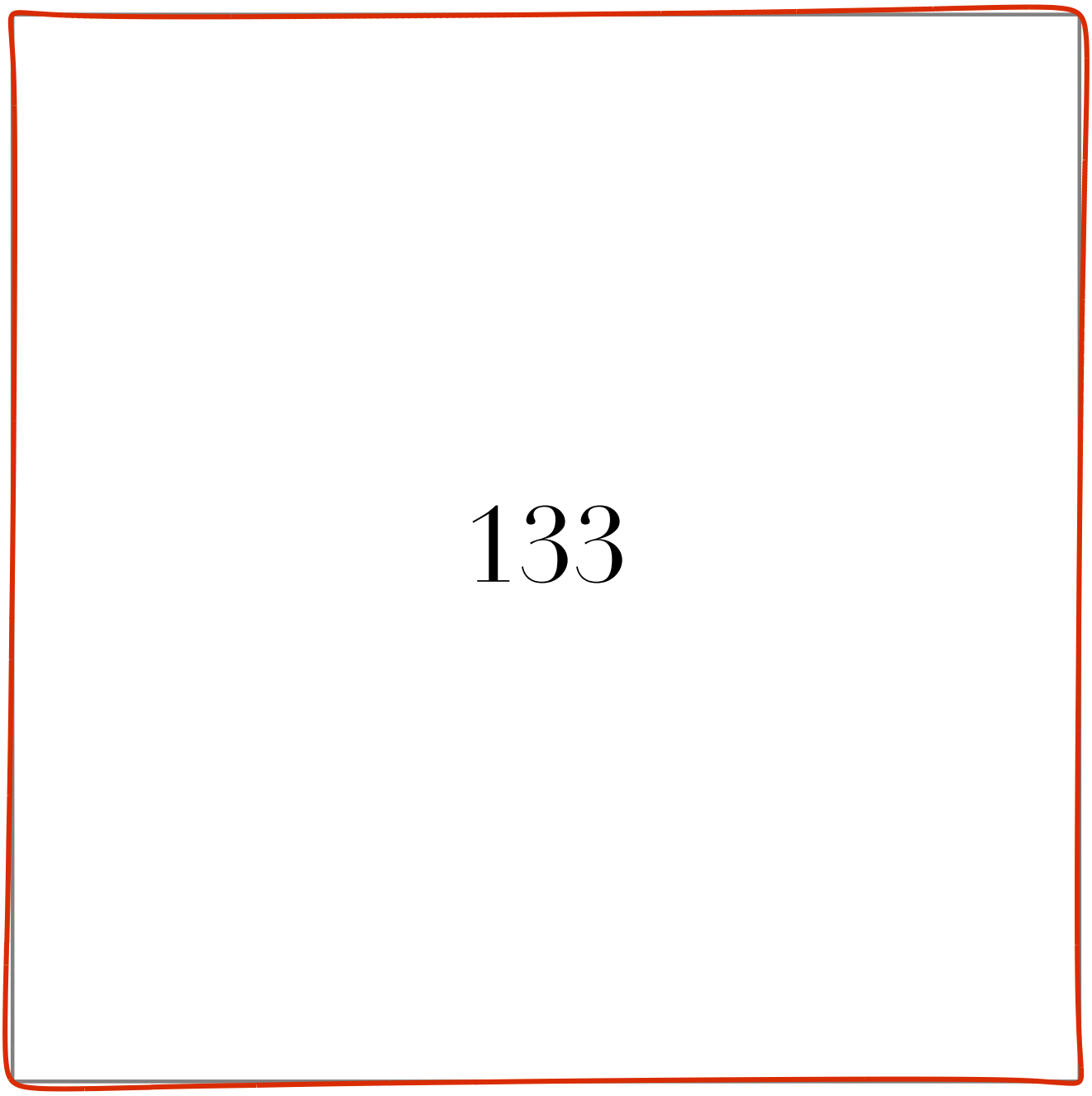
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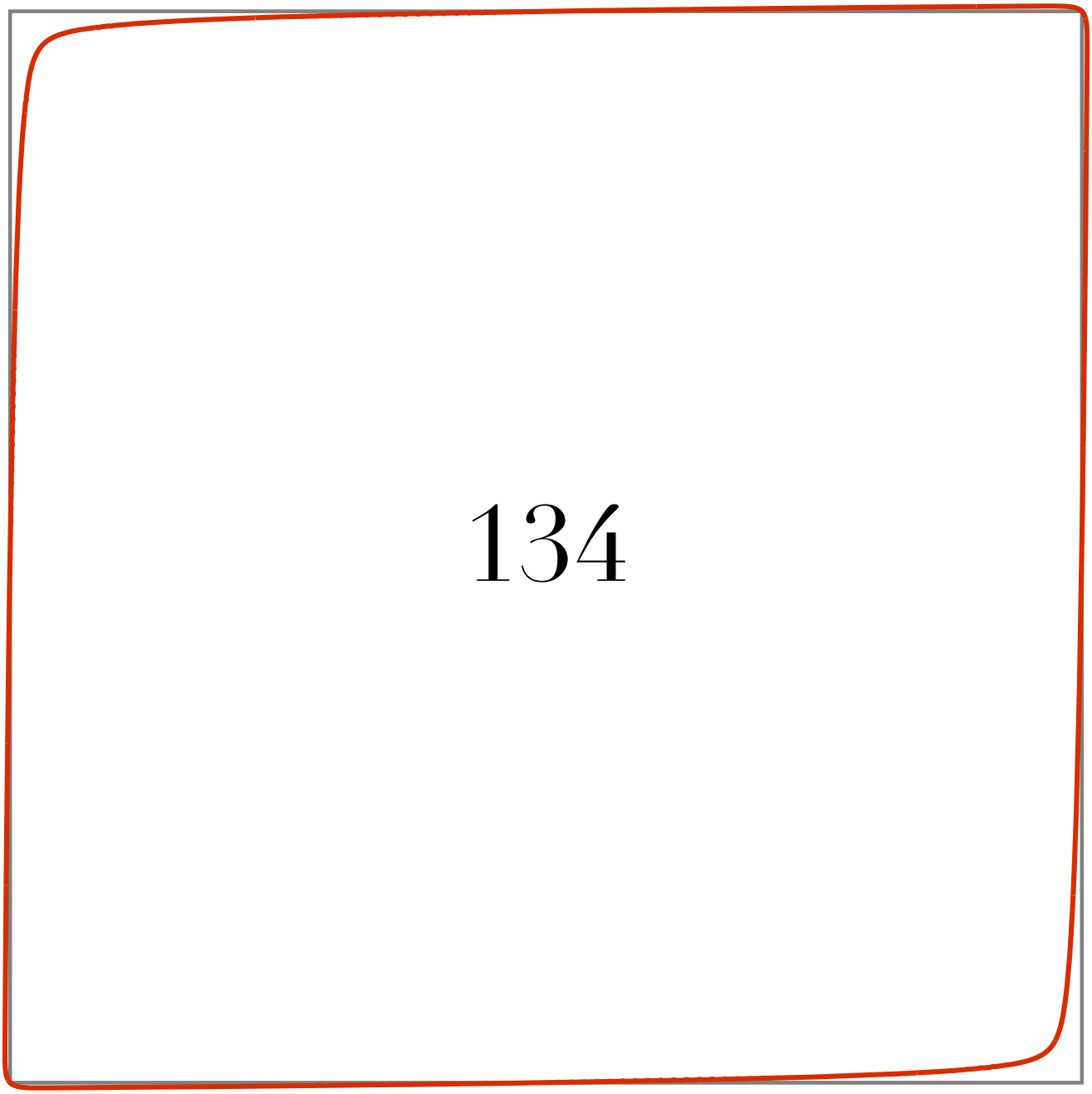

