

“Selected topics in Arithmetic Algebraic Geometry”

Institute of Mathematics of the Vietnam Academy of Science and Technology (VAST).

Course: *Representation of integers by binary forms.*

Michel Waldschmidt

References

- [1] Étienne Fouvry, Claude Levesque, and Michel Waldschmidt. Representation of integers by cyclotomic binary forms. *Acta Arith.*, 184(1):67–86, 2018.
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- [2] Étienne Fouvry and Michel Waldschmidt. On the representation of integers by cyclotomic forms with large degree. *Bull. Soc. Math. Fr.*, 148(2):253–282, 2020.
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- [4] Étienne Fouvry and Michel Waldschmidt. Number of integers represented by families of binary forms. II: Binomial forms. *Acta Arith.*, 214:271–287, 2024.
doi <https://doi.org/10.4064/aa230525-6-9> arxiv <https://arxiv.org/abs/2306.02462>

Fundamental cyclotomic domain

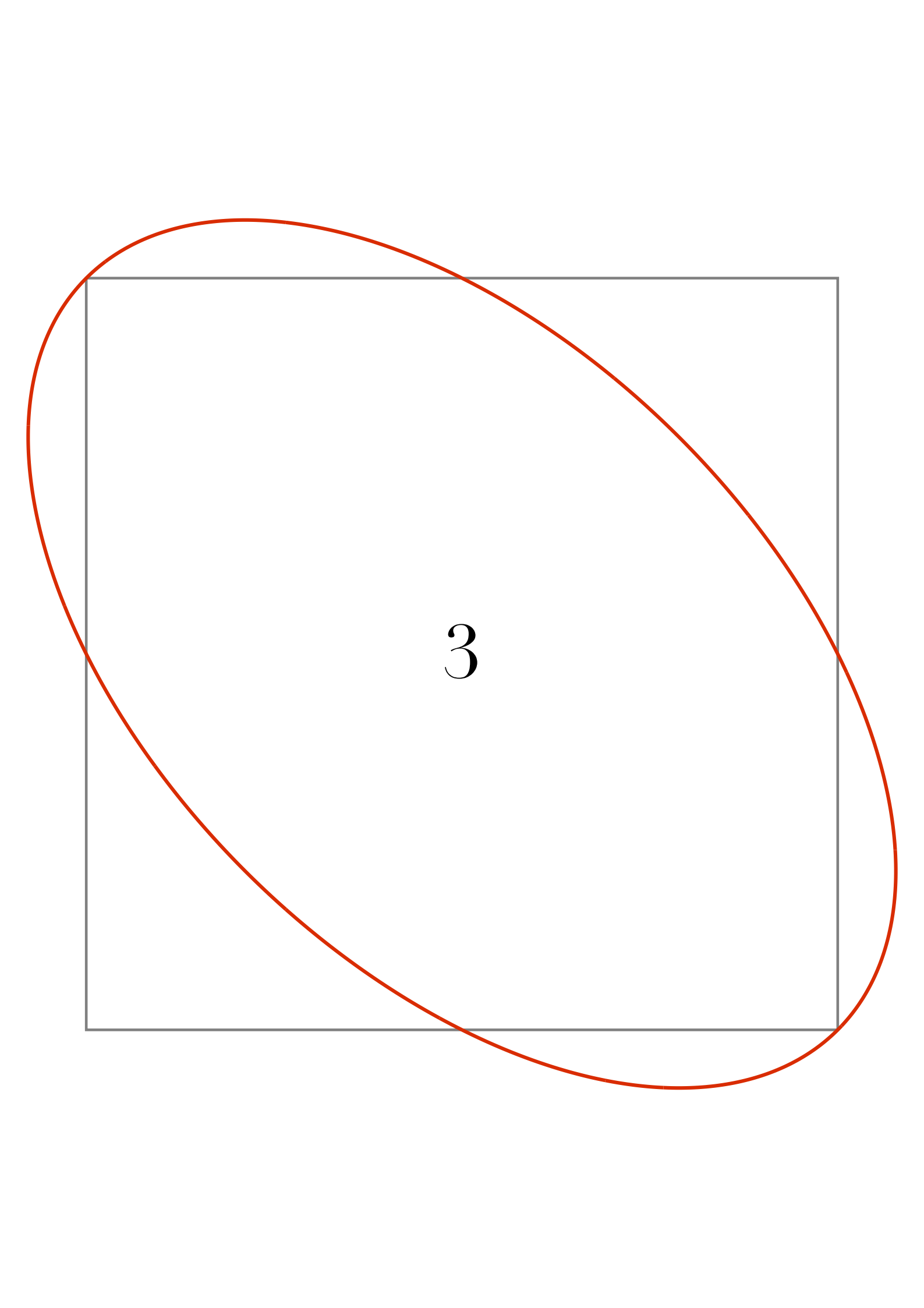
The following pages display for $n = 3$ to 150 the fundamental domain

$$\mathcal{O}_n := \{(x, y) \in \mathbb{R}^2 \mid \Phi_n(x, y) \leq 1\}$$

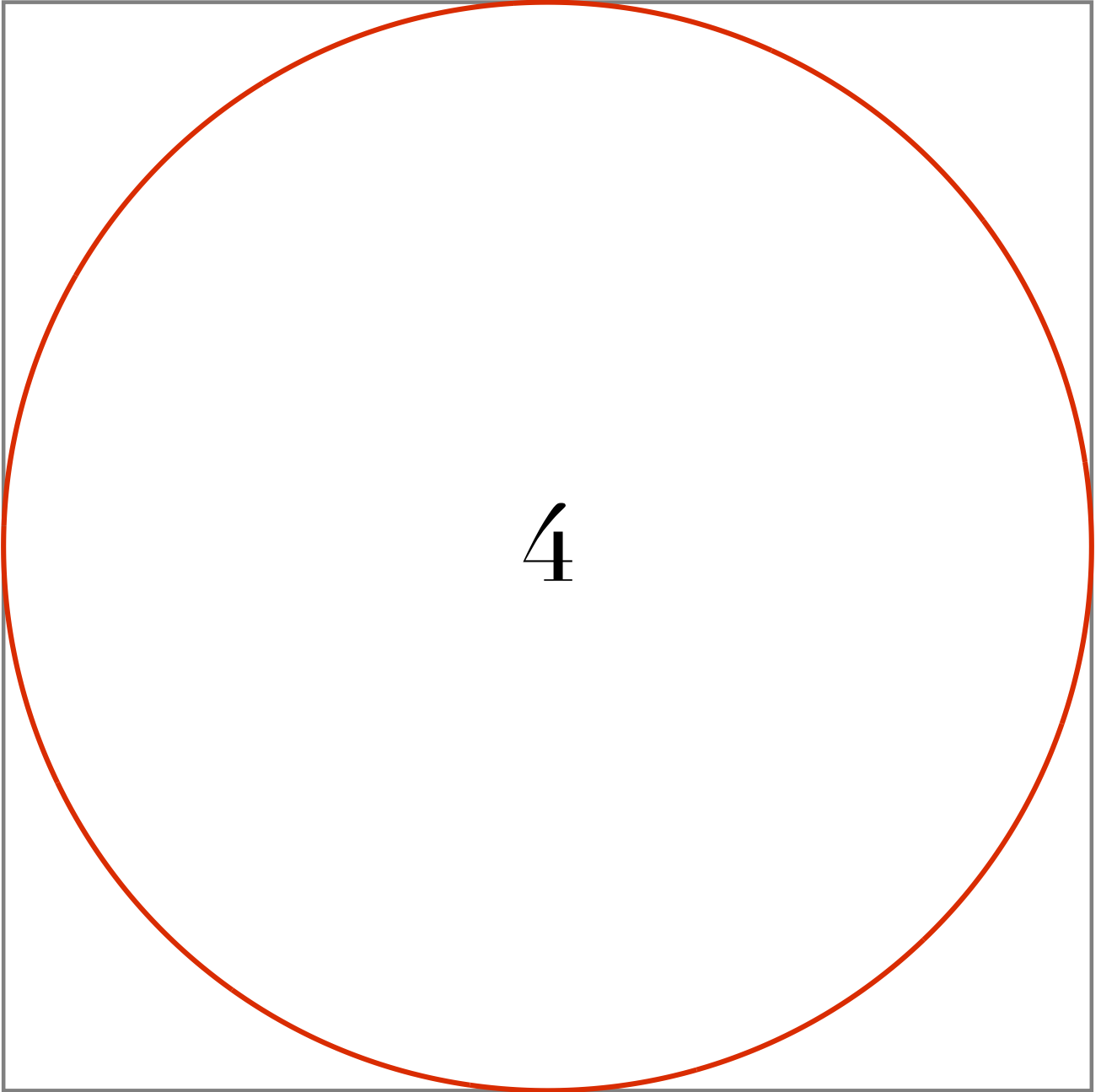
of the n -th cyclotomic form $\Phi_n(X, Y)$ defined by induction for $n \geq 1$ by

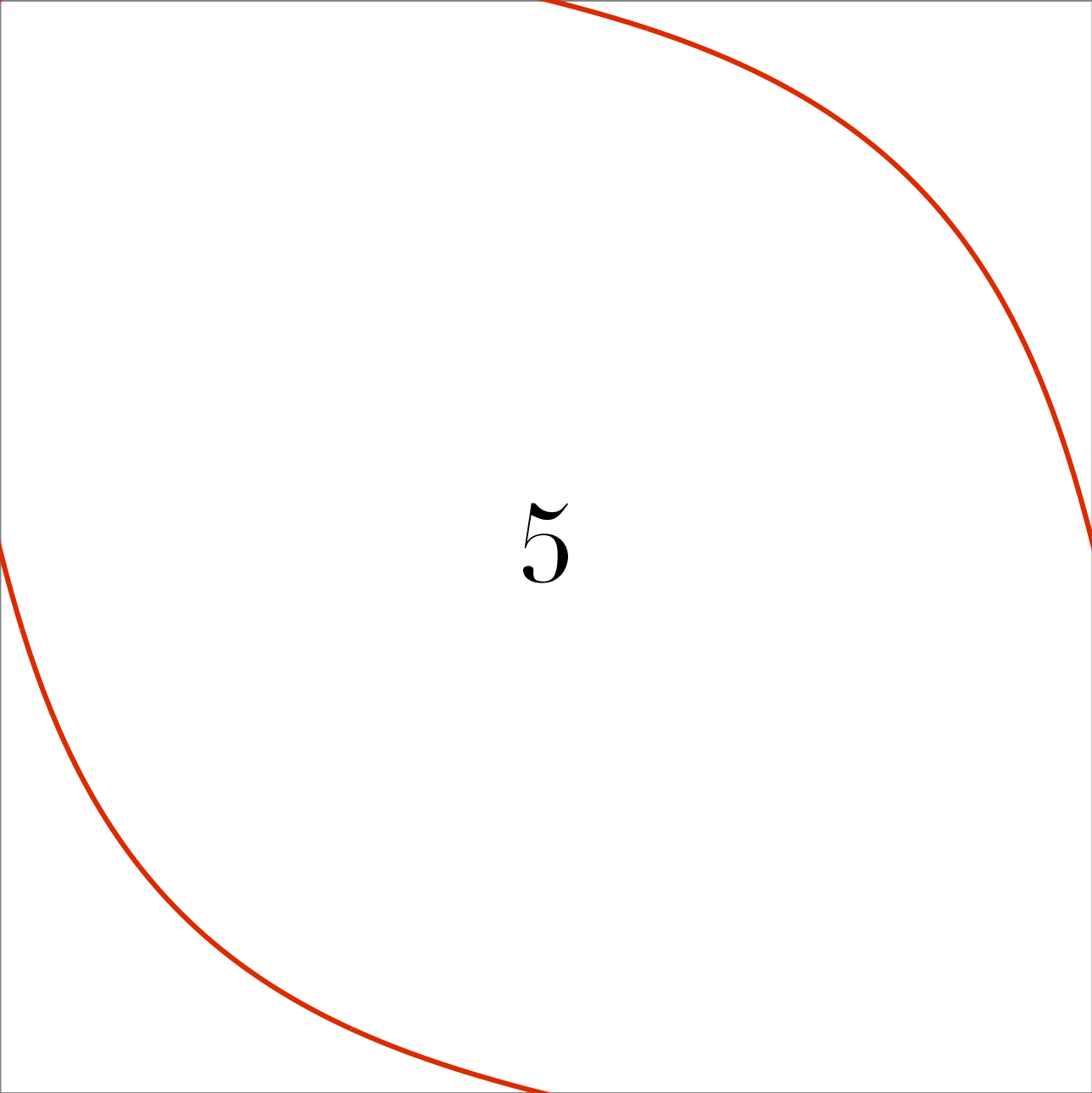
$$X^n - Y^n = \prod_{n|d} \Phi_d(X, Y).$$

Exercise. For which $n \geq 3$ is $\mathcal{O}_n$ convex?

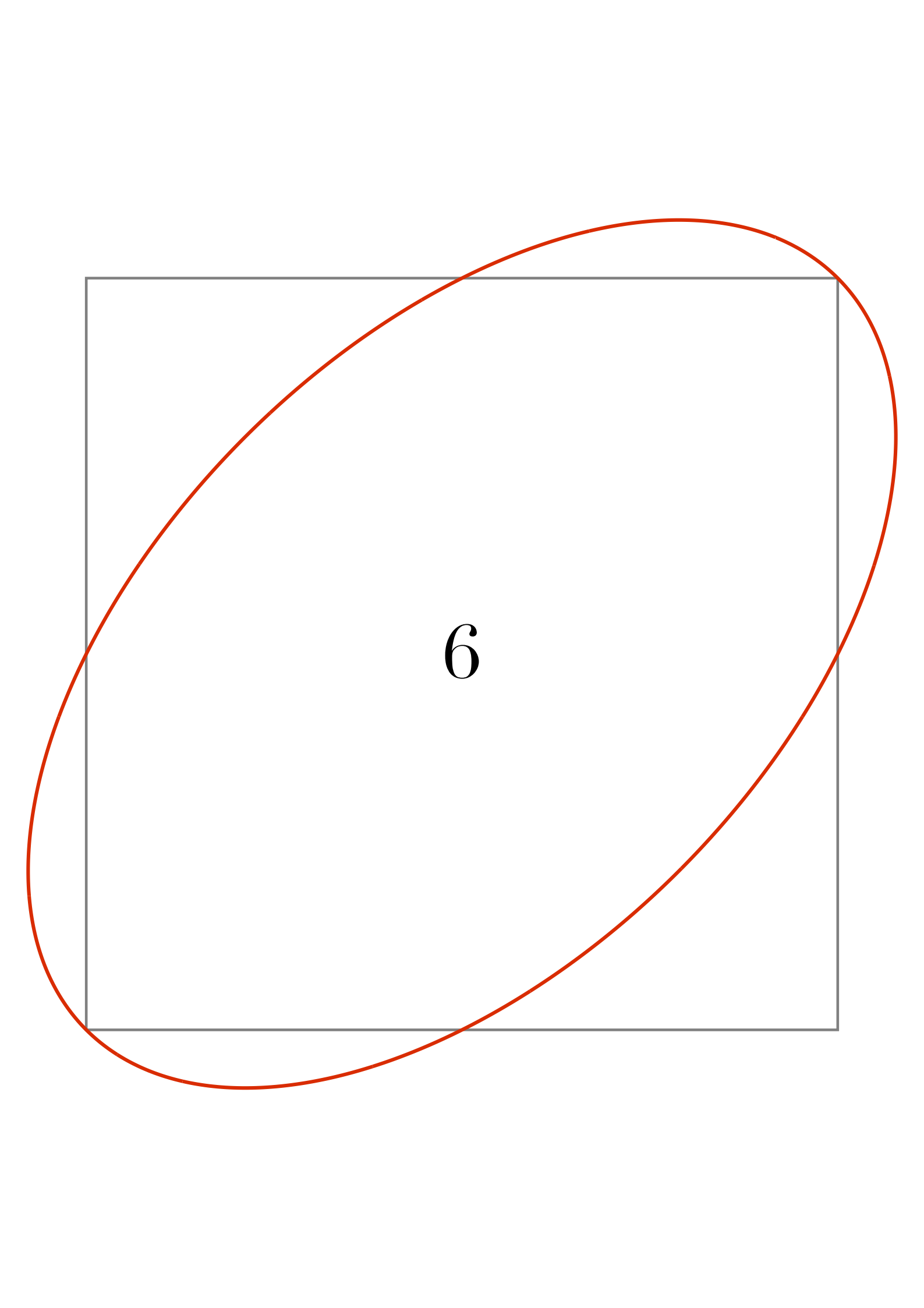


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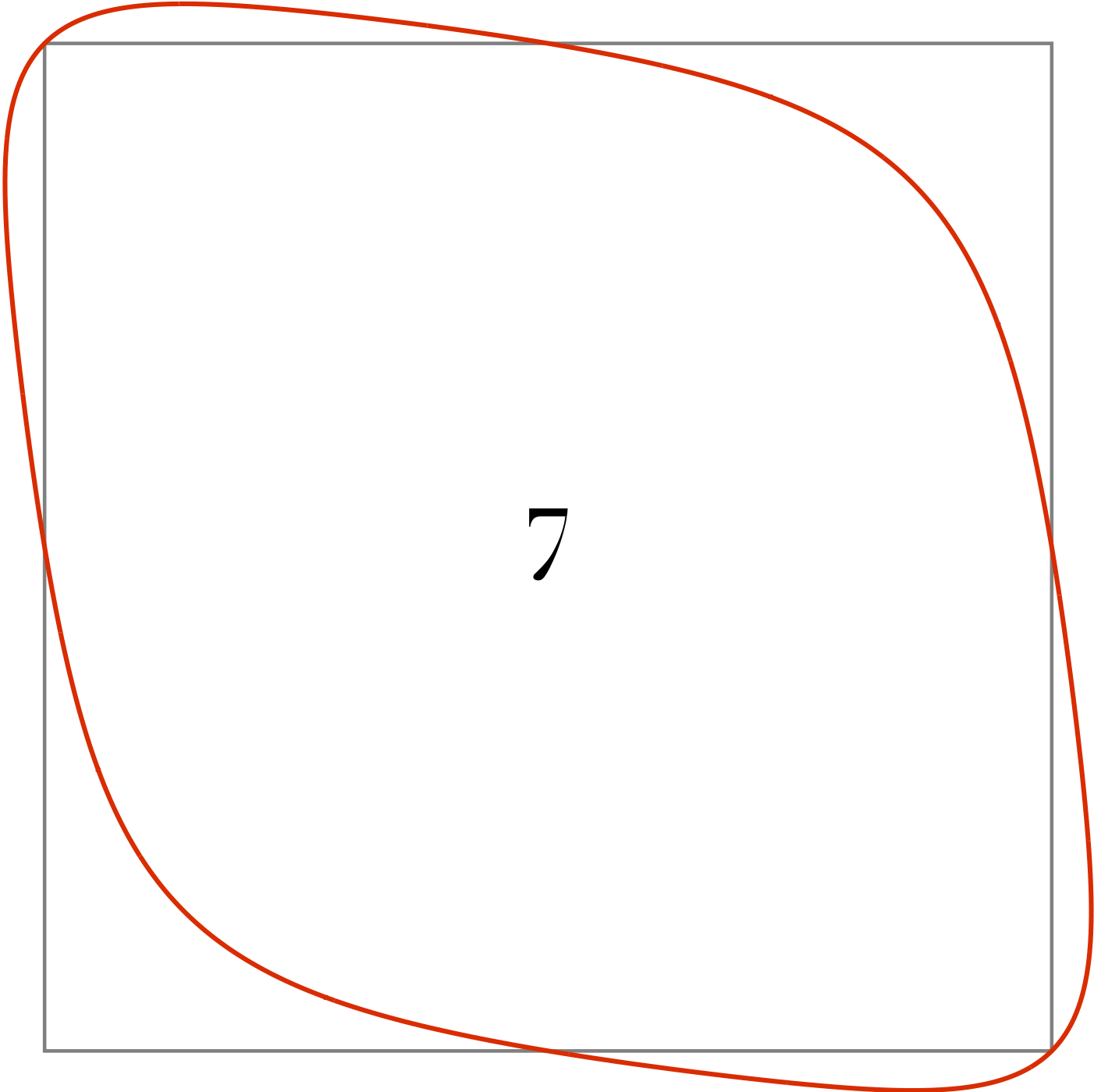




