1
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Zbl pre05074489
Waldschmidt, Michel
Transcendence of periods: the state of the art. (English)
[J] Pure Appl. Math. Q. 2, No. 2, 435-463 (2006). ISSN 1558-8599; ISSN 1558-8602
MSC 2000:
*11J81 Transcendence (general theory)
11J86 Linear forms in logarithms; Baker's method
11J89 Transcendence theory of elliptic and abelian functions
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2
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Zbl pre05054470
Waldschmidt, Michel
Further variations on the six exponentials theorem. (English)
[J] Hardy-Ramanujan J. 28, 1-9 (2005).
MSC 2000:
*11J72 Irrationality
PDF XML ASCII DVI PS BibTeX Online Ordering Link to Serial

3
OpenURL

Zbl pre05004722
Roy, Damien
Simultaneous approximation by conjugate algebraic numbers in fields of transcendence degree one. (English)
[J] Int. J. Number Theory 1, No. 3, 357-382 (2005). ISSN 1793-0421

Der vorliegende Aufsatz setzt -- auch beweismethodisch -- die Arbeit von Verf. und {\it M. Waldschmidt} [Compos. Math. 140, 593--612 (2004; Zbl 1055.11043)] fort. Die in deren Referat erklärte Notation wird im folgenden beibehalten. \par Das Hauptergebnis lautet wie folgt. Es sei K ein algebraischer Zahlkörper mit $d:=[K:\mathbbQ]$. Mit $t\in \mathbbN$ seien $\xi_1, \dots,\xi_t\in K_w$ so gegeben, dass der Körper $K(\xi_1,\dots,\xi_t)$ über K Transzendenzgrad ≤ 1 hat; s sei die Anzahl der verschiedenen Glieder in der Folge $\xi_1,\dots,\xi_t$ und m sei die größte Vielfachheit, mit der ein ξ in dieser Folge auftritt. Es sei $\wp$ ein Primideal in $K[x_1,\dots,x_t]$ vom Rang $t-1$, dessen Elemente an $(\xi_1,\dots,\xi_t)$ sämtlich verschwinden; D sei der Grad von $\wp$. Schließlich genüge $n\in \mathbbN$ der Bedingung $n\ge 4Dst$ und für $i=1,\dots,t$ gelte $[K(\xi_i):K]\ge n/(Dt)$. Dann gibt es unendlich viele über K algebraische α vom Grad n (über K), die paarweise verschiedene Konjugierte $\alpha_1,\dots,\alpha_t$ in K_w besitzen, für die $(\ast): |\xi_i-\alpha_i|\le H(\alpha)^{-n/(4dDmst)} (i=1,...,t)$ gilt. Dabei ist der Rang eines $\wp$ (wie oben) das größte $r\ge 0$, zu dem es eine streng aufsteigende Kette von $r+1$ Primidealen in $K[x_1,\dots,x_t]$ gibt, die mit $\wp$ endet; unter dem Grad von $\wp$ versteht man den Grad des entsprechenden homogenen Primideals in $K[x_0,x_1,\dots,x_t]$. \par Sind obige $\xi_1,\dots,\xi_t$ paarweise verschieden und alle in $K+K\xi_1$ gelegen, so kann $D=m=1, s=t$ genommen werden und der Exponent rechts in $(\ast)$ wird $-n/(4dt^2)$. Das zweite Hauptergebnis betrifft eine Situation, in der hier der Faktor t^2 im Nenner durch t ersetzt werden kann. Als Nebenresultat ergibt sich eine Version des Gelfondschen Transzendenzkriteriums, ausgedrückt durch Polynome beschränkten Grades, die auf einer festen arithmetischen (bzw. geometrischen) Punktfolge mit rationaler Differenz (bzw. rationalem Quotienten) kleine Werte annehmen.
[Peter Bundschuh (Köln)]
MSC 2000:
*11J13 Simultaneous homogeneous approximation, linear forms
11J82 Measures of irrationality and of transcendence
Citations: Zbl 1055.11043
PDF XML ASCII DVI PS BibTeX Online Ordering Link to Serial

4
Zbl pre02226145
Waldschmidt, Michel
Variations on the Six Exponential Theorem. (English)
[A] Tandon, Rajat (ed.), Algebra and number theory. Proceedings of the silver jubilee conference, Hyderabad, India, December 11--16, 2003. New Delhi: Hindustan Book Agency. 338-355 (2005). ISBN 81-85931-57-7/hbk
MSC 2000:
*11J81 Transcendence (general theory)
11J86 Linear forms in logarithms; Baker's method
11J89 Transcendence theory of elliptic and abelian functions
PDF XML ASCII DVI PS BibTeX Online Ordering

5
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[Zbl 1101.16032](#)
[Waldschmidt, Michel](#)
Introduction to Hopf algebras for Diophantine problems. (Introduction aux algèbres de Hopf pour des questions diophantiennes.) (French)
[J] [Ann. Sci. Math. Qué.](#) 28, No. 1-2, 199-218 (2004). ISSN 0707-9109

This paper describes, without much proof, two applications of Hopf algebras to Diophantine problems.\par For the first, let $k=\overline\bbfQ$ be an algebraic closure of $\bbfQ$. Let ${\frak C}_1$ be the category whose objects are triples (G,W,Γ), where G is the algebraic group $\bbfG_a^{d_0}\times\bbfG_m^{d_1}$ (where $\bbfG_a$ and $\bbfG_m$ are the additive and multiplicative group, respectively), $W\subset T_e(G) $ a rational subspace over $\overline\bbfQ$, and Γ a torsion-free subgroup of $G(\overline\bbfQ)$ of finite type. The morphisms in ${\frak C}_1$ consist of maps $f\colon G_1\to G_2$ such that $f(\Gamma_1)\subset\Gamma_2$ and $df(W_1)\subset W_2$, where df is the map on tangent spaces $df\colon T_e(G_1)\to T_e(G_2)$ induced by f. Write ℓ_0 for the dimension of W and ℓ_1 for the dimension of Γ. Additionally, let ${\frak C}_2$ be the category whose objects are triples $(H,H',\langle\cdot,\cdot\rangle)$, where H and H' are bicommutative (that is, commutative and cocommutative) Hopf algebras which are integral domains and $\langle\cdot,\cdot\rangle$ is a bilinear product $H\times H\to\overline\bbfQ$ such that $\langle x,yy'\rangle=\langle\Delta x,y\otimes y'\rangle$ and $\langle xx',y\rangle=\langle x\otimes x',\Delta y\rangle$, where Δ is used to denote the comultiplication on each Hopf algebra. The morphisms in this category $(f,g)\colon(H_1,H_1',\langle\cdot,\cdot\rangle_1)\to(H_2,H_2',\langle\cdot,\cdot\rangle_2)$ consist of $f\colon H_1\to H_2$, $g\colon H_1'\to H_2'$ such that $\langle x_1,g(x_2')\rangle_1= \langle f(x_1),x_2'\rangle_2$. Then a theorem of Fischler states that ${\frak C}_1$ and ${\frak C}_2$ are equivalent; furthermore the equivalence is contravariant with respect to d_0,d_1,ℓ_0,ℓ_1. This provides ``new interpolation lemmas which play a role in transcendental number theory''. \par For the second, the author uses Hopf algebras which are not bicommutative to discuss questions related to Multiple Zeta Values. Let $\frak S$ be the set of sequences $\underline s=(s_1,\dots,s_k)$ of natural numbers with $k\geq 0$, $s_i\geq 1$ for $i\neq 1$ and $s_1\geq 2$. For each $\underline s\in{\frak S}$ let $\zeta(\underline s)=\sum_{n_i>n_{i+1}}n_1^{-s_1}\cdots n_k^{-s_k}$. This is clearly a generalization of the Riemann zeta function on the natural numbers, and are known as ``Multiple Zeta Values''. Let ${\frak z}\subset\bbfC$ be the $\bbfQ$-vector space generated by $(2\pi i)^{-(s_1+\cdots +s_k)}\zeta(\underline s)$ for $\underline s\in{\frak S}$. This space $\frak z$ is in fact a $\bbfQ$-subalgebra of $\bbfC$, filtered by k as well as by $s_1+\cdots +s_k$. For $C_\bullet$ a graded Lie algebra, let ${\frak U}C_\bullet$ be the universal enveloping algebra for $C_\bullet$ and ${\frak U}C_\bullet^\vee=\bigoplus_{n\geq 0}({\frak U}C_\bullet^\vee)_n^*$ the graded dual. Then ${\frak U}C_\bullet^\vee$ is a commutative Hopf algebra. A conjecture of Goncharov states that ${\frak z}\cong{\frak U}C_\bullet^\vee$ for some choice of $C_\bullet$.
[[Alan Koch (Decatur)](#)]
MSC 2000:
[*16W30](#) Hopf algebras (assoc. rings and algebras)
[11J81](#) Transcendence (general theory)
[11M41](#) Other Dirichlet series and zeta functions
Keywords: Hopf algebras; multiple zeta values; Diophantine problems
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7
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[Zbl 1066.11030](#)
[Waldschmidt, Michel](#)
Open Diophantine problems. (English)
[J] [Mosc. Math. J.](#) 4, No. 1, 245-305 (2004). ISSN 1609-3321

\par This paper gives a very detailed survey of the most important conjectures and problems in Transcendental Number Theory and Diophantine Approximation. Its 61 pages contain a lot of information. It is almost impossible to give a complete review of it, I shall only present a list of the main topics treated in this survey. The different sections are: 1. Diophantine equations 2. Diophantine Approximation 3. Transcendence 4. Heights 5. Further Problems: Metric Problems, Function Fields. The paper contains more than 70 problems or conjectures. This is a splendid road-map for the present millenium.
[[Maurice Mignotte (Strasbourg)](#)]
MSC 2000:
[*11Jxx](#) Diophantine approximation
[11-02](#) Research monographs (number theory)
[11Dxx](#) Diophantine equations
[11G50](#) Heights
[14G05](#) Rationality questions, rational points
Keywords: Transcendental number theory; Diophantine approximation; Diophantine equations
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9
[OpenURL](#)

[Zbl 1055.11043](#)
[Roy, Damien](#); [Waldschmidt, Michel](#)
Diophantine approximation by conjugate algebraic integers. (English)
[J] [Compos. Math.](#) 140, No. 3, 593-612 (2004). ISSN 0010-437X; ISSN 1570-5846

Um die Hauptresultate der vorliegenden Arbeit beschreiben zu können, sei K ein algebraischer Zahlkörper mit $d:=[K:\BbbQ]$. Für jede Stelle v von K sei K_v die Vervollständigung von K an v und d_v sein lokaler Grad an v. Der entsprechende Absolutbetrag $|\cdot|_v$ sei so normiert, dass $|p|_v = p^{-d_v/d}$ gilt, falls v über einer Primzahl p von $\BbbQ$ liegt, bzw. dass $|x|_v =|x|^{d_v/d}$ für jedes $x\in \BbbQ$ gilt, falls v archimedisch ist. \par Damit lautet der zentrale {\it Approximationssatz} der Verfasser: n,t mögen $t\le n/4$ genügen. Sei w eine Stelle von K, $\xi \in K_w$ sei nicht algebraisch über K von einem Grad $\le (n+1)/(2t)$; weiter sei $|\xi|_w \le 1$ bei nichtarchimedischem w. Dann gibt es unendlich viele algebraische α mit Grad $n+1$ über K und mit Grad $d(n+1)$ über $\BbbQ$, die paarweise verschiedene über K Konjugierte $\alpha_1,\dots ,\alpha_t\in K_w$ mit $$|\xi-\alpha_i|_w\le cH(\alpha)^{-(n+1)/(4dt^2)} \quad i=1,\dots,t \tag*$ $$ besitzen, $H(\alpha)$ die klassische Höhe von α, wo $c>0$ nur von K,n,w und ξ abhängt. \par Im Fall $t=1,K=\BbbQ,K_w=\BbbR$ vergleiche man dies Resultat mit Theorem 2 von {\it H. Davenport} und {\it W. M. Schmidt} [Acta Arith. 15, 393--416 (1969; [Zbl 0186.08603](#))], deren Beweisstrategie die Verfasser folgen. Sie beruht auf einem Dualitätsargument, kombiniert mit der folgenden {\it Version des Gel'fond-Kriteriums} für algebraische Unabhängigkeit: Seien t,n,K,w,ξ wie im Approximationssatz. Dann existiert ein $c>0$ wie oben mit folgender Eigenschaft. {\it Wenn} es zu jedem genügend großen $X\in \BbbR_+$ ein $Q\in K[T]\setminus\{0\}$ höchstens vom Grad n gibt, so dass an jeder Stelle $v \ne w$ von K der v-adische Absolutbetrag aller Koeffizienten von Q nicht größer als 1 ist und für die j-ten Ableitungen von Q die Ungleichungen $$|Q^{(j)}(\xi)|_w \le cX^{-t/([n/4]+1-t)}\quad0\le j\le n-t,\qquad |Q^{(j)}(\xi)|_w \le X\quad n-t<j\le n\tag$**$ $$ gelten, {\it dann} ist $

\xi$ über K algebraisch von einem Grad $\le(n-[n/4]+1)/(2t)$. \par Zur Schärfe der erzielten Ergebnisse wird gezeigt: Der Approximationsexponent in ($*$) ist bis auf den Faktor $1/4$ von $(n+1)/(dt^2)$ bestmöglich, der nicht durch eine reelle Zahl >2 ersetzt werden kann. Des weiteren kann in ($**$) der Exponent von X (bei sonst gleicher Aussage) nicht auf $-t/(n+1-t)$ vergrößert werden.
[Peter Bundschuh (Köln)]
MSC 2000:
*11J13 Simultaneous homogeneous approximation, linear forms
11J61 Approximation in non-Archimedean valuations
11J82 Measures of irrationality and of transcendence
Citations: Zbl 0186.08603

Cited in: Zbl pre05004722 Zbl 1064.11049

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10
Zbl 1093.11054
Waldschmidt, Michel
Linear independence measures for logarithms of algebraic numbers. (English)
[A] Amoroso, Francesco (ed.) et al., Diophantine approximation. Lectures given at the C. I. M. E. summer school, Cetraro, Italy, June 28--July 6, 2000. Berlin: Springer. Lect. Notes Math. 1819, 249-344 (2003). ISBN 3-540-40392-2/pbk

Let $\alpha_1,\dots,\alpha_n$ be non zero (complex) algebraic numbers and let $\lambda_1,\dots,\lambda_n$ be complex numbers such that $e^{\lambda_i}=\alpha_i$ for all $i=1,\dots,n$. A celebrated theorem of {\it A. Baker} [``Linear forms in the logarithms of algebraic numbers: I, II, III, IV''. Mathematika Lond. 13, 204--216 (1966; Zbl 0161.05201); ibid. 14, 102--107 (1967; Zbl 0161.05202); ibid. 14, 220--228 (1967; Zbl 0161.05301); ibid. 15, 204-216 (1968; Zbl 0169.37802)] asserts that if $\lambda_1,\dots,\lambda_n$ are linearly independent over the rationals, then they are linearly independent over the field of algebraic numbers. More important, Baker's method provides a non-trivial lower bound for the quantity $\vert b_0+b_1\lambda_1+\dots+b_n\lambda_n \vert $, where $b_0,\dots,b_n$ are algebraic numbers. Much of what is known about effective solutions of Diophantine equations ultimately depends on such lower bounds. For this reason the subject has been much developed in the last forty years. The paper under review consists of detailed notes of six lectures delivered by the author on this theory. The first result (Theorem 1.1) states that for every positive integer n the real number e^n is not an integer. The author shows two proofs of this fact and leaves the open question of proving the lower bound $\vert m-e^n\vert >n^{-c}$ for a positive constant c and all integers $m,n\geq 2$. Starting from this basic result, he proceeds to prove the newest sophisticated lower bounds for linear forms in logarithms. Many results are proved by using the method of interpolation determinant, introduced by {\it M. Laurent} in this context [``Sur quelques résultats récents de transcendence. Journées arithmétiques'', (Luminy 1989). Astérisque 198--200, 209--230(1991; Zbl 0762.11027)]. The outcome is a self-contained text presenting both classical results in transcendence theory and some of the most recent advances in the theory of linear forms in logarithms, carefully written by a leading contributor to this theory.
[Pietro Corvaja (Udine)]
MSC 2000:
*11J82 Measures of irrationality and of transcendence
11-02 Research monographs (number theory)
Keywords: linear forms in logarithms; Baker's method; lower bounds