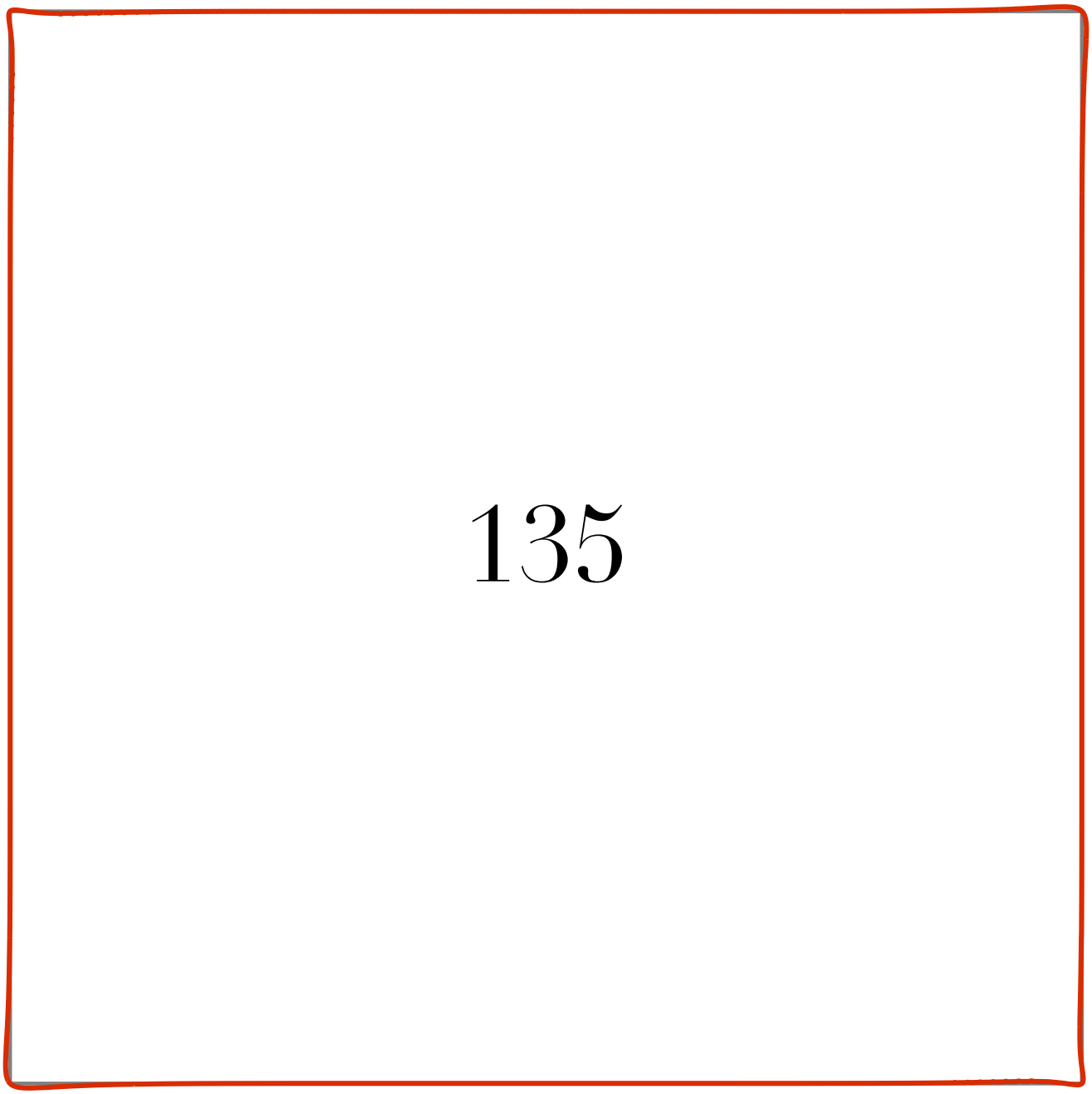
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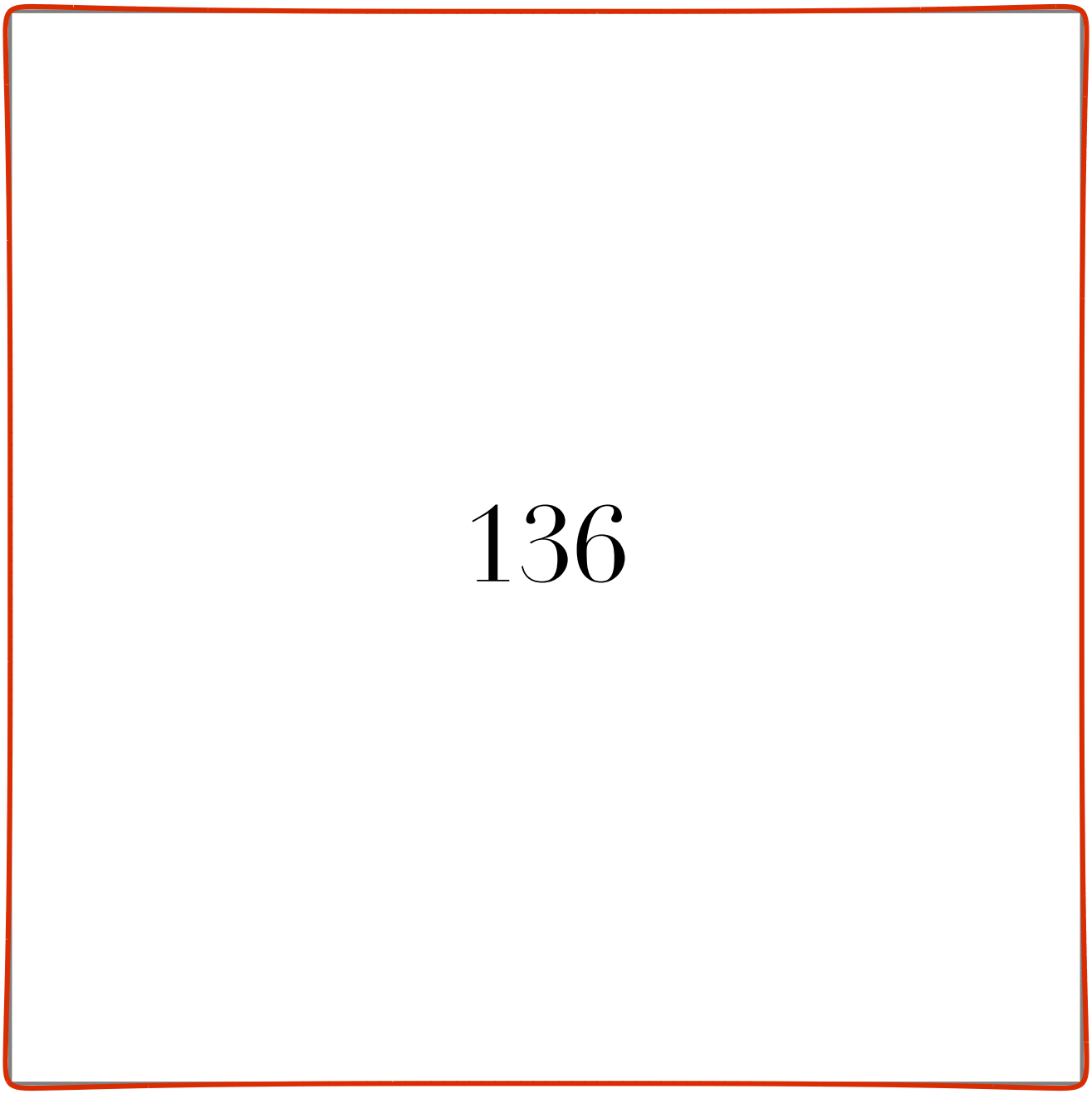
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