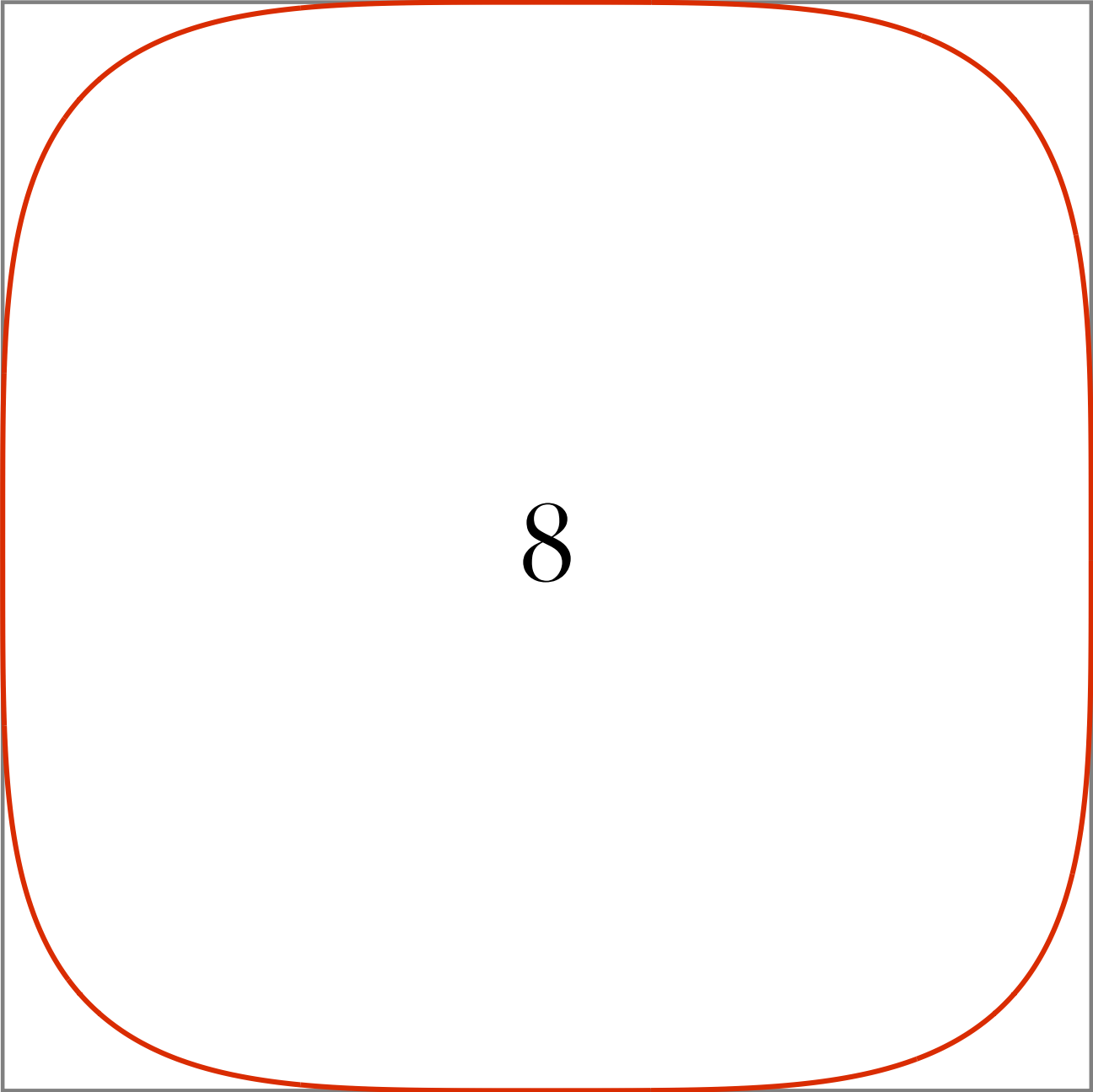
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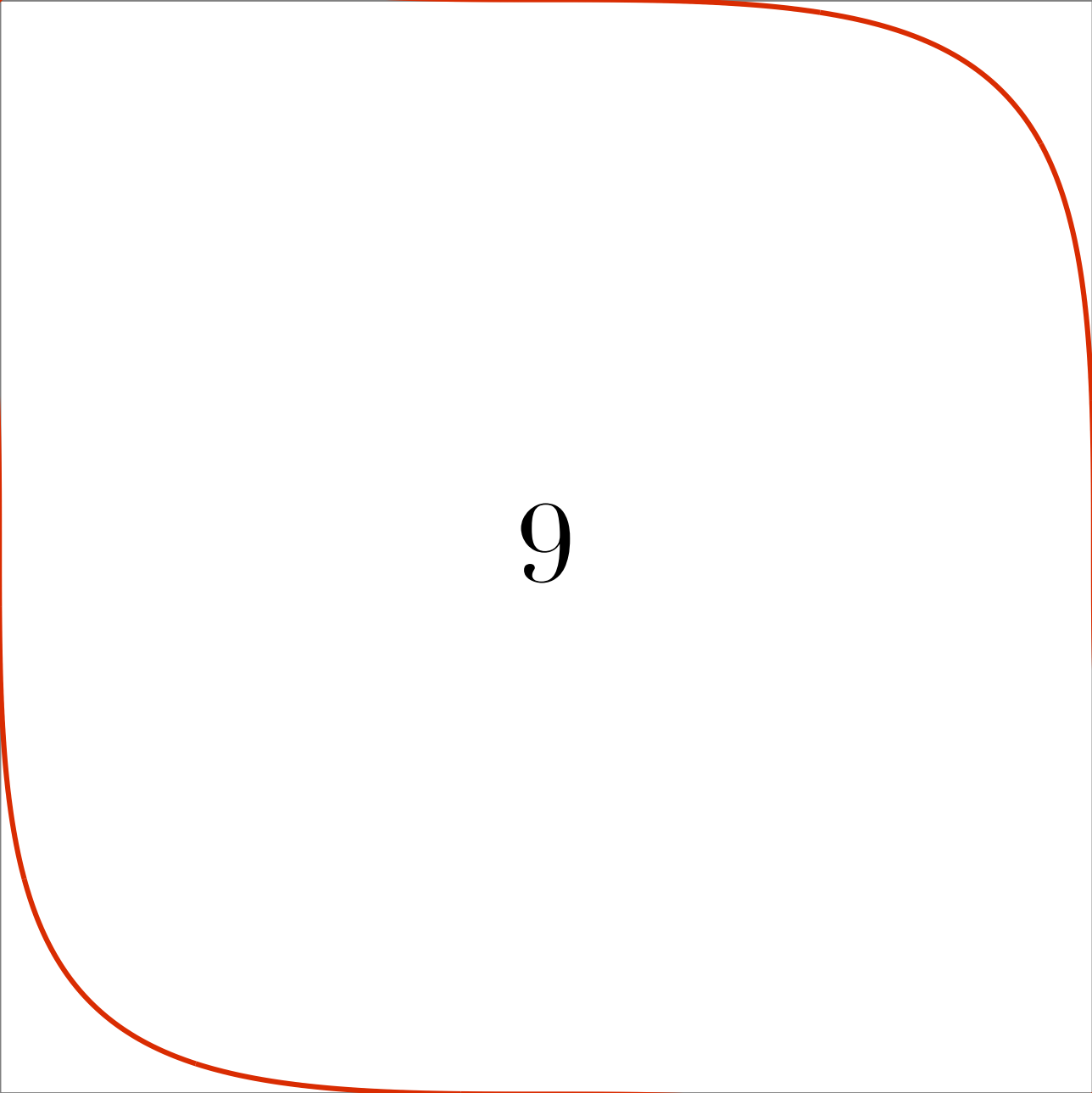
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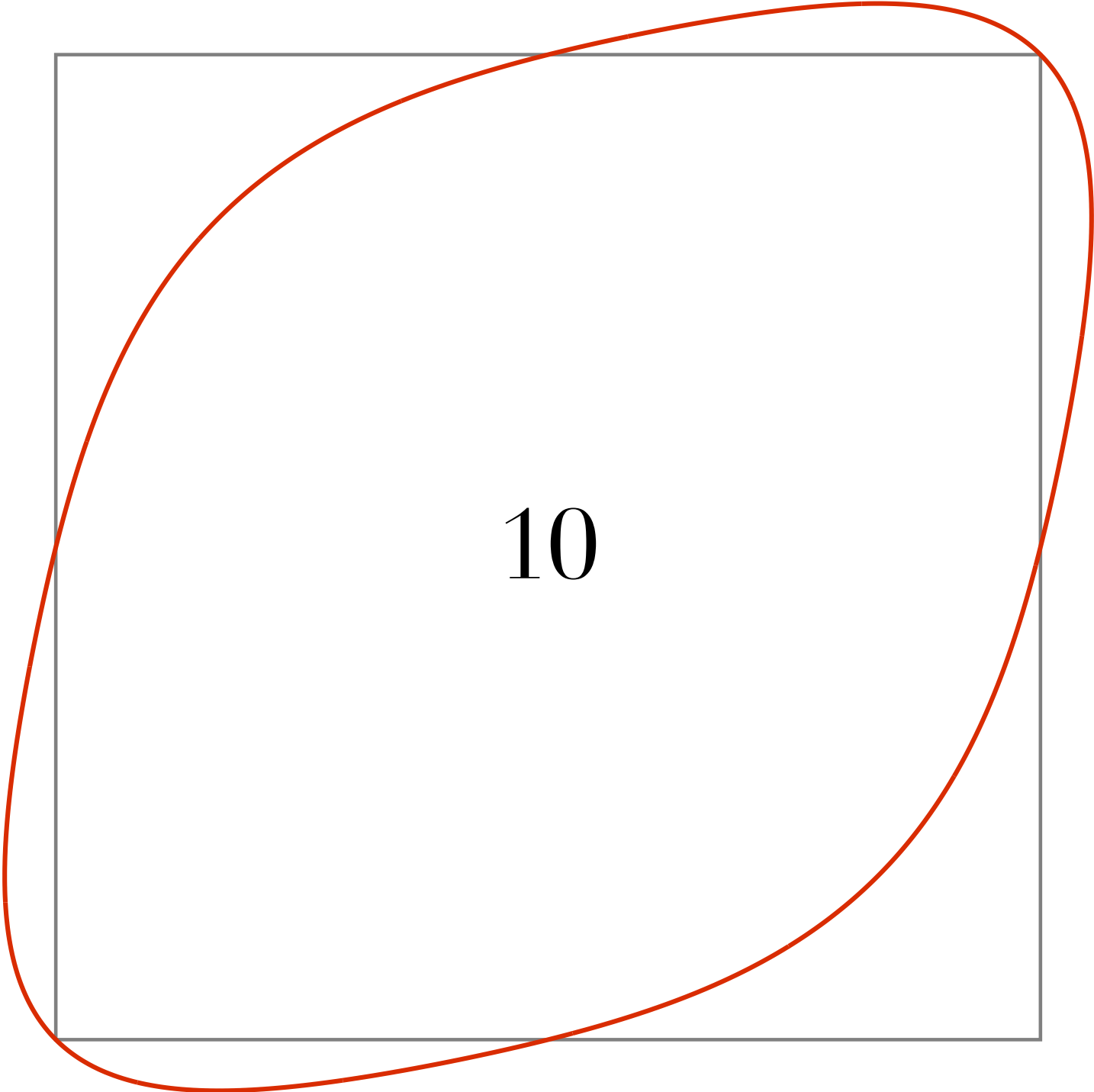
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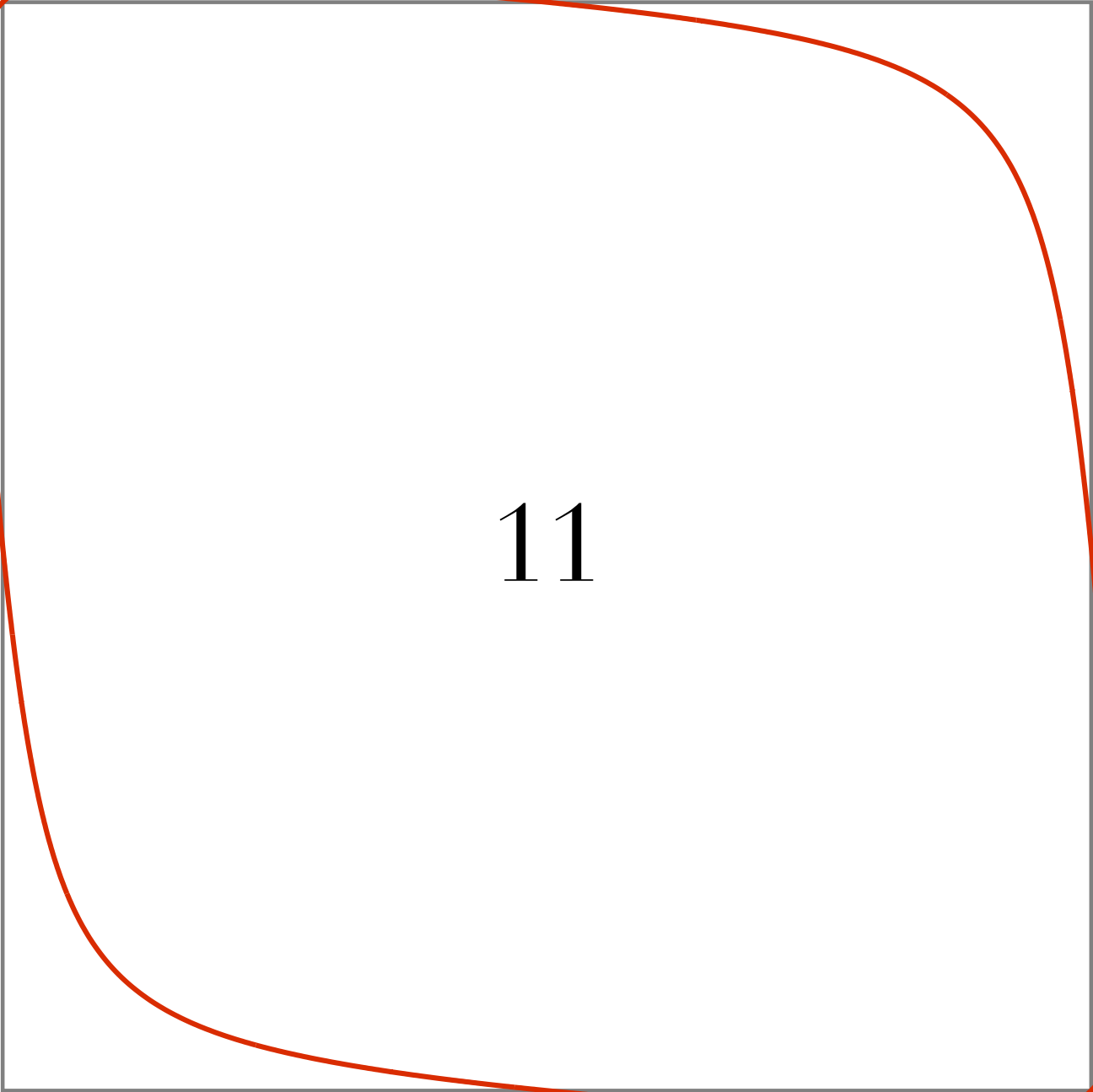
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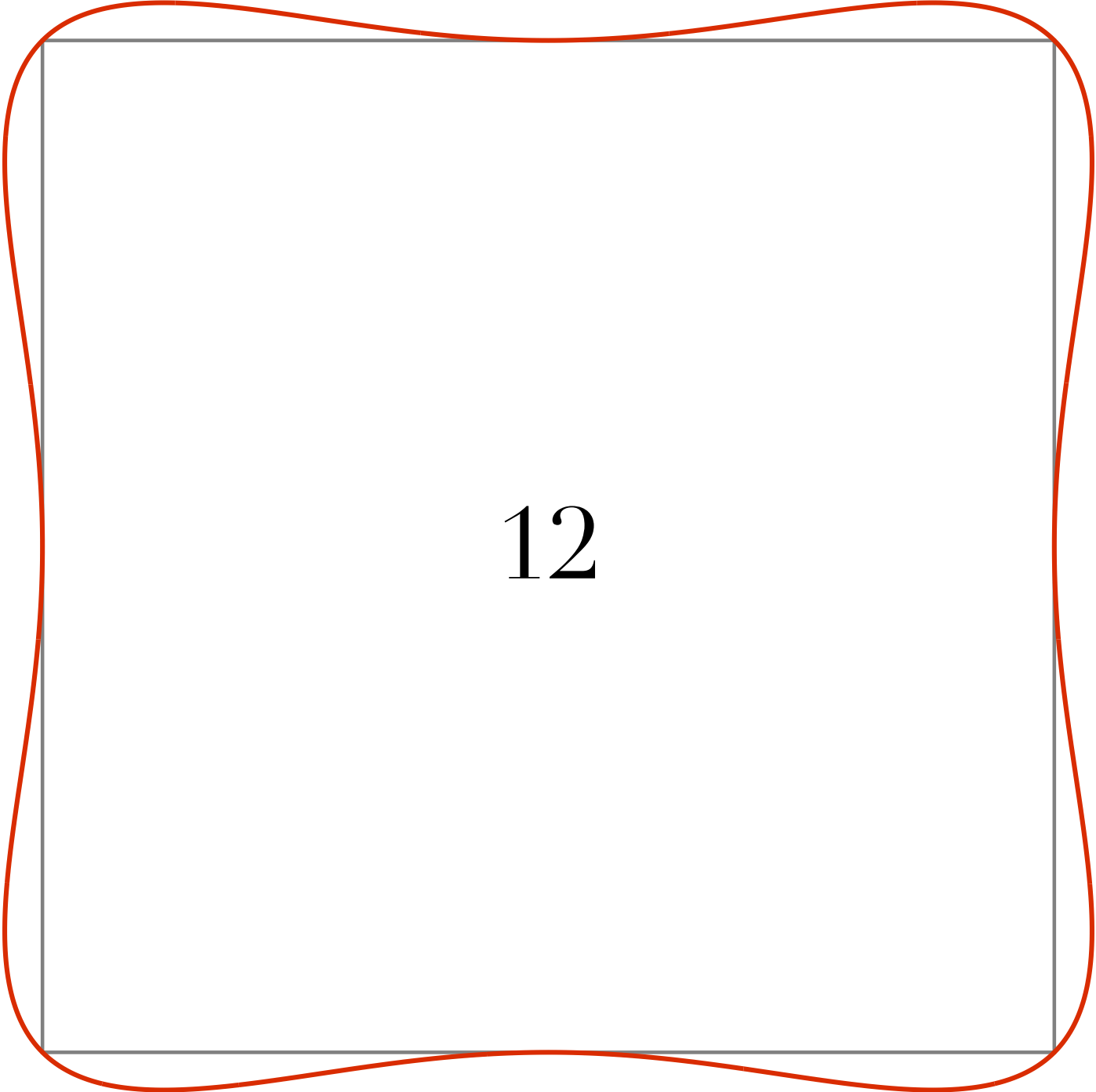
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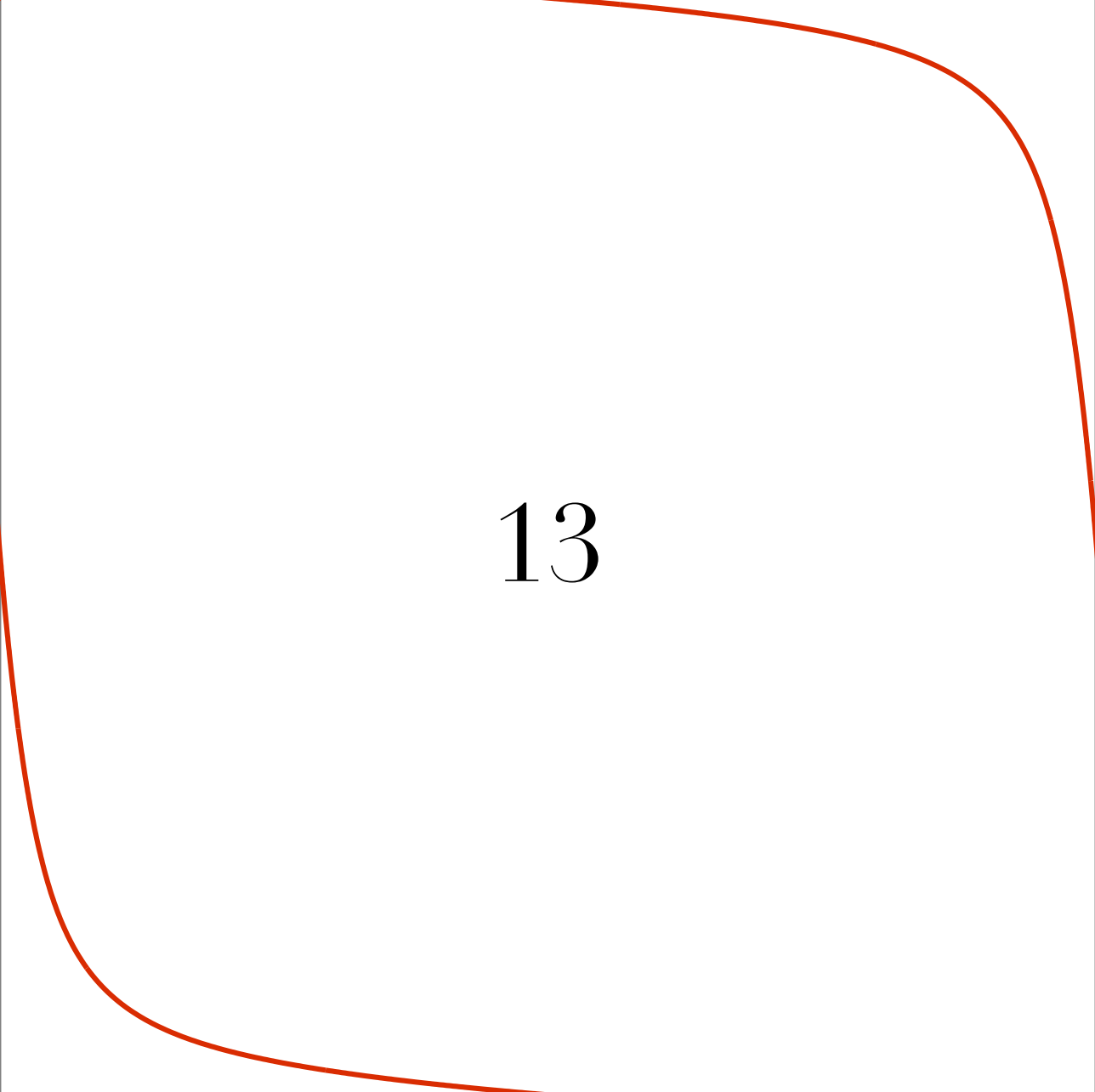
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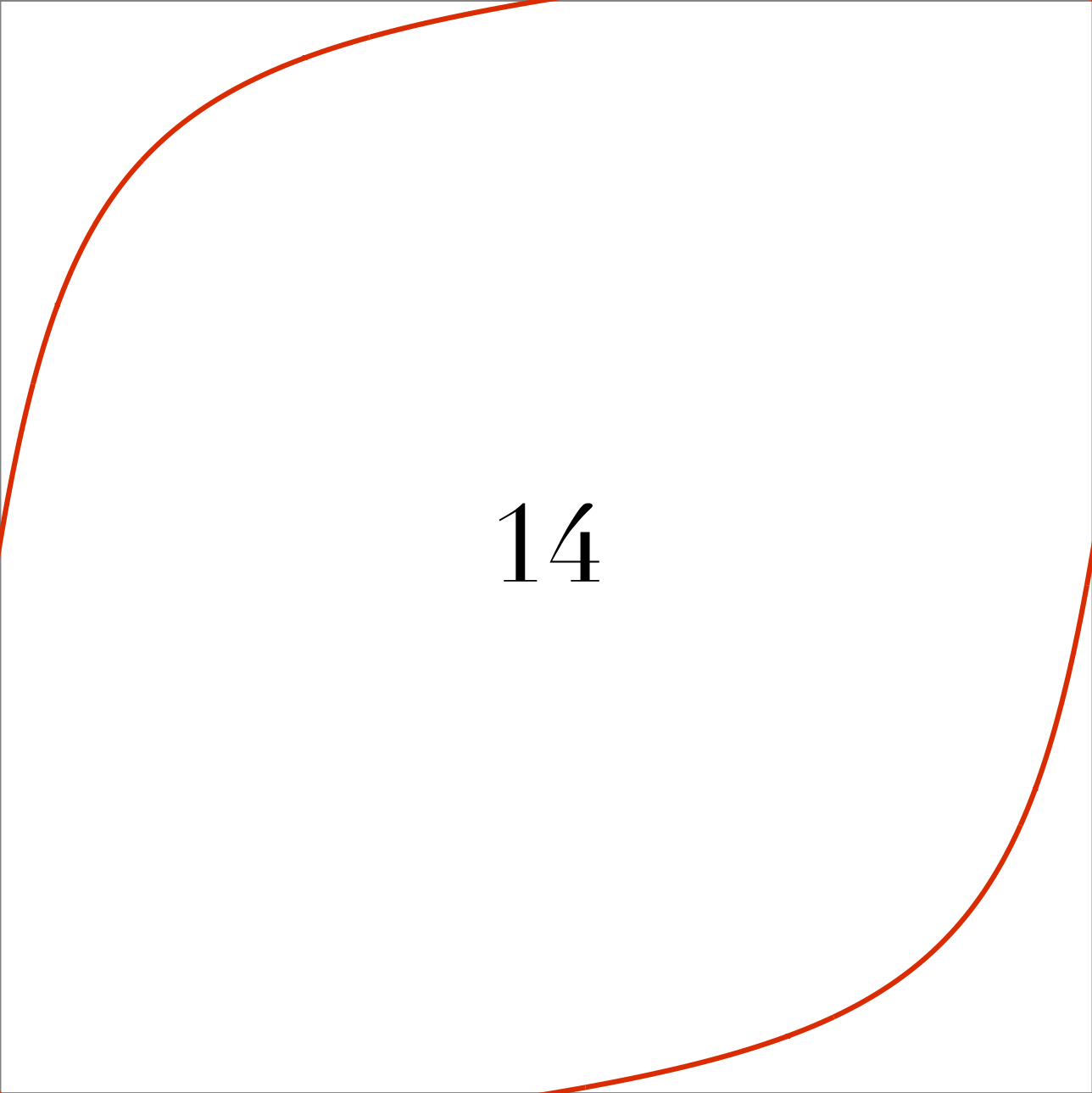
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A rectangular frame with a thin gray border. Surrounding this frame is a thick, wavy red line that follows the general shape of the rectangle but with irregular, hand-drawn curves.