Citations: Zbl 0161.05201; Zbl 0161.05301; Zbl 0169.37802; Zbl 0762.11027; Zbl 0161.05202

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11
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Zbl 1062.11049
Waldschmidt, Michel
Algebraic values of analytic functions. (English)
[J] J. Comput. Appl. Math. 160, No. 1-2, 323-333 (2003). ISSN 0377-0427

L'Auteur rend compte de résultats recents, et de leur contexte, dans le domaine des valeurs algébriques prises par des fonctions analytiques transcendantes en des points algébriques:\par {\it A.~Surroca} [C. R. Math., Acad. Sci. Paris 334, No.~9, 721--725 (2002; Zbl 1013.11037)] estime le nombre de nombres algébriques de degré et hauteur bornés en lesquels une fonction analytique transcendante prend des valeurs algébriques de degré et hauteur bornés. D. Delbos établit des critères de transcendance pour des fonctions entières d'une ou plusieurs variables complexes, qui sont des variantes du théorème de {\it E. Bombieri} [Invent. Math. 10, 267--287 (1970; Zbl 0214.33703)] généralisant le critère de Schneider-Lang. Enfin, l'Auteur montre comment ramener la preuve de l'indépendance algébrique de logarithmes de nombres algébriques à celle de l'indépendance linéaire de valeurs de polylogarithmes multiples en des points algébriques.
[François Gramain (Saint-Etienne)]
MSC 2000:
*11J81 Transcendence (general theory)
33B30 Higher logarithm functions
Keywords: algebraic values; analytic transcendental functions; transcendence criterion; diophantine analysis

Citations: Zbl 1013.11037; Zbl 0214.33703

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12
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Zbl 1046.01516
Waldschmidt, Michel
The French Mathematical Society. (Société Mathématique de France.) (English)
[J] Aust. Math. Soc. Gaz. 30, No. 5, 284-286 (2003). ISSN 0311-0729
MSC 2000:
*01A74 History of mathematics at institutions and academies (nonuniversity)
Keywords: SMF

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14
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Zbl 1085.11036
Roy, Damien
Interpolation on pertubations of Cartesian products. (Interpolation sur des perturbations d'ensembles produits.) (French)

[J] Bull. Soc. Math. Fr. 130, No. 3, 387-408 (2002). ISSN 0037-9484

In 1970, E. Bombieri and S. Lang used analytic results of P. Lelong to establish a Schwarz lemma for a well distributed set of points in $\BbbC^n$. Their result was extended to an interpolation lemma, first by D. W. Masser in the case of polynomials, then by M. Waldschmidt for analytic functions. J.-C. Moreau gave an analog of it over the real numbers and P. Robba in the p-adic realm. Robba also conjectured a p-adic interpolation lemma for the case where the set of points of interpolation is what he calls a perturbation of a product set, a situation that includes both the case of a well distributed set and the case of a cartesian product. In this paper, the author presents an algebraic proof of Robba's conjecture together with a generalization of it over the complex numbers.
[Martin Chuaqui (Santiago de Chile)]
MSC 2000:
*11J99 Diophantine approximation
32E30 Holomorphic approximation (several variables)
Keywords: interpolation; polynomials; Schwarz's lemma; analytic functions; p-adic analysis; Cartesian products; well distributed set.

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16
Zbl 1035.11033
Waldschmidt, M.
Multiple polylogarithms: an introduction. (English)
[A] Agarwal, A. K. (ed.) et al., Number theory and discrete mathematics. Proceedings of the international conference in honour of Srinivasa Ramanujan, Chandigarh, India, October 2--6, 2000. Basel: Birkhäuser. Trends in Mathematics, 1-12 (2002). ISBN 3-7643-6720-2/hbk

This paper surveys the recent progresses that have been made toward the complete description of all algebraic relations over ${\Bbb Q}$ between the Multiple Zeta Values (MZV) defined by $\zeta(s_1,\dots,s_k)=\sum_{n_1>\cdots>n_k\ge 1} n_1^{-s_1}\cdots n_k^{-s_k}$ where $k,s_1,\dots,s_k$ are positive integers with $s_1\ge 2$. It explains clearly how such relations arise from the shuffle and stuffle products of multiple polylogarithms, with the consideration of divergent series as well. The main conjecture (Zagier, Goncharov,\dots) is that these relations generate all algebraic relations between MZV over ${\Bbb Q}$, then reducing the problem to that of finding all linear relations between MZV of a given weight (the sum $s_1+\cdots+s_k$). \par More precisely, a conjecture of Zagier asserts that the dimension of the vector space over ${\Bbb Q}$ generated by all MZV of weight p is given by the recurrence relation $d_p=d_{p-2}+d_{p-3}$ with initial conditions $d_0=1$, $d_1=0$ and $d_2=1$. That the actual dimension of this space is bounded above by d_p has been proved by {\it T. Terasoma} in [Invent.\ Math. 149, 339--369 (2002; Zbl 1042.11043)], who expresses multiple zeta values as periods of relative cohomologies and uses mixed Tate Hodge structures (see also the work of {\it A. G. Goncharov} [European Congress of Mathematics, Vol. I, 361--392 (2001; Zbl 1042.11042)] referred to in the same paper). It is hoped that this stream of research will eventually lead to a proof of the long standing conjecture to the effect that the values $\zeta(2)$ and $\zeta(3), \zeta(5), \zeta(7), \dots$ of the usual Riemann zeta function $\zeta(s) = \sum_{n\ge 1} n^{-s}$ are algebraically independent over ${\Bbb Q}$. At present, little is known about the later except that $\zeta(2) = \pi^2/6$ is transcendental, that $\zeta(3)$ is irrational, and, thanks to recent work of K.~Ball and T.~Rivoal, that these numbers generate over ${\Bbb Q}$ a vector space of infinite dimension.
[Damien Roy (Ottawa)]
MSC 2000:
*11J91 Transcendence theory of other special functions
11G55 Polylogarithms and relations with K-theory
11M41 Other Dirichlet series and zeta functions
33B30 Higher logarithm functions
Keywords: Riemann zeta-function; multiple zeta values; multiple polylogarithms; Euler-Zagier numbers; polyzeta; shuffle product; stuffle product; irrationality; linear independence; algebraic independence

Citations: Zbl 1042.11043; Zbl 1042.11042

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18
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Zbl 0997.00018
Agarwal, A.K. (ed.); Berndt, Bruce C. (ed.); Krattenthaler, Christian F. (ed.); Mullen, Gary L. (ed.); Ramachandra, K. (ed.); Waldschmidt, Michel (ed.)
Number theory and discrete mathematics. Proceedings of the international conference in honour of Srinivasa Ramanujan, Chandigarh, India, October 2--6, 2000. (English)
[B] Trends in Mathematics. Basel: Birkhäuser. xvi, 314 p. EUR 88.79/net; sFr. 142.00 (2002). ISBN 3-7643-6720-2/hbk

The articles of this volume will be reviewed individually.
MSC 2000:
*00B25 Proceedings of conferences of miscellaneous specific interest
11-06 Proceedings of conferences (number theory)
05-06 Proceedings of conferences (combinatorics)
14-06 Proceedings of conferences (algebraic geometry)
Keywords: Conference; Proceedings; Number theory; Discrete mathematics; Dedication; Chandigarh (India)

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28
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Zbl 0966.11032
Nesterenko, Yu.V. (ed.); Philippon, Patrice (ed.)
(Amoroso, F.; Bertrand, D.; Brownawell, W.D.; Diaz, G.; Laurent, M.; Nishioka, K.; Rémond, G.; Roy, D.; Waldschmidt, M.)
Introduction to algebraic independence theory. With contributions from F. Amoroso, D. Bertrand, W. D. Brownawell, G. Diaz, M. Laurent, Yu. V. Nesterenko, K. Nishioka, P. Philippon, G. Rémond, D. Roy, M. Waldschmidt. (English)
[B] Lecture Notes in Mathematics. 1752. Berlin: Springer. xiii, 256 p. DM 74.00; öS 541.00; sFr. 65.50; \sterling 25.50; \$ 52.80 (2001). ISBN 3-540-41496-7/pbk

Ce joli petit livre (environ 250 pages) est issu d'un cours donné à l'automne 1997 au CIRM (Luminy) pour des postdoctorants sous la direction de M. Waldschmidt, R. Tijdeman et Yu. Nesterenko. Prenant prétexte de la preuve par Yu. Nesterenko (1996) de

l'indépendance algébrique de π, e^π et $\Gamma(1/4)$ (plus généralement de valeurs prises par les séries d'Eisenstein $E_2(\tau)$, $E_4(\tau)$, $E_6(\tau)$ et l'exponentielle $e^{2i\pi\tau}$), le livre expose en détails les techniques actuelles utilisées pour l'indépendance algébrique. Chaque chapitre est rédigé par un chercheur spécialiste de la question traitée.\par Le livre se compose de quatre parties: après l'exposition des derniers résultats de transcendance obtenus pour les fonctions et formes modulaires (chapitres 1 à 4), une deuxième partie donne les notions nécessaires d'algèbre commutative (chapitres 5 à 9: théorie de l'élimination, théorème de Bézout, critères d'indépendance algébrique), la troisième partie présente des lemmes de zéros, les uns plus spécifiques aux fonction modulaires, les autres plus généraux (chapitres 10 et 11). Enfin la dernière partie (chapitres 12 à 16) développe des applications des outils présentés auparavant, retrouvant, en particulier, des résultats classiques et posant des questions ouvertes. \par C'est un livre indispensable pour qui s'intéresse à l'indépendance algébrique.
[F.Gramain (Saint-Etienne)]
MSC 2000:
*11J85 Algebraic independence results
11-02 Research monographs (number theory)
11F03 Modular and automorphic functions
11J91 Transcendence theory of other special functions
Keywords: algebraic independence of numbers; modular functions; Eisentein series; elimination theory; Bézout theorem; zero estimates
Cited in: Zbl 1047.14003
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30
Zbl 1072.11510
Waldschmidt, Michel
Algebraic independence of transcendental numbers: a survey. (English)
[A] Bambah, R. P. (ed.) et al., Number theory. Basel: Birkhäuser. Trends in Mathematics, 497-527 (2000). ISBN 3-7643-6259-6/hbk

In this survey article, the author covers the important progress made in the algebaic independence of transcendental numbers during the period 1984--1997. Due to the enormous number of results available, the author confines his discussion to results related to Gel'fond's method. This article will be very beneficial for experts and non-experts. Section 1 deals with various conjectures on algebraic independence beginning with those of Schanuel and of Gel'fond-Schneider. The author proposes another conjecture in place of the generalisation of Schanuel's conjecture suggested in his previous survey covering the progress on the subject before 1984. In view of a result of Bijlsma, the old conjecture is not valid without a technical hypothesis. An important special case of Schanuel's conjecture says that if $ l_1, \cdots, l_n$ are $\Bbb Q$-linearly independent logarithms of algebraic numbers, then they are algebraically independent. The author discusses the work of D. Roy which yields a new approach to the conjecture. Section 2 discusses the methods in this theory. The main method in this subject revolves around the following: Given functions $f_1, \cdots, f_{\alpha}$ and points $u_1, \cdots, u_d$, under suitable conditions give a lower bound for the transcendence degree of the field generated over $\Bbb Q$ by the $d l$ numbers $f_i (u_j),\ 1 \leq i \leq d, 1 \leq j \leq l$, denoted by θ_r, say, for $1 \leq r \leq d l$. The basic method involves the construction of some number γ in $K=\Bbb Q (\theta_1, \cdots, \theta_n)$. Assuming $\theta_1, \cdots, \theta_n$ are algebraic we have γ also algebraic. By zero estimates one shows that γ does not vanish. Using an arithmetic estimate (Liouville-type argument), one gets a lower bound for $|\gamma|$ and by analytic estimates one obtains an upper bound for $|\gamma|$. Gel'fond's transcendence criterion enables one to prove that some fields have transcendence degree ≥ 2 over $\Bbb Q$. Criteria for algebraic independence have been worked out by several authors, the sharp estimate being that of Philippon. The results on large transcendence degree involving tools from commutative algebra are described. Some recent approaches to algebraic independence via simultaneous approximation are described. The basic result here is due to Wirsing in 1960. In Section 3 the author describes the progress made using Gel'fond's method. This covers the works of G. V. Chudnovsky and Y. André. The sharp measure of algebraic independence of π/ω and η/ω due to Philippon is stated. The results on large transcendence degree are described. The sharpest estimate in this direction is due to G. Diaz and says that the transcendence degree over $\Bbb Q$ of $\Bbb Q (\alpha^{\beta}, \alpha^{\beta^2} , \cdots, \alpha^{\beta^{d-1}})$ is greater than $(d-1)/2$, where α is a nonzero complex number with log\,α a nonzero logarithm and β an algebraic number of degree $d \geq 2$. Several results on the quantitative refinements of the Lindemann-Weierstrass theorem and its elliptic analog are mentioned. Also, the elliptic analog of Diaz's theorem due to Ably and Shestakov is quoted. Section 4 deals with results concerning values of modular functions. In 1996, {\it K. Barré-Sirieix} et al. [Invent. Math. 124, No. 1-3, 1--9 (1996; Zbl 0853.11059)] used modular functions to solve a problem of Mahler in the complex case and of Manin in the p-adic case by showing that $J(q)$ is transcendental when q is algebraic and J is modular invariant. This was followed by the remarkable result of Nesterenko that if q is a complex or p-adic number satisfying $0 < | q | <1$, then at least three of the numbers $q, P(q), Q(q), R(q)$ are algebraically independent, where P, Q, R are the Eisenstein series of weights $2, 4$ and 6, respectively. In the complex case, this yields that at least 3 of $g_2, g_3, \omega_1/\pi, \eta_1/\pi,\ e^{2 \pi i w_1/w_2}$ are algebraically independent, where w_1, w_2 are the fundamental periods of the elliptic curve $y^2=4x^3-g_2 x-g_3$ with $g^3_2 \neq 27g^2_3$ and η_1, η_2 are the corresponding quasi-periods. As a consequence, we obtain the algebraic independence of π, e^{π} and $\Gamma (\frac14)$. The last section is devoted to the concepts of Diophantine rings which were introduced in work of Philippon, where he proposed an axiomatic method for transcendence and algebraic independence by which he could cover Gel\cprime fond's method, Baker's theorem, the Lindemann-Weierstrass theorem, Mahler's method, the Carlitz exponential and Drinfel'd modules. This article ends with numerous references in chronological order, which will be extremely useful for researchers in this field.
[N. Saradha (Mumbai)]
MSC 2000:
*11J85 Algebraic independence results
11-02 Research monographs (number theory)
Citations: Zbl 0853.11059
PDF XML ASCII DVI PS BibTeX Online Ordering

31
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Zbl 1002.11061
Waldschmidt, Michel
On a problem of Mahler concerning the approximation of exponentials and logarithms. (English)
[J] Publ. Math. 56, No.3-4, 713-738 (2000). ISSN 0033-3883

Seien $\lambda_1,\ldots,\lambda_m\in{\bbfC}$ so, daß $\alpha_i := e^{\lambda_i}\in \overline{\bbfQ}$ für $i = 1,\ldots,m$ gilt; seien $\beta_0,\ldots,\beta_m\in\overline{\bbfQ}$ und $D := [{\bbfQ}(\alpha_1,\ldots,\alpha_m, \beta_0,\ldots,\beta_m):{\bbfQ}]$. Bezeichnet $h(\gamma)$ die absolute logarithmische Höhe von $\gamma\in\overline{\bbfQ}$, so sei $h\in{\bbfR}_+$ nicht kleiner als $(1/D)\max(1,|\lambda_1|,\ldots,|\lambda_m|)$ bzw. als alle $h(\alpha_i)$ und alle $h(\beta_j)$. Dann vermutet Verf., daß es absolute Konstanten $c_1,c_2\in{\bbfR}_+$ gibt mit folgenden Eigenschaften. \par (1) ($\Lambda := \beta_0+\sum_{i=1}^m \beta_i \lambda_i\not= 0$ impliziert $|\Lambda|\geq \exp(-c_1mD^2h)$.\par (2) Falls $\lambda_1,\ldots,\lambda_m$ über ${\bbfQ}$ linear unabhängig sind, gilt $\sum_{i=1}^m|\lambda_i-\beta_i|\geq \exp(-c_2mD^{(m+1)/m}h)$. \par Nach ausführlicher