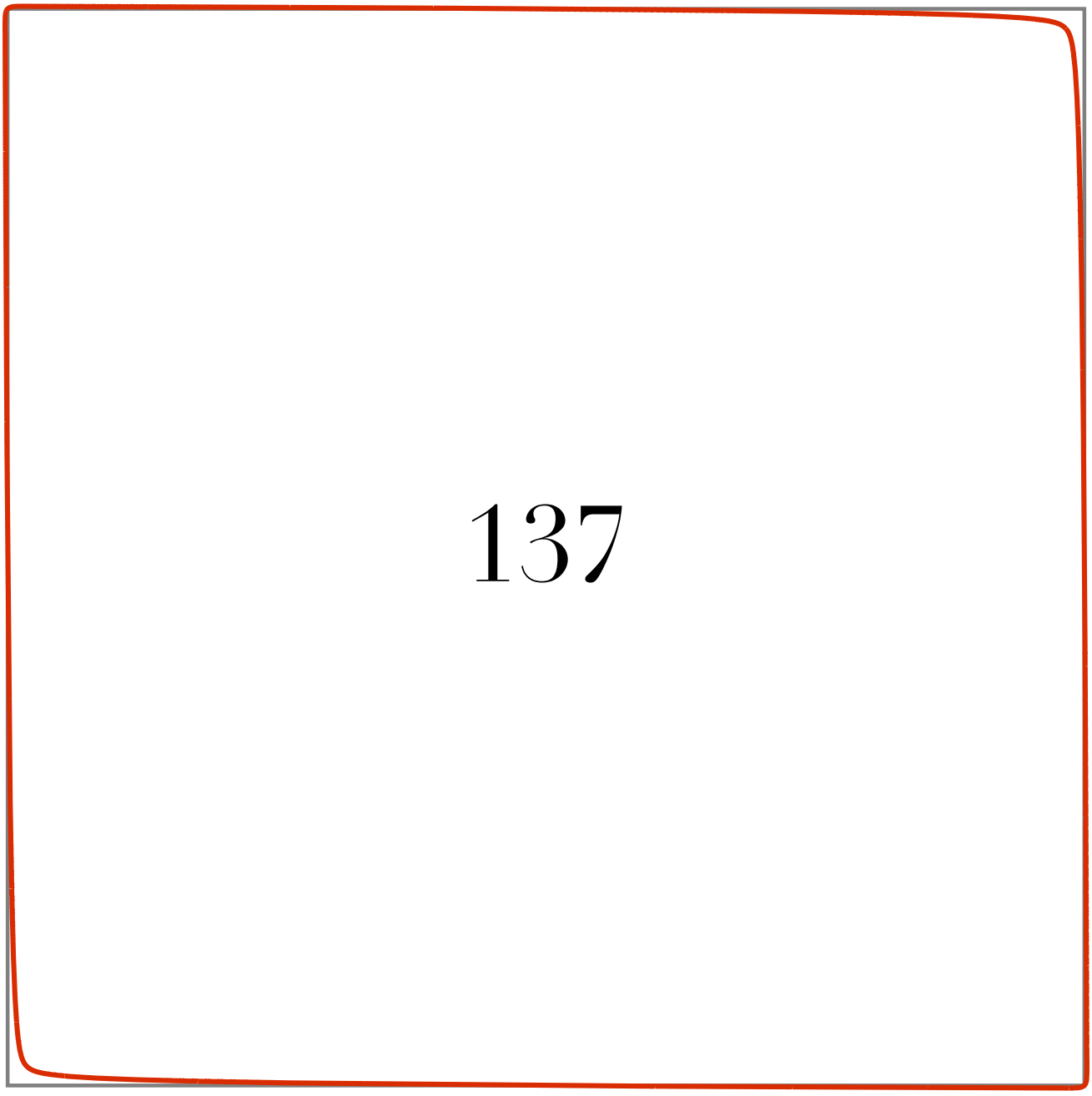

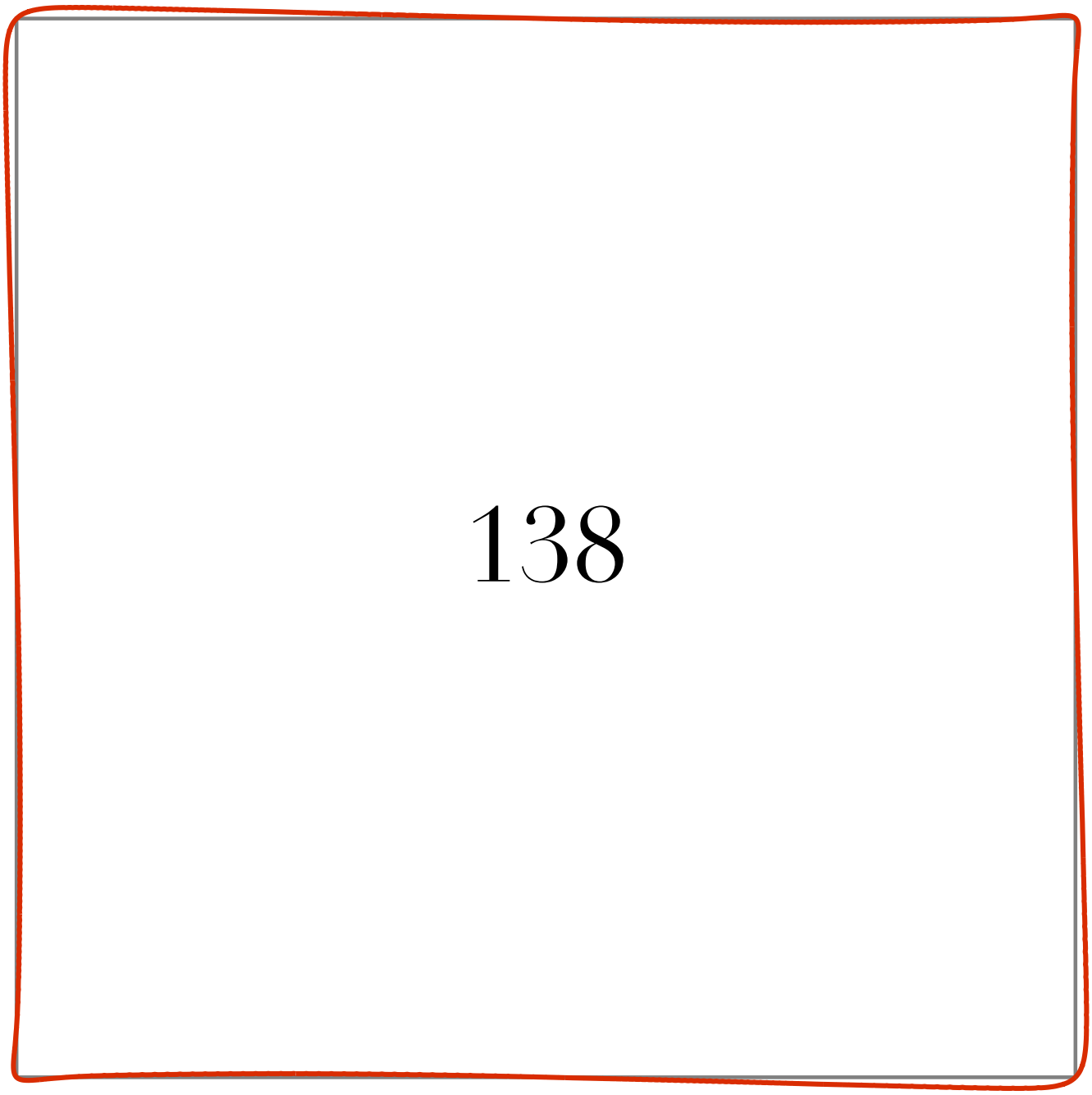
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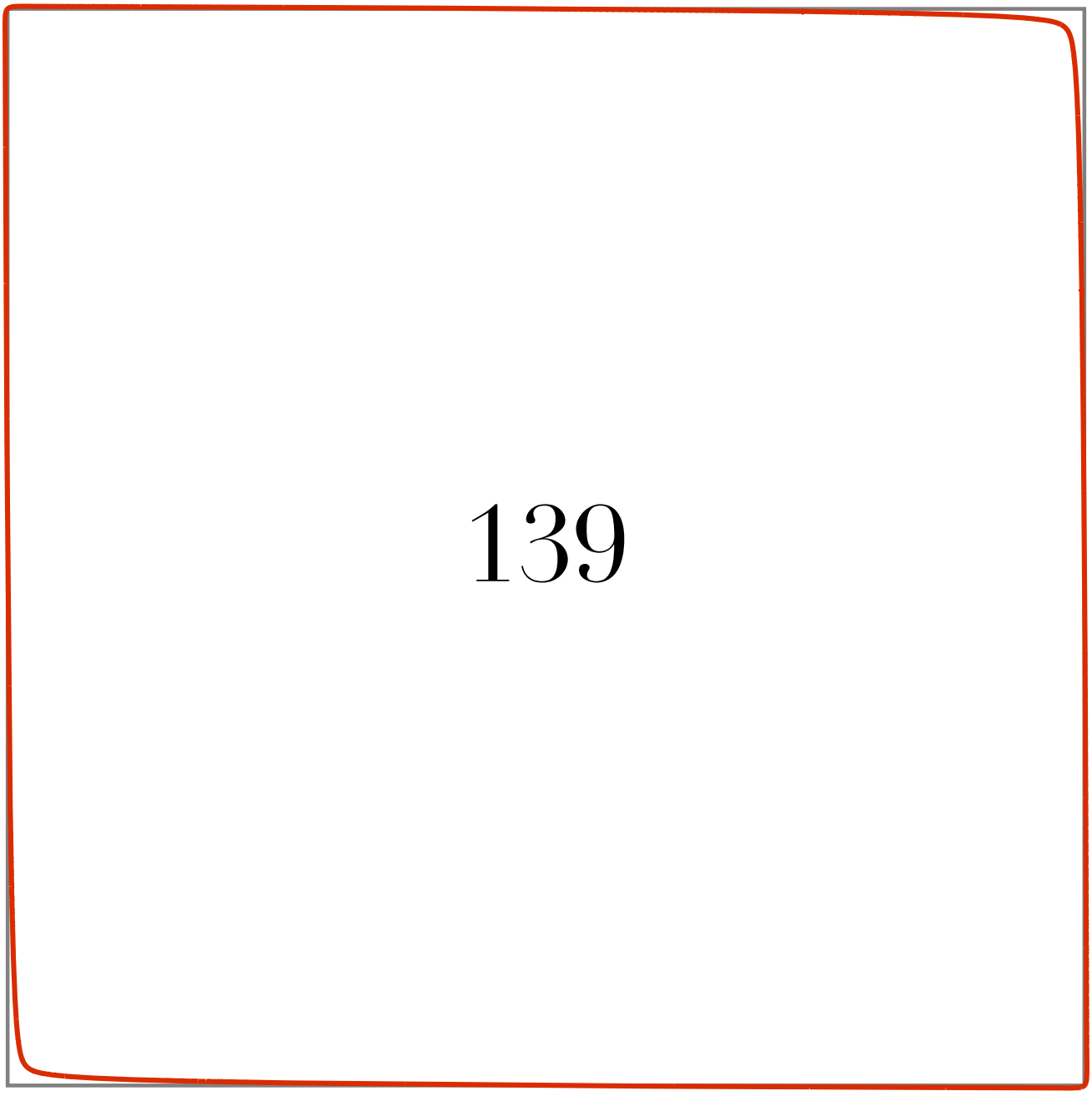
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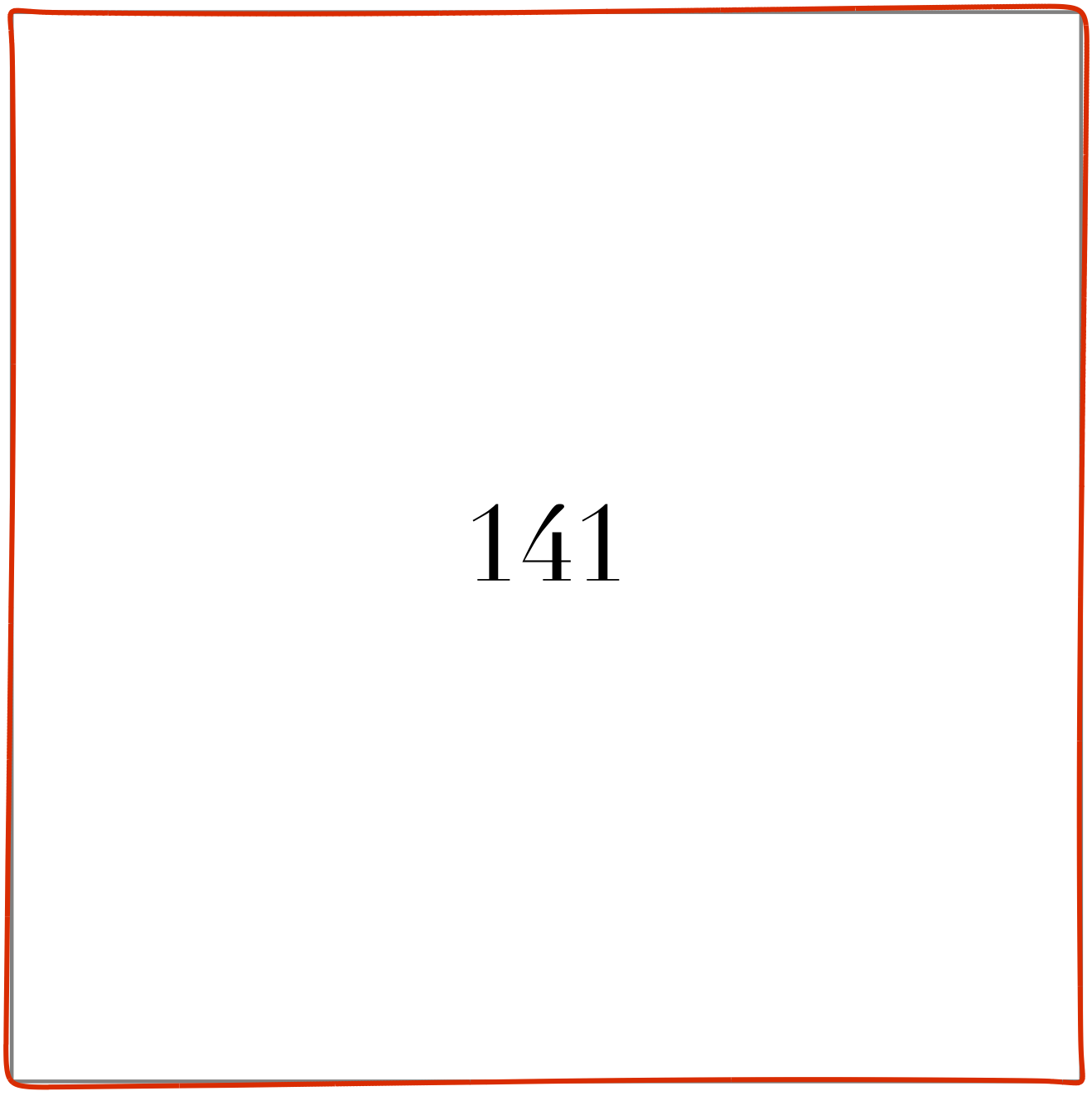

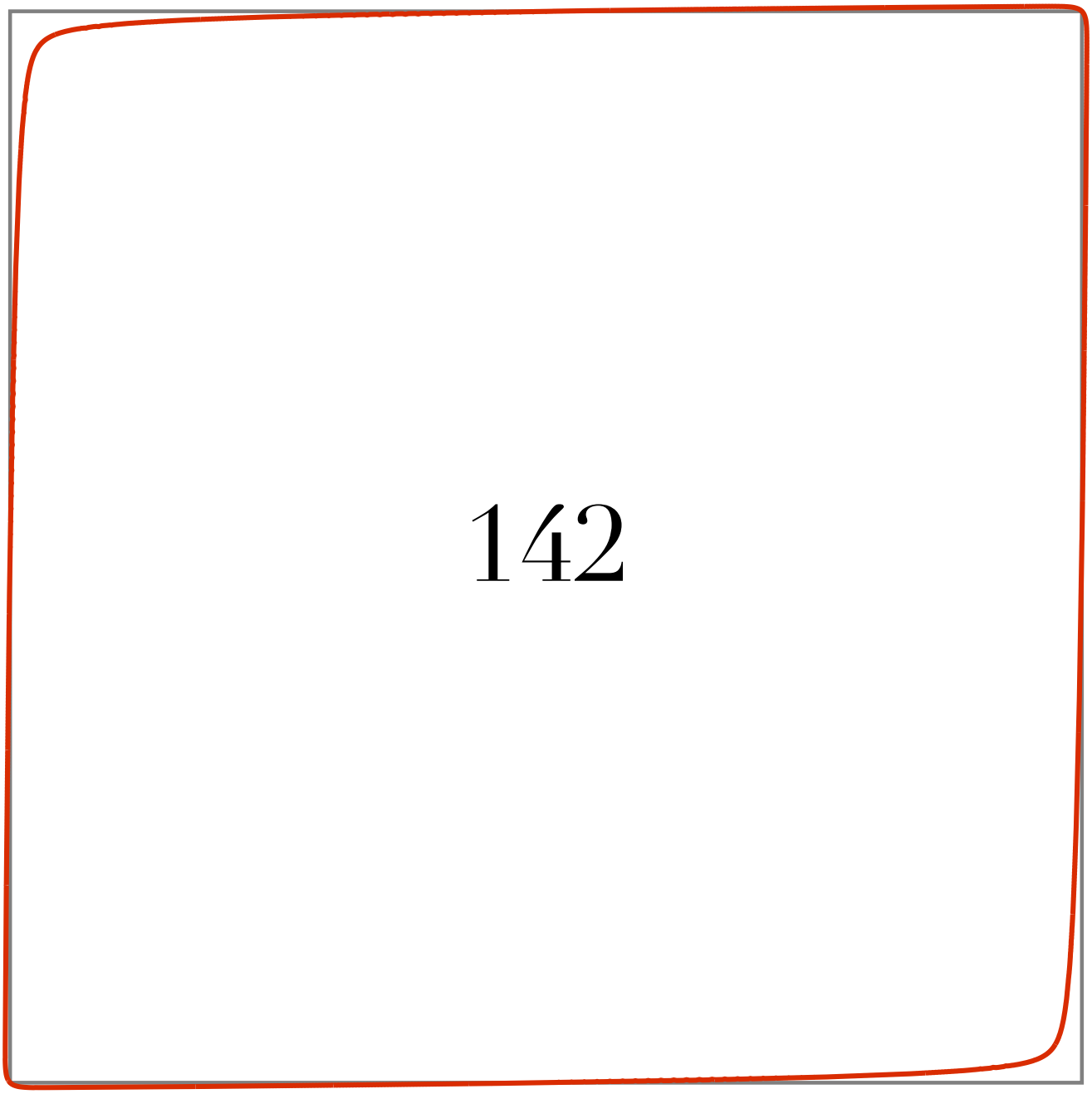

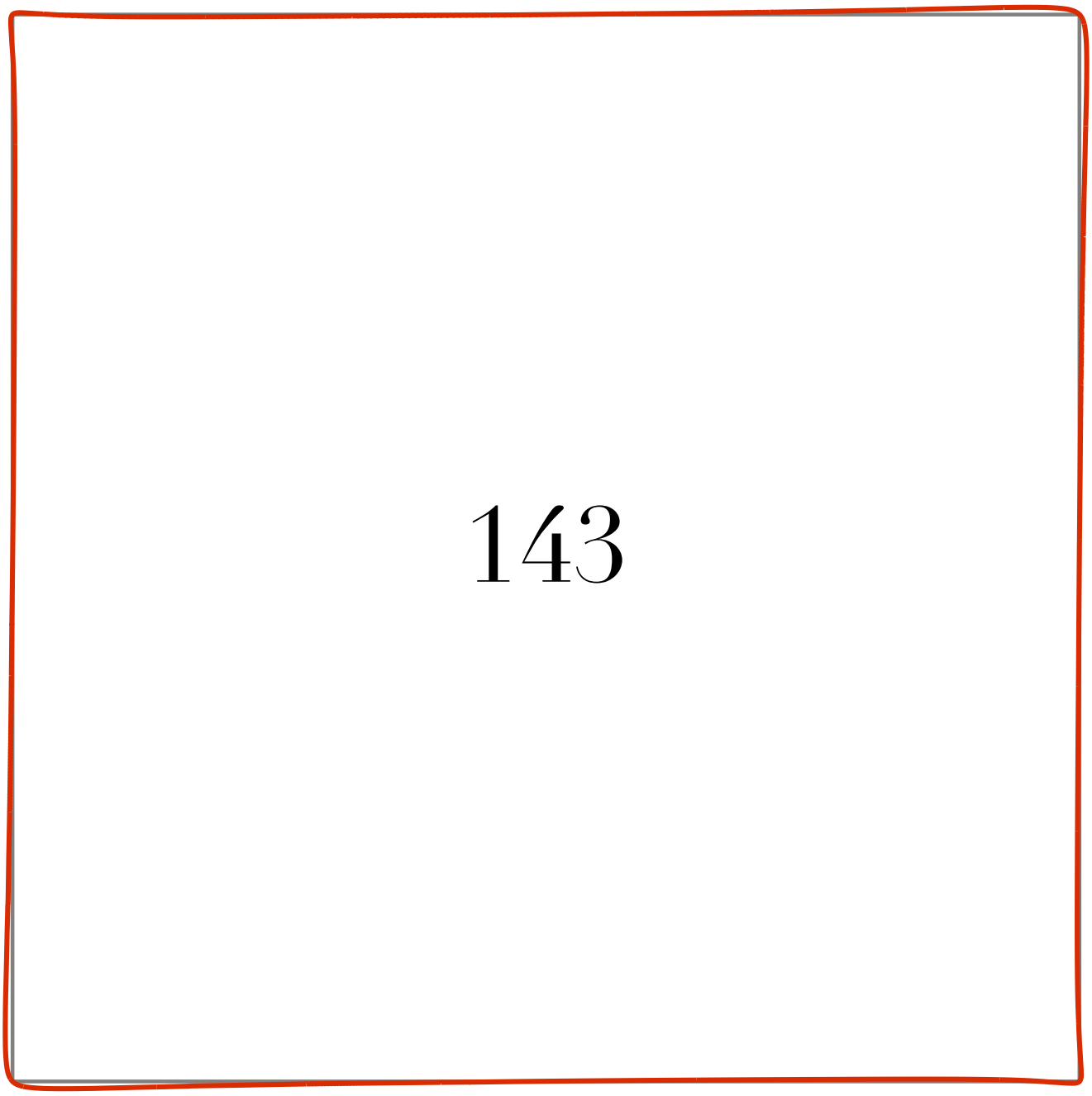
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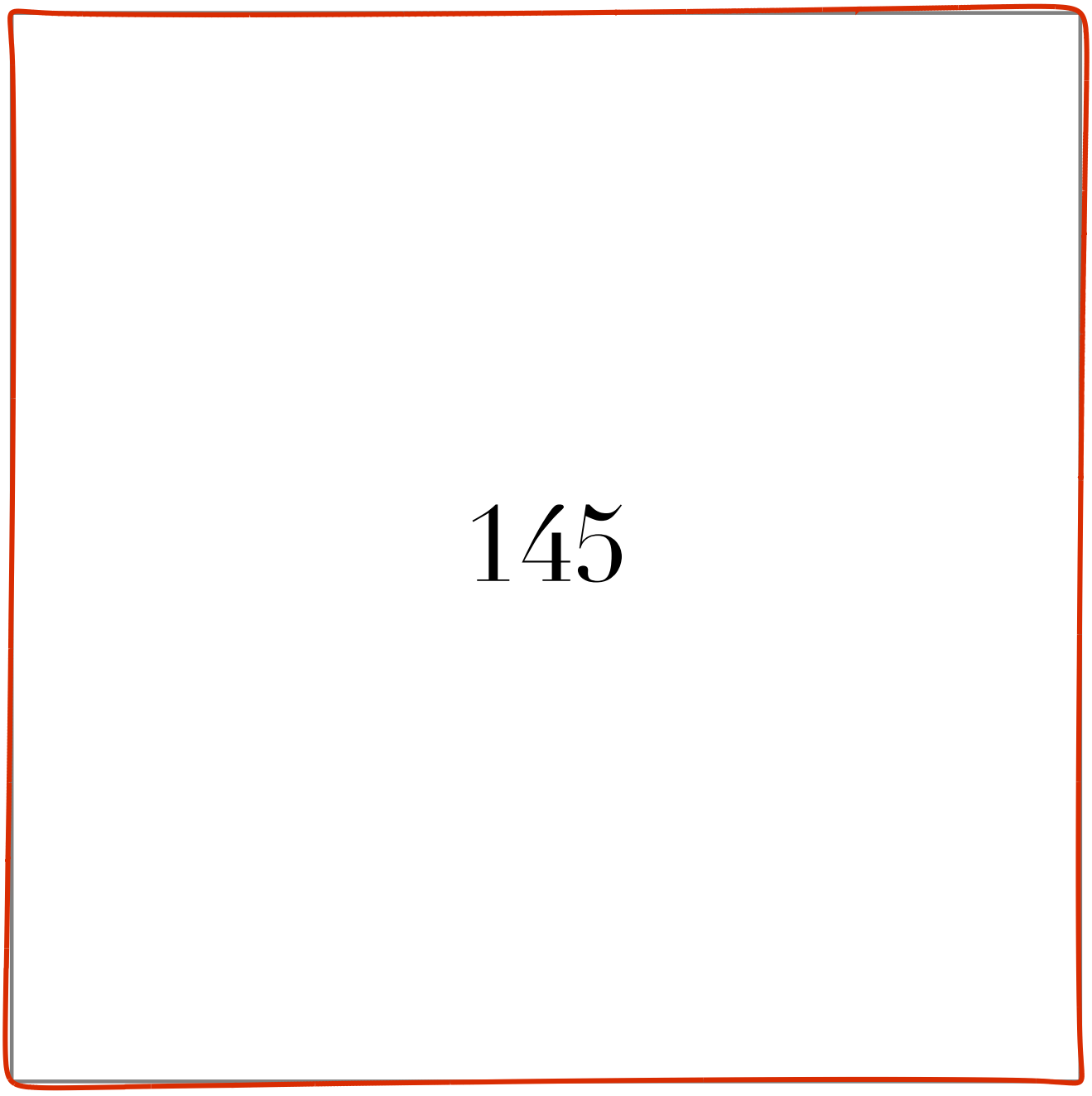
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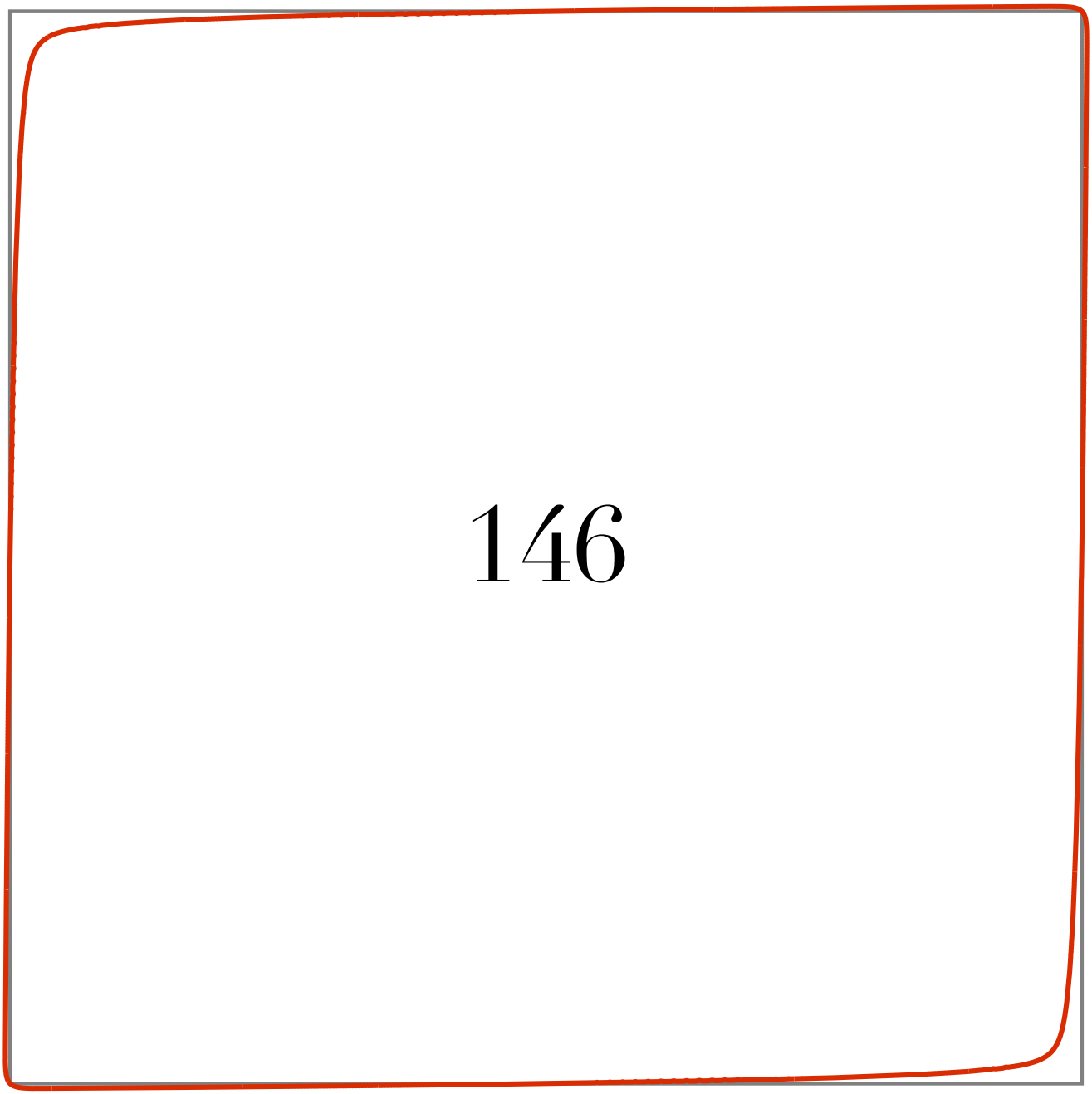
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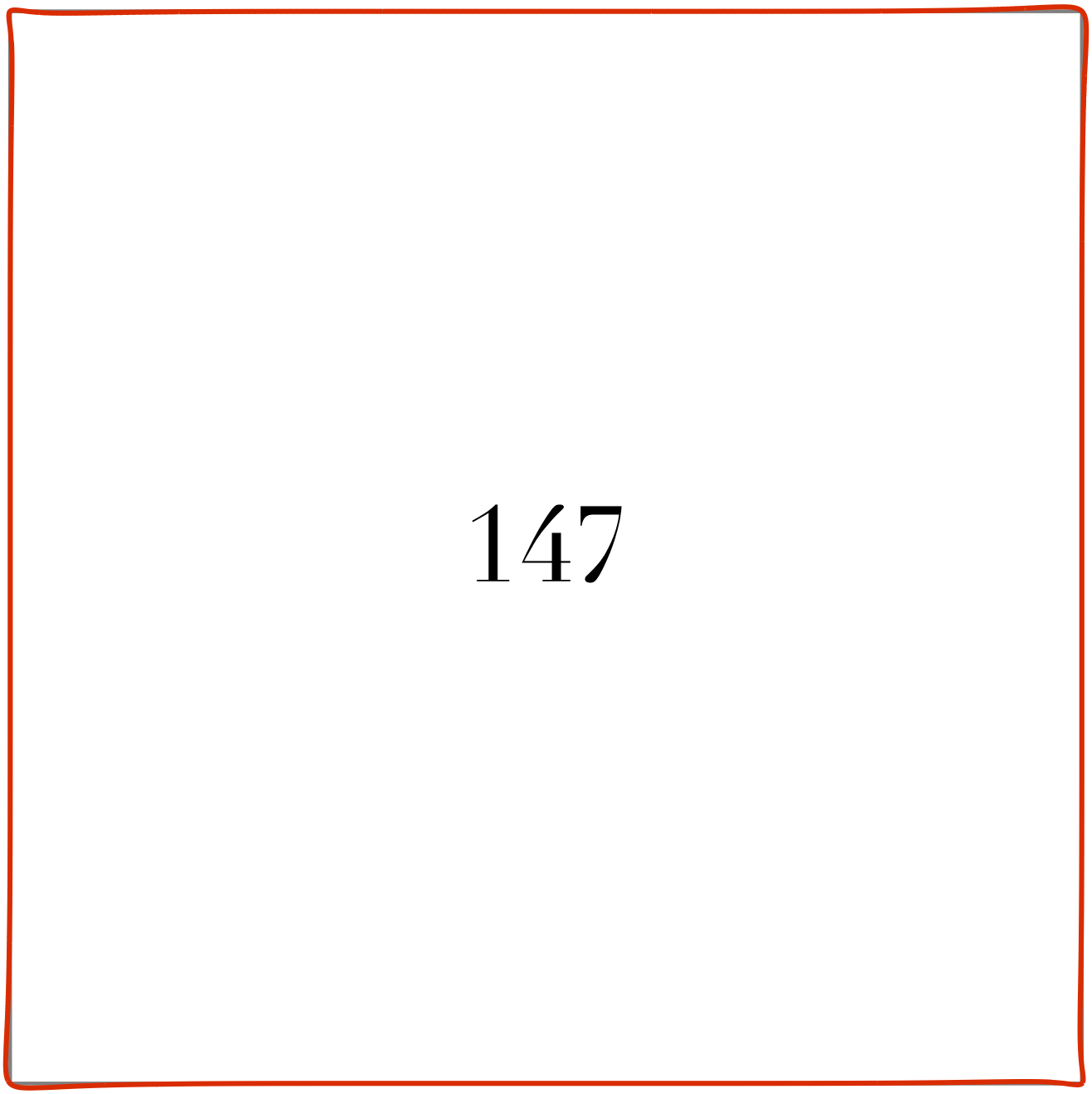
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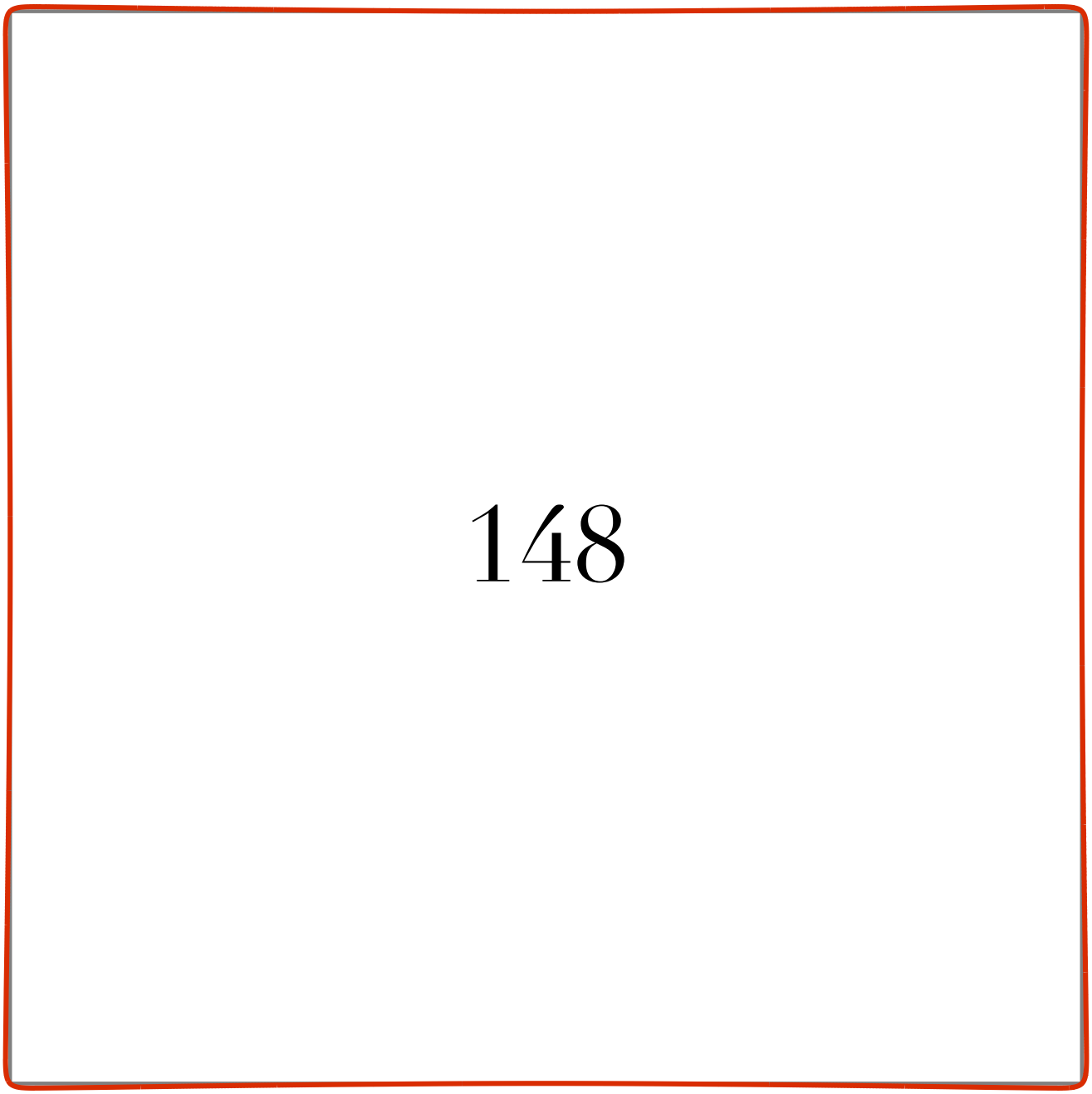

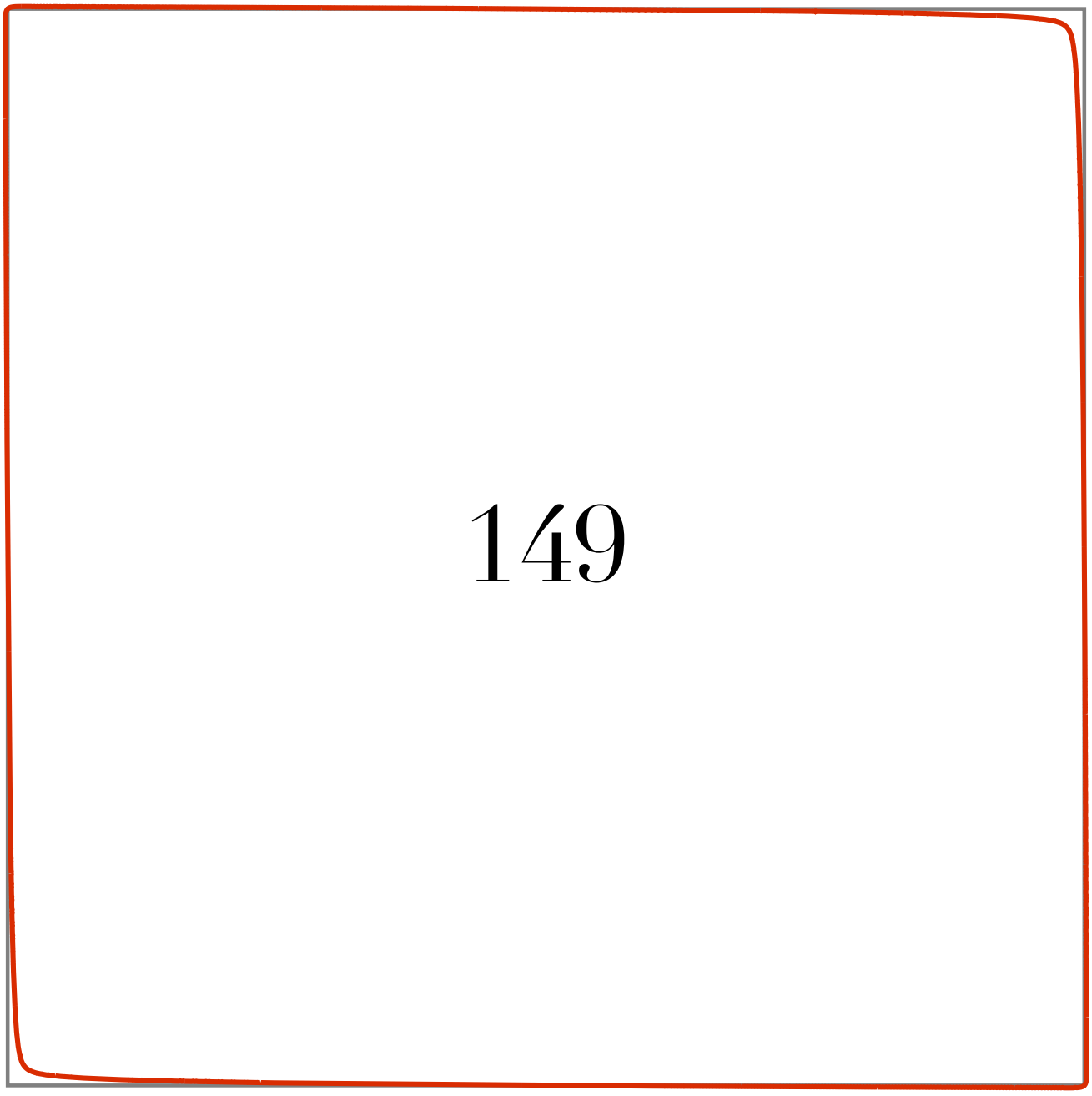

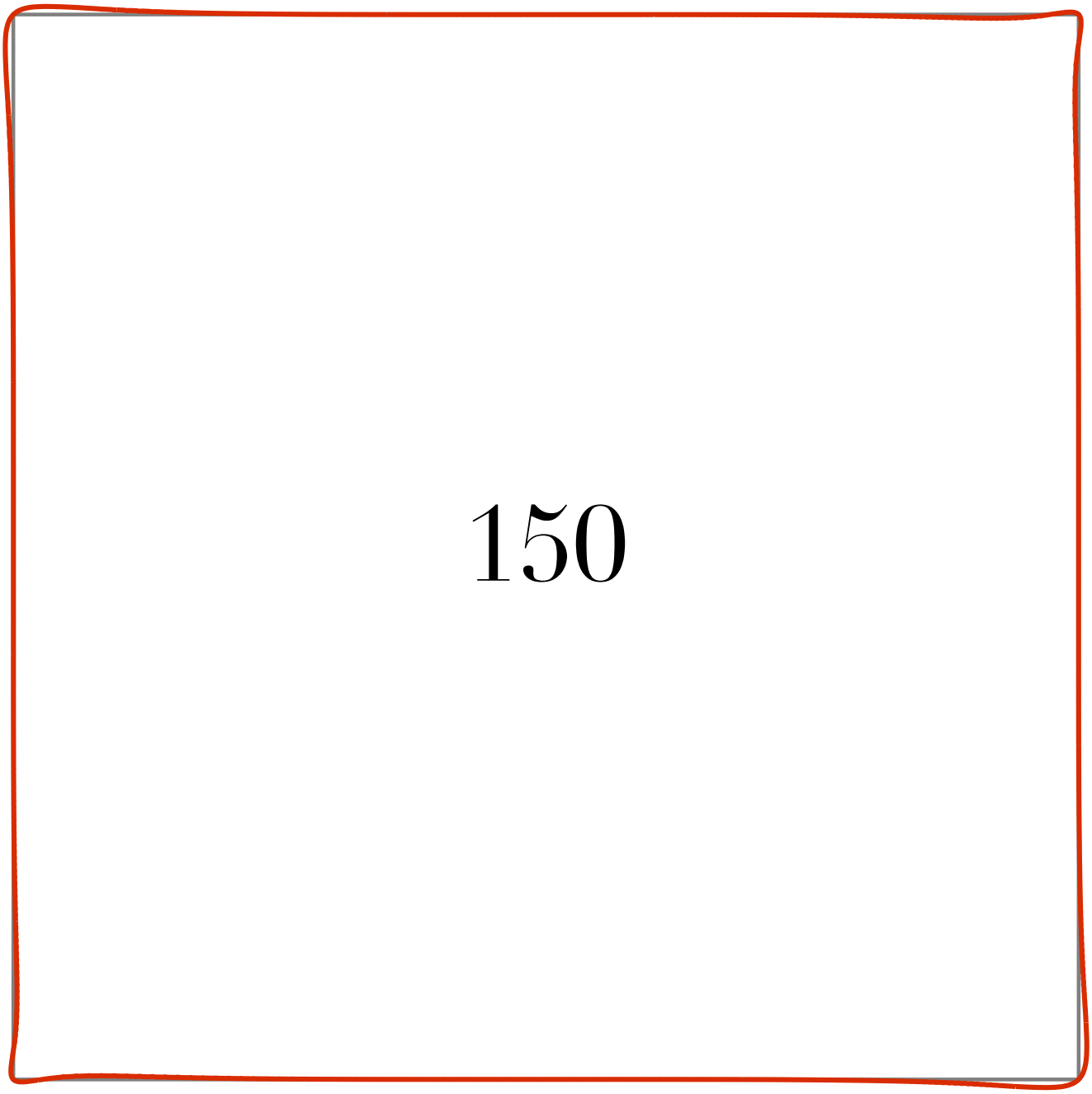
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