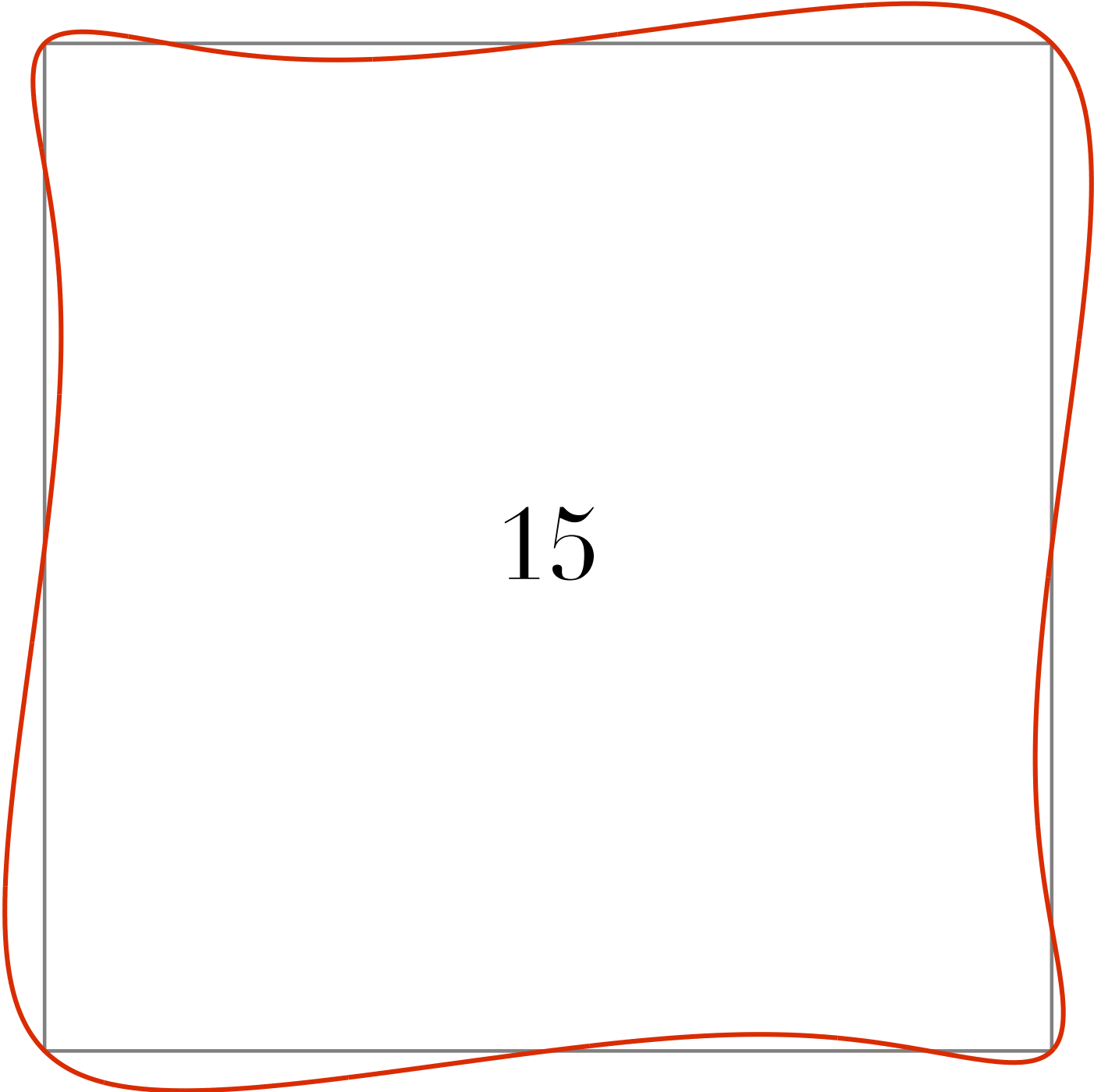
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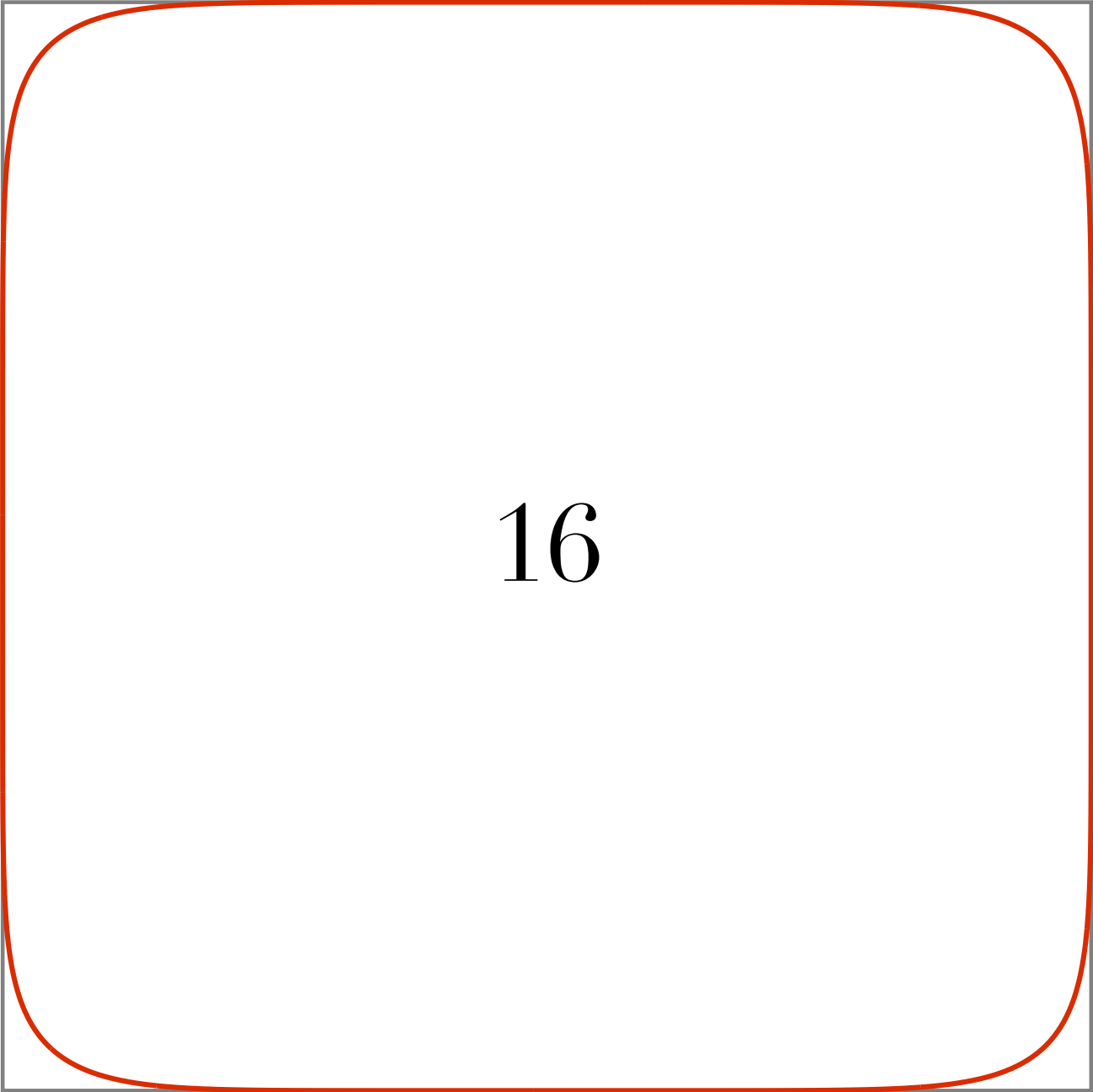
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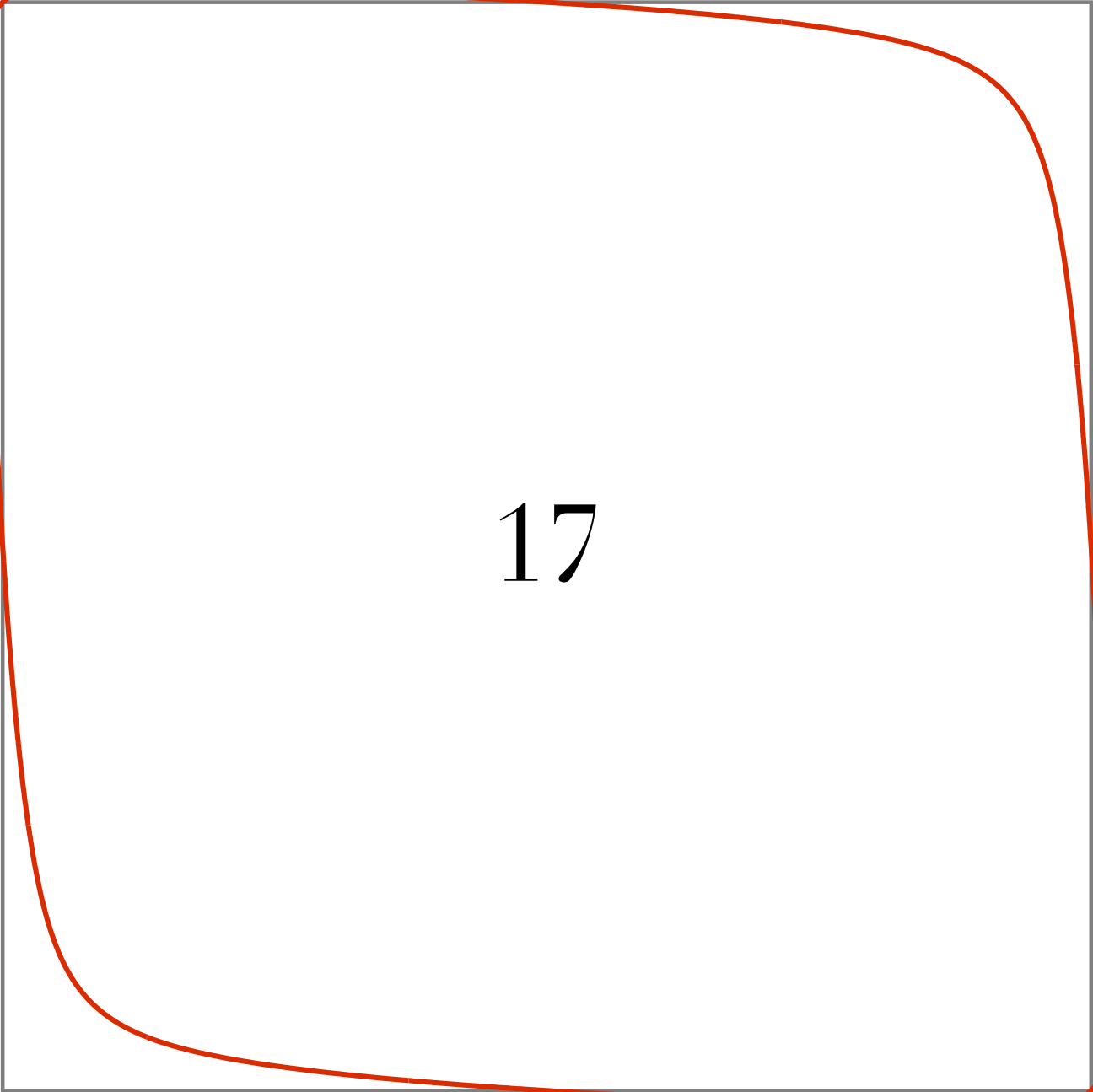
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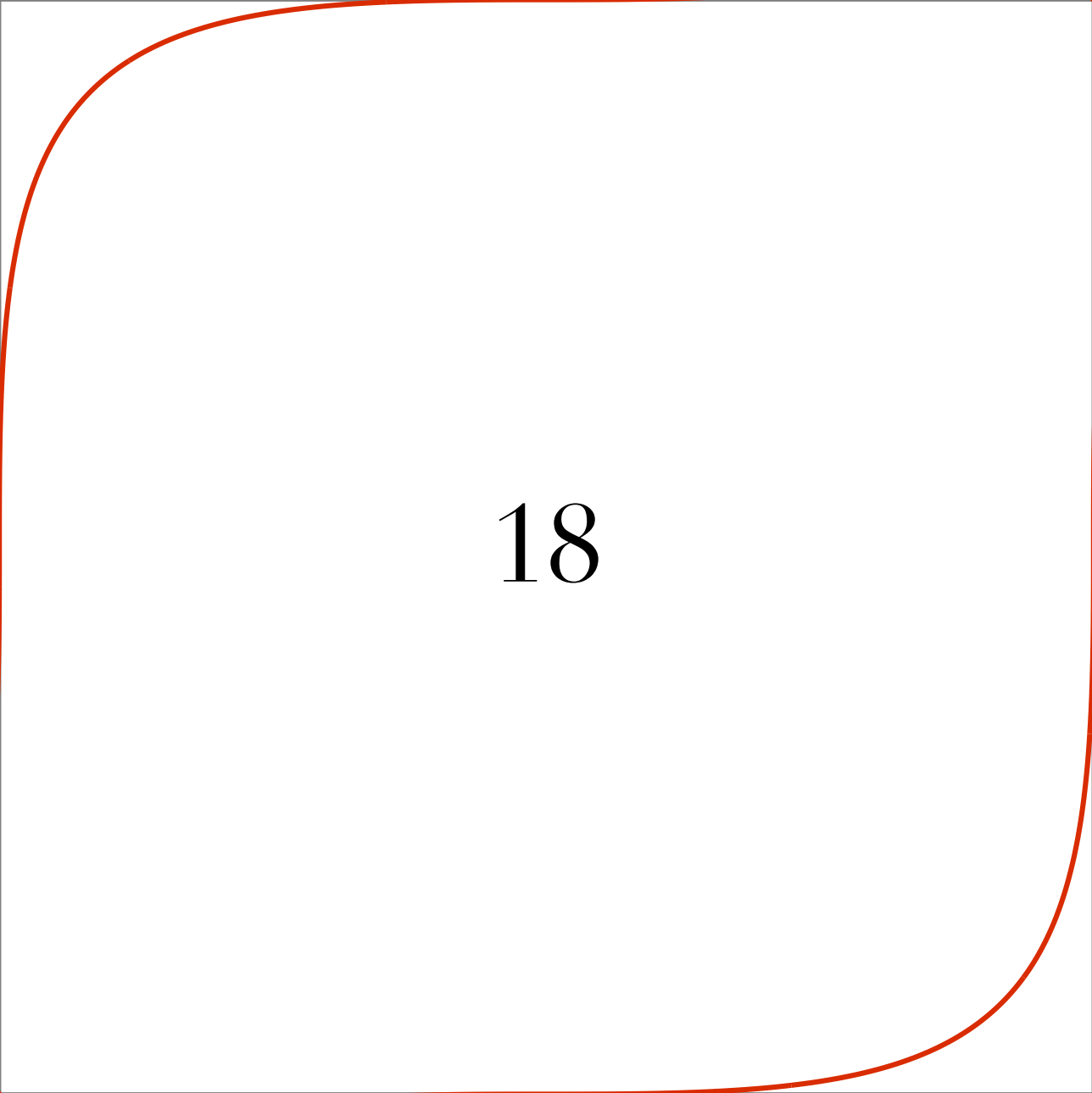
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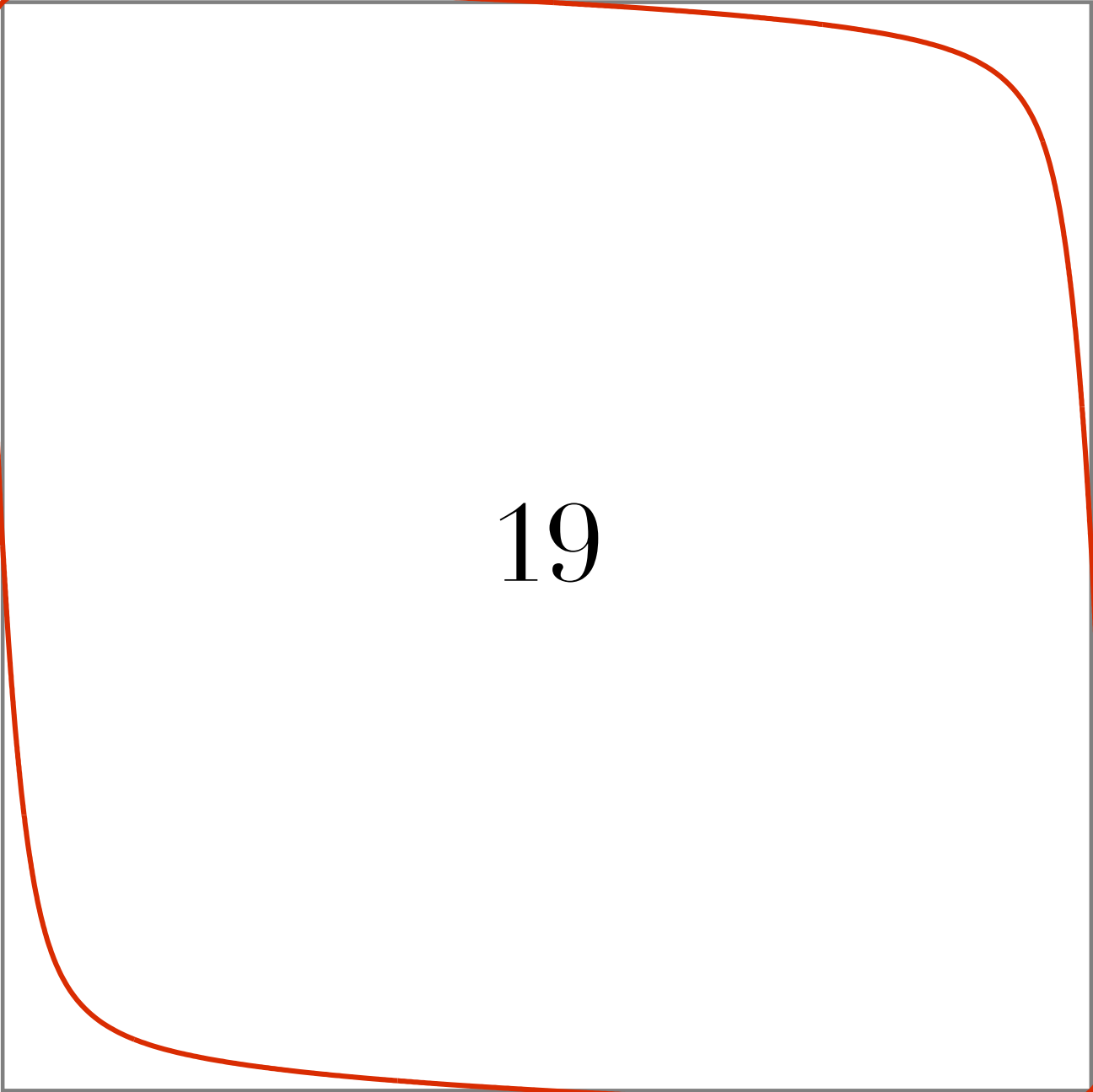
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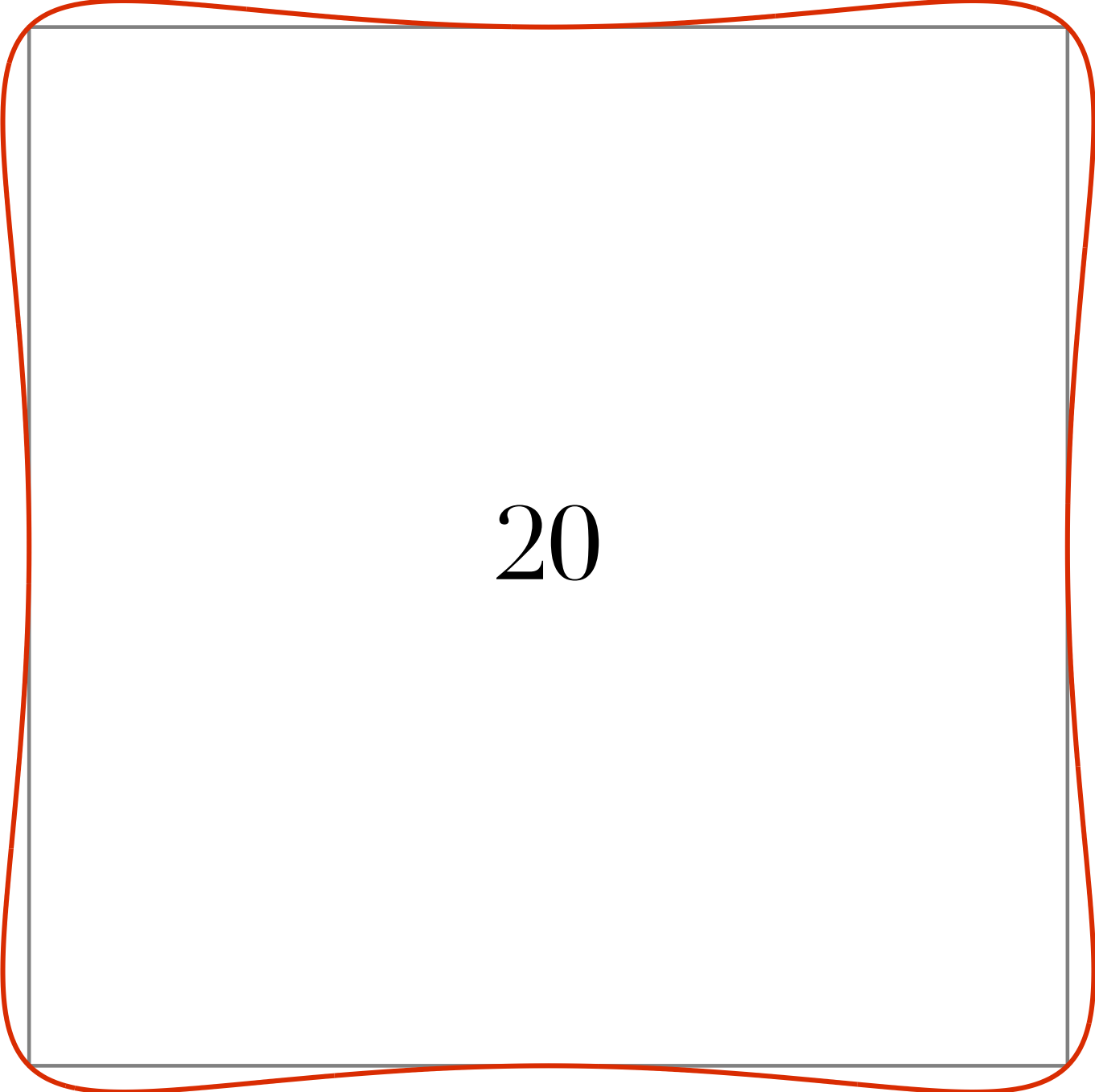
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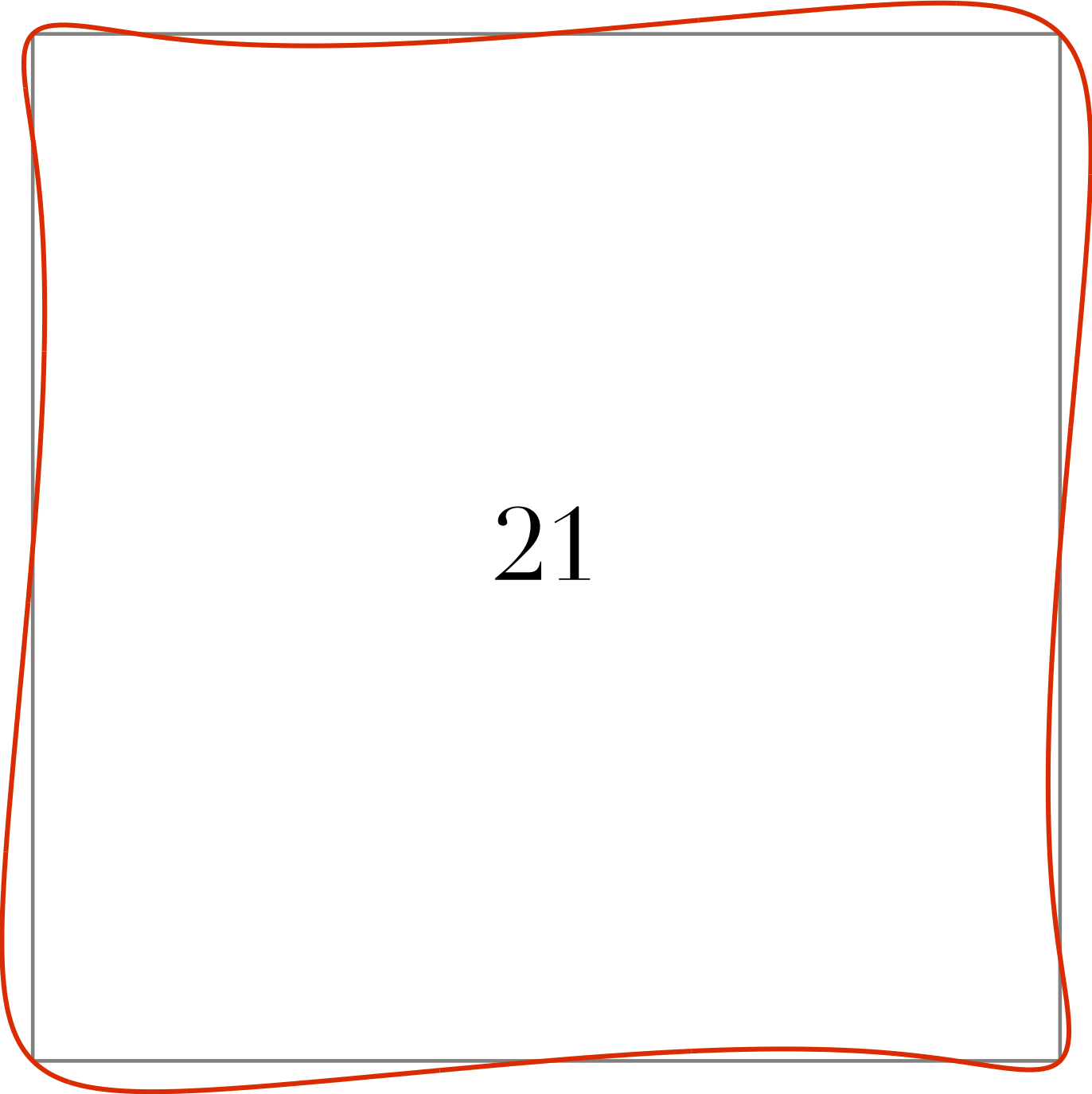
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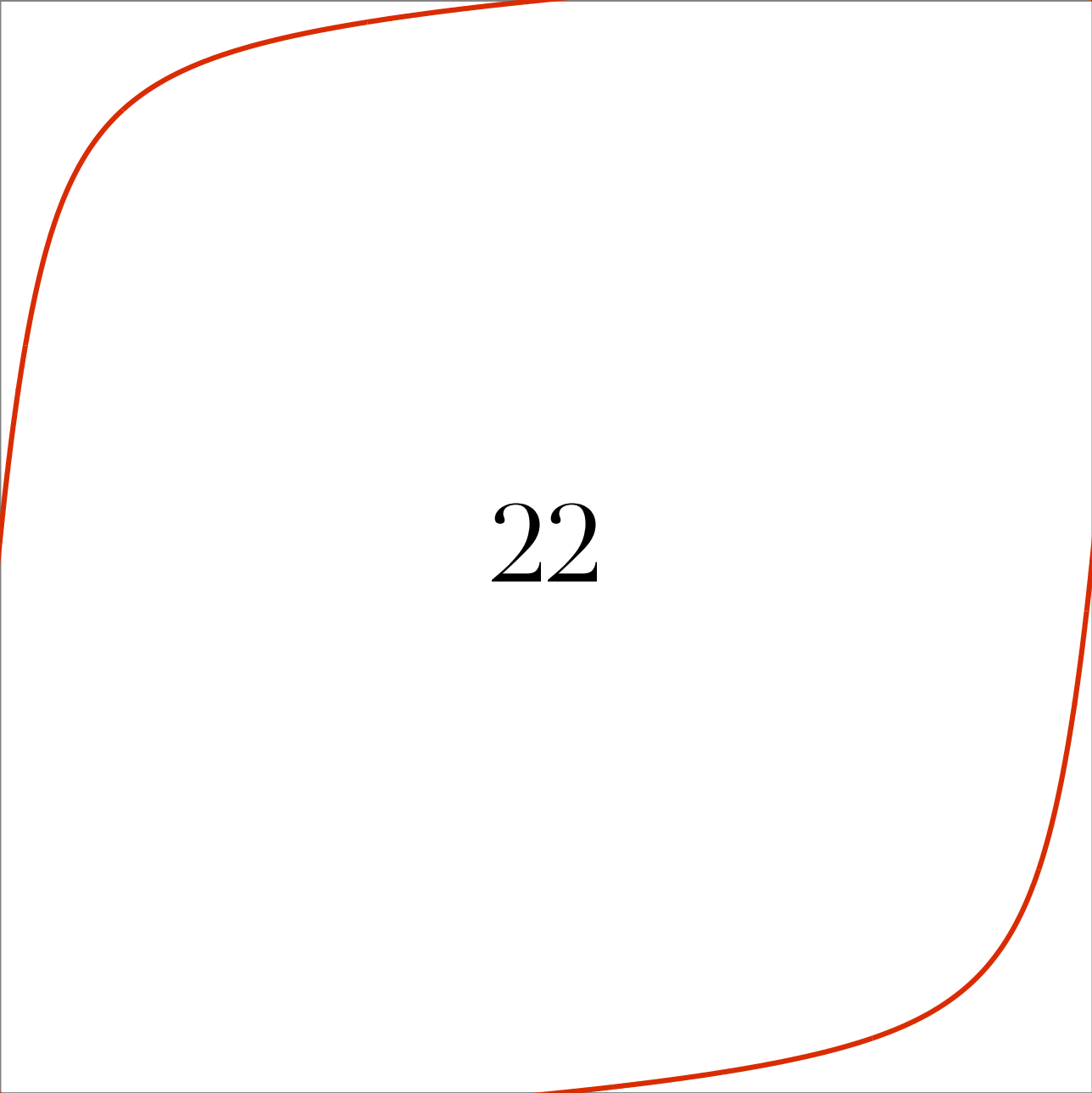
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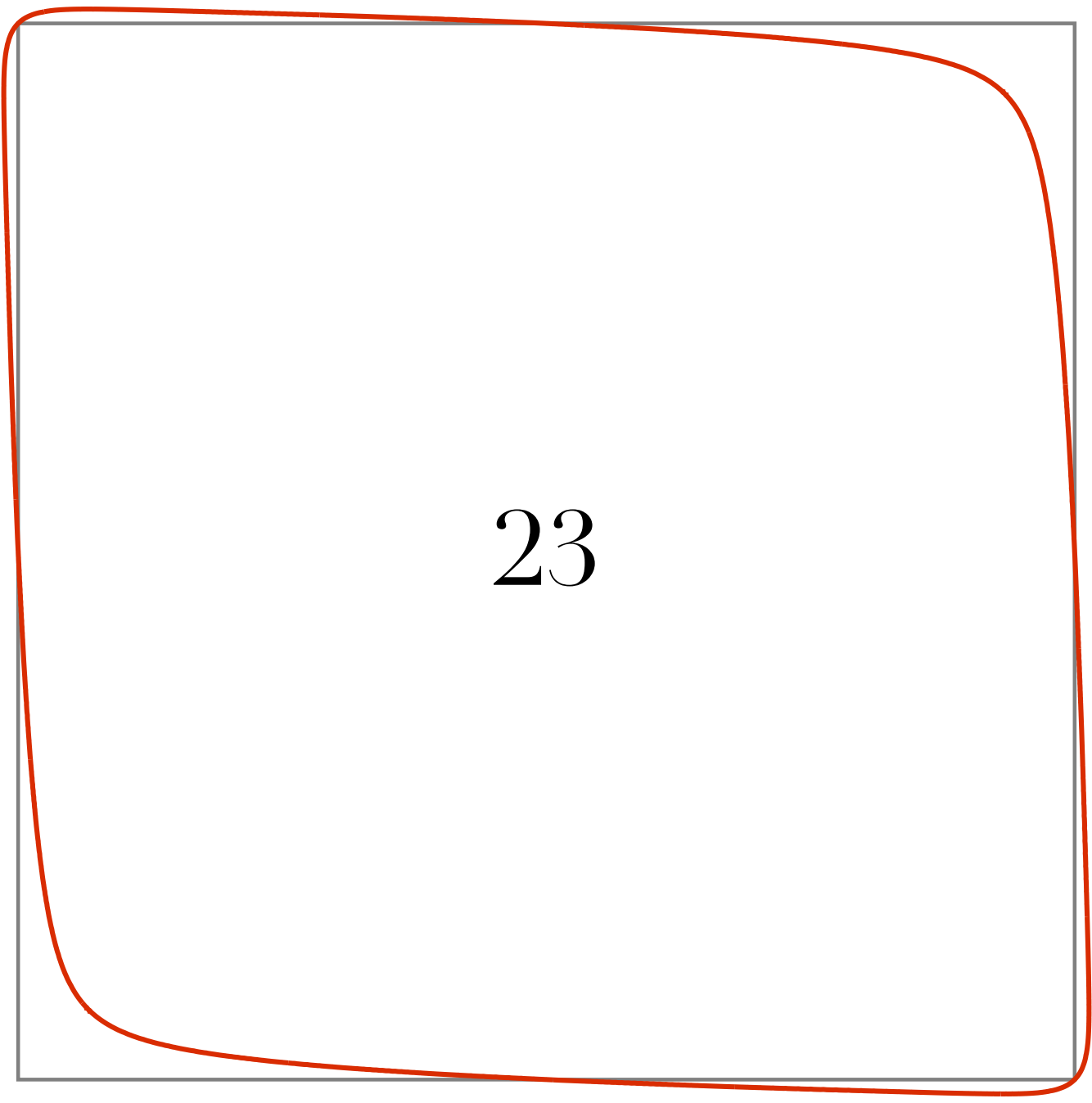
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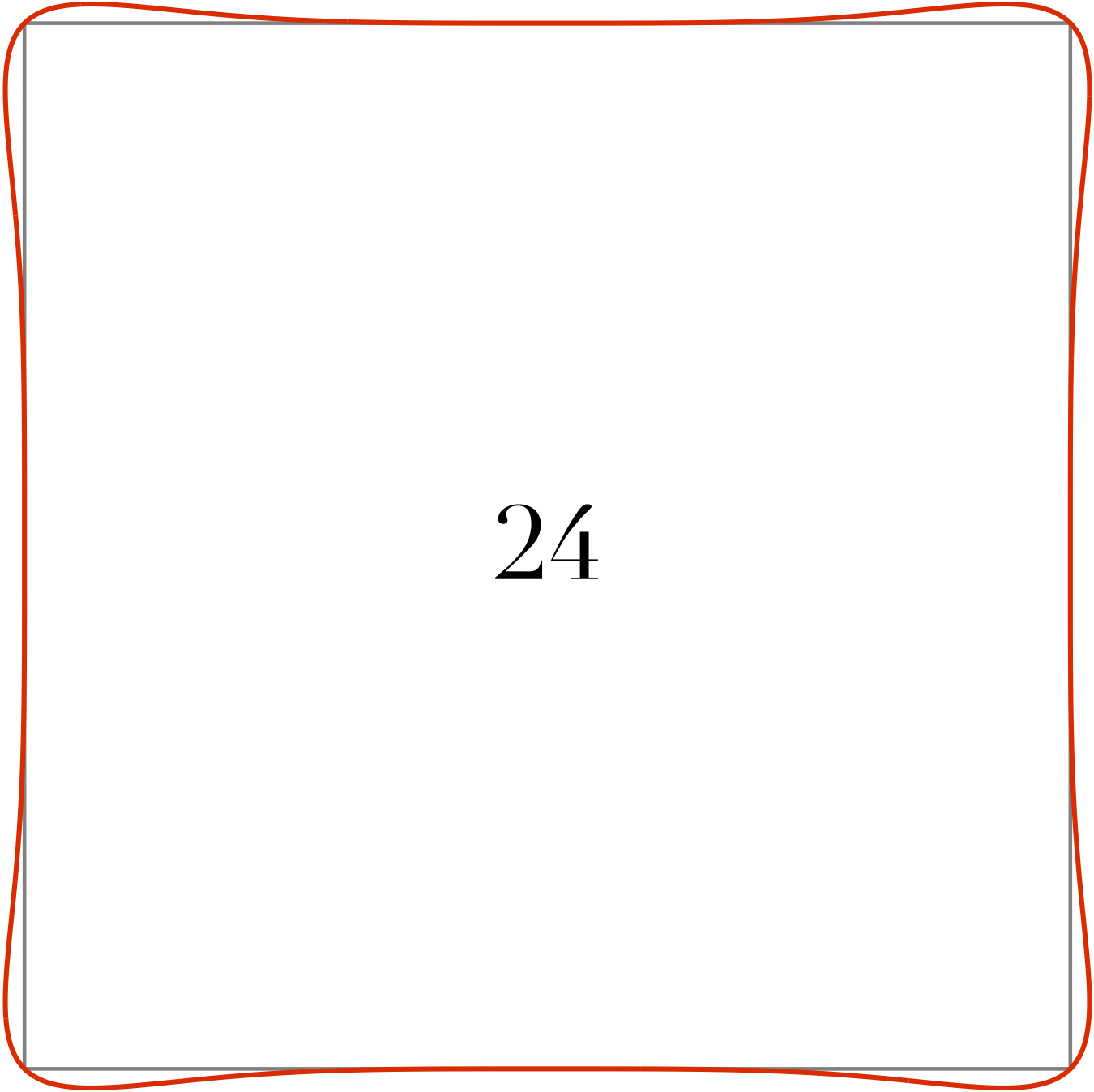
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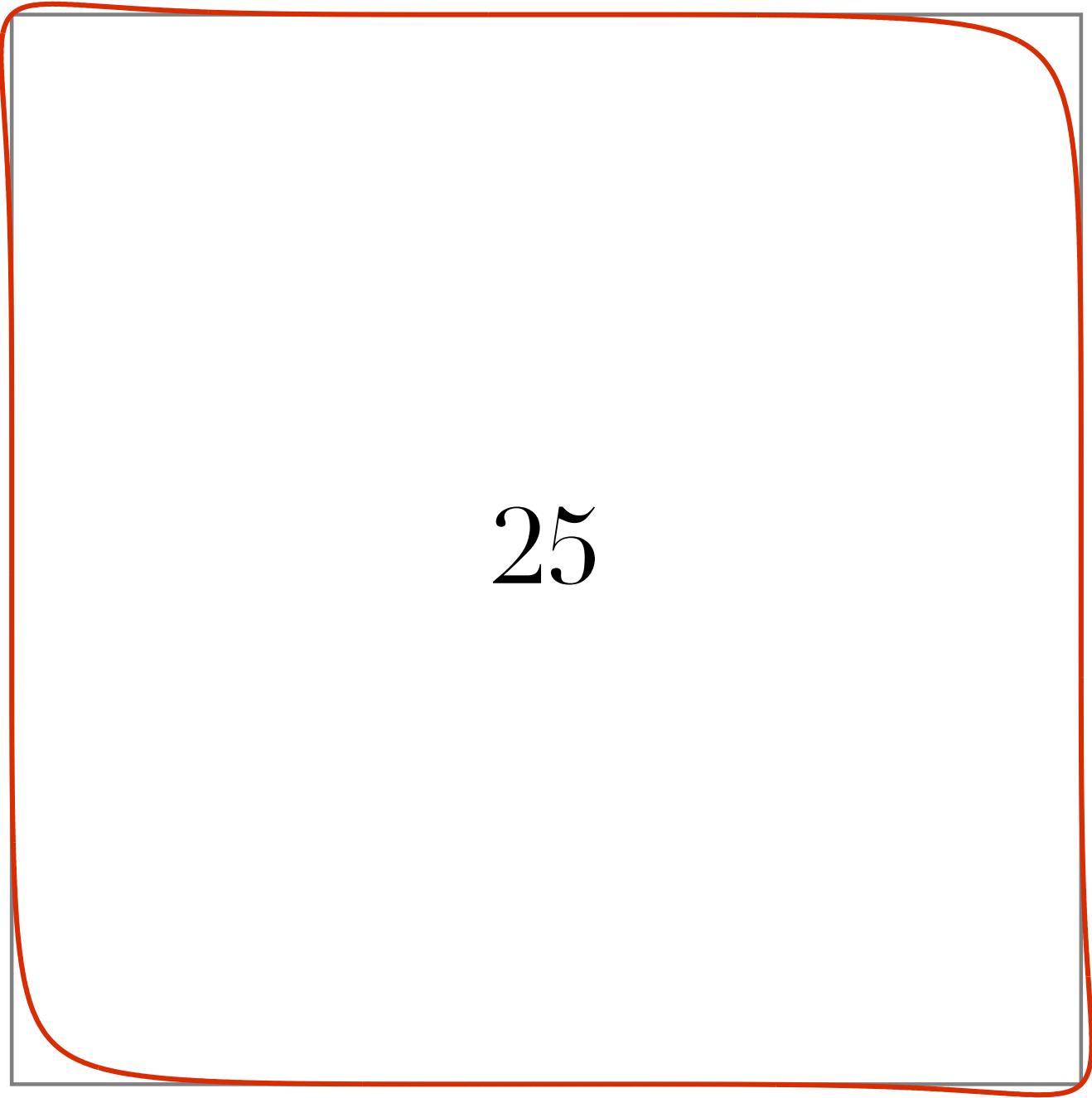
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A thick red line forms a rounded rectangular border around the central text. The corners are smoothly curved, and the line is uniform in thickness.

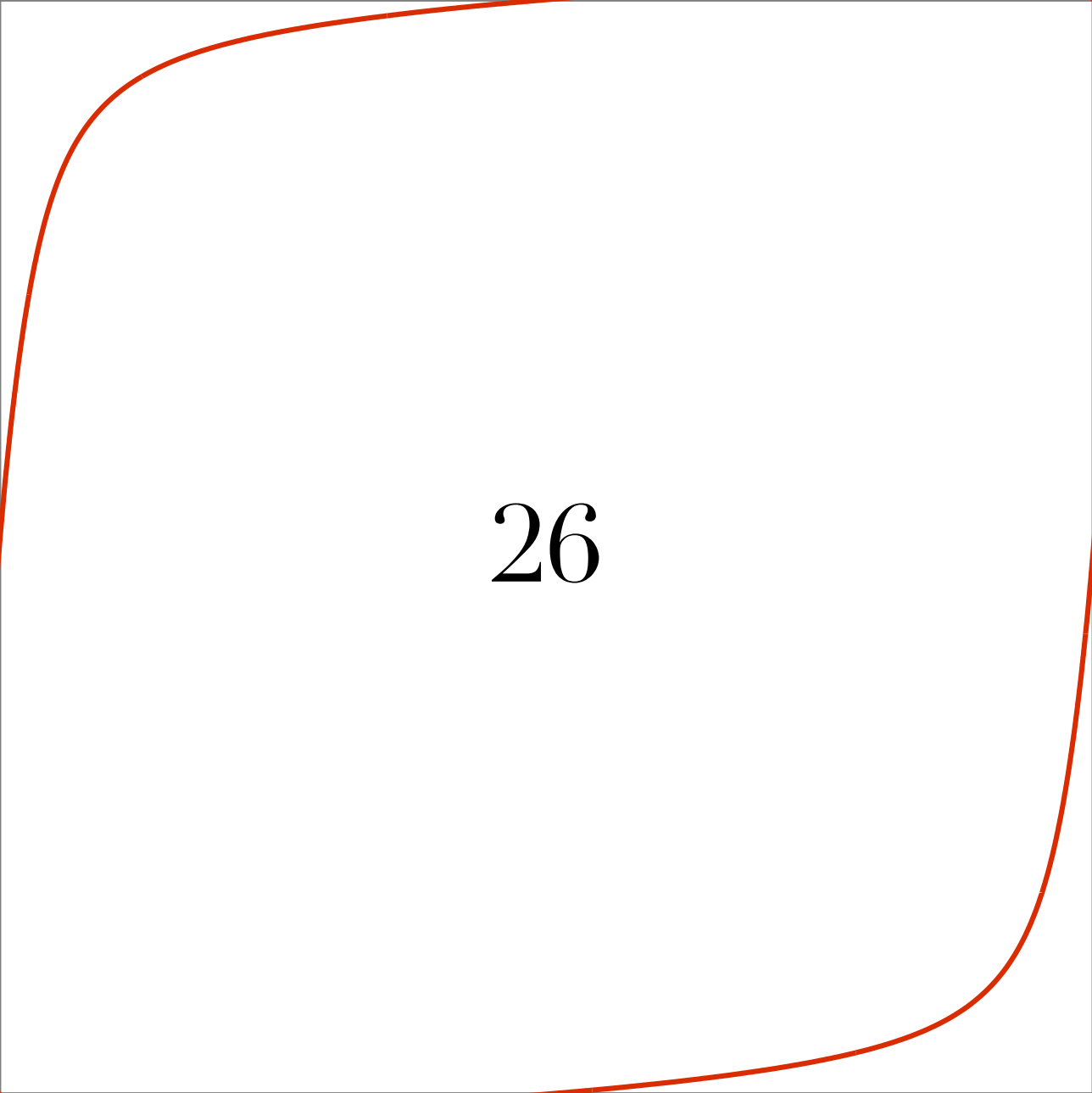
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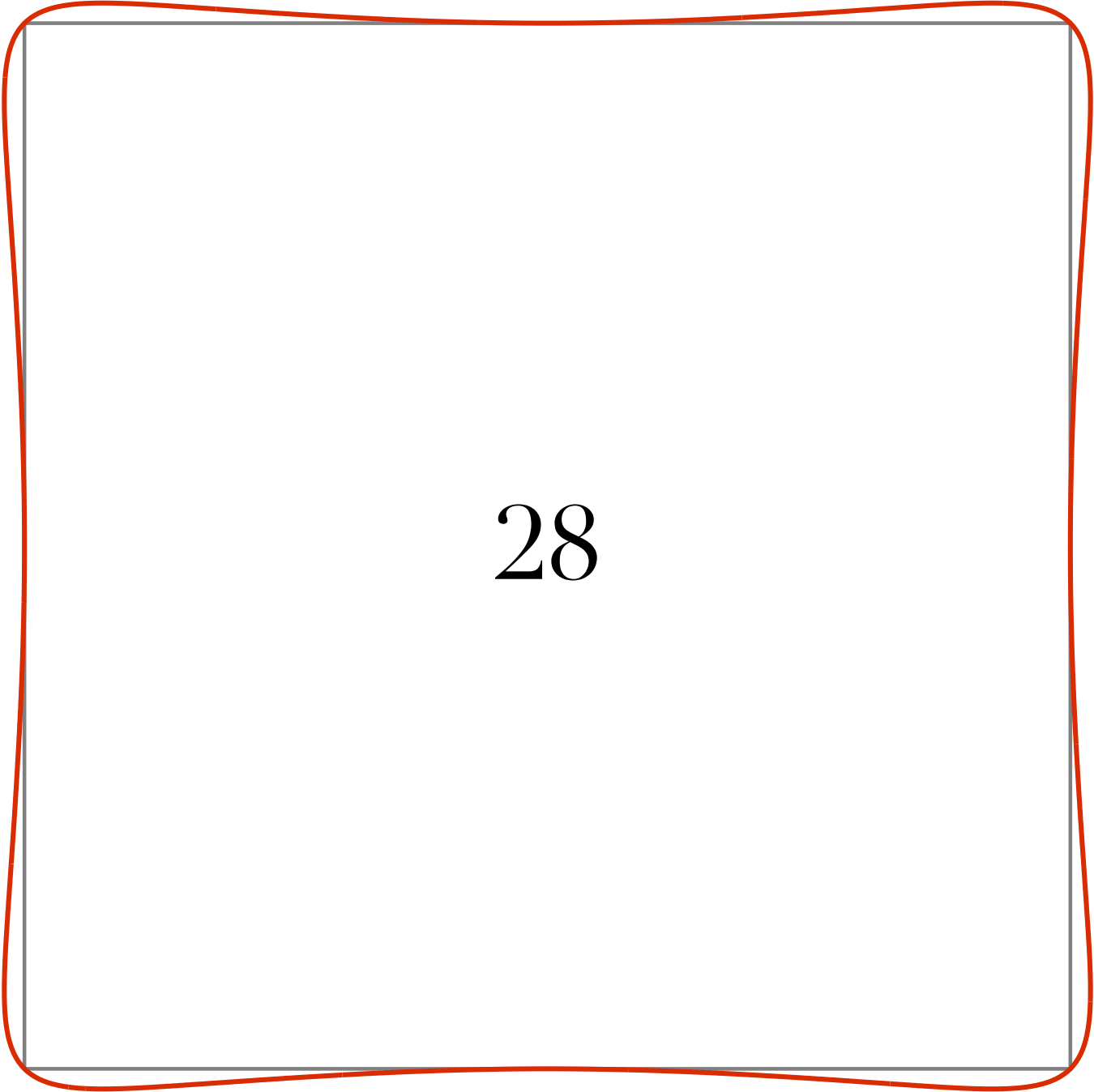


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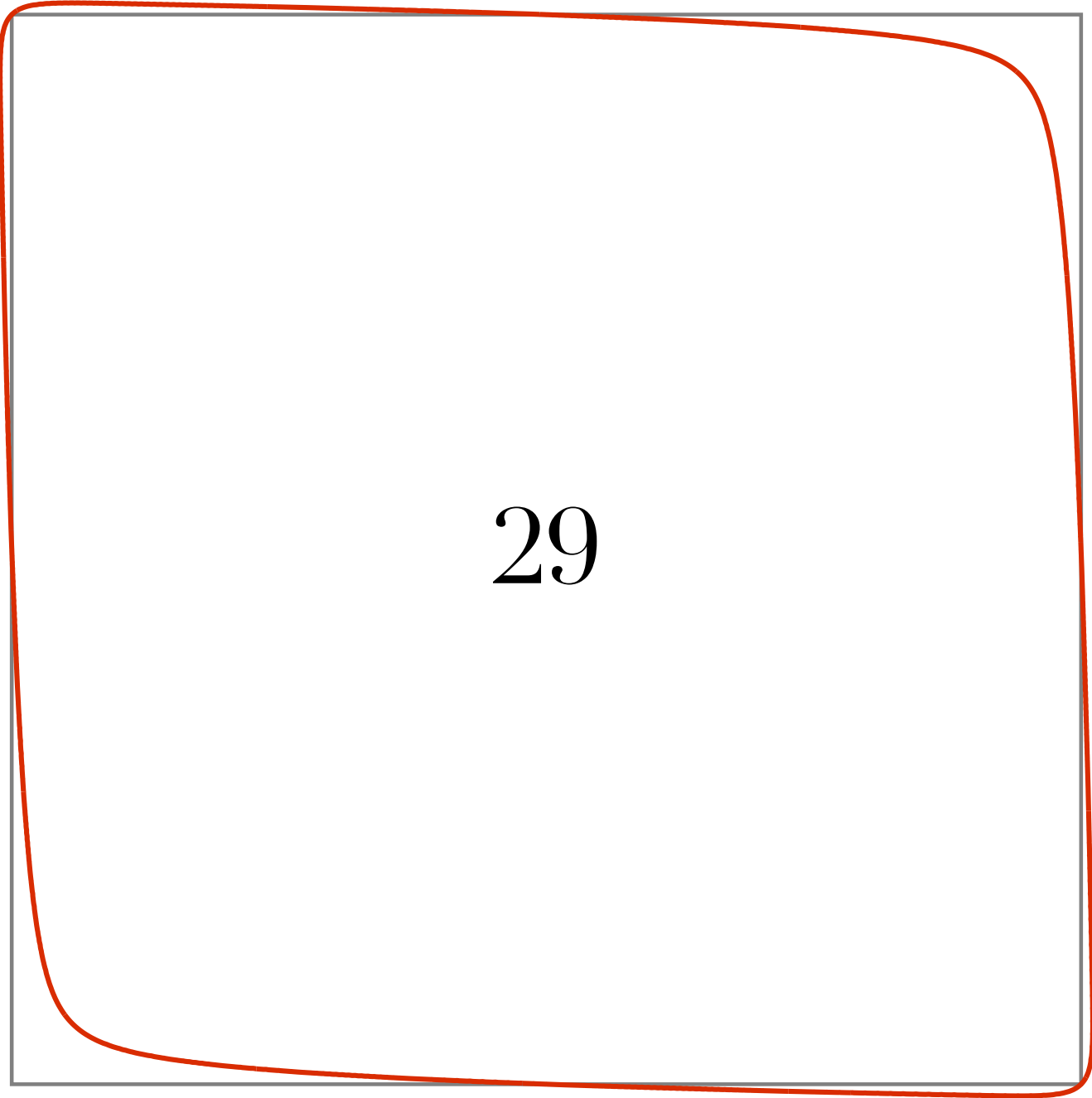


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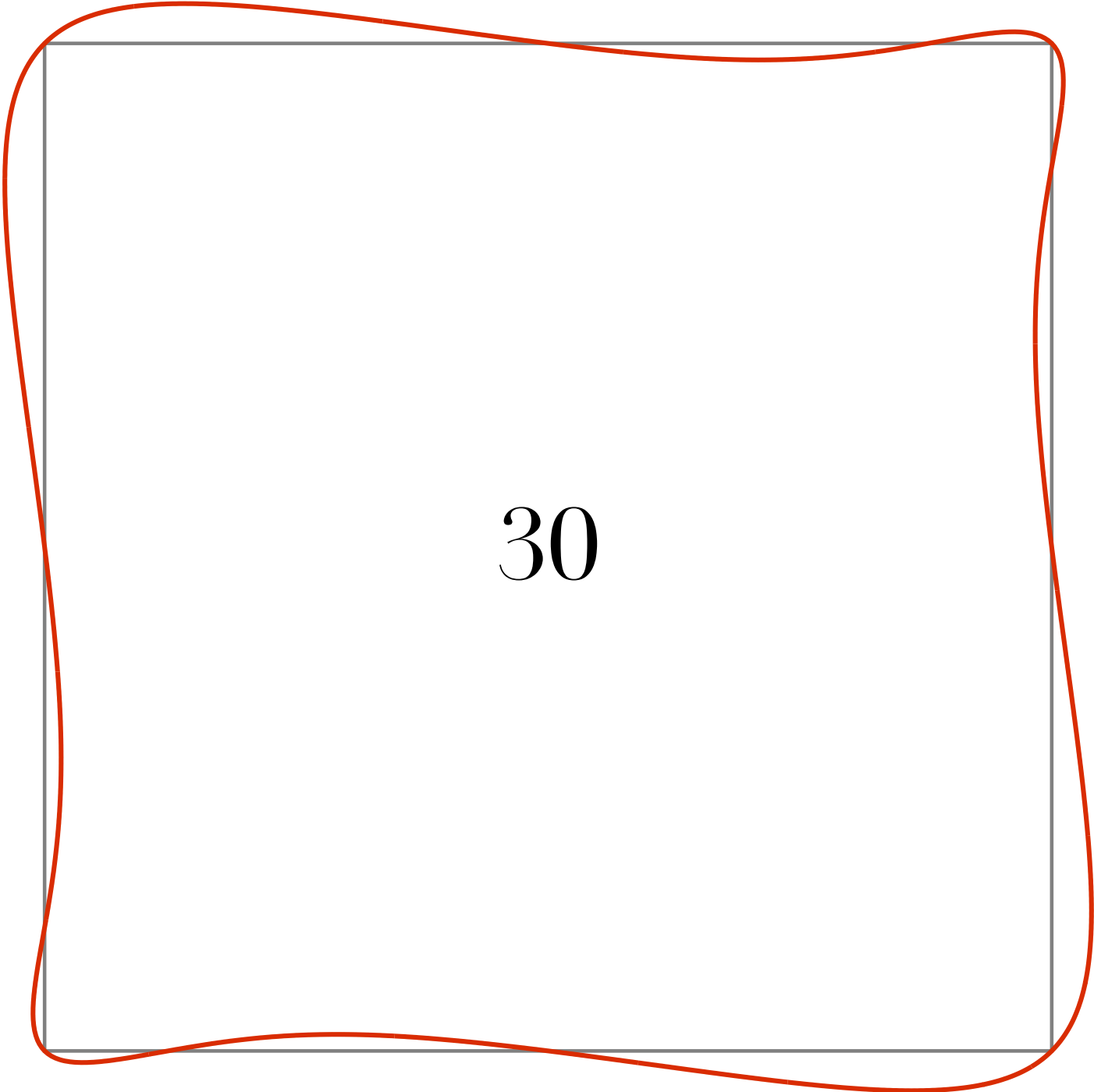
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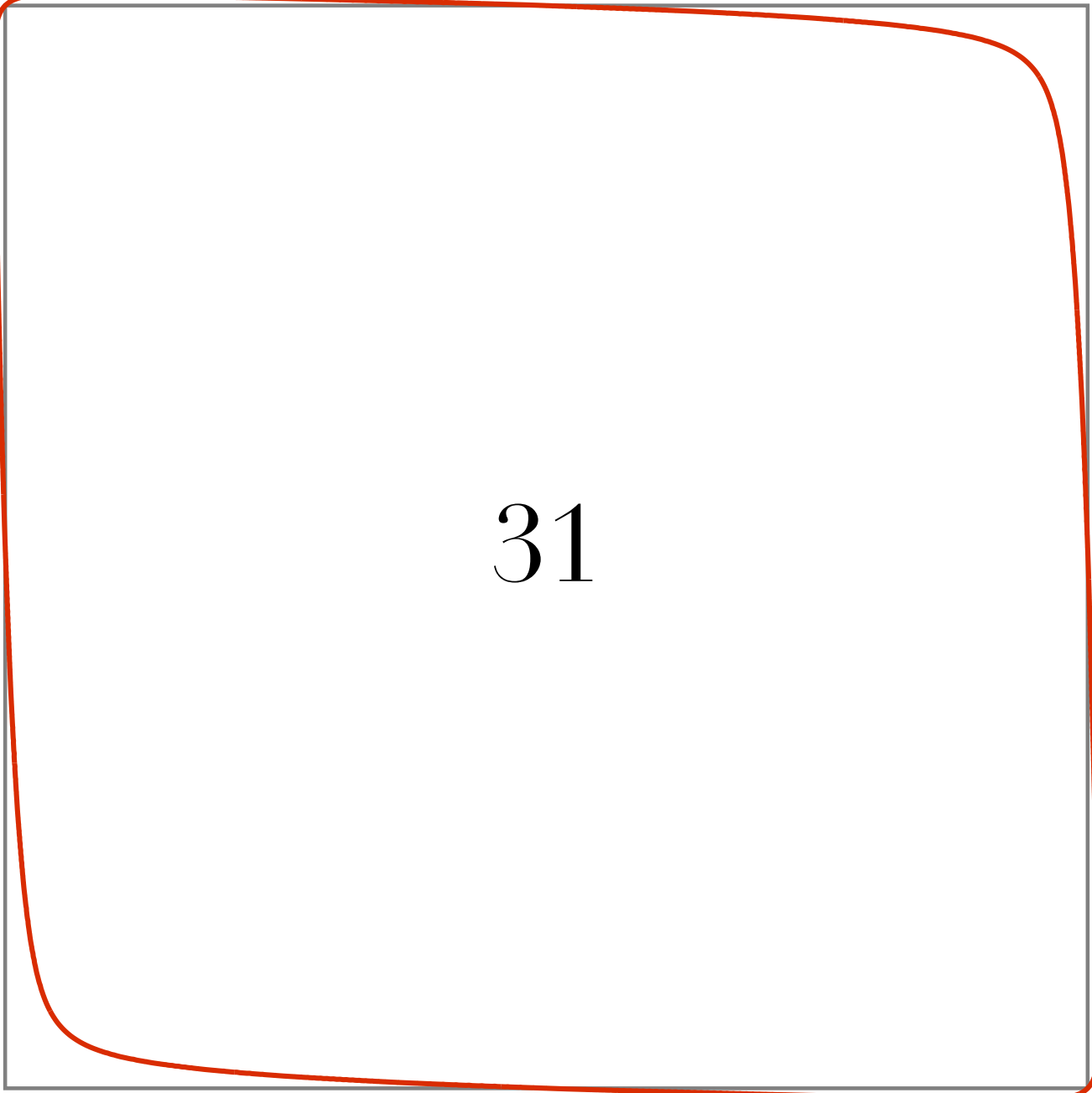
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A thick red line forms a rounded rectangular border around the central text. The corners are smoothly curved, and the line is uniform in thickness.

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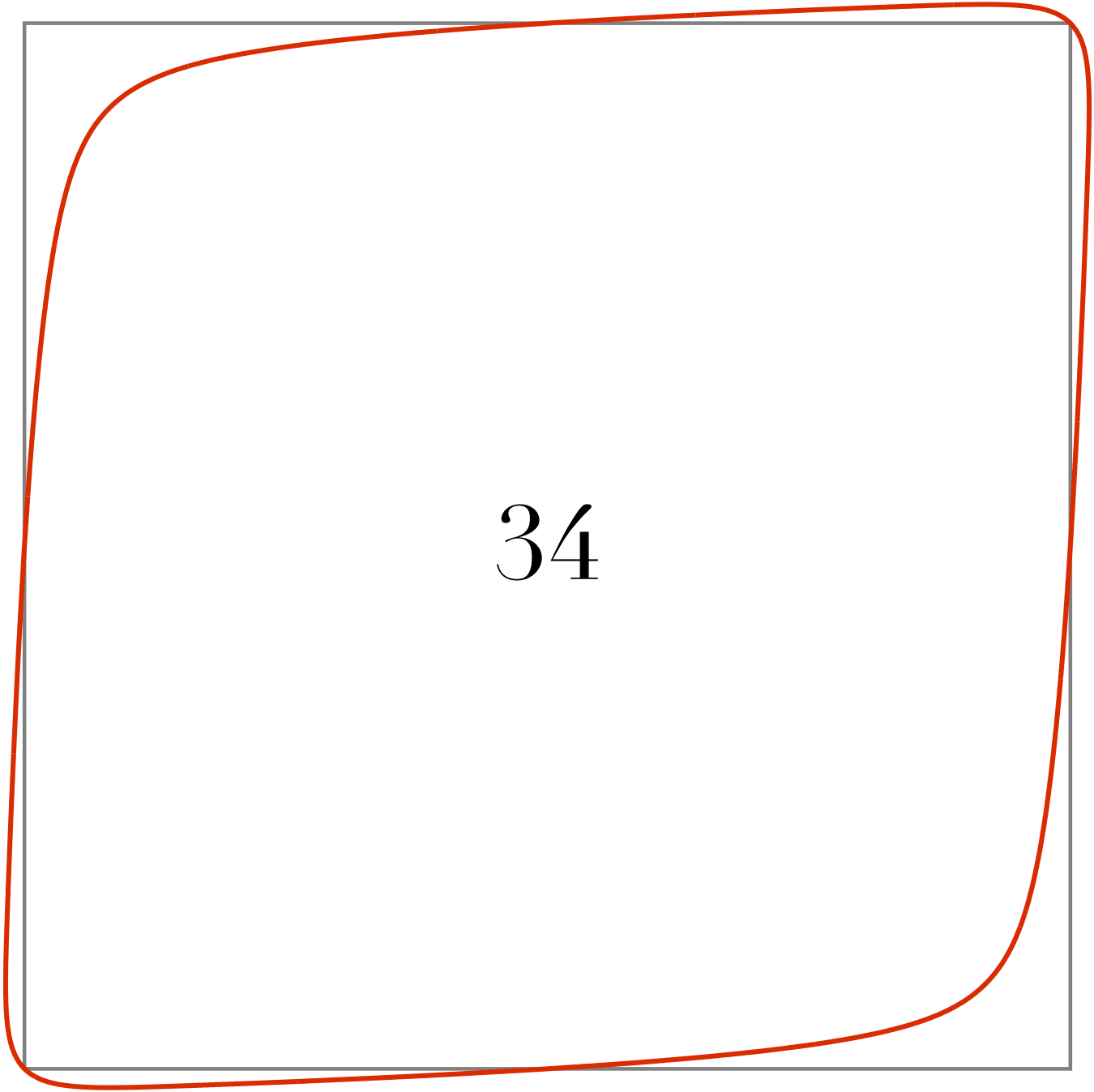
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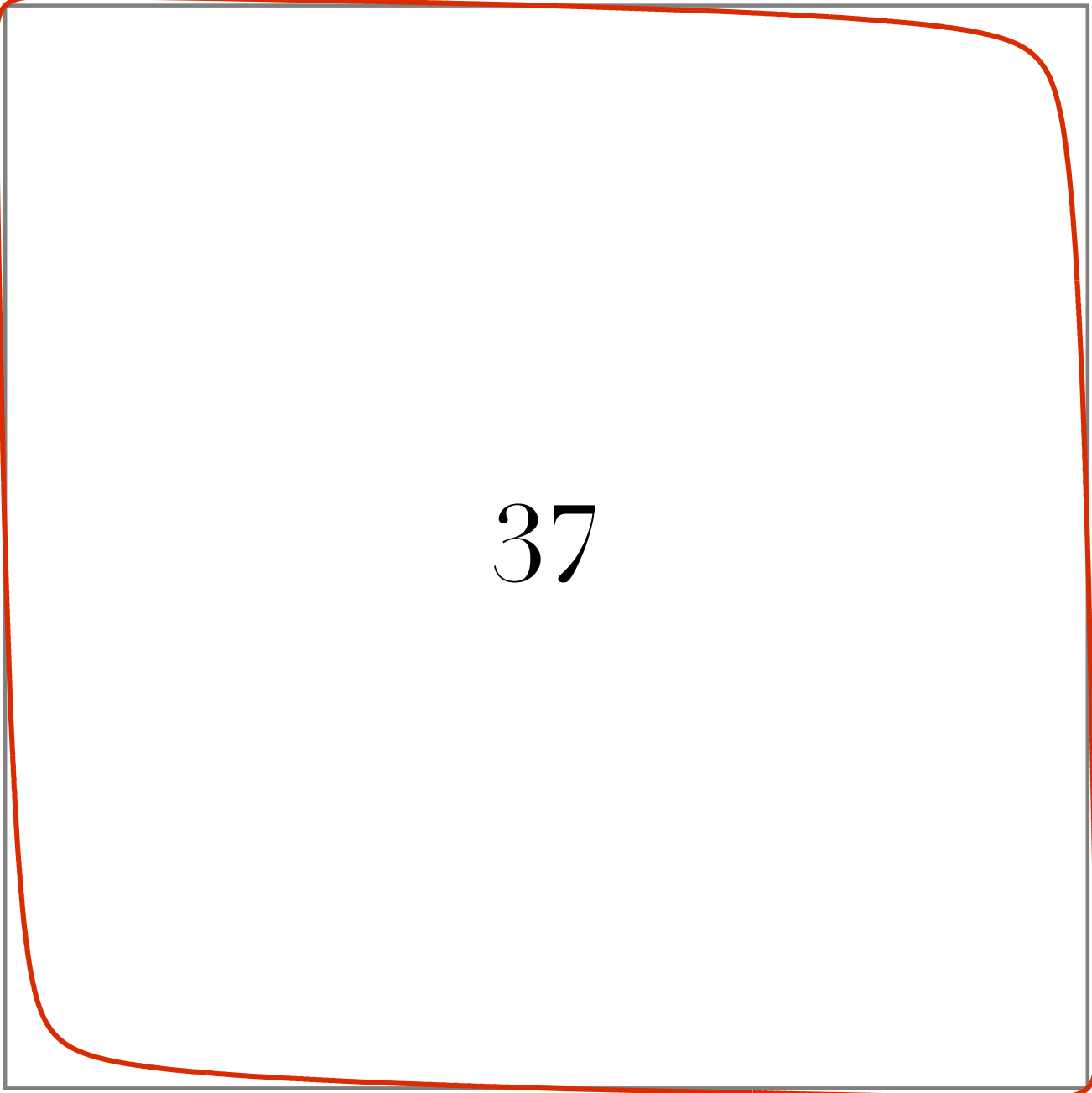
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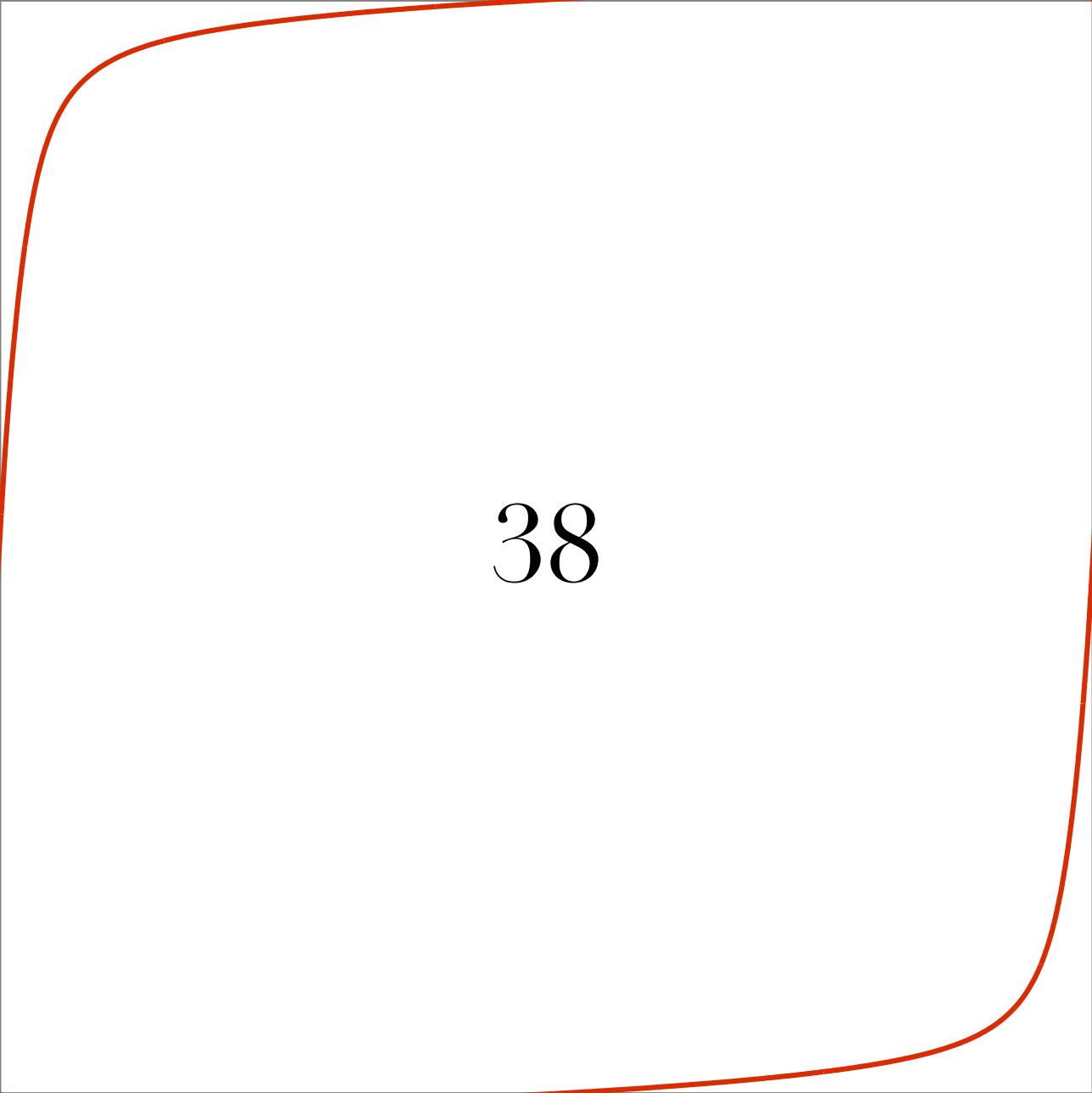
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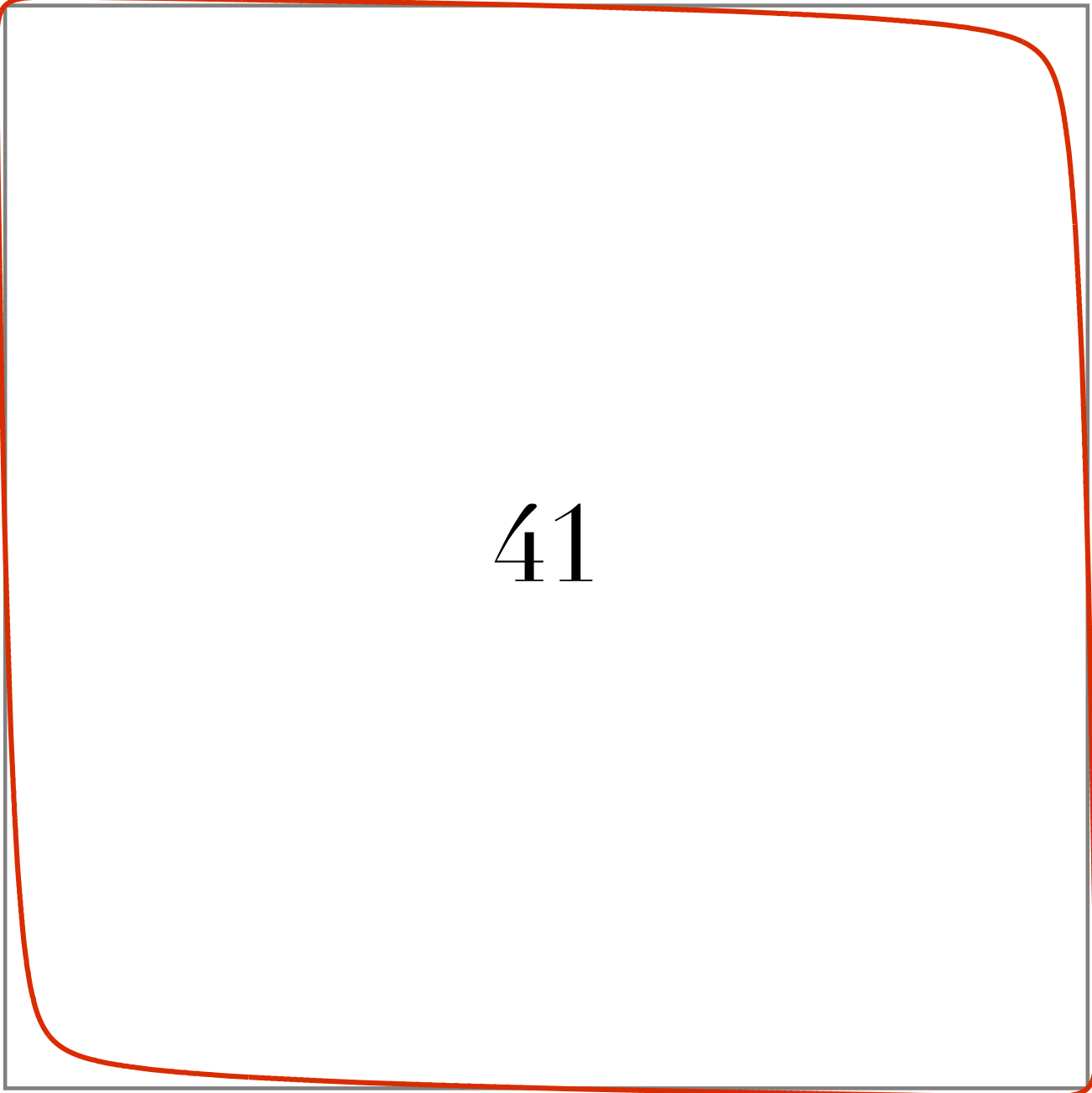
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A square frame with a thin gray border. Inside the frame, there is a red line that forms a rounded square shape, following the inner edge of the gray border with rounded corners.

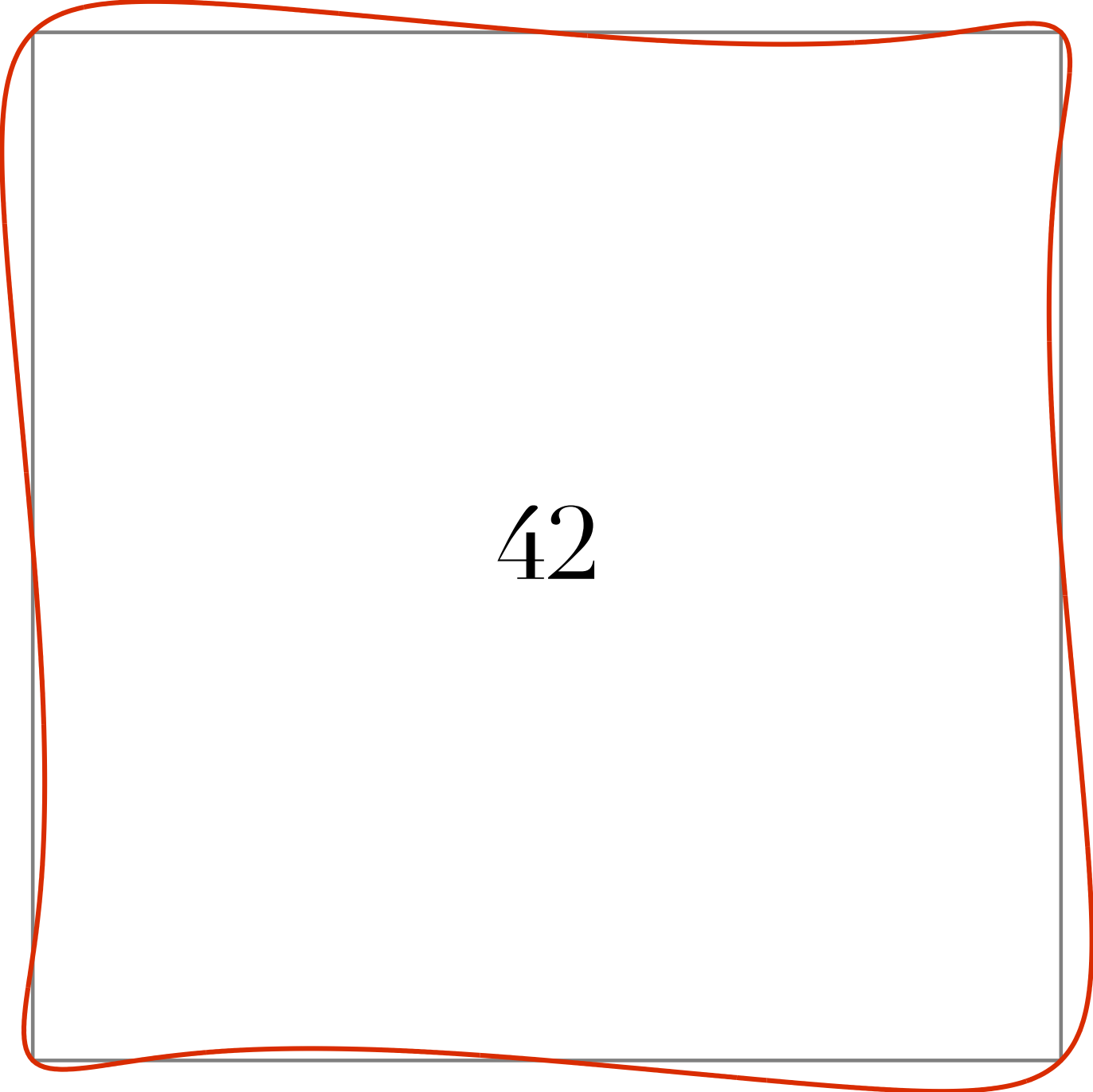
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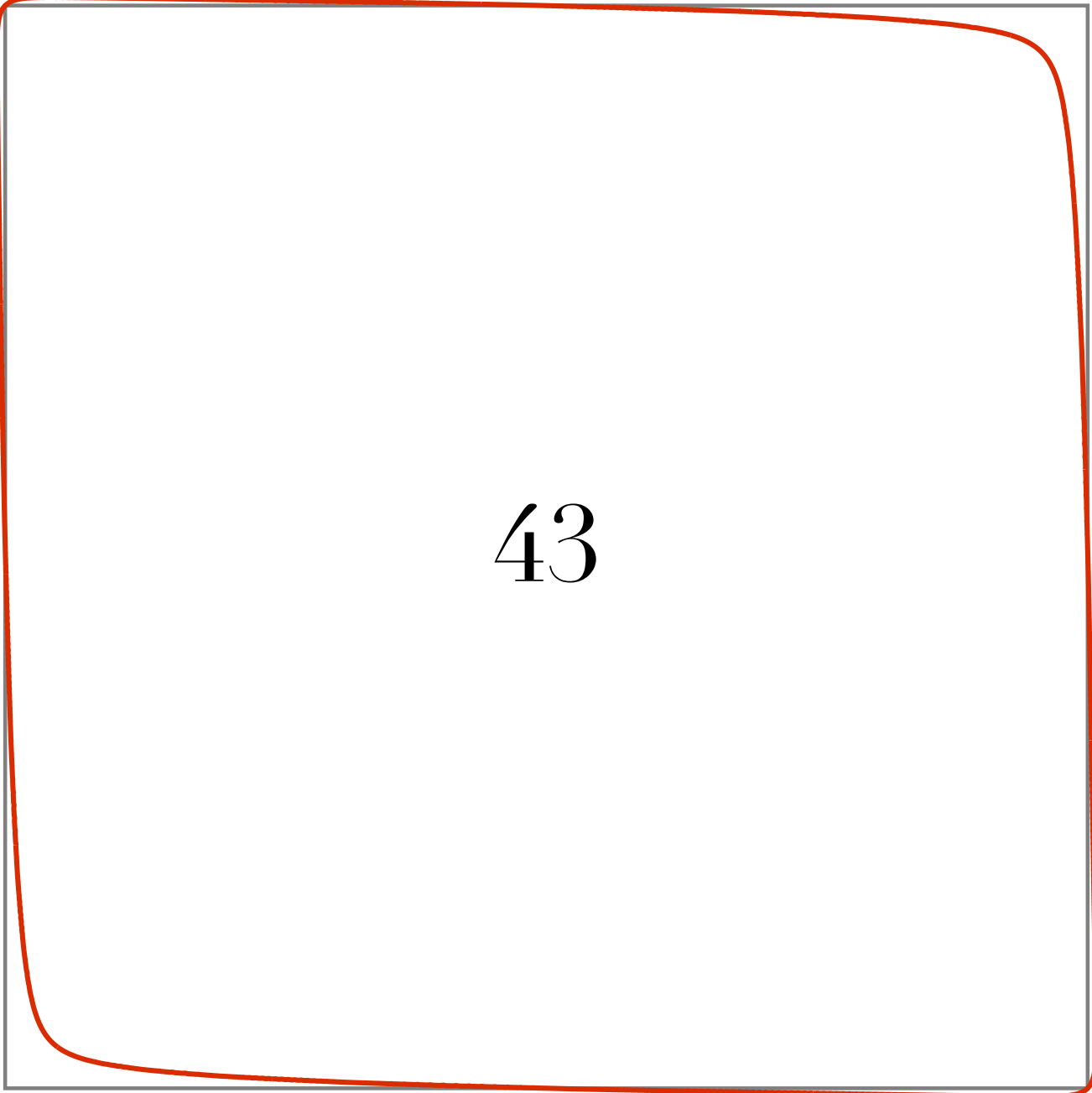
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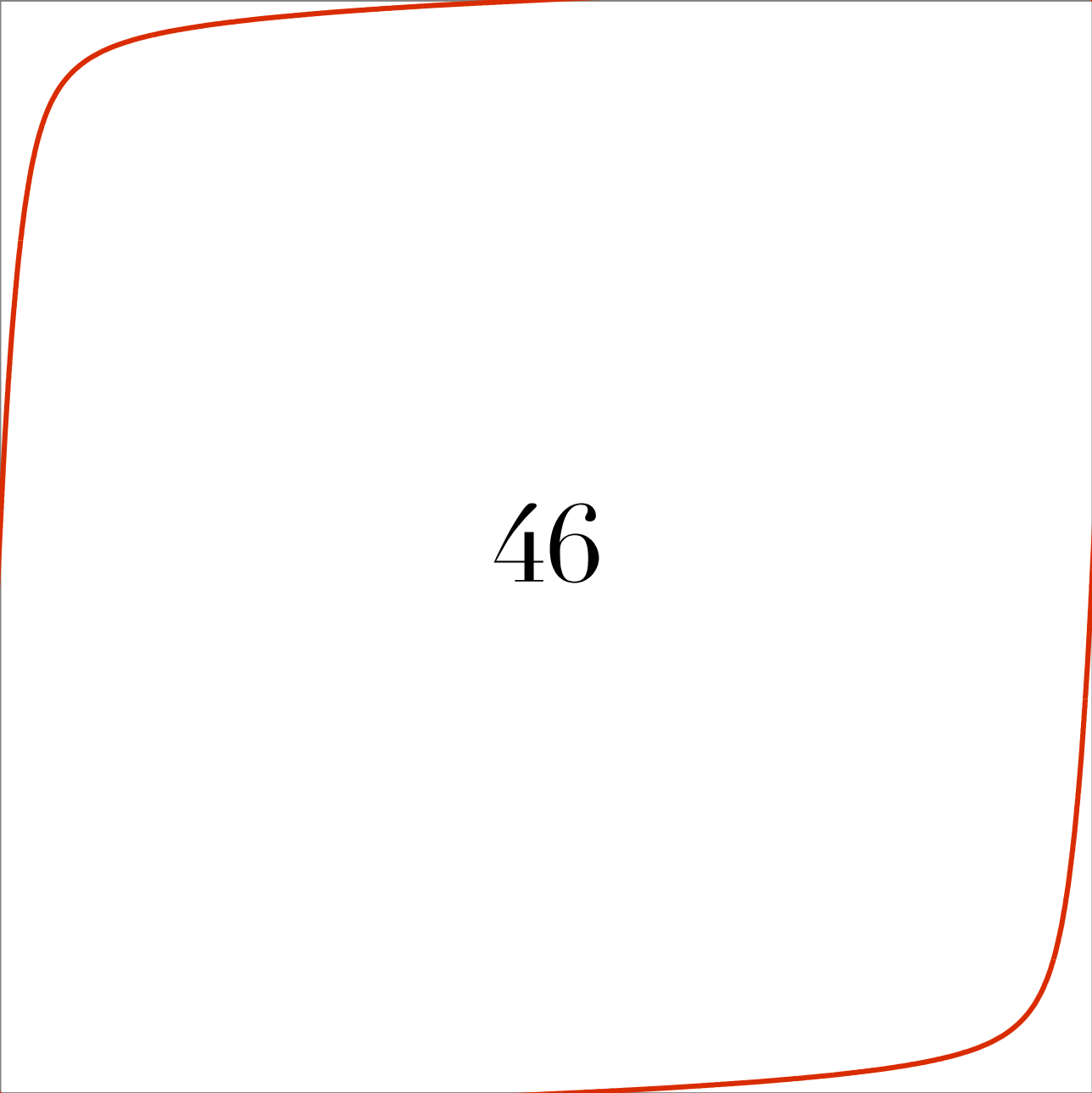
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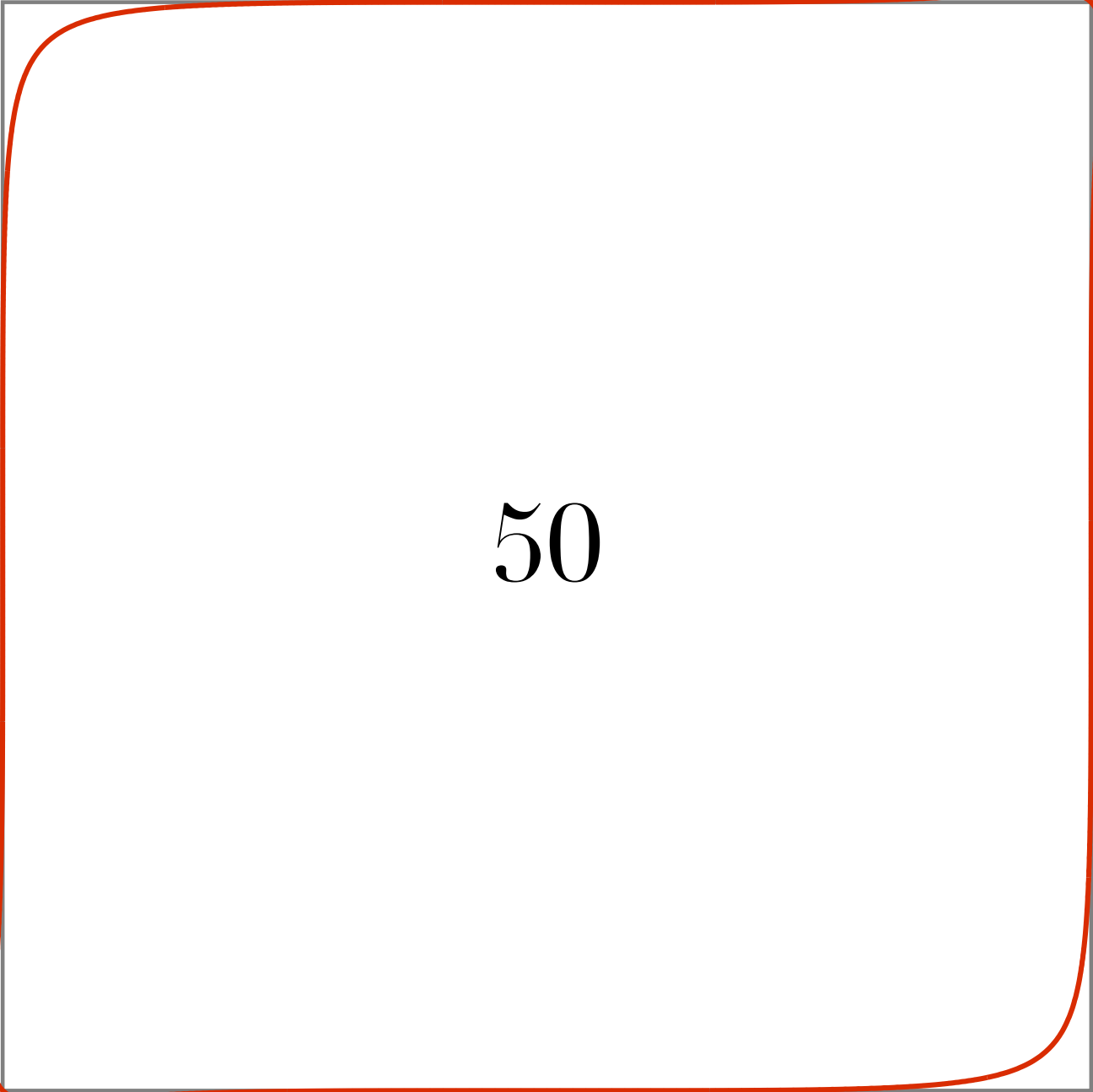


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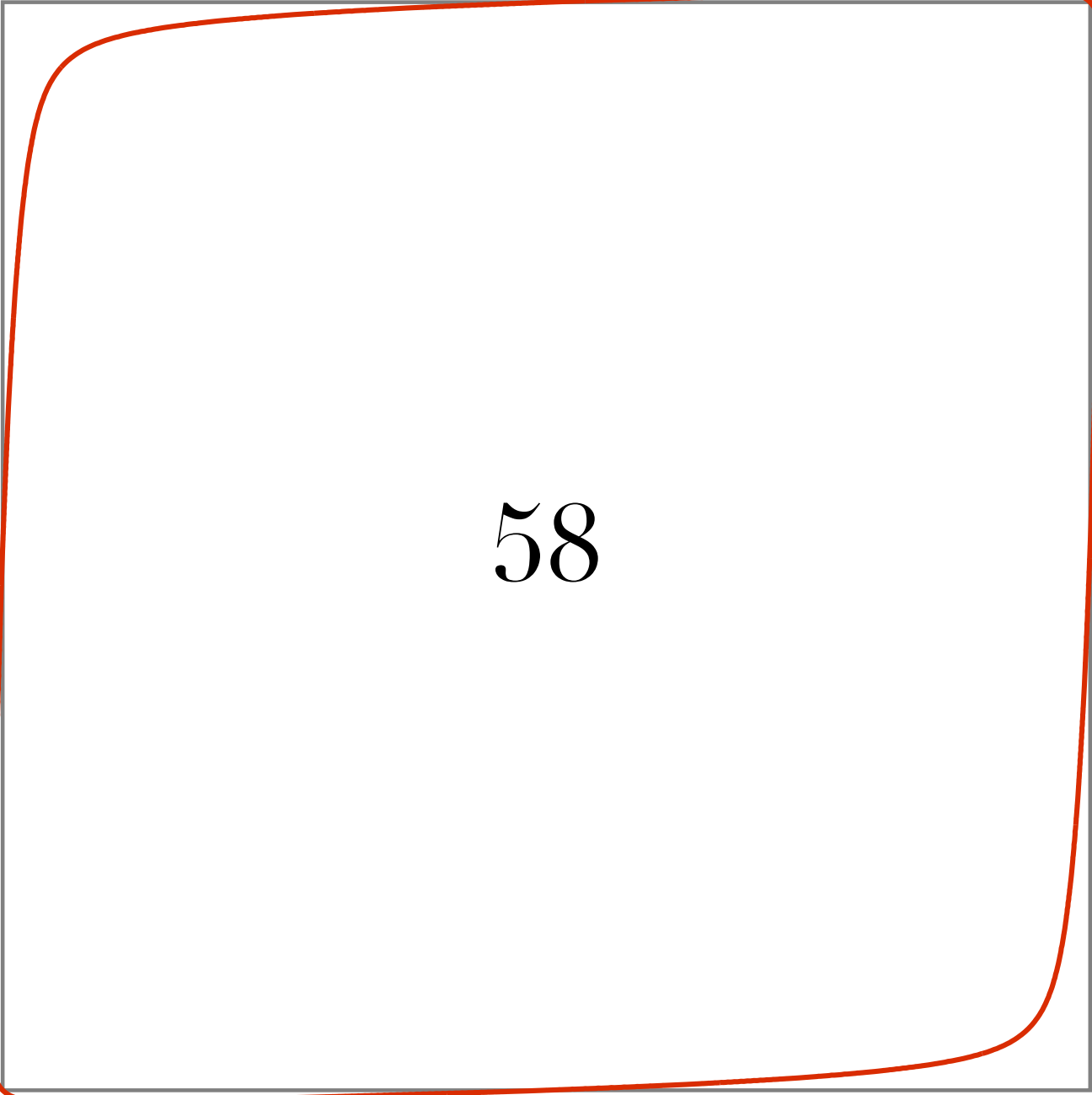
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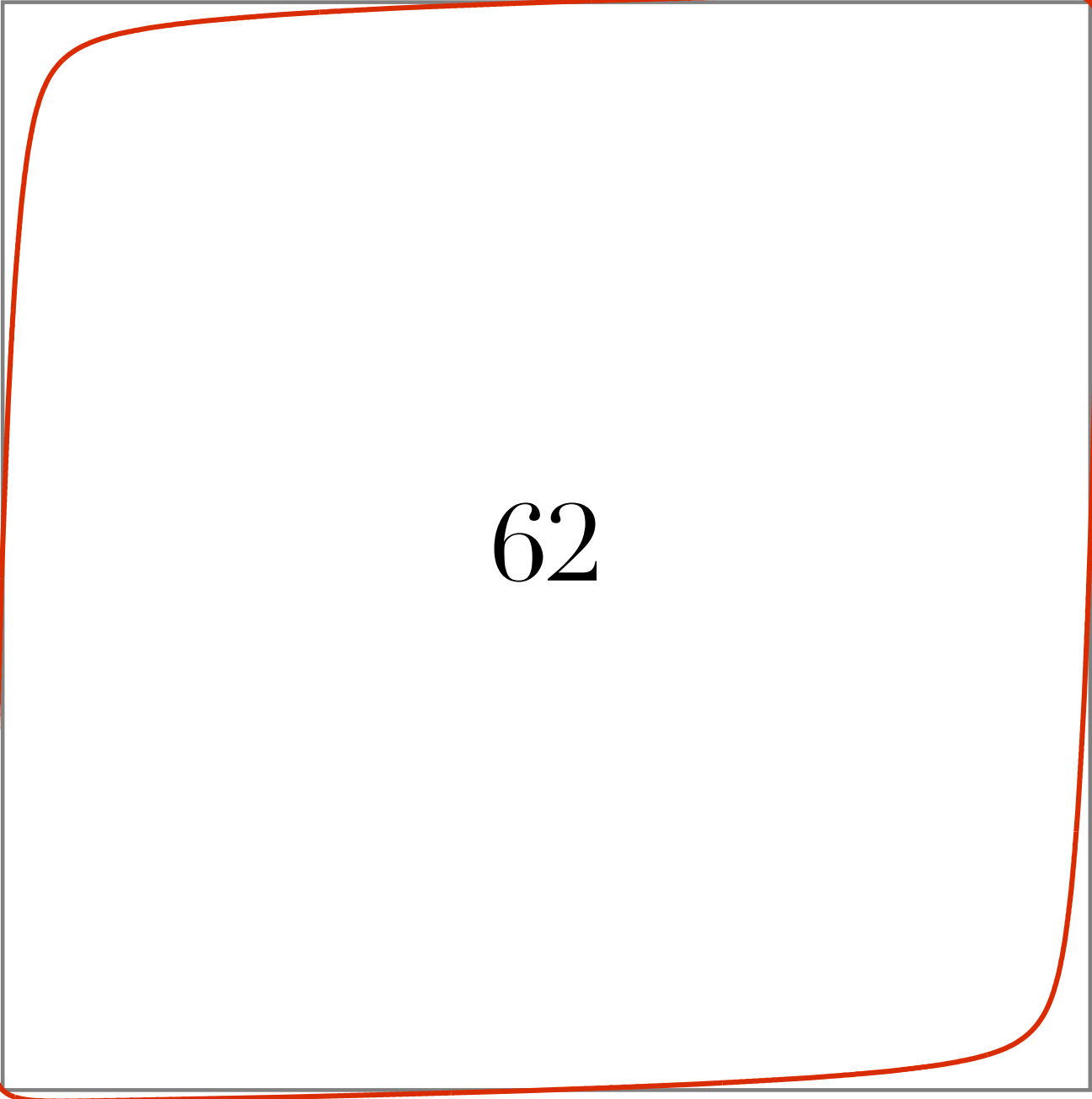


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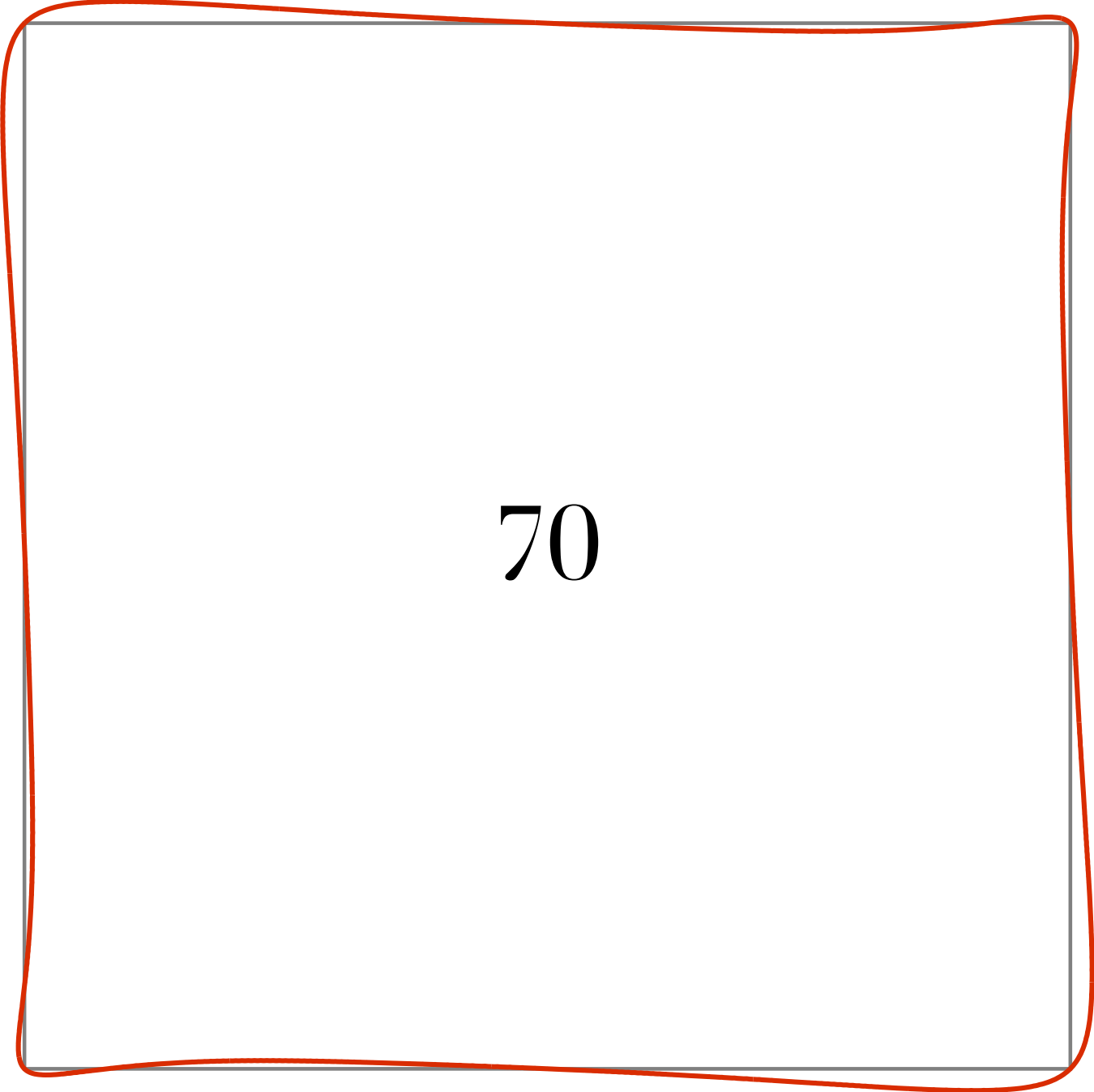
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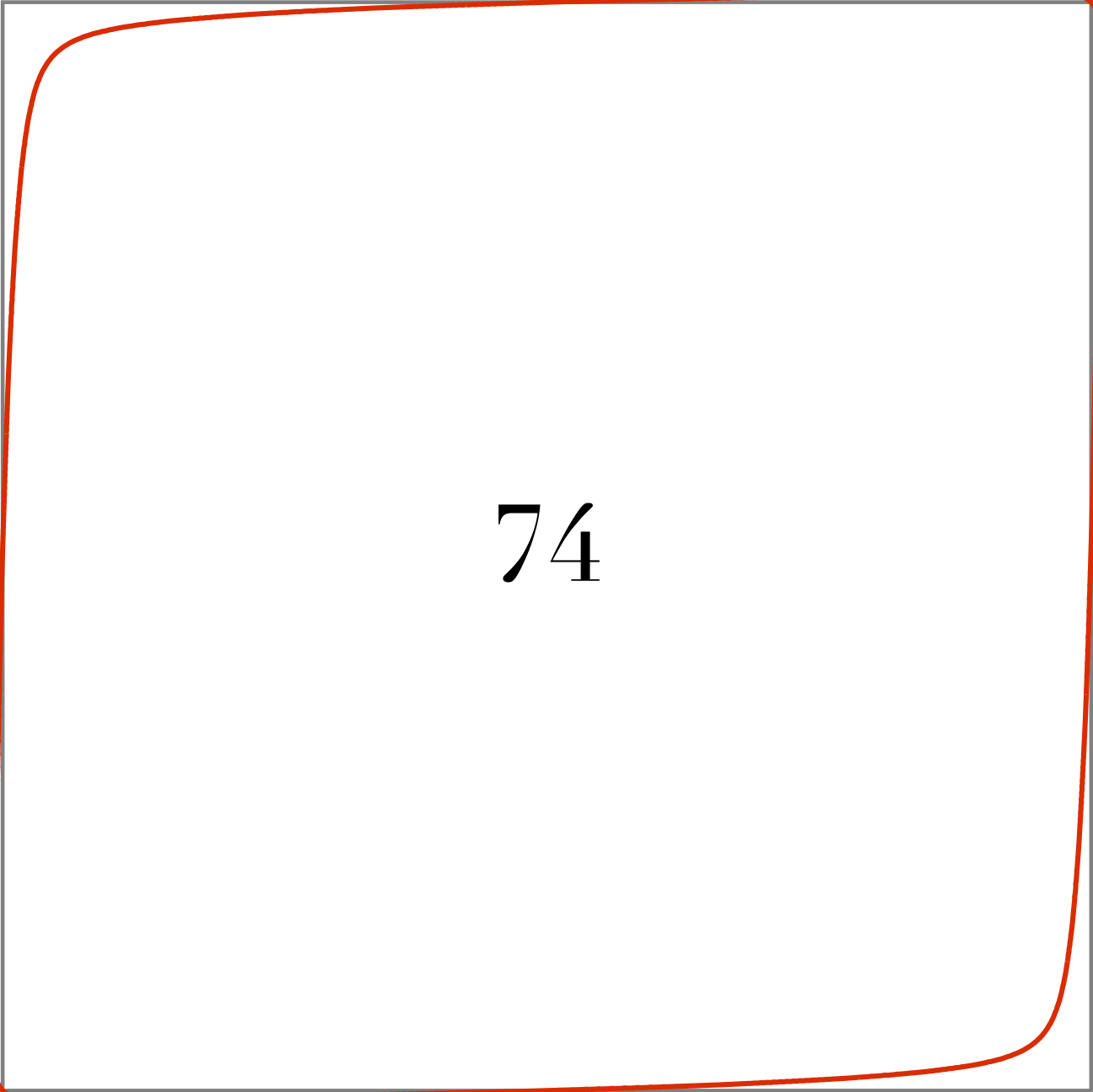


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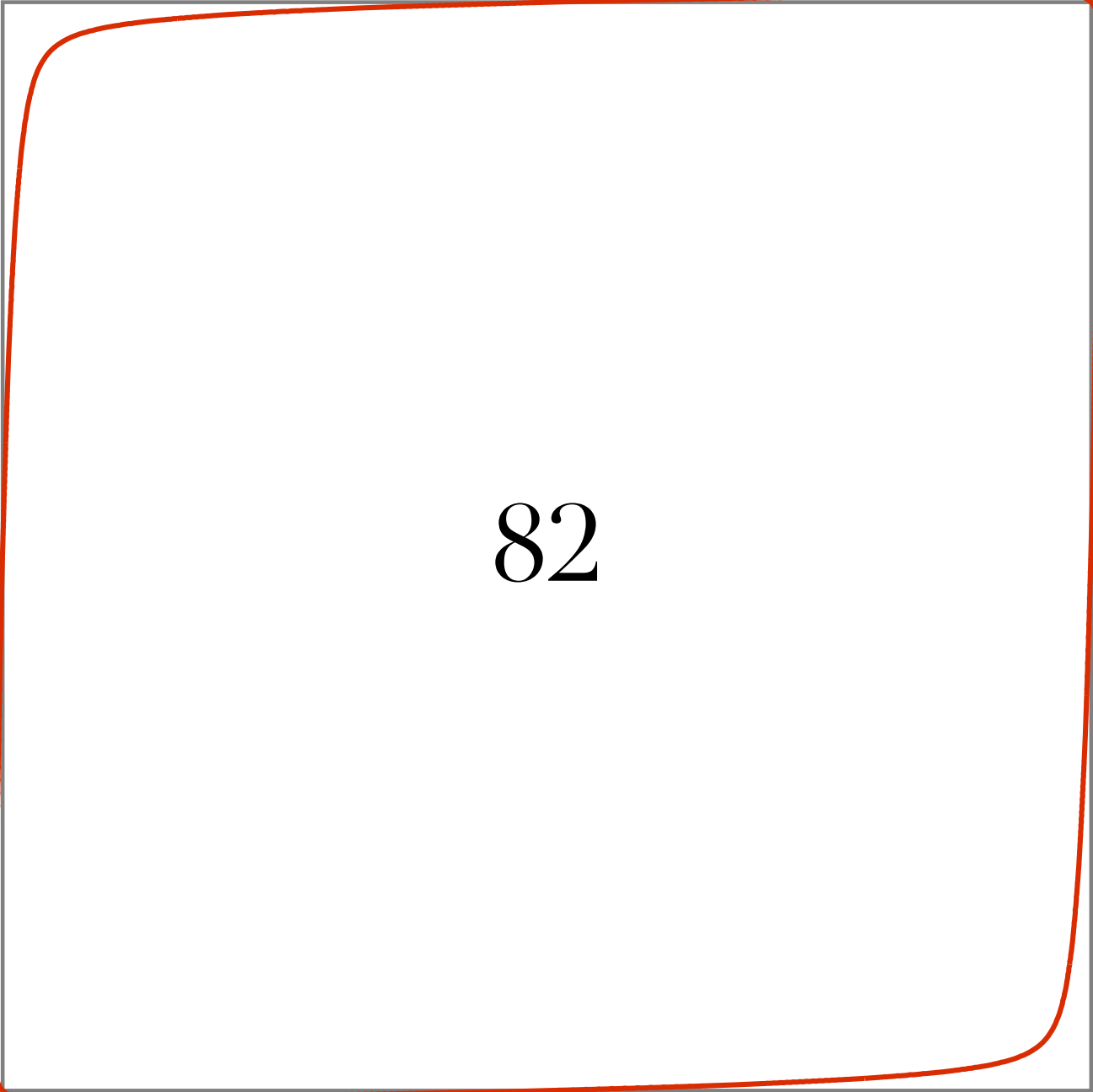
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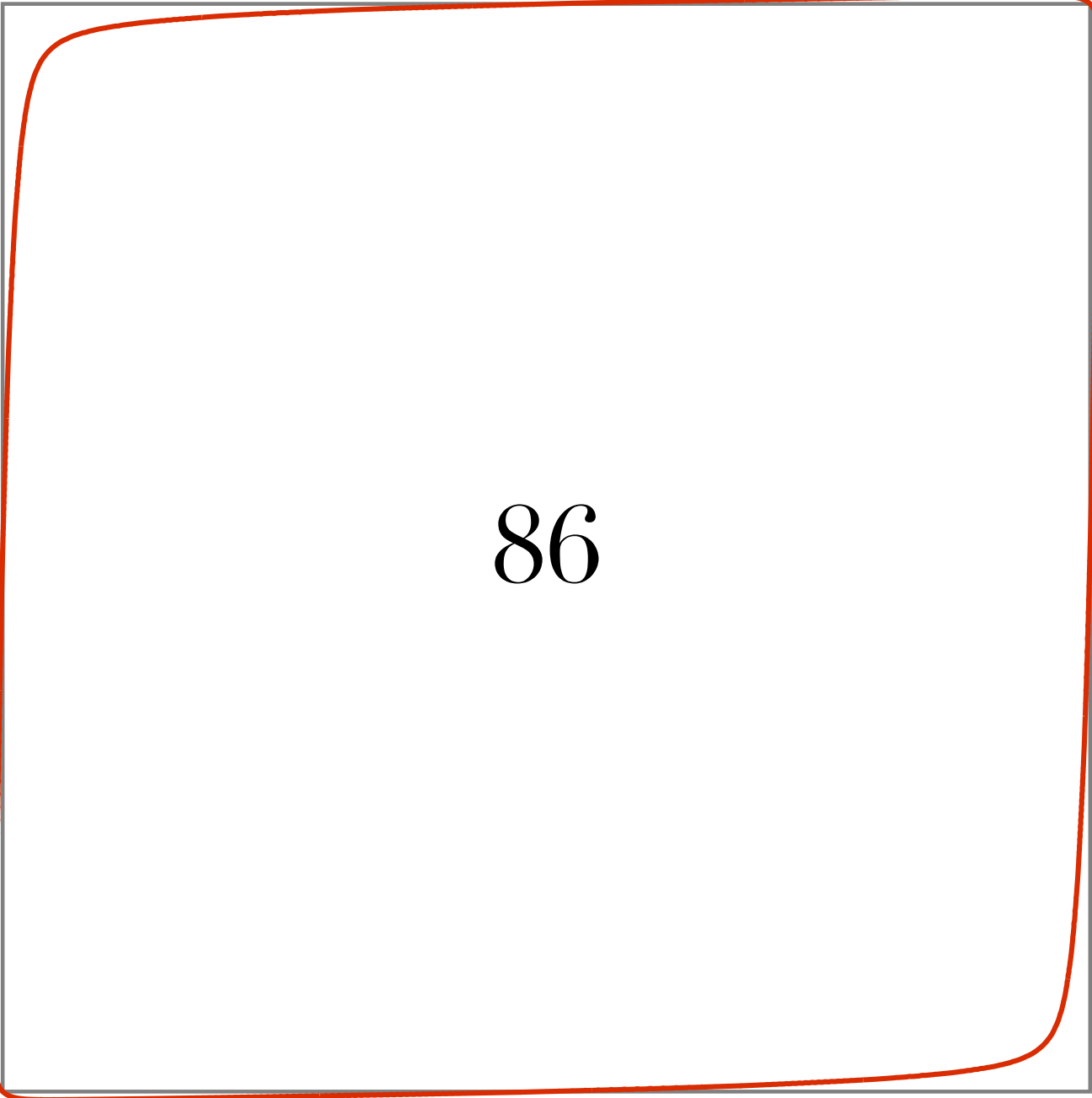


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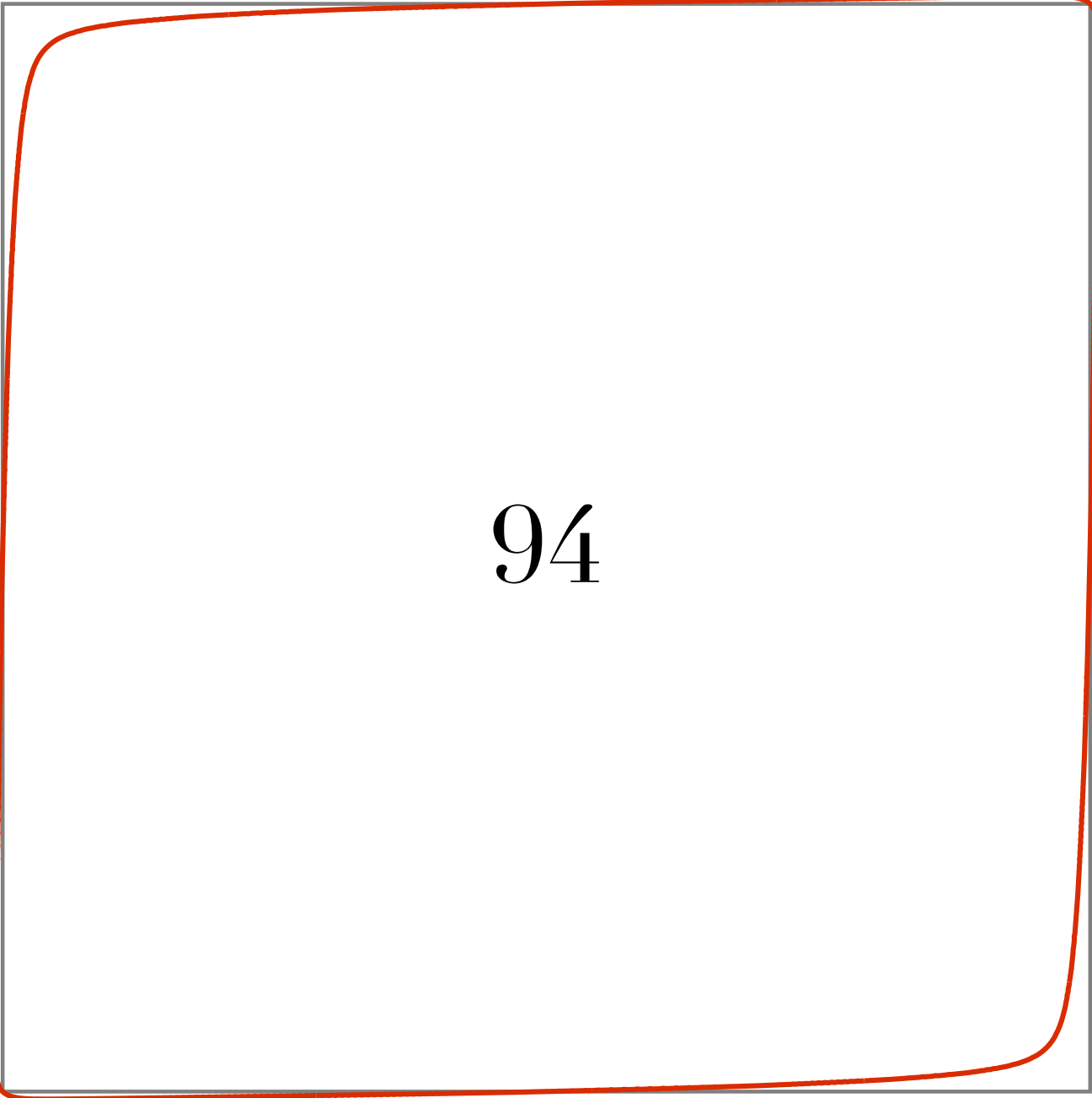
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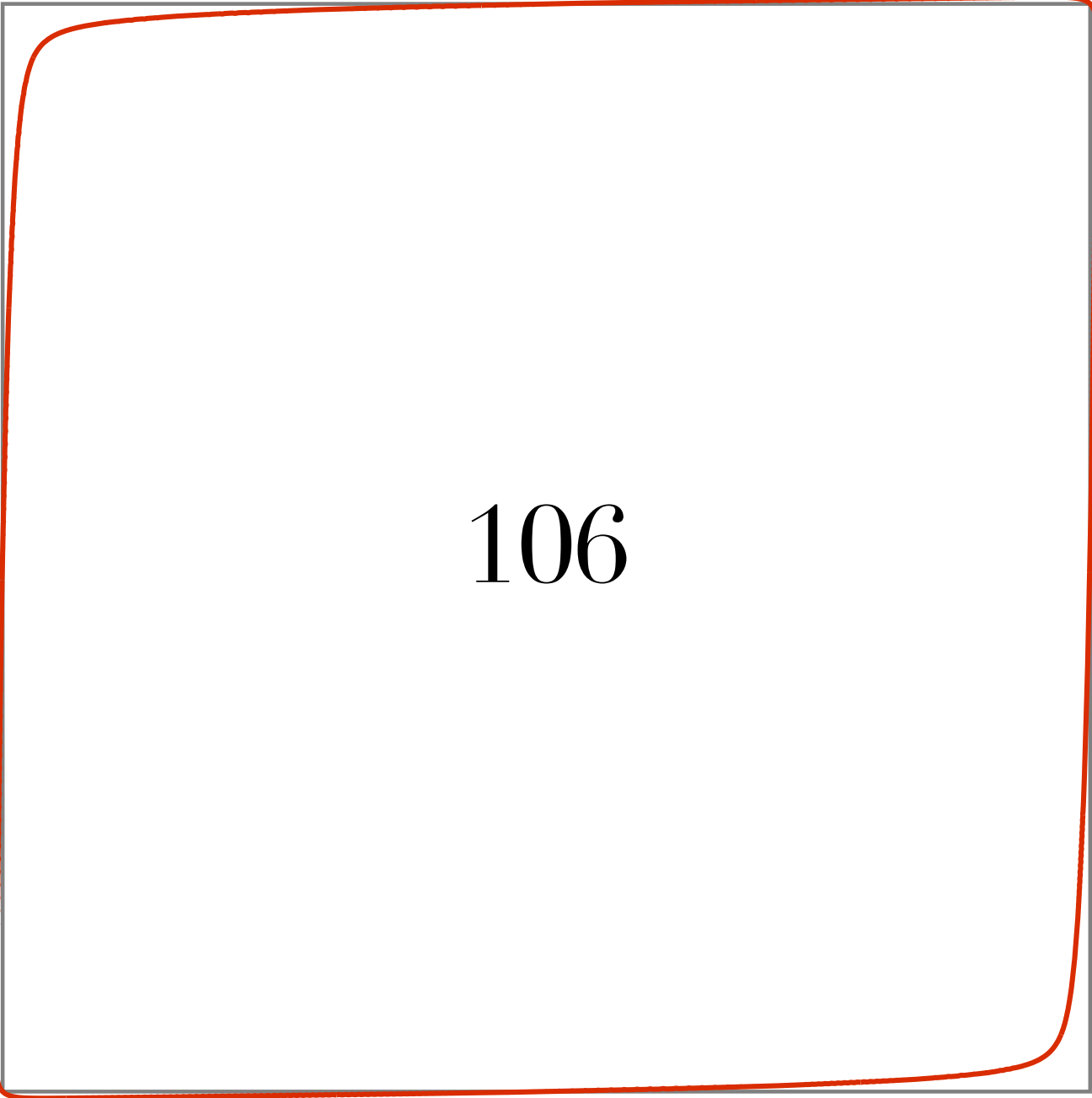
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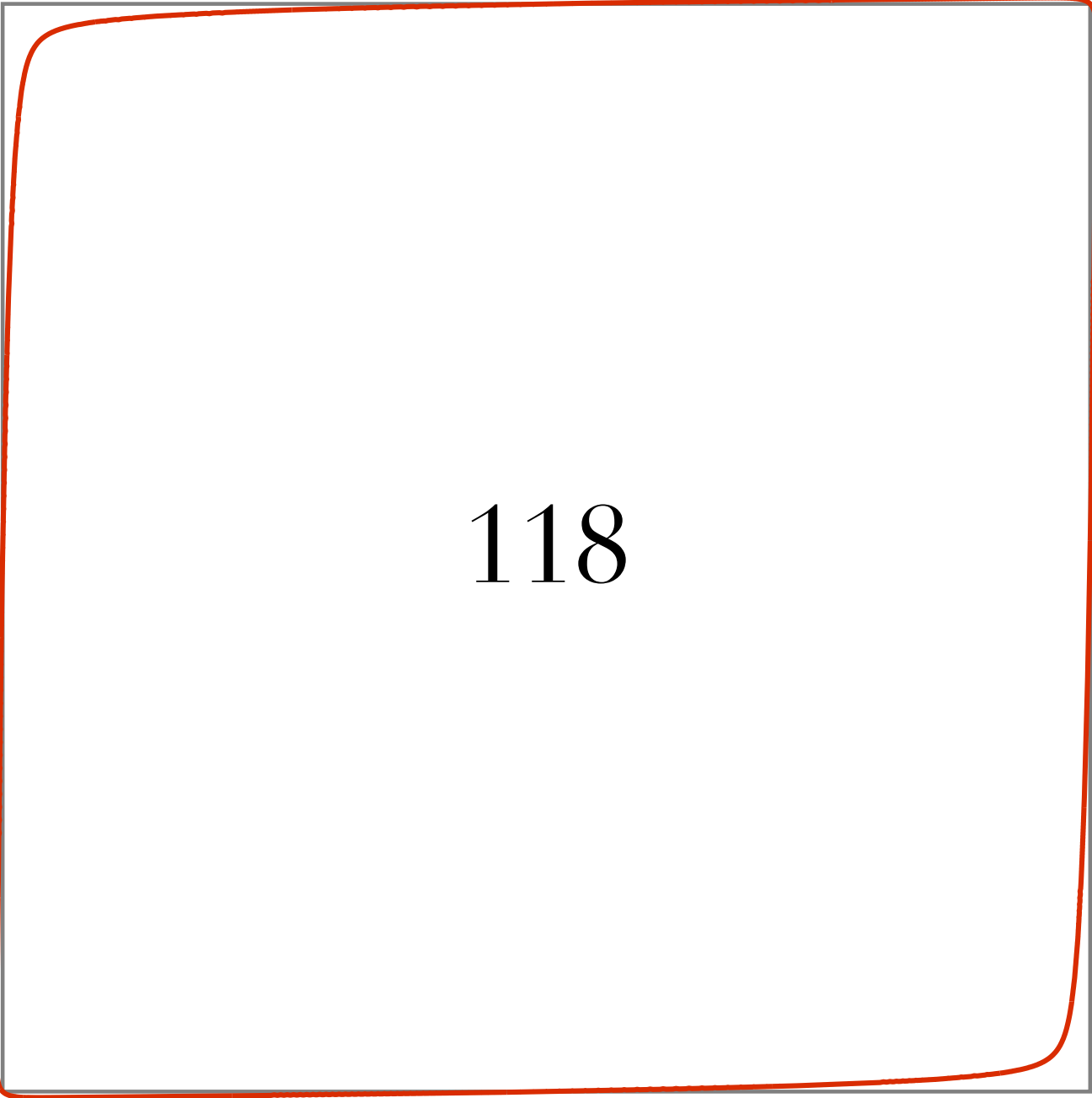
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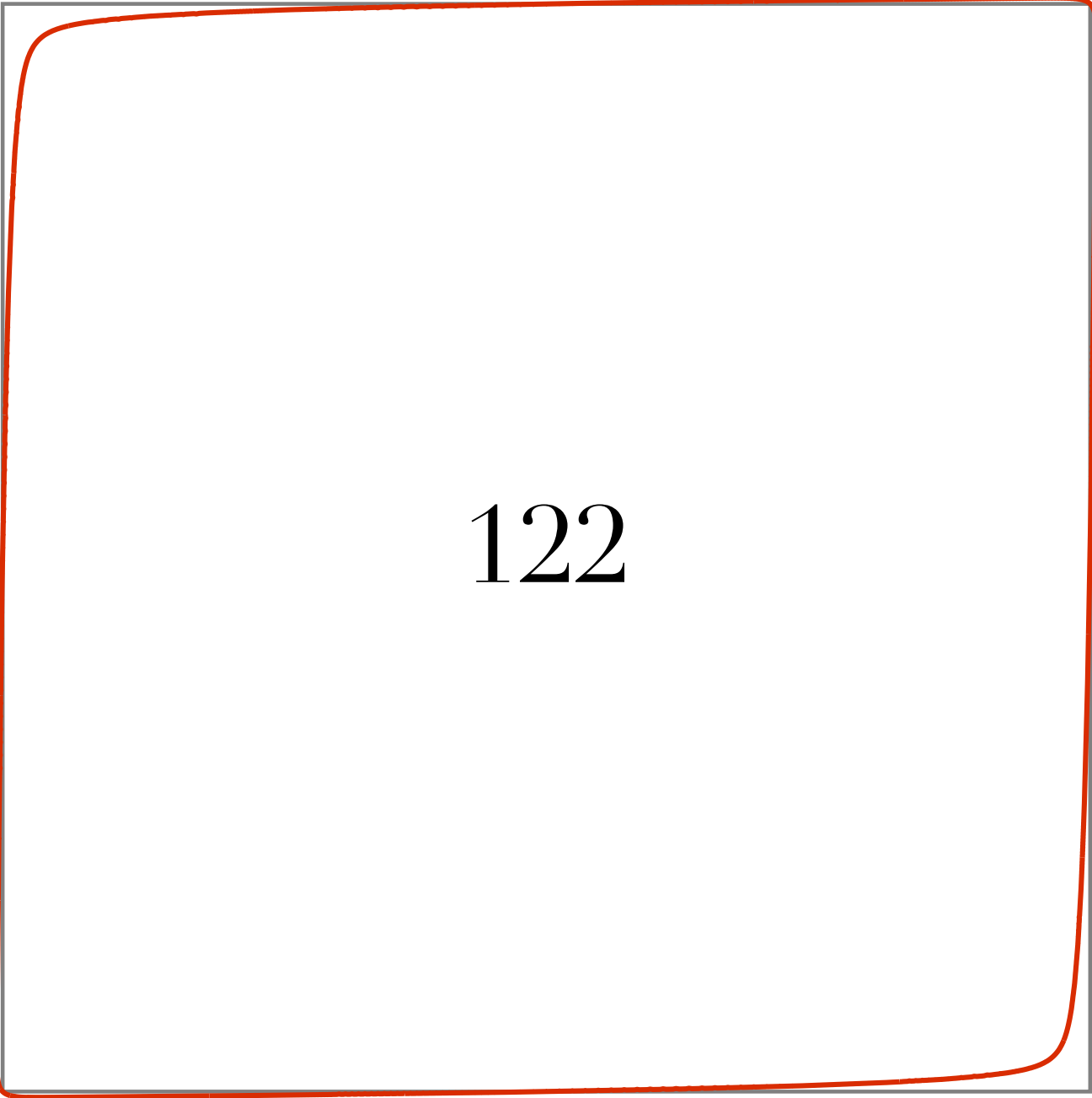


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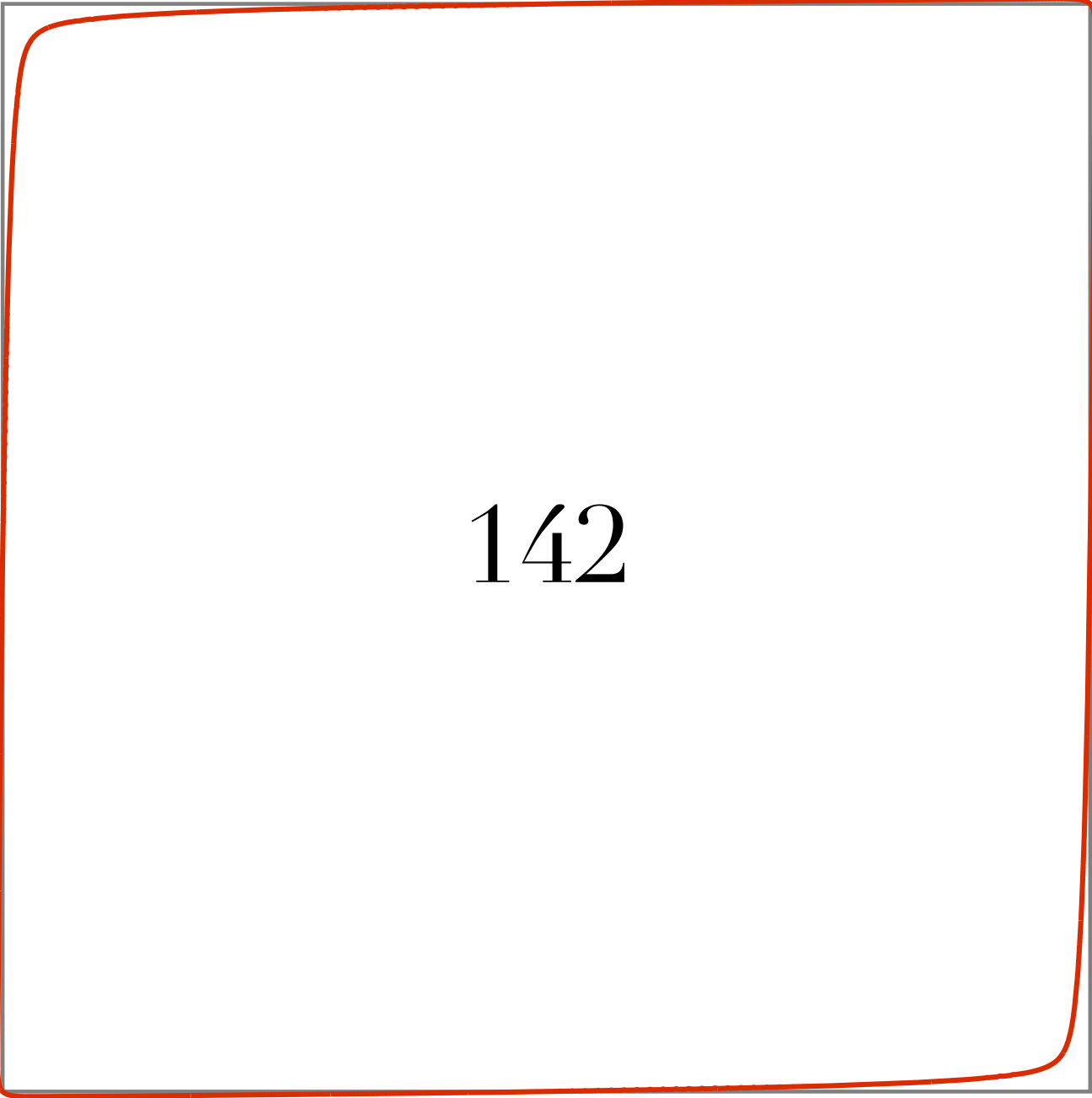
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