Diskussion der beiden Vermutungen beweist Verf. zwei partielle Resultate in Richtung auf Vermutung (2). Um diese bequem formulieren zu können, sei folgende Definition vorangestellt: $(\lambda_{i,j})\in \textMat_{m\times n}({\bbfC})$ genügt der linearen Unabhängigkeits-Bedingung (kurz: LUB), wenn $\sum_{i=1}^m \sum_{j=1}^n t_i s_j\lambda_ij \not= 0$ für jedes nichttriviale $(t_1, \ldots,t_m)\in {\bbfZ}^m$ und jedes nichttriviale $(s_1,\ldots,s_n)\in{\bbfZ}^n$ gilt. Damit hat man Satz 1, eine Variante von Theorem 10.1 aus {\it D. Roy} und Verf. [Ramanujan J. 1, 379-430 (1997; Zbl 0916.11042)]: \par Mit $m,n,r\in{\bbfN}$ sei $ \theta := r(m+n)/mn$. Dann gibt es ein $c_1\in{\bbfR}_+$ mit folgender Eigenschaft: Hat $(\beta_ij)\in \textMat_{m\times n} (K)$ einen Rang $\leq r$, wobei K ein algebraischer Zahlkörper ist, und ist $L := (\lambda_ij)\in \textMat _{m\times n}({\bbfC})$ so, daß alle $\alpha_ij := \exp(\lambda_ij)\in K ^\times$ sind und daß L der LUB genügt, so gilt $$\sum_{i=1}^m \sum_{j=1}^n| \lambda_ij-\beta_ij|\geq \exp(-c_1\varphi)$$ mit $\varphi := Dh_1(Dh_2)^\theta$ bei $Dh_1 \geq (Dh_2)^{1-\theta}$ bzw. $ \varphi := (Dh_1)^{1/(1-\theta)}$ bei $Dh_1 \leq (Dh_2)^{1-\theta}$. Dabei ist $D := [K:{\bbfQ}]$ und die $h_1,h_2\in{\bbfR}_+$ genügen den Ungleichungen $h_1 \geq$ alle $h(\alpha_ij), \ Dh_1 \geq$ alle $|\lambda_ij|$, $Dh_1 \geq 1$; $h_2 \geq$ alle $h (\beta_ij)$, $h_2 \geq \log(Dh_1)$, $h_2 \geq \log D$, $h_2 \geq 1$. \par Satz 2: Sind m,n,r,K,D wie in Satz 2, aber $\theta < 1$, so gibt es ein $c_2\in{\bbfR}_+$ mit folgender Eigenschaft. Sind alle Elemente von $L = (\lambda_ij)\in \textMat_{m\times n} ({\bbfC})$ Logarithmen algebraischer Zahlen, genügt L der LUB und liegen alle $\alpha_ij := \exp(\lambda_ij)$ in K, so gilt für jedes $(x_ij)\in \textMat_{m\times n}({\bbfC})$ vom Rang $\leq r$ die Ungleichung $$\sum_{i=1}^m \sum_{j=1}^n|\lambda_ij-x_ij| \geq \exp(-c_2(Dh)^{1/(1-\theta)}).$$ Dabei ist $h \geq$ alle $h(\alpha_ij)$ und Dh ist ≥ 1 und $\geq$ alle $|\lambda_ij|$.
[Peter Bundschuh (Köln)]
MSC 2000:
*11J82 Measures of irrationality and of transcendence
Keywords: exponential function; Diophantine approximation of logarithms of algebraic number

Citations: Zbl 0916.11042

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32
Zbl 0977.11030
Waldschmidt, Michel
Half a century of transcendence. (Un demi-siècle de transcendance.) (French)
[A] Pier, Jean-Paul (ed.), Development of mathematics 1950-2000. Basel: Birkhäuser. 1121-1186 (2000). ISBN 3-7643-6280-4/hbk

This paper gives a very detailed survey of Transcendental Number Theory between 1950 and 2000. It is written in a very lively and lucid style and its 66 pages contain a lot of information. As it is almost impossible to give a complete review of it, I shall only present a list of the main topics treated in this survey. \par The different sections are: 1) Irrationality and rational approximations. 2) Around Mahler's method. 3) The Gelfond-Schneider's method: transcendence and algebraic independence. 4) The method of Siegel-Shidlovskii: E-functions and G-functions, hypergeometric functions. 5) The Gelfond-Baker's method. 6) Zero lemmas, 7) Transcendence in function fields.\par The paper ends with a list of references concerning books, surveys and proceedings of conferences. \par The first chapter on irrationality and rational approximations begins with Liouville's theorem on the approximation of algebraic numbers. It contains a section on irrationality measures (examples: π, $\zeta(3)$, e^π, \dots). Lehmer's problem is studied in a second section. The next sections present the Thue-Siegel theorem culminating with Roth's theorem and W. Schmidt's subspace theorem. The last two sections of this chapter are devoted to Padé approximations and to the irrationality of certain numbers related to the Fibonacci sequence. \par The chapter on Mahler's method begins with a section on arithmetical entire functions. The second section presents functional equations as studied first by Mahler. The next section indicates the link of these functional equations with fractals and dynamical systems. The last section deals with modular functions, a beautiful example of this theory is: π, e^π and $\Gamma(1/4)$ are algebraically independent (Yu.~Nesterenko, 1996). \par The third chapter begins with a survey on algebraic independence. The second section is ``algebraic groups and the four exponential conjecture'', for example the following problem remains unsolved: {\sl does there exist an irrational real number x such that 2^x and 3^x are both rational integers}? In the next section the case of functions of several variables is studied. This chapter ends with a short section on the classification of transcendental numbers.\par The E-functions and G-functions which are the theme of the fourth chapter were introduced by Siegel in 1929. The results on E-functions are presented in the first section and those on G-functions in the next one. For example, they lead to transcendence results for hypergeometric functions. The last section is devoted to Hilbert's irreducibility theorem. \par The first section of the chapter on the Gelfond-Baker method gives a survey on lower bounds for linear forms in logarithms, including the effective refinement of Liouville's theorem and presenting some conjectures (``abc'' and the Lang-Waldschmidt conjecture). The second section deals with Diophantine equations, more precisely the consequences of Baker's method: bounds for the solutions of certain Diophantine equations and the possibility to solve some of them completely, which is for me the greatest achievement of this theory. This section is very detailed and contains many examples. The third section is ``algebraic groups and Diophantine geometry'' and presents numerous deep works which began in the 80's. \par Chapter six is about zero lemmas, maybe the deepest question in this theory where to get a proof we almost always construct some number $ \gamma$ which is small for analytic reasons and which cannot be too small because of arithmetical reasons (Liouville estimate)\dots \ if it is nonzero, and very often the main problem is to prove (by some ``zero-lemma'') that $\gamma\not=0$. This subject is also related to effective versions of Hilbert Nullstellensatz. \par The last chapter is a survey on transcendence in function fields. The field of Laurent series $K((T^{-1}))$ has numerous similarities with the field of real numbers, this is the reason for this study. The case of nonzero characteristic leads to many problems. This theory has also links with automata theory. \par To conclude I translate (freely) the conclusion of the author: ``Very important progress has been made during the last fifty years, but the amount of open problems shows that this theory is far from being completely mature. Many questions, very easy to state, remain unsolved. But, the present methods are rich of varied and deep applications. I am sure that the future of this theory will be beautiful''.
[Maurice Mignotte (Strasbourg)]
MSC 2000:
*11Jxx Diophantine approximation
11-02 Research monographs (number theory)
11-03 Historical (number theory)
11Dxx Diophantine equations
Keywords: transcendental number theory; survey; irrationality; rational approximations; Mahler's method; Gelfond-Schneider's method; transcendence; algebraic independence; method of Siegel-Shidlovskii; E-functions; G-functions; hypergeometric functions; Gelfond-Baker's method; zero lemmas; transcendence in function fields

Cited in: Zbl 0977.11035

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33
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Zbl 0976.11037
Waldschmidt, Michel
Multiple zeta values: an introduction. (Valeurs zêta multiples. Une introduction.) (French)
[J] J. Théor. Nombres Bordx. 12, No.2, 581-595 (2000). ISSN 1246-7405

For positive integers $s_1,\ldots,s_k$ with $s_1\ge 2$, the multiple zeta values are $$\zeta(s_1,\ldots,s_k)=\sum_{n_1>\ldots>n_k

\ge 1} n_1^{-s_1}\cdots n_k^{-s_k}.$$ The product of two zeta values is a sum of zeta values which is obtained by multiplying out the two series (e.g. $\zeta(s)\zeta(s')=\zeta(s,s')+\zeta(s',s)+\zeta(s+s')$). Another linear relation arises from the iterated integral representation for $\zeta({\bold s})$. Write $x_{\bold s}=x_{\varepsilon_1}\cdots x_{\varepsilon_p}$ where ${\bold s}=(s_1,\ldots,s_k)$, $p=s_1+\cdots+s_k$ and $\varepsilon=0$ or 1. Set $\omega_0(t)=dt/t$ and $\omega_1(t)=dt/(1-t)$ and let Δ_p denote the simplex $\{{\bold t}:1>t_1>\cdots>t_p>0\}$ in $\bbfR^p$. Then $$\zeta({\bold s})=\int_{\Delta_p}\omega_{\varepsilon_1}(t_1)\cdots \omega_{\varepsilon_p}(t_p).$$ Writing this as an iterated integral leads to linear relations such as $\zeta(2)\zeta(3)=\zeta(2,3)+3\zeta(3,2)+6\zeta(4,1)$. The main conjecture is that these two types of relations are sufficient to describe all algebraic relations between these numbers. The paper goes on to explore some algebraic implications of these ideas. There are intriguing connections with the theories of polylogarithms of {\it A. B. Goncharov} [Math. Res. Lett. 5, 497-516 (1998; Zbl 0961.11040)] and {\it D. B. Zagier} [Proc. First European Congress of Mathematicians, Vol. 2, Prog. Math. 120, 497-512 (1994; Zbl 0822.11001)].
[John H.Loxton (North Ryde)]
MSC 2000:
*11M41 Other Dirichlet series and zeta functions
33B30 Higher logarithm functions
Keywords: multiple zeta values; algebraic relations; polylogarithms

Citations: Zbl 0961.11040; Zbl 0822.11001

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35
Zbl 0962.11028
Waldschmidt, Michel
Conjectures for large transcendence degree. (English)
[A] Halter-Koch, Franz (ed.) et al., Algebraic number theory and diophantine analysis. Proceedings of the international conference, Graz, Austria, August 30-September 5, 1998. Berlin: Walter de Gruyter. 497-520 (2000). ISBN 3-11-016304-7/hbk

In letzter Zeit sind verschiedentlich Zusammenhänge zwischen algebraischer Unabhängigkeit und diophantischen Approximationen gefunden worden. In der vorliegenden Arbeit werden zwei zentrale Vermutungen in dieser Richtung aufgestellt und deren wichtigste Folgerungen diskutiert. Die erste bringt den Transzendenzgrad eines über ${\Bbb Q}$ endlich erzeugten Körpers mit diophantischen Approximationseigenschaften einer Transzendenzbasis dieser Erweiterung in Verbindung. Die zweite behauptet eine untere Abschätzung für den Abstand gewisser transzendenter Zahlen von algebraischen, wobei hier der Grad (und nicht wie gewöhnlich die Höhe) der approximierenden algebraischen Zahlen die wesentliche Rolle spielt. Die Kombination beider Vermutungen würde u.a. folgende Vermutung über die algebraische Unabhängigkeit von Logarithmen algebraischer Zahlen erledigen: Sind $\lambda_1,\ldots,\lambda_n$ über ${\Bbb Q}$ linear unabhängige Logarithmen algebraischer Zahlen $\ne 0$, so sind diese λ's über ${\Bbb Q}$ algebraisch unabhängig. \par Genau lauten die beiden Vermutungen folgendermaßen: \par 1) Für $\underline \Theta :=(\Theta_1,\ldots,\Theta_m)\in{\Bbb C}^m$ sei $t :={\text{tr deg}}_{{\Bbb Q}} {\Bbb Q}(\underline\Theta) \geq 1$ und es gebe Konstanten $c_1,c_2\in{\Bbb R}_+$ mit folgenden Eigenschaften: Genügen monoton wachsende (D_ν), $(\mu_\nu)\in{\Bbb R}^{{\Bbb N}_0}$ den Bedingungen $c_1 \leq D_\nu \leq \mu_\nu$, $D_{\nu+1} \leq 2D_\nu$, $\mu_{\nu+1} \leq 2\mu_\nu$ für alle $\nu\geq 0$ und ist (μ_ν) unbeschränkt, so existiert zu unendlich vielen $\nu\in{\Bbb N}_0$ ein $\underline\gamma=(\gamma_1,\ldots,\gamma_m)\in\overline{\Bbb Q}^m$ mit $[{\Bbb Q}(\underline\gamma):= {\Bbb Q}] \leq D_\nu$, $\mu(\underline\gamma) : [{\Bbb Q}(\underline\gamma) : {\Bbb Q}] \max \{h(\gamma_1),\ldots,h(\gamma_m)\}\leq \mu_\nu$ (h die absolute logarithmische Höhe) und $\log|\Theta_j-\gamma_j|\leq -c_2D_\nu^{1/t} \mu_\nu$ für $j= 1,\ldots,m$. Eine heuristische Motivation für diese Vermutung gab M. Laurent [Györy, K. (ed.) et al., Number theory. Diophantine, computational and algebraic aspects. Proc. Internat. Conf. Eger/Hungary 1996, 311-330 (1998; Zbl 0933.11039)]. Den Fall $m= 1$ haben {\it D. Roy} und {\it M. Waldschmidt} [Ann. Sci. Ecole Norm. Sup. (4) 30, 753-796 (1997; Zbl 0895.11030)] bewiesen. \par 2) Zu jedem $n\in{\Bbb N}$ existiert ein $c(n)\in{\Bbb R}_+$ mit folgenden Eigenschaften. Seien $\lambda_1,\ldots,\lambda_n$ wie oben, $\alpha_i := e^{\lambda i}$ $(i= 1,\ldots,n)$, seien $\beta_1,\ldots,\beta_n\in\overline{\Bbb Q}$ und $\underline\gamma := (\alpha_1,\ldots,\alpha_n, \beta_1,\ldots,\beta_n)$. Sei $D\in{\Bbb N}$, $\mu\in{\Bbb R}$ mit $\mu\geq D$, so daß $[{\Bbb Q}(\underline\gamma) :{\Bbb Q}] \leq D$, $\mu(\underline\gamma) \leq \mu$ und $|\lambda_i|\leq \mu$ für $i= 1,\ldots,n$ gelten. Dann ist $$\sum^n_{i=1}|\lambda_i-\beta_i|\geq \exp(-c(n)D^{1/n}\mu).$$
[Peter Bundschuh (Köln)]
MSC 2000:
*11J82 Measures of irrationality and of transcendence
11J85 Algebraic independence results
Keywords: algebraic independence of logarithms of algebraic numbers; transcendence basis; transcendence degree

Citations: Zbl 0933.11039; Zbl 0895.11030

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37
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Zbl 0944.11024
Waldschmidt, Michel
Diophantine approximation on linear algebraic groups. Transcendence properties of the exponential function in several variables. (English)
[B] Grundlehren der Mathematischen Wissenschaften. 326. Berlin: Springer. xxiii, 633 p. DM 169.00; öS 1234.00; sFr. 153.00; \sterling 58.50; \$ 94.00 (2000). ISBN 3-540-66785-7/hbk

This extensive monograph gives an excellent report on the present state of the art of some of the most important parts in modern transcendence theory. In the following, it will be tried to provide the reader with an idea of what he may find in this book. \par The first four chapters (1: introduction and historical survey, 2: transcendence proofs in one variable, 3: heights of algebraic numbers, 4: criterion of Schneider-Lang) may serve as an introduction to the subject. For instance, the first three of these do not require much preliminary knowledge and include already complete proofs of a number of classical transcendence results. The first (of three) proof(s) of Baker's theorem on linear independence of logarithms of algebraic numbers, in Ch. 4, follows an argument of Bertrand and Masser who derived it from the Schneider-Lang criterion concerning algebraic values of meromorphic functions on cartesian products. \par Part II (linear independence of logarithms and measures) comprises chapters 5-7 on zero estimate (by D. Roy), linear independence of logarithms of algebraic numbers, homogeneous measures for linear independence. In particular, in Ch. 6 Schneider's method is extended (as in Ch. 9 for the inhomogeneous case), and in Ch. 7 a relatively simple proof for a linear independence measure is explained. \par Part III (multiplicities in higher dimension) contains multiplicity estimates (again by D. Roy), refined measures, Baker's method as chapters 8-10 where in the last one Baker's argument extending Gelfond's solution of Hilbert's seventh problem is explained. \par A central result in this book, entitling Part IV, is the so-called linear subgroup theorem (LST) which occurs in two forms. The qualitative one (Ch. 11: points whose coordinates are logarithms of algebraic numbers) gives a lower bound for the dimension of the ${\Bbb C}$-vector subspace of ${\Bbb C}^d$ spanned by points whose coordinates are either algebraic numbers or logarithms of such numbers. Finally, lower bounds for the rank of matrices (title of Ch. 12) are deduced from the LST. \par The (last) Part V, simultaneous approximation of values of the exponential function in several variables, starts with Ch.

13: a quantitative version of the LST. This version, by no means a simple statement, includes a lot of diophantine estimates, as shown in Ch. 14: applications to diophantine approximation. Ch. 15 deals with algebraic independence (criteria, small and large transcendence degrees). \par Altogether, the author's emphasis is not only on the results, but also on the methods; this is why he gives sometimes several proofs of the same result. An original feature is certainly the systematic use of Laurent's interpolation determinants instead of the classical construction of auxiliary functions via Thue-Siegel's lemma. \par What is excluded from the presentation? Not considered are elliptic curves, abelian varieties, and more generally nonlinear algebraic groups. Not discussed is Nesterenko's breakthrough concerning the algebraic independence of π and e^π; excluded are also elliptic, theta, and abelian functions as well as all kinds of non-archimedean considerations. \par The reader having enough time and energy may learn from this carefully written book a great deal of modern transcendence theory from the very beginning. In this process, the many included exercises may be very helpful. Everybody interested in transcendence will certainly admire the author's achievement to present such a clear and complete exposition of a topic growing so fast. The value of Waldschmidt's new monograph for the further development of the subject cannot be overestimated.
[P.Bundschuh (Köln)]
MSC 2000:
*11J81 Transcendence (general theory)
11-02 Research monographs (number theory)
20G15 Linear algebraic groups over arbitrary fields
20G30 Linear algebraic groups over global fields and their integers
Keywords: transcendence theory; heights of algebraic numbers; Baker's theorem; linear independence of logarithms of algebraic numbers; Schneider-Lang criterion; zero estimate; homogeneous measures for linear independence; Schneider's method; multiplicity estimates; linear subgroup theorem; simultaneous approximation; values of the exponential function in several variables; algebraic independence; small transcendence degree; large transcendence degree; Laurent's interpolation determinants; exercises

Cited in: Zbl 1011.11049

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38
Zbl pre01790495
Waldschmidt, Michel
Algebraic dynamics and transcendental numbers. (English)
[A] Planat, Michel (ed.), Noise, oscillators and algebraic randomness. From noise communication system to number theory. Lectures of a school, Chapelle des Bois, France, April 5--10, 1999. Berlin: Springer. Lect. Notes Phys. 550, 372-378 (2000). ISBN 3-540-67572-8
MSC 2000:
*37F10 Polynomials; rational maps; entire and meromorphic functions
11J81 Transcendence (general theory)
37F99 Complex dynamical systems
PDF XML ASCII DVI PS BibTeX Online Ordering

40
Zbl 1009.11003
Waldschmidt, Michel
The beginning of the theory of transcendental numbers. (Les débuts de la théorie des nombres transcendants.) (French)
[A] Serfati, Michel (ed.), La recherche de la vérité. Paris: ACL-Les Éditions du Kangourou. L'Écriture des Mathématiques. 73-96 (1999). ISBN 2-87694-057-4/pbk

Es handelt sich um eine ausführliche und interessante historische Einführung in die mathematischen Abhandlungen zur arithmetischen Natur der Zahlen. Neben Hinweisen auf Fragen der Antike, insbesondere derjenigen nach der Quadratur des Kreises, wird vor allem die Epoche von 1844 bis 1900 behandelt. In diese Zeit fallen die grundlegenden Arbeiten zur Irrationalität und Transzendenz von Liouville, Cantor, Hermite und Lindemann, deren mathematische Grundideen sehr gut herausgearbeitet werden. Einige Hinweise auf neuere Entwicklungen und auf offene Fragestellungen werden ebenfalls gegeben.
[R.Wallisser (Freiburg i.Br.)]
MSC 2000:
*11-03 Historical (number theory)
01A55 Mathematics in the 19th century
11K60 Diophantine approximation (probabilistic number theory)
11J81 Transcendence (general theory)
Keywords: Liouville; Hermite; Lindemann-Weierstraß; irrational; transcendence; 19th century
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41
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Zbl 0971.11041
Waldschmidt, Michel
Transcendence and algebraic independence of Liouville-like numbers. (English)
[J] Bull. Greek Math. Soc. 42, 119-133 (1999). ISSN 0072-7466

In den letzten Jahren sind verschiedentlich Sätze folgenden Typs bewiesen worden: Wenn $\theta_1,\ldots,$ $\theta_m\in{\bbfC}$ über ${\bbfQ}$ einen Körper vom ``kleinen'' Transzendenzgrad erzeugen, können sie simultan ``gut'' durch algebraische Zahlen approximiert werden, vgl. etwa {\it D. Roy} und Verf. [Ann. Sci. Éc. Norm. Supér., IV. Sér. 30, 753-796 (1997; Zbl 0895.11030), Ramanujan J. 1, 379-430 (1997; Zbl 0916.11042)]. Solche Resultate können Sätze über algebraische Unabhängigkeit nach sich ziehen: Wenn man eine genügend gute untere Schranke für die simultane Approximation von $\theta_1,\ldots,\theta_m$ zeigen kann, führt dies zu einer ``guten'' Unterschranke für $\texttrdeg_{\bbfQ} {\bbfQ}(\theta_1,\ldots,\theta_m)$. \par Es ist eine naheliegende Frage, ob sich solche Sätze umkehren lassen. In der vorliegenden Arbeit zeigt Verf., daß dem {\it nicht} so ist: Er konstruiert algebraisch unabhängige komplexe Zahlen, die sich simultan ``gut'' durch algebraische Zahlen beschränkter Höhe annähern lassen. Diese Zahlen sind von der Form $\theta_i = \alpha^{\xi_i}$ mit $\alpha\in{\overline{\bbfQ}}^\times$ (hier $\alpha= 2$), wobei die ξ_i gewisse Liouville-Zahlen sind. Genauer lautet das Hauptergebnis folgendermaßen. \par Seien $\varphi,\Phi: {\bbfN} \to {\bbfN}$ streng wachsend mit $\varphi(D)/D \to \infty$ bei $D \to \infty$ bzw. mit $\Phi(n+1) \geq 2 \varphi(2^{n\Phi(n)})$ für alle $n\in{\bbfN}$. Für reelles $a\in[0,1/2[$ sei $\xi_a:= \sum_{n\geq 1} [n^a]2^{-\phi(n)}$ und $\theta_a:= 2^{\xi_a}$. Dann ist die Familie $\{\theta_a\mid a\in[0,1/2[\}$ algebraisch unabhängig. Genauer gilt sogar: Zu jedem $m \in{\bbfN}_+$ und jedem $(a_1,\ldots,a_m)\in [0,1/2[^m$ existieren unendlich viele $D\in{\bbfN}_+$ mit folgender Eigenschaft: Es gibt $(\gamma_1,\ldots,\gamma_m)\in{\overline{\bbfQ}}^m$ mit allen $h(\gamma_i)\leq 1$, so daß $[{\bbfQ}(\gamma_1,\ldots, \gamma_m): {\bbfQ}]\leq D$ und $\sum^m_{i=1}|\theta_{a_i}-\gamma_i|\leq e^{-\varphi(D)}$; $h(\gamma)$ ist die absolute logarithmische Höhe von $\gamma\in{\overline{\bbfQ}}$.
[Peter Bundschuh (Köln)]
MSC 2000:

*11J85 Algebraic independence results
11J13 Simultaneous homogeneous approximation, linear forms
Keywords: transcendence; algebraic independence; Liouville-like numbers; simultaneous good approximation by algebraic numbers

Citations: Zbl 0895.11030; Zbl 0916.11042

PDF XML ASCII DVI PS BibTeX Online Ordering

44
Zbl 0941.11028
Waldschmidt, Michel
Integer valued entire functions on Cartesian products. (English)
[A] Győry, Kálmán (ed.) et al., Number theory in progress. Proceedings of the international conference organized by the Stefan Banach International Mathematical Center in honor of the 60th birthday of Andrzej Schinzel, Zakopane, Poland, June 30-July 9, 1997. Volume 1: Diophantine problems and polynomials. Berlin: de Gruyter. 553-576 (1999). ISBN 3-11-015715-2/hbk

Integer valued entire function of several complex variables are studied by techniques going back essentially to Siegel, Schneider and the author. Two far reaching results are derived. The applications concern, for example, integer valued functions of a single variable on a set of sums x_1+x_2 or products $x_1x_2, x_1\in X_1\subset \bbfC$, $x_2\in X_2\subset \bbfC$, and a new proof of the Six Exponentials Theorem. Finally some new insight in integer valued entire functions on geometric progressions is given.
[R.Wallisser (Freiburg i.Br.)]
MSC 2000:
*11J81 Transcendence (general theory)
32A15 Entire functions (several variables)
30D15 Special classes of entire functions
Keywords: integer valued entire function; six exponential theorem; several complex variables; single variable; geometric progressions

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46
Zbl 0934.11014
Brindza, Béla; Pintér, Ákos; van der Poorten, Alfred J.; Waldschmidt, Michel
On the distribution of solutions of Thue's equation. (English)
[A] Győry, Kálmán (ed.) et al., Number theory in progress. Proceedings of the international conference organized by the Stefan Banach International Mathematical Center in honor of the 60th birthday of Andrzej Schinzel, Zakopane, Poland, June 30-July 9, 1997. Volume 1: Diophantine problems and polynomials. Berlin: de Gruyter. 35-46 (1999). ISBN 3-11-015715-2/hbk

Consider the Thue equation $F(x,y)=m$, where $F(x,y)\in \bbfZ [x,y]$ is an irreducible binary form which has degree $n\geq 3$ and Mahler's measure M. Assume that m has s distinct prime factors. Finally, put $\mu(n)= (n-2)^{-1}+ (n-1)^{-2}$. It can be easily proved that all solutions (x,y) satisfy $\max(|x|,|y|)\geq \frac 12 M^{-1/2} m^{1/n}$. By a result of the first named author, at most $6n$ solutions satisfy $\max(|x|,|y|)\geq m^{1/(n-2)+ 1(n-2)}$. \par In this paper it is proved that at most $2n^2(2+1)+ 13n$ solutions satisfy $$\max(|x|,|y|)\geq 21n^2 M^5 m^{\mu(n)}.$$ Roughly speaking, this result also shows that, if m is large compared to M, then ``almost all'' solutions are ``small'' and around $m^{1/n}$. The proof makes use of Baker's theory and a gap principle.
[N.Tzanakis (Iraklion)]
MSC 2000:
*11D59 Thue-Mahler equations
Keywords: Baker's method; Thue equation; gap principle

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47
OpenURL

Zbl 0931.11020
Waldschmidt, Michel
Density measure of rational points on Abelian varieties. (English)
[J] Nagoya Math. J. 155, 27-53 (1999). ISSN 0027-7630

Let ${\cal A}$ be a simple abelian variety of dimension g over ${\Bbb Q}$, and let ℓ be the rank of the Mordell-Weil group ${\cal A}({\Bbb Q})$. Assume $\ell\ge 1$. A conjecture of Mazur [{\it B.~Mazur}, Exp. Math. 1, No. 1, 35-45 (1992; Zbl 0784.14012)] asserts that the closure of ${\cal A}({\Bbb Q})$ into ${\cal A}({\Bbb R})$ for the real topology contains the neutral component ${\cal A}({\Bbb R})^0$ of the origin. This is known only under the extra hypothesis $\ell\ge g^2-g+1$ [{\it M.~Waldschmidt}, Exp. Math. 3, 329-352 (1994; Zbl 0837.11040) and 4, 255 (1995; Zbl 0853.11057)]. \par We investigate here a quantitative refinement of this question: for each given positive h, the set of points in ${\cal A}({\Bbb Q})$ of Néron-Tate height $\le h$ is finite, and we study how these points are distributed into the connected component ${\cal A}({\Bbb R})^0$. More generally we consider an abelian variety ${\cal A}$ over a number field K embedded in ${\Bbb R}$, and a subgroup Γ of ${\cal A}(K)$ of sufficiently large rank. The effective result of density we obtain relies on an estimate of diophantine approximation, namely a lower bound for linear combinations of determinants involving abelian logarithms.
[M.Waldschmidt (Paris)]
MSC 2000:
*11G10 Abelian varieties of dimension >1
14G05 Rationality questions, rational points
11J89 Transcendence theory of elliptic and abelian functions
14K15 Arithmetic ground fields (abelian varieties)
11G50 Heights
Keywords: density measure; rational points; abelian variety; rank; Mordell-Weil group; Néron-Tate height

Citations: Zbl 0874.14012; Zbl 0837.11040; Zbl 0853.11057; Zbl 0784.14012

PDF XML ASCII DVI PS BibTeX Online Ordering Link to Serial

48
Zbl 0930.11055
Waldschmidt, Michel
Transcendence and algebraic independence of values of modular functions. (Transcendance et indépendance algébrique de valeurs de fonctions modulaires.) (French)
[A] Gupta, Rajiv (ed.) et al., Number theory. Fifth conference of the Canadian Number Theory Association, Ottawa, Ontario, Canada, August 17--22, 1996. Providence, RI: American Mathematical Society. CRM Proc. Lect. Notes. 19, 353-375 (1999). ISBN 0-8218-0964-4/pbk

The paper gives new and simpler proofs for two important recent transcendence results:\par (1) the ``théorème stéphanois'' that

$q=e^{2\pi i\tau}$ or $J(q)$ is transcendent for the elliptic modular function J [{\it K. Barré-Sirieix}, {\it G. Diaz} {\it F. Gramain} and {\it G. Philibert}, Invent. Math. 124, 1-9 (1996; Zbl 0853.11059)], using ideas of D. Bertrand;\par (2) Nesterenko's result about the algebraic independence of q, $P(q)$ and $\Delta(q)$, here under the additional hypothesis that $J(q)$ is algebraic where Δ denotes Dedekind's discriminant and P Ramanujan's Eisenstein series of weight 2.\par As a third point, the author discusses highly interesting consequences of the four exponential conjecture and its elliptic counterpart. For example, $w=1$ should be the only algebraic point on the curve $w=e^{2\pi iz}$, $|z|=1$.
[Jürgen Wolfart (Frankfurt /Main)]
MSC 2000:
*11J85 Algebraic independence results
11J89 Transcendence theory of elliptic and abelian functions
11J91 Transcendence theory of other special functions
11F03 Modular and automorphic functions
Keywords: transcendence; elliptic modular function; Nesterenko's theorem; algebraic independence; Dedekind's discriminant; Ramanujan's Eisenstein series; consequences of the four exponential conjecture

Citations: Zbl 0853.11059

PDF XML ASCII DVI PS BibTeX Online Ordering

49
Zbl 0930.11056
Waldschmidt, Michel
From simultaneous approximations to algebraic independence. (English)
[A] Yildirim, Cem Y. (ed.) et al., Number theory and its applications. Proceedings of a summer school at Bilkent University, Ankara, Turkey. New York, NY: Marcel Dekker. Lect. Notes Pure Appl. Math. 204, 283-305 (1999). ISBN 0-8247-1969-7

Der vorliegende Aufsatz ist im wesentlichen die Zusammenfassung eines Sommerschul-Kurses, den Verf. über den im Titel genannten Gegenstand abgehalten hat. \par In {\S}1 werden diophantische Approximationen mittels der "gewöhnlichen" Höhe (eines Polynoms oder einer algebraischen Zahl) diskutiert: Liouville-Abschätzung und Verallgemeinerungen; Existenz eines $f\in{\Bbb Z}[X]\setminus\{0\}$ vom Grad $\leq D$ und gewöhnlicher Höhe $\leq H$, so daß $|f(\theta)|\leq cH^{-D}$ mit explizitem $c = c(\theta, D)\in{\Bbb R}_+$ bei vorgegebenem $\theta\in{\Bbb R}$ gilt; die analoge Frage für $\theta\in{\Bbb C}\setminus {\Bbb R}$. Schließlich wird auf das eng verwandte Problem eingegangen, wie klein (in Abhängigkeit von D und H) bei festem $\theta\in {\Bbb C}\setminus \overline{\Bbb Q}$ das Infimum von $|\theta-\alpha|$ werden kann, wenn $\alpha\in\overline{\Bbb Q}$ Grad $\leq D$ und Höhe $\leq H$ hat. In dieser Richtung wird als aktuellstes Resultat Theorem 6 von {\it M. Laurent} und {\it D. Roy} [Trans. Am. Math. Soc. 351, 1845-1870 (1999; Zbl 0923.11016)] zitiert. \par In {\S}2 wird die absolute log(arithmetische) Höhe eingeführt und ihr Zusammenhang mit der gewöhnlichen Höhe besprochen. Sodann wird ein Approximationsmaß für $\theta\in{\Bbb C}$ als Abbildung $\varphi: {\Bbb N} \times {\Bbb R}_+ \to{\Bbb R}_+\cup \{\infty\}$ definiert, so daß reelle $D_0, h_0 \geq 1$ mit der Eigenschaft existieren, daß für alle $D\geq D_0$, $ h\geq h_0$ und für alle $\gamma\in\overline{\Bbb Q}$ mit Grad $\leq D$ und $\log$.Höhe $\leq h$ die Ungleichung $|\theta-\gamma|\geq \exp(-\varphi(D,h))$ gilt. Als Antwort auf die Frage, wie gut ein Approximationsmaß ausfallen kann, wird ein Ergebnis von {\it D. Roy} und Verf. [Ann. Sci. Ec. Norm. Supér., IV. Sér. 30, 753-796 (1997; Zbl 0895.11030)] angegeben: Für jedes Approximationsmaß φ eines $\theta\in{\Bbb C}\setminus\overline{\Bbb Q}$ gilt $\limsup_{D\to\infty} D^{-2}\varphi(D,h)\geq 10^{-7}h$, wenn nur h genügend groß ist. \par Analog zu {\S}2 wird im {\S}3 ein Maß φ für die simultane Approximation eines $(\theta_1,\ldots,\theta_m)\in{\Bbb C}^m$ durch die Bedingung $\max_{1\leq i\leq m} |\theta_i-\gamma_i|\geq \exp(-\varphi(D,h))$ für alle $(\gamma_1,\ldots,\gamma_m)\in\overline{\Bbb Q}^m$ mit $[{\Bbb Q}(\gamma_1,\ldots,\gamma_m):{\Bbb Q}]\leq D$ und $h(\gamma_i)\leq h$ $(i=1,\ldots,m)$ eingeführt. Als ersten Zusammenhang zwischen simultanen Approximationsmaßen und algebraischer Unabhängigkeit (a.U.) zweier Zahlen wird ein Resultat von D. Roy und Verf. [loc. cit.] angegeben: Ist $\varphi(D,h)= o(D^2)$ bei $D\to \infty$ (für alle genügend großen h) ein simultanes Approximationsmaß für $(\theta_1,\ldots,\theta_m)\in{\Bbb C}^m$, $m\geq 2$, so sind unter den Zahlen $\theta_1,\ldots,\theta_m$ mindestens zwei algebraisch unabhängige. Ein allgemeineres und neueres Resultat von M. Laurent und D. Roy wird ebenfalls vorgestellt. \par In {\S}4 geht es um a.U., die aus genügend guten simultanen Approximationsmaßen abgeleitet werden kann. Z.B. wird das Ergebnis von G. V. Chudnovsky (1976) über die a.U. von π und $\Gamma(1/4)$ aus der von {\it D. Roy} und Verf. [Ramanujan J. 1, 379-430 (1997; Zbl 0916.11042)] bewiesenen Tatsache gefolgert, daß diese beiden Zahlen ein simultanes Approximationsmaß der Form $cD^{3/2}(h+\log D)^{3/2}$ zulassen. Auch das simultane Approximationsmaß von {\it P. Philippon} [J. Reine Angew. Math. 497, 1-15 (1998; Zbl 0887.11032)] der Form $c(\varepsilon)(Dh)^{(4/3)+\varepsilon}$, $\varepsilon\in{\Bbb R}_+$ beliebig, für $\pi, e^\pi, \Gamma(1/4)$ wird genannt, welches die von {\it Yu. V. Nesterenko} [Mat. Sb. 187, 65-96 (1996); English translation in Sb. Math. 187, 1319-1348 (1996; Zbl 0898.11031)] gefundene a.U. dieser drei Zahlen auf anderem Wege begründet.
[P.Bundschuh (Köln)]
MSC 2000:
*11J85 Algebraic independence results
11J13 Simultaneous homogeneous approximation, linear forms
11-02 Research monographs (number theory)
11G50 Heights
Keywords: measure of simultaneous approximation; algebraic independence; height; Liouville estimates; absolute logarithmic height

Citations: Zbl 0923.11016; Zbl 0895.11030; Zbl 0916.11042; Zbl 0887.11032; Zbl 0898.11031

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50
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Zbl 0926.11019
Schlickewei, Hans Peter; **Schmidt, Wolfgang M.**; **Waldschmidt, Michel**
Zeros of linear recurrence sequences. (English)
[J] Manuscr. Math. 98, No.2, 225-241 (1999). ISSN 0025-2611; ISSN 1432-1785

Let $f(x)= P_0(x) \alpha_0^x+\cdots+ P_k(x) \alpha_k^x$ be an exponential polynomial over a field $\bbfK$ of zero characteristic. For $i=0,\dots, k$, $0\ne P_i\in \bbfK[x]$ and $0\ne \alpha_i\in \bbfK$ such that for each pair (i,j) with $i\ne j$, α_i/α_j is not a root of unity. Further, let $\Delta= \Delta(f)= \sum_{i=0}^k (\deg P_i+1)$. The sequence $(f(n))_{n=0}^\infty$ is a nondegenerate recurrence sequence of order Δ, and every such sequence of elements of $\bbfK$ can be represented by an exponential polynomial as given above. \par This paper is concerned with the study of $x\in \bbfZ$ for which $f(x)=0$, in particular with the number of such solutions. According to an old conjecture, this number is bounded by a function that depends on Δ only. After a brief discussion of the known facts thus far, the paper proceeds by stating the main result. A partition of $\{\alpha_0,\dots, \alpha_k\}$ into subsets $\{\alpha_{i0},\dots, \alpha_{ik_i}\}$ $(1\leq i\leq m)$ is introduced that induces a decomposition $f= f_1+\cdots+ f_m$, so that for $i=1,\dots, m$, $(\alpha_{i0}:\cdots: \alpha_{ik_i}\in \bbfP_{k_i} (\overline{\bbfQ})$, while for $i,u= 1,\dots, m$ with $i\ne u$, the number α_{i0}/ α_{u0} either is transcendental or else algebraic with not too small a height. It is shown that for all but at most $\exp (\Delta (5\Delta)^{5\Delta})$ solutions $x\in \bbfZ$ of $f(x)=0$ it is true that $f_1(x)= \cdots= f_m(x)= 0$. In particular, this result says that the conjecture can only fail when all α_i belong to the algebraic

closure of $\bbfQ$ in $\bbfK$. \par PS: After the paper was written, the conjecture was settled by the second author [{\it W. M. Schmidt}, The zero multiplicity of linear recurrence sequences. Acta Math. 182, No. 2, 243-282 (1999)].
[R.J.Stroeker (Rotterdam)]
MSC 2000:
*11D61 Exponential diophantine equations
11B37 Recurrences
Keywords: exponential diophantine equation; exponential polynomial; recurrence sequence
PDF XML ASCII DVI PS BibTeX Online Ordering Link to Full Text Link to Serial
52
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Zbl 1011.11049
Waldschmidt, Michel
On the numbers e^e, e^{e^2} and e^{π^2}. (English)
[J] Hardy-Ramanujan J. 21, 27-34 (electronic) (1998).

Author's summary: ``We give two measures of simultaneous approximation by algebraic numbers, the first one for the triple (e,e^e,e^{e^2}) and the second one for (π, e, e^{π^2}). We deduce from these measures two transcendence results which had been proved in the early 70's by {\it W. D. Brownawell} [J. Number Theory 6, 22-31 (1974; Zbl 0275.10020)] and the author [J. Number Theory 5, 191-202 (1973; Zbl 0262.10021)]." \par The formulas given in the paper are truncated. They should read as follows: \par There exist two positive constants c_1 and c_2 such that, if γ_0, γ_1, γ_2 are algebraic numbers in a field of degree D, then $$\multline |e-\gamma_0|+|e^e- \gamma_1|+|e^{e^2}- \gamma_2|>\\ \exp \{-c_1D^2 (h_0+h_1+h_2)^{1/2} (h_1+h_2)^{1/2} (h_0+\log D)(\log D)^{-1}\}, \endmultline$$ $$|\pi- \gamma_0|+|e-\gamma_1|+|e^{\pi^2}-\gamma_2|> \exp \{-c_2D^2 (h_0+ \log(Dh_1h_2)) h_1^{1/2} h_2^{1/2} (\log D)^{-1}\},$$ where $h_i= \max\{e,h(\gamma_i)\}$ for $i=0,1,2$. \par The proof of the measures appears in Chapter 14 of {\it M. Waldschmidt's} ``Diophantine approximation on linear algebraic groups" (2000; Zbl 0944.11024).
MSC 2000:
*11J82 Measures of irrationality and of transcendence
Keywords: measures of simultaneous approximation by algebraic numbers
Citations: Zbl 0944.11024; Zbl 0275.10020; Zbl 0262.10021
PDF XML ASCII DVI PS BibTeX Online Ordering Link to Full Text Link to Serial
53
Zbl 0938.11040
Waldschmidt, Michel
Extrapolation with interpolation determinants. (English)
[A] Srinivasa Rao, K. (ed.) et al., Special functions and differential equations. Proceedings of a workshop, WSSF '97, Madras, India, January 13-24, 1997. New Delhi: Allied Publishers Private Limited. 356-366 (1998). ISBN 81-7023-764-5/pbk

One of the major tools in transcendence theory is Siegel's lemma, which states that under certain hypotheses, a system of homogeneous linear equations with integer coefficients has a non-trivial solution consisting of small integers. In many diophantine approximation proofs, Siegel's lemma is used to prove the existence of a suitable auxiliary function with high order of vanishing at certain points. \par In this survey paper, the author discusses an alternative approach, in which one uses, instead of a non-explicit auxiliary function as above, an explicit so-called interpolation determinant. Such determinants already occurred in work of Cantor and Straus related to Lehmer's problem of estimating from below the Mahler measure of an algebraic number. Laurent observed that interpolation determinants could be used to prove many classical transcendence results, such as the six exponentials theorem of Lang and Ramachandra and Gel'fond's and Baker's lower bounds for linear forms in logarithms. The author discusses the applications of interpolation determinants. As an illustration, he sketches the proofs of a weak version of the six exponentials theorem and of Polyá's theorem about the growth of an entire function $f(z)$ with $f({\Bbb N})\subset{\Bbb Z}$.
[Jan-Hendrik Evertse (Leiden)]
MSC 2000:
*11J99 Diophantine approximation
11-02 Research monographs (number theory)
Keywords: interpolation determinants; transcendence; Siegel's lemma; survey; Lehmer's problem; Mahler measure; six exponentials theorem; Polyá's theorem
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58
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Zbl 0878.00049
Murty, V.Kumar (ed.); Waldschmidt, Michel (ed.)
Number theory. Proceedings of the international conference on discrete mathematics and number theory, Tiruchirapalli, India, January 3--6, 1996 on the occasion of the 10th anniversary of the Ramanujan Mathematical Society. (English)
[B] Contemporary Mathematics. 210. Providence, RI: American Mathematical Society (AMS). vii, 399 p. \$ 69.00 (1998). ISBN 0-8218-0606-8/pbk

The articles of this volume will be reviewed individually.
MSC 2000:
*00B25 Proceedings of conferences of miscellaneous specific interest
11-06 Proceedings of conferences (number theory)
14-06 Proceedings of conferences (algebraic geometry)
Keywords: Number theory; Ramanujan Mathematical Society; Discrete Mathematics; Tiruchirapalli (India); Proceedings; Conference
Cited in: Zbl 0929.00101
PDF XML ASCII DVI PS BibTeX Online Ordering
60
OpenURL

Zbl 0916.11042
Roy, Damien; Waldschmidt, Michel
Simultaneous approximation and algebraic independence. (English)
[J] Ramanujan J. 1, No.4, 379-430 (1997). ISSN 1382-4090; ISSN 1572-9303

Rappelons la notion de mesure d'approximation simultanée (MAS): soit $\theta= (\theta_1,\dots,\theta_n)\in \bbfC^n$; une application $\varphi: \bbfN\times [0,+\infty[\to [0,+\infty]$ est une MAS pour θ si il existe $D_0\in\bbfN$ et $h_0\ge 1$

tels que, pour tout entier $D\ge D_0$, tout nombre réel $h\ge h_0$ et tout n-uplet $(\gamma_1,\dots, \gamma_n)$ de nombres algébriques satisfaisant $[\bbfQ(\gamma_1,\dots, \gamma_n): \bbfQ]\le D$ et $\max_i h(\gamma_i)\le h$, on a $\max_i|\theta_i-\gamma_i|\ge\exp(- \varphi(D,h))$, où $h(.)$ est la hauteur logarithmique absolue.\par Il y a dans ce texte deux thèmes largement indépendants.\par Le premier thème concerne la façon d'obtenir des MAS pour des familles de nombres liés aux fonctions exponentielle et elliptiques. Il s'agit d'exploiter le théorème du sous-groupe algébrique (version effective) de {\it M. Waldschmidt} [J. Reine Angew. Math. 493, 61-113 (1997; Zbl 0880.11054)]; ici les AA. explicitent ce théorème dans le cas d'un groupe algébrique de la forme $\bbfG^{d_0}_a\times \bbfG^{d_1}_m\times E^{d_2}$, où E est une courbe elliptique d'invariants algébriques (Theorem 5.1) et tous les cas traités sauf un (Theorem 4.1: MAS de $(\pi, \Gamma(1/4))$) sont obtenus comme corollaires de ce Theorem 5.1. Sous ce théorème du sous-groupe algébrique est donc cachée la ``machine transcendante'', qui utilise principalement deux ingrédients: la technique des déterminants d'interpolation de M. Laurent et les lemmes de zéros pour les groupes algébriques commutatifs de P. Philippon. Bien sûr, l'idée d'obtenir des MAS via les constructions transcendantes associées à la méthode de Gel'fond-Schneider-Baker n'est pas nouvelle; ce qui est nouveau, c'est de proposer un résultat général (le théorème de Waldschmidt (loc. cit.)) ayant vocation à contenir tout ce que l'on sait faire dans le cadre des groupes algébriques commutatifs en matière d'approximation diophantienne (avec quelques nuances: voir le \S 7 de Waldschmidt (loc. cit.)). Comme exemples de MAS obtenues par les AA. notons une MAS de $(a,a^\beta,\dots, a^{\beta^{d-1}})$ où $a\in \bbfC\setminus\{0\}$ et β est algébrique de degré $d\ge 2$; une MAS de $(e^\beta,\dots, e^{\beta_n})$ où $(\beta_1,\dots, \beta_n)$ est une famille $\bbfQ$-linéairement indépendante de nombres algébriques; une MAS de $(\log \alpha,\alpha^\beta)$ où β est un nombre quadratique et $\log \alpha$ un logarithme non nul du nombre algébrique α.\par Le second thème concerne les liens qui existent entre les propriétés d'approximation diophantienne de $\underline\theta= (\theta_1,\dots, \theta_n)\in \bbfC^n$ et $t(\underline\theta)= \text{deg tr}_{\bbfQ}\bbfQ(\theta_1,\dots, \theta_n)$: que peut-on dire de $t(\underline\theta)$ lorsque l'on dipose d'une MAS φ de θ? Plus précisément, peut-on donner des conditions suffisantes sur φ qui permettent de minorer $t(\underline\theta)$? L'idée sous-jacente est que si l'on a une ``bonne'' MAS de $\underline\theta$ alors $t(\underline\theta)$ doit être ``grand'' (cela ne fonctionne pas dans l'autre sens: $t(\underline\theta)$ peut être ``grand'' sans que $\underline\theta$ possède une ``bonne'' MAS; voir par exemple {\it M. Waldschmidt} [Transcendence and algebraic independence of Liouville-like numbers, Bull. Greek Math. Soc. (à paraître)]. Les AA. donnent une condition suffisante pour avoir $t(\underline\theta)\ge 2$ (Corollary 1.2); c'est un corollaire d'un résultat d'approximation obtenu par {\it D. Roy} et {\it M. Waldschmidt} [Ann. Sci. Éc. Norm. Supér. (4) 30, No. 6, 753-796 (1997; Zbl 0895.11030)]. Ce résultat d'approximation (théorème 3.2 de l'article précité) complète des résultats antérieurs de E. Wirsing et de A. Durand, il a suscité une série de nouveaux travaux (M. Laurent et D. Roy, P. Philippon, Y. Bugeaud). Ces résultats d'approximation sont liés à la question suivante: étant donné un nombre transcendant θ, avec quelle précision peut-on approcher θ par des nombres algébriques dont on veut contrôler le degré et/ou la hauteur?\par Le Corollary 1.2 (condition suffisante pour avoir $t(\underline\theta)\ge 2$) peut être vu comme une alternative au classique ``critère de Gel'fond''. Mais il est clair que, si l'objectif principal est d'obtenir $t(\underline\theta)\ge 2$, le passage par une MAS peut être pénalisant en ce sens qu'il oblige à introduire des ``hypothèses techniques'' indésirables (voir un exemple avec le Theorem 2.3). Par ailleurs on ne dispose pas de résultat général donnant une minoration $t(\underline\theta)\ge k$ (en dehors du cas $k=2$). Il existe une conjecture générale de {\it M. Waldschmidt} [Conjectures for large transcendence degree, Number theory, Conference Graz 1998 (à paraître)] et {\it P. Philippon} [Approximations algébriques des points dans les espaces projectifs (soumis)], avec un point de vue légèrement différent, annonce des résultats qui devraient pouvoir conduire à $t(\underline\theta)\ge 3$, $t(\underline\theta)\ge 4$.
[Guy Diaz (Saint-Etienne)]
MSC 2000:
*11J85 Algebraic independence results
11J82 Measures of irrationality and of transcendence
11J89 Transcendence theory of elliptic and abelian functions
11J91 Transcendence theory of other special functions
Keywords: algebraic approximation; algebraic independence; simultaneous approximation; exponential functions; elliptic functions; Waldschmidt's algebraic subgroup theorem; logarithmic independence; transcendental methods; heights

Citations: Zbl 0880.11054; Zbl 0895.11030

Cited in: Zbl 1030.11033 Zbl 1002.11061 Zbl 0971.11041 Zbl 0930.11056 Zbl 0923.11105 Zbl 0933.11039 Zbl 0895.11030

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62
Zbl 0908.11029
Waldschmidt, Michel
On the arithmetic nature of the values of modular functions. (Sur la nature arithmétique des valeurs de fonctions modulaires.)
(French)
[A] Séminaire Bourbaki. Volume 1996/97. Exposés 820--834. Paris: Société Mathématique de France, Astérisque. 245, 105-140, Exp. No.824 (1997).

This is a very useful survey -- including some new proofs -- about old and new transcendence and algebraic independence results concerning automorphic functions. The author starts with {\it Th. Schneider's} [Math. Ann. 113, 1--13 (1936; Zbl 0014.20402)] transcendence result concerning $j(\tau)$ where j is the elliptic modular function and τ algebraic but not imaginary quadratic. Then he describes a proof of the Mahler-Manin conjecture that $$J(q):= J(e^{2\pi i\tau}):= j(\tau)$$ has transcendental values at algebraic arguments $q= e^{2\pi i\tau}$, $0<| q|< 1$, proved recently by {\it K. Barré-Sirieix}, {\it G. Diaz}, {\it F. Gramain} and {\it G. Philibert} [Invent. Math. 124, 1--9 (1996; Zbl 0853.11059)].\par The second part treats results about algebraic independence of periods of elliptic curves and the values of modular forms and functions at the corresponding arguments in the upper half plane, beginning with the results of {\it G. V. Chudnovsky} [Proc. Int. Cong. Math., Helsinki 1978, Vol. 1, 339--350 (1980; Zbl 0431.10019)]. It culminates with {\it Yu. Nesterenko's} result [Sb. Math. 187, No. 9, 1319--1348 (1996); translation from Mat. Sb. 187, 65--96 (1996; Zbl 0898.11031)] that the transcendence degree of $\bbfQ(q,P(q),Q(q), R(q))$ is at least 3 if $0<| q|<1$ and P, Q, R denote the Eisenstein series of weight 2, 4, 6 in Ramanujan's notation. Nestenko's theorem has many spectacular consequences, e.g. the algebraic independence of the three numbers π, e^π, $\Gamma(1/4)$ or π, $e^{\pi \sqrt 3}$, $\Gamma(1/3)$, several corollaries about values of theta functions, and the transcendence of the series $\sum F^{-2}_n$ for the Fibonacci numbers F_n. All these results have effective versions!\par Besides classical methods of transcendence, the main new ingredients are zero estimates established in the last years by Nesterenko, Philippon and Philibert. These are described in Section 3.\par Waldschmidt's survey ends with a rich commented bibliography (of 83 titles). Many conjectures spread all over the text show that transcendence has made remarkable progress but did not at all arrive at an end.
[Jürgen Wolfart (Frankfurt am Main)]
MSC 2000:
*11J89 Transcendence theory of elliptic and abelian functions
11J85 Algebraic independence results
11J91 Transcendence theory of other special functions
11F11 Modular forms, one variable
33B15 Gamma-functions, etc.
11F03 Modular and automorphic functions
Keywords: survey; transcendence; algebraic independence; automorphic functions; elliptic modular function; Mahler-Manin conjecture; Eisenstein series

Citations: Zbl 0014.20402; Zbl 0853.11059; Zbl 0431.10019; Zbl 0898.11031

Cited in: Zbl 0889.11027

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64

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Zbl 0895.11030

Roy, Damien; Waldschmidt, Michel

Diophantine approximation and algebraic independence of logarithms. (Approximation diophantienne et indépendance algébrique de logarithmes.) (French)

[J] Ann. Sci. Éc. Norm. Supér. (4) 30, No. 6, 753-796 (1997). ISSN 0012-9593

In der vorliegenden Arbeit stellen Verff. einen neuartigen Zugang zur algebraischen Unabhängigkeit bei kleinen Transzendenzgraden vor. Statt des üblichen Gel'fondschen Kriteriums verwenden sie ein neues Resultat über diophantische Approximationen, das frühere Ergebnisse von {\it E. Wirsing} [J. Reine Angew. Math. 206, 67-77 (1961; Zbl 0097.03503)] und {\it A. Durand} [C. R. Acad. Sci., Paris, Sér. A 287, 595-597 (1978; Zbl 0389.10028)] ergänzt. Dieses Resultat garantiert die Existenz guter algebraischer Approximationen an Familien von Zahlen aus einem Teilkörper von $\Bbb C$ vom Transzendenzgrad 1 über $\Bbb Q$. Mittels dieses Resultats geben Verff. in Fortführung ihrer Note [{\it D. Roy} und {\it M. Waldschmidt}, Proc. Jap. Acad., Ser. A 71, 151-153 (1995; Zbl 0860.11040)] in der vorliegenden Arbeit Beweise für algebraische Unabhängigkeit, die sich letztlich auf die von {\it M. Laurent} [Astérisque 198/200, 209-230 (1991; Zbl 0762.11027)] eingeführte Technik der Interpolationsdeterminanten stützt. \par Das Hauptresultat, das Verff. mit ihrer Methode beweisen, ist völlig neuartig und kann hier aufgrund des zu seiner Formulierung nötigen umfangreichen Begriffsapparates nicht reproduziert werden. Eine Folgerung aus diesem Hauptresultat sei zitiert: Sind $\log\alpha_1,\ldots,\log \alpha_n$ über $\Bbb Q$ linear unabhängige Logarithmen nichtverschwindender algebraischer Zahlen, die einen Körper vom Transzendenzgrad 1 über $\Bbb Q$ erzeugen, so gilt $q(\log\alpha_1,\ldots,\log\alpha_n)\not= 0$ für jede quadratische Form $q \in{\Bbb Q}[X_1,\ldots,X_n] \setminus \{0\}$. \par Verff. haben die vorliegende Studie inzwischen im Ramanujan J. 1, 379-430 (1997; Zbl 0916.11042) fortgesetzt.

[P.Bundschuh (Köln)]

MSC 2000:

*11J85 Algebraic independence results

Keywords: diophantine approximation; algebraic independence of logarithms; small transcendence degree; algebraic approximations; real quadratic forms

Citations: Zbl 0097.03503; Zbl 0389.10028; Zbl 0860.11040; Zbl 0762.11027; Zbl 0916.11042

Cited in: Zbl 0962.11028 Zbl 0971.11041 Zbl 0930.11056 Zbl 0933.11039 Zbl 0916.11042

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66

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Zbl 0885.11048

Waldschmidt, Michel

Integer valued functions on products. (English)

[J] J. Ramanujan Math. Soc. 12, No.1, 1-24 (1997). ISSN 0970-1249

Le théorème principal de l'article est le suivant: Soient a, b, r, c des nombres réels >0 tels que $1/a+ 1/b\le 1/r$. Alors il existe un nombre réel $d>0$ ayant la propriété suivante: si X et Y sont des parties de $\bbfC$ telles que $\textcard(X\cap \{|z|\le R\})\ge c R^a$ et $\textcard(Y\cap \{|z|\le R\})\ge cR^b$ pour tout R suffisamment grand, et si $f:\bbfC\to \bbfC$ est une fonction entière telle que $f(xy)\in\bbfZ$ pour tout $(x,y)\subset X\times Y$ et $\log\max\{|f(z)|;|z|\le R\}\le dR^r$ pour tout R suffisamment grand, alors f est un polynôme.\par Sa démonstration est assez inattendue. L'A. utilise un théorème auxiliaire concernant une fonction entière de 2 variables: Soient X et Y des suites non bornées de nombres complexes. Si $g: \bbfC^2\to \bbfC$ est une fonction entière telle que $g(X\times Y)\subset\bbfZ$ et ne croissant pas trop vite (la condition assez compliquée fait intervenir X et Y), alors les $\bbfQ$-espaces vectoriels engendrés par $\{z\mapsto g(x,z); x\in X\}$ d'une part et $\{z\mapsto g (z,y); y\in Y\}$ d'autre part sont de même dimension finie.\par La démonstration utilise un lemme de Schwarz (en plusieurs variables) pour les déterminants d'interpolation. Le théorème principal a pour corollaire un cas particulier du théorème des 6 exponentielles. \par Dans ``Integer valued entire functions on Cartesian products'' [Proc. Zakopane Number Theory Conf. June-July 1997 (à paraître)], l'A. a développé des corollaires du théorème auxiliaire (fonctions prenant des valeurs entières sur des sous-groupes additifs de $\bbfC$ ou multiplicatifs de $\bbfC^\times$). Il y obtient aussi des résultats comparables par un procédé plus naturel et plus proche de la méthode classique de Schneider en plusieurs variables.

[F.Gramain (Saint-Etienne)]

MSC 2000:

*11J81 Transcendence (general theory)

30D15 Special classes of entire functions

32A15 Entire functions (several variables)

Keywords: integer valued functions; entire functions; polynomials; Schwarz lemma; interpretation determinants; special case of six exponentials theorem

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67

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Zbl 0880.11054

Waldschmidt, Michel

Diophantine approximation in commutative algebraic groups. I: An effective version of the algebraic subgroup. (Approximation diophantienne dans les groupes algébriques commutatifs. I: Une version effective du sous-groupe algébrique.) (French)

[J] J. Reine Angew. Math. 493, 61-113 (1997). ISSN 0075-4102; ISSN 1435-5345

The aim of this paper is to produce a general estimate of diophantine approximation. The main statement (which is too long to be explicitly stated here) includes several effective transcendence results for numbers related with the exponential map of a commutative algebraic group which is defined over the field of complex algebraic numbers. \par Applications (which are not considered in the paper under review) include transcendence measures, lower bounds for linear combinations of logarithms or rational points on algebraic groups, density statements and distribution measure of rational points on an algebraic group (for instance on an abelian variety), as well as results of algebraic independence arising from measures of simultaneous approximation. Concerning this last topic, see the two following papers by {\it D. Roy} and {\it M. Waldschmidt} [Approximation diophantienne et indépendance algébrique de logarithmes, Ann. Sci. Éc. Norm. Supér., IV. Sér. 30, 753-796 (1997)] and [Simultaneous approximation and algebraic independence, Ramanujan J. 4, 379-430 (1997)].

[M.Waldschmidt (Paris)]
MSC 2000:
*11J81 Transcendence (general theory)
11G10 Abelian varieties of dimension >1
14G05 Rationality questions, rational points
14L10 Group varieties
Keywords: numbers related with exponential map; effective transcendence results; commutative algebraic group

Cited in: Zbl 0916.11042

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70

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Zbl 0863.11044
Waldschmidt, Michel
Simultaneous approximation by products of powers of algebraic numbers. (Approximation simultanée par des produits de puissances de nombres algébriques.) (French)
[J] Acta Arith. 79, No.2, 137-162 (1997). ISSN 0065-1036; ISSN 1730-6264

Let $\zeta_1, \dots, \zeta_d$ be real or complex numbers and let α_ij be nonzero algebraic numbers $(1 \leq i\leq d$, $1 \leq j\leq l)$. We consider the simultaneous approximation of the numbers ζ_i by algebraic numbers of the form $\alpha_{i1}^{t_1} \cdots \alpha_il^{t_l}$, with $(t_1, \dots, t_l)\in \bbfZ^l$. A transference theorem reduces this question to a problem of diophantine approximation of logarithms of algebraic numbers, which we solve by means of an explicit version of the linear subgroup theorem.
[M.Waldschmidt (Paris)]
MSC 2000:
*11J13 Simultaneous homogeneous approximation, linear forms
11J81 Transcendence (general theory)
11H60 Mean value and transfer theorems
Keywords: transcendental numbers; transference theorem; logarithms of algebraic numbers; linear subgroup theorem

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71

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Zbl 0883.11036
Waldschmidt, Michel
Dependence of logarithms on commutative algebraic groups. (English)
[J] Rocky Mt. J. Math. 26, No.3, 1199-1223 (1996). ISSN 0035-7596

Eine wohlbekannte Vermutung besagt, daß über $\bbfQ$ linear unabhängige Logarithmen von 0 verschiedener algebraischer Zahlen schon über $\bbfQ$ algebraisch unabhängig sein müssen. Einerseits ist bisher nicht einmal die Existenz {\itzweier} algebraisch unabhängiger Logarithmen algebraischer Zahlen gesichert. Andererseits hat {\it D. Roy} [Acta Math. 175, 49-73 (1995; Zbl 0833.11031)] gezeigt, daß obige Vermutung äquivalent ist zu einer (hier nicht kurz zu beschreibenden) Vermutung über einen exakten Ausdruck für den Rang gewisser Matrizen, deren Eingänge entweder algebraische Zahlen oder Logarithmen von solchen sind. Bei der genaueren Formulierung dieser letzteren Vermutung kommt eine spezielle lineare algebraische Gruppe in natürlicher Weise ins Spiel. \par Verf. geht nun den Weg, algebraische Unabhängigkeit von Logarithmen allgemein in kommutativen algebraischen Gruppen zu untersuchen in einer Weise, daß sich daraus im Fall linearer algebraischer Gruppen in Richtung auf die letztere Vermutung ergibt: Der fragliche Matrizenrang ist mindestens halb so groß wie vermutet.
[P.Bundschuh (Köln)]
MSC 2000:
*11J81 Transcendence (general theory)
14L10 Group varieties
11J85 Algebraic independence results
11J89 Transcendence theory of elliptic and abelian functions
Keywords: commutative algebraic groups; logarithmic dependence; algebraic independence; logarithms of algebraic numbers

Citations: Zbl 0833.11031

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76

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Zbl 0859.11012
Balasubramanian, R.; Langevin, M.; Shorey, T.N.; Waldschmidt, M.
On the maximal length of two sequences of integers in arithmetic progressions with the same prime divisors. (English)
[J] Monatsh. Math. 121, No.4, 295-307 (1996). ISSN 0026-9255; ISSN 1436-5081

It was conjectured by Erdös and Woods [see {\it R. Balasubramanian}, {\it T. N. Shorey}, {\it M. Waldschmidt}, Acta Math. Hung. 54, 225-236 (1989; Zbl 0689.10049)] that there exists a positive integer k with the property: if $x,y\in \bbfZ^+$ are such that for $1 \le i\le k$ the sets of prime divisors of $x+i$ and $y+i$ coincide, then $x=y$. In the present paper an analogue of this problem for arithmetic progressions is considered.\par Let $u(n)$ denote the greatest squarefree divisor of the positive integer n. For each quadruple (x,y,d,d') of positive integers satisfying $$(x,d)\ne(y,d') \quad \text and \quad \textgcd(x,d) = \textgcd(y,d')=1, \tag 1 $$ let $K=K(x,y,d,d')$ be the largest positive integer K for which $u(x+id) = u(y+id')$ for $0\le i\le K-1$. The authors prove the following propositions, assuming the abc-conjecture of Masser and Oesterlé to be true:\par For each pair (d,d') of two positive integers, the set of pairs (x,y) satisfying (1) and $K(x,y,d,d')$$>2$ is finite; and\par the set of quadruples (x,y,d,d') satisfying (1) and $K(x,y,d,d')>4$ is finite.\par Also some interesting partial, unconditional results are obtained.
[R.J.Stroeker (Rotterdam)]
MSC 2000:
*11B25 Arithmetic progressions
11N25 Distribution of integers with specified multiplicative constraints
11J86 Linear forms in logarithms; Baker's method
Keywords: maximal length of sequences; linear forms in logarithms; abc-conjecture; prime divisors; arithmetic progressions; greatest squarefree divisor

Citations: Zbl 0689.10049

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80
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Zbl 0860.11040
Roy, Damien; Waldschmidt, Michel
Quadratic relations between logarithms of algebraic numbers. (English)
[J] Proc. Japan Acad., Ser. A 71, No.7, 151-153 (1995). ISSN 0386-2194

Bisher ist die Vierexponenten-Vermutung lediglich in folgendem Spezialfall von {\it W. D. Brownawell} [J. Number Theory 6, 22-31 (1974; Zbl 0275.10020)] bzw. {\it M. Waldschmidt} [J. Number Theory 5, 191-202 (1973; Zbl 0262.10021)] bewiesen worden: Seien x_1, x_2 komplexe, über $\bbfQ$ linear unabhängige Zahlen und y_1,y_2 ebenso; der Transzendenzgrad von $\bbfQ(x_1, x_2,y_1, y_2)$ über $\bbfQ$ sei 1. Dann ist mindestens eine der vier Zahlen $\exp(x_j y_k)$ transzendent.\par Hierfür skizzieren Verff. einen neuen Beweis, der sich nicht mehr wie die früheren Ansätze auf Gelfonds Transzendenzkriterium stützt. Stattdessen verwenden Verff. um die von {\it M. Laurent} [Astérisque 198/200, 209-230 (1991; Zbl 0762.11027)] eingeführten Interpolationsdeterminanten kombiniert mit einem Resultat von {\it E. Wirsing} [J. Reine Angew. Math. 206, 67-77 (1961; Zbl 0097.03503)] über die Approximierbarkeit transzendenter Zahlen durch algebraische von beschränktem Grad. Schließlich wird ein Spezialfall einer Nullstellenabschätzung von {\it P. Philippon} [Bull. Soc. Math. Fr. 114, 355-383 (1986; Zbl 0617.14001) bzw. 115, 397-398 (1987; Zbl 0634.14001)] herangezogen.\par Ohne Beweis kündigen die Verfasser folgende Verallgemeinerung des obigen Brownawell-Waldschmidtschen Satzes an: Sei $V\subset\bbfC^n$ die Nullstellenmenge eines nicht konstanten Polynoms $P\in\bbfQ [X_1,\dots,X_n]$ eines Grades ≤ 2 und sei $(\lambda_1, \dots, \lambda_n) \in V \cap L^n$, wo $L:= \{z\in \bbfC \mid e^z\in \overline\bbfQ\}$ gesetzt ist. Der Transzendenzgrad von $\bbfQ (\lambda_1, \dots, \lambda_n)$ über $\bbfQ$ sei 1. Dann liegt $ (\lambda_1, \dots, \lambda_n)$ in einem über $\bbfQ$ definierten Teilraum von $\bbfC^n$, der seinerseits in V enthalten ist. (Nimmt man hier $n=4$ und $P=X_1X_4-X_2 X_3$, so erhält man gerade den oben zitierten Satz).
[P.Bundschuh (Köln)]
MSC 2000:
*11J81 Transcendence (general theory)
11J85 Algebraic independence results
Keywords: logarithms of algebraic numbers; four exponents conjecture; transcendence degree; interpolation determinants; transcendental approximation; Brownawll-Waldschmidt theorem; zero estimates

Citations: Zbl 0275.10020; Zbl 0262.10021; Zbl 0762.11027; Zbl 0097.03503; Zbl 0617.14001; Zbl 0634.14001

Cited in: Zbl 0895.11030

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83
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Zbl 0853.11057
Waldschmidt, Michel
Density of rational points on an algebraic group. Errata. (Densité des points rationnels sur un groupe algébrique (errata).) (French)
[J] Exp. Math. 4, No.3, 255 (1995). ISSN 1058-6458

An additional condition is added to a statement made on p. 346 of the original [see ibid. 3, 329-352 (1994; Zbl 0837.11040)], and a remark on p. 344 is corrected.
MSC 2000:
*11J81 Transcendence (general theory)
14G05 Rationality questions, rational points
Citations: Zbl 0837.11040

Cited in: Zbl 0931.11020

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89
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Zbl 0837.11040
Waldschmidt, Michel
Density of rational points on an algebraic group. (Densité des points rationnels sur un groupe algébrique.) (French)
[J] Exp. Math. 3, No.4, 329-352 (1994). ISSN 1058-6458

Let G be a commutative algebraic group defined over a number field K, and let Γ be a finitely generated subgroup of $G(K)$. The author first supposes that K is embedded in $\bbfR$ and studies under which conditions does Γ contain a dense subgroup of the neutral component $G(\bbfR)^0$ of $G(\bbfR)$ for the real topology. He gives sufficient conditions for this to hold, and defines a class of groups G for which necessary and sufficient conditions can be given. This study is motivated by a conjecture of {\it B. Mazur} about the density of rational points on a nonsingular algebraic variety defined over $\bbfQ$ [Exp. Math. 1, 35-45 (1992; Zbl 0784.14012)].\par Write G in the form $\bbfG_a^{d_0} \times \bbfG_m^{d_1} \times G_2$ where G_2 is defined over $\overline \bbfQ$ and has no linear factor of positive dimension. Put $\alpha (G) = d_1 + 2d_2$ and denote by $ \kappa_\bbfR (G)$ the rank of the kernel of $\exp_G$ in $T_G (\bbfR)$. Define $m_\bbfR (G)$ to be the smallest rank which can have a finitely generated dense subgroup of $G (\bbfR)^0$, and put $$m_\bbfR' (G) = \alpha (G) (d - 1) + \min \{2,d + 1 - d \kappa_\bbfR (G) \}.$$ Then, the main result of the paper takes the following form:\par (a) Assume that, for any algebraic subgroup G' of G defined over K with $\dim G' < \dim G$, one has $\text rank_\bbfZ (\Gamma/ \Gamma \cap G'(K)) \ge m_\bbfR' (G/ G')$. Then, Γ contains a dense subgroup of $G(\bbfR)^0$.\par (b) Assume that, for any algebraic subgroup G' of G defined over K with $\dim G' < \dim G$, one has $\text rank_\bbfZ (\Gamma/ \Gamma \cap G'(K)) \ge m_\bbfR' (G/G') + d - 1$. Then, Γ contains a dense subgroup of rank $m_\bbfR (G)$ of $G(\bbfR)^0$.\par The author defines G to have the density property if, independently of the choice of Γ, the following two conditions are equivalent:\par (i) the topological closure of Γ in $G(\bbfR)$ contains $G(\bbfF)^0$;\par (ii) if $\{\gamma_1, \dots, \gamma_l\} \subset \Gamma$ generate a subgroup of Γ of finite index and if H denotes the Zariski closure of the point $(\gamma_l, \dots, \gamma_l)$ in $G^l $, then there is a point $(\eta_1, \dots, \eta_l)$ in $H(\bbfR)$ such that $\{\eta_1, \dots, \eta_l\}$ generate a dense subgroup of $G (\bbfR)^0$.\par The main result and the density property are studied in more details in the case of linear groups, abelian varieties and extensions of an elliptic curve by a multiplicative group. It is hoped that the density property holds for those families of groups, and this leads to many interesting conjectures. In the case of the linear groups, it is shown that the assumption that $\bbfG_m^d$ has the density property for all positive integers d and all number fields $K \subset \bbfR$ is equivalent to a standard conjecture about logarithms of algebraic numbers.\par The author also studies the case where K is embedded in $\bbfC$, and considers the topological closure of Γ in $G(\bbfC)$. This question is reduced to the first by considering the group $\widetilde G$ gotten from G by extending the scalars from $\bbfR$ to $\bbfC$.\par A short errata concerning $\S 4$ should appear in the same

journal.
[D.Roy (Ottawa)]
MSC 2000:
*11J81 Transcendence (general theory)
14G05 Rationality questions, rational points
Keywords: group of rational points; Mazur's conjecture; commutative algebraic group; density property; linear groups; abelian varieties; extensions of an elliptic curve

Citations: Zbl 0784.14012

Cited in: Zbl 0931.11020 Zbl 0859.11046 Zbl 0853.11057

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92

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Zbl 0801.11033
Mignotte, Maurice; Waldschmidt, Michel
On algebraic numbers of small height: Linear forms in one logarithm. (English)
[J] J. Number Theory 47, No.1, 43-62 (1994). ISSN 0022-314X

The authors establish a lower bound for $\vert\alpha -1\vert$ where α is a complex algebraic number $\ne 1$. This lower bound is completely explicit in terms of the degree D of α and its Mahler measure $M(\alpha)$.\par Given any number $\mu>0$ with $\mu\geq \log M(\alpha)$, the authors prove $$\vert\alpha -1\vert\geq \exp \Bigl\{- \bigl(\textstyle{3\over 2} \sqrt{D\mu \log\sb + (D/\mu)}+ 2\mu+ \log\sb + (D/\mu) \bigr) \Bigr\},$$ where $\log\sb + x= \max\{ \log x,0\}$. From this inequality, they deduce $$\vert \alpha-1 \vert\geq \max \{2, M(\alpha) \}\sp{-3 \sqrt{D\log D}},$$ which refines a result of {\it M. Mignotte} [Ann. Fac. Sci. Toulouse 1, 165-170 (1979; Zbl 0421.10022)] and improves on Liouville's inequality concerning the dependency in D.\par The above mentioned result follows from a more general theorem from which the authors also deduce an application to Lehmer's problem as well as the conjecture of {\it A. Schinzel} and {\it H. Zassenhaus} [Mich. Math. J. 12, 81-85 (1965; Zbl 0128.034)]. A corollary of this theorem which is sufficient for these applications is the following:\par Let E be a number field of degree D over $\bbfQ$, let $\alpha\sb 1, \dots, \alpha\sb K$ be distinct elements of E with $K\geq 2$, and let G' be a set of embeddings of E into $\bbfC$. For each $\sigma\in G'$, choose $\phi\sb \sigma\in i\bbfR$ and determinations of $\log (\sigma \alpha\sb k)$ for $k=1,\dots, K$. Then the average values $$\Lambda\sb \sigma= \textstyle {1 \over K} \sum\sb{k=1}\sp K \vert \log (\sigma \alpha\sb k)+ \phi\sb \sigma\vert$$ satisfy $$- \sum\sb{\sigma\in G'} \log \Lambda\sb \sigma \leq {{2D} \over K} \sum\sb{k=1}\sp K h(\alpha\sb k)+ {D\over {K-1}} \biggl(2+\log \biggl({{2K+1} \over 4} \biggr) \biggr)+ {K\over {K-1}} \vert G'\vert,$$ where $h(\beta)$ stands for the absolute logarithmic Weil height of β and $\vert G'\vert$ denotes the cardinality of G'. The method of proof uses the technique of interpolation determinants of {\it M. Laurent} [Acta Arith. 66, 181- 199 (1994; see the review below)].
[D.Roy (Ottawa)]
MSC 2000:
*11J86 Linear forms in logarithms; Baker's method
Keywords: linear forms in logarithms; Mahler measure; height; interpolation determinants

Citations: Zbl 0801.11034; Zbl 0421.10022; Zbl 0128.034

Cited in: Zbl 1032.11031 Zbl 0927.11050 Zbl 0892.11023 Zbl 0866.11060 Zbl 0848.11047 Zbl 0802.11011

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94
Zbl 1014.11502
Waldschmidt, Michel
On the difference between two products of powers of algebraic numbers. (English)
[A] Nagasaka, Kenji (ed.), Analytic number theory and related topics. Proceedings of the symposium, Tokyo, Japan, November 11-13, 1991. Singapore: World Scientific. 127-138 (1993). ISBN 981-02-1499-5/hbk

Summary: Let k and n be two positive integers with $1\leq k< n$, let $\alpha_1,\dots, \alpha_n$ be real algebraic numbers, and let $h_1,\dots, h_n$ be positive integers such that $\alpha_1^{h_1}\cdots \alpha_k^{h_k}\ne \alpha_{k+1}^{h_{k+1}}\cdots \alpha_n^{h_n}$. Our aim is to give explicit lower bounds for the distance between these two numbers. Until recently, all effective nontrivial estimates valid for any n used Gel'fond-Baker's method. The new estimates we give here rest on another transcendence method, which is related to Schneider's solution of Hilbert's seventh problem.
MSC 2000:
*11J86 Linear forms in logarithms; Baker's method
Keywords: explicit lower bounds; Schneider's solution of Hilbert's seventh problem

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97

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Zbl 0813.11041
Roy, Damien; Waldschmidt, Michel
On the algebraic subgroup theorem. (Autour du théorème du sous-groupe algébrique.) (French)
[J] Can. Math. Bull. 36, No.3, 358-367 (1993). ISSN 0008-4395; ISSN 1496-4287

Sei G eine über $\overline \bbfQ$ definierte kommutative algebraische Gruppe positiver Dimension, $\exp\sb G : T\sb G (\bbfC) \to G (\bbfC)$ die Exponentialabbildung der Liegruppe $G(\bbfC)$, sei $\Omega (G)$ deren Kern und $\Lambda (G) : = \exp\sb G \sp{-1} (G (\overline \bbfQ))$ der aus den Logarithmen der über G algebraischen Punkte gebildete $\bbfQ$- Vektorraum. Für algebraische Zahlkörper K bezeichne $\Omega\sb K(G)$ (bzw. $\Lambda\sb K (G))$ den von $\Omega (G)$ (bzw. $\Lambda (G))$ erzeugten K-Untervektorraum von $T\sb G (\bbfC)$. Ist $\beta : = (\beta\sb 1, \dots, \beta\sb n)$ eine $\bbfQ$-Basis von K und identifiziert man $T\sb{G\sp n} (\bbfC)$ mit $T\sp n\sb G (\bbfC)$, so wird durch $(z\sb 1, \dots, z\sb n) \mapsto \beta\sb 1 z\sb 1 + \cdots + \beta\sb n z\sb n$ eine $\bbfC$-lineare Surjektion $p\sb \beta : T\sb{G\sp n} (\bbfC) \to T\sb G (\bbfC)$ definiert. Sei $S\sb K(G)$ die (von der Basiswahl in K unabhängige) Menge der Teilräume von $T\sb G (\bbfC)$ der Form $p\sb \beta (T\sb{G'} (\bbfC))$, wo G' eine über $\overline \bbfQ$ definierte algebraische Untergruppe von $G\sp n$ ist.\par Bezeichnet schließlich $d\sb 0 (G)$ (bzw. $d\sb 1 (G))$ die größte natürliche Zahl d, zu der es einen über $\overline \bbfQ$ definierten Epimorphismus von G in $G\sb a\sp d$ (bzw. in $G\sp d\sb m)$ gibt $(G\sb a$ bzw. $G\sb m$ additive bzw. multiplikative Gruppe), so setzt man $\alpha (G) : = 2 \dim (G) - 2d\sb 0 (G) - d\sb 1 (G)$. Damit definiert man $\alpha\sb K (G) : = {1 \over n} \alpha (G\sp n/H)$, wobei H die größte zusammenhängende algebraische Untergruppe von $G\sp n$ bezeichnet, die $p\sb \beta (T\sb H (\bbfC)) = 0$ genügt.\par Verff. beweisen in dieser Terminologie nun folgenden Satz: Sei E ein mit einer $\overline \bbfQ $-Struktur versehener $\bbfC$-Vektorraum endlicher Dimension, $\psi : T\sb G (\bbfC) \to E$ eine lineare, über $\overline \bbfQ$ rationale Abbildung und seien V, W zwei Teilräume von E mit $W \subset V$ und W rational über $\overline \bbfQ$, so

sind folgende drei Aussagen äquivalent. (i) V enthält keinen Teilraum $\ne \{0\}$ der Gestalt $\psi (S)$ mit $S \in S\sb K (G)$; (ii) der K-Vektorraum $V \cap \psi (\Lambda\sb K (G))$ hat endliche Dimension; (iii) es gilt $\dim\sb K (V \cap \psi (\Lambda\sb K (G)) \le (\alpha\sb K (G) - \kappa) \dim\sb \bbfC (V/W)$, wo $\kappa : = \dim\sb K (V \cap \psi (\Omega\sb K (G)))$ gesetzt ist. \par Spezialfälle dieses Satzes wurden verschiedentlich gezeigt, insbesondere von beiden Verff. Man vergleiche etwa {\it D. Roy} [J. Number Theory 41, 22-47 (1992; Zbl 0763.11030)], dessen Schlußweisen die hier vorliegenden Verallgemeinerung inspiriert haben.
[P.Bundschuh (Köln)]
MSC 2000:
*11J81 Transcendence (general theory)
11R99 Algebraic number theory over global fields
14L99 Algebraic groups
Keywords: algebraic subgroup theorem; commutative algebraic groups

Citations: Zbl 0763.11030

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100
Zbl 0791.11032
Waldschmidt, Michel
Transcendental numbers and functions of several variables. (English)
[A] Gouvêa, Fernando Q. (ed.) et al., Advances in number theory. The proceedings of the third conference of the Canadian Number Theory Association, held at Queen's University, Kingston, Canada, August 18-24, 1991. Oxford: Clarendon Press. 67-80 (1993). ISBN 0-19-853668-2/hbk

This paper presents a survey of recent applications of the theorem of the algebraic subgroup [the author, New advances in transcendence theory 1986, 375-398 (1988; Zbl 0659.10035)] together with a new proof of this result using interpolation determinants.\par The first two applications concern density questions. One is the existence of finitely generated subgroups of the multiplicative group $k\sp \times$ of a number field k whose image is dense in $(k\otimes \bbfR)\sp{\times}$, together with an answer to the question of J.-J. Sansuc about the smallest rank of such a subgroup [{\it D. Roy}, Invent. Math. 109, 547-556 (1992; Zbl 0780.11060)]. The other is a step towards a conjecture of B. Mazur. It says that if the Mordell-Weil group $A(\bbfQ)$ of a simple Abelian variety A defined over $\bbfQ$ is sufficiently large then the topological closure of $A(\bbfQ)$ in $A(\bbfR)$ contains the neutral component of $A(\bbfR)$.\par The author also gives four different ways to recover Baker's theorem from the algebraic subgroup theorem. They are dual by pairs associated to A. Baker's method and N. Hirata's method. The duality in question is explained in the paper and is reflected by the fact that the corresponding interpolation matrices are transposed of one another [see the author, J. Anal. Math. 56, 231-254, 255-279 (1991; Zbl 0742.11035 and Zbl 0742.11036)].\par The last application is motivated by Leopold's conjecture and concerns an arithmetic lower bound for the rank of matrices whose coefficients are linear combinations of logarithms of algebraic numbers with algebraic coefficients [the reviewer, J. Number Theory 41, 22-47 (1992; Zbl 0763.11030)]. There is also mention of a generalization of this type of result in the context of algebraic groups [{\it D. Roy} and {\it M. Waldschmidt}, Autour du théorème du sous-groupe algébrique, Can. Math. Bull. 36, 358-367 (1993)].
[D.Roy (Ottawa)]
MSC 2000:
*11J81 Transcendence (general theory)
11R99 Algebraic number theory over global fields
Keywords: transcendence; Mazur's conjecture; linear forms in logarithms; survey; interpolation determinants; density; finitely generated subgroups of the multiplicative group; algebraic subgroup theorem; Baker's method; Hirata's method; duality; Leopold's conjecture; rank of matrices

Citations: Zbl 0659.10035; Zbl 0780.11060; Zbl 0742.11035; Zbl 0742.11036; Zbl 0763.11030

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105

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Zbl 0774.11036
Waldschmidt, Michel
Lower bounds for linear forms in logarithms of algebraic numbers. (Minorations de combinaisons linéaires de logarithmes de nombres algébriques.) (French)
[J] Can. J. Math. 45, No.1, 176-224 (1993). ISSN 0008-414X; ISSN 1496-4279

Es ist wohlbekannt, daß Schneiders klassische Transzendenzmethode (für Funktionen einer komplexen Variablen) zu sehr guten unteren Abschätzungen für Linearkombinationen in $n=2$ Logarithmen algebraischer Zahlen $\alpha\sb j\ne 0$ mit algebraischen Koeffizienten $\beta\sb j$ führt, vgl. etwa {\it M. Mignotte} und Verf. [Ann. Fac. Sci. Toulouse, V. Sér., Math. 1989, Spec. Issue, 43-75 (1989; Zbl 0702.11044)].\par In der vorliegenden Arbeit werden überaus detaillierte, hier aber keinesfalls reproduzierbare untere Abschätzungen für $\vert\beta\sb 1\log \alpha\sb 1+\dots+ \beta\sb n \log \alpha\sb n\vert$ bei beliebigem $n\geq 2$ durch eine Verallgemeinerung der Schneiderschen Methode gewonnen nach dem vom Verf. in [J. Anal. Math. 56, 231-279 (1991; Zbl 0742.11035 und 36)] und \S1 von [Sémin. Théor. Nombres Bordx., Sér. II 3, 129-185 (1991; Zbl 0733.11020)] dargestellten Beweisprinzip. U. a. ist in den erzielten Ergebnissen eine Verbesserung des Theorems 2 von {\it J. H. Loxton}, {\it M. Mignotte}, {\it A. J. van der Poorten} und Verf. [C. R. Math. Acad. Sci., Soc. R. Can. 11, 119-124 (1987; Zbl 0623.10023)] enthalten.
[P.Bundschuh (Köln)]
MSC 2000:
*11J86 Linear forms in logarithms; Baker's method
Keywords: lower bounds; linear forms in logarithms of algebraic numbers; Schneider's method in several variables

Citations: Zbl 0742.11036; Zbl 0702.11044; Zbl 0742.11035; Zbl 0733.11020; Zbl 0623.10023

Cited in: Zbl 0963.11021 Zbl 0876.11037 Zbl 0849.11031 Zbl 0774.11034 Zbl 0774.11035

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111

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Zbl 0809.11038
Waldschmidt, Michel
Linear independence of logarithms of algebraic numbers (with an appendix by M. Laurent). (English)
[B] IMSc Report. 116. Madras: Institute of Mathematical Sciences, 168 p. (1992).

Die vorliegende Monographie ging aus einer Vortragsreihe hervor, die Verf. im Frühjahr 1992 in Indien, insbesondere in Madras, gehalten hat. Sein erstes Ziel war ein möglichst einfacher Beweis des Bakerschen Satzes über Linearformen in Logarithmen algebraischer Zahlen mit algebraischen Koeffizienten (homogener Fall). Dabei werden die früheren Techniken von Baker, d.h. Ableitungen einer geeigneten Hilfsfunktion in mehreren Variablen und Extrapolation, nicht mehr benützt. Stattdessen stützt sich Verf.

auf die von M. Laurent in diesem Kontext eingeführten Interpolationsdeterminanten: Vergleich analytischer oberer Abschätzungen derselben mit arithmetischen unteren führen zum Ziel, wenn man noch eine Nullstellenabschätzung investiert, die auf einem Satz von Bézout über die als endlich vorausgesetzte Anzahl der gemeinsamen Nullstellen einer endlichen Anzahl von Polynomen in mehreren Variablen beruht.\par Der zweite Teil beginnt mit einer quantitativ präzisierten Durchsicht der Beweisanordnung des ersten Teils, um Maße für die lineare Unabhängigkeit der Logarithmen algebraischer Zahlen zu gewinnen. Diese Maße erreichen zunächst nicht die heute bekannten besten, genügen aber schon zur effektiven Behandlung gewisser diophantischer Gleichungen, ein hier nicht weiter vertieftes Thema. Sodann werden schärfere Maße abgeleitet, indem eine von P. Philippon stammende verbesserte Nullstellenabschätzung ebenso wie genauere Aussagen über Interpolationsdeterminanten herangezogen werden. Der Fall inhomogener Linearformen wird zum Schluß ebenso behandelt wie der Sechsexponentensatz mit Verallgemeinerungen.\par Ein Anhang über Vermutungen und einer von {\it M. Laurent} über Interpolationsdeterminanten und den für viele Anwendungen so bedeutsamen Spezialfall von Linearformen in zwei Logarithmen beschließt diese Vorlesungsausarbeitung. Das Werk erhält seinen Wert nicht zuletzt dadurch, daß jedem Kapitel eine Reihe von Übungsaufgaben ebenso wie eine bis in die neueste Zeit reichende einschlägige Literaturliste beigefügt sind.
[P.Bundschuh (Köln)]
MSC 2000:
*11J86 Linear forms in logarithms; Baker's method
11-02 Research monographs (number theory)
Keywords: Baker's theorem; linear forms in logarithms of algebraic numbers with algebraic coefficients; interpolation determinant; zero estimates; homogeneous linear forms; inhomogeneous linear forms; six exponentials theorem; linear forms in two logarithms

Cited in: Zbl 0829.11017

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123

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Zbl 0784.00021
Waldschmidt, Michel (ed.); Moussa, Pierre (ed.); Luck, Jean-Marc (ed.); Itzykson, Claude (ed.)
From number theory to physics. Lectures of a meeting on number theory and physics held at the Centre de Physique, Les Houches (France), March 7-16, 1989. (English)
[B] Berlin: Springer-Verlag. xiii, 690 p. DM 128.00 (1992). ISBN 3-540-53342-7/hbk

Contents: Pierre Cartier, An introduction to zeta functions (1--63); Jean-Benoît Bost, Introduction to compact Riemann surfaces, Jacobians, and abelian varieties (64--211); Henri Cohen, Elliptic curves (212--237); Don Zagier, Introduction to modular forms (238--291); Robert Gergondey, Decorated elliptic curves: modular aspects (292--312); Harold M. Stark, Galois theory, algebraic number theory, and zeta functions (313--393); Èric Reyssat, Galois theory for coverings and Riemann surfaces (394--412); Frits Beukers, Differential Galois theory (413--439); Gilles Christol, p-adic numbers and ultrametricity (440--475); Marjorie Sénéchal, Introduction to lattice geometry (476--495); André Katz, A short introduction to quasicrystallography (496--537); Jean Béllissard, Gap labelling theorems for Schrödinger operators (538--630); Predrag Cvitanovic, Circle maps: irrationally winding (631--658); Jean-Christophe Yoccoz, An introduction to small divisors problems (659--679). The articles of this volume will be reviewed individually.
MSC 2000:
*00B25 Proceedings of conferences of miscellaneous specific interest
11-06 Proceedings of conferences (number theory)
11Z05 Miscellaneous appl. of number theory
12-06 Proceedings of conferences (field theory)
14-06 Proceedings of conferences (algebraic geometry)
37-06 Proceedings of conferences (Dynamical systems and ergodic theory)
52-06 Proceedings of conferences (convex and discrete geometry)
81-06 Proceedings of conferences (quantum theory)
82-06 Proceedings of conferences (statistical mechanics)
Keywords: Number theory; Physics; Les Houches (France); Meeting; Lectures

Citations: Zbl 0702.00008

Cited in: Zbl 1104.11003 Zbl 1106.11002 Zbl 0852.17027

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127

Zbl 0770.11038
Waldschmidt, Michel
Constructions de fonctions auxiliaires. (Constructions of auxiliary functions). (French)
[A] Approximations diophantiennes et nombres transcendants, C.-R. Colloq., Luminy/ Fr. 1990, 285-307 (1992). ISBN 3-11-013486-1/hbk

[For the entire collection see Zbl 0745.00060.]\par Verf. stellt eine Transzendenzmethode vor, bei der die einzige eingehende analytische Schlußweise sich sogleich im ersten Beweisschritt findet. Dies ist bis heute der einzig gangbare Weg, um Funktionen mehrerer Variablen zu behandeln, für die weder ein Schwarzsches Lemma noch Interpolationsformeln vorliegen. Er konstruiert eine Hilfsfunktion, die er benutzt, um einen Beweis des Bakerschen Linearformensatzes (sowohl nach Gelfonds als auch nach Schneiders Methode) zu geben. Allerdings führt diese Hilfsfunktion auf dem quantitativen Sektor nicht zu den derzeit besten Abschätzungen. \par Sodann bringt Verf. eine erste Verbesserung ein, die sich auf Bakers Methode bezieht; eine entsprechende Verfeinerung von Schneiders Methode schließt sich an. Um dies Programm durchzuführen, entwickelt er den Begriff der Dualität, bei dem analytische Funktionale ins Spiel kommen. Als Beispiel gibt er ein neues Beweisschema für Bakers Satz mittels des dualen Analogons der Methode von {\it N. Hirata} [Invent. Math. 104, 401- 433 (1991; Zbl 0704.11016)]. Abschließend kombiniert er die angedeuteten Ergebnisse und baut eine sehr allgemeine Hilfsfunktion auf, deren Relevanz für Fragen der algebraischen Unabhängigkeit er aufzeigt.\par Es sei darauf hingewiesen, daß Verf. die hier nur angerissene Problematik der Konstruktion von Hilfsfunktionen und -funktionalen ebenso wie sein Dualitätsprinzip in zwei umfangreichen Arbeiten ausführlich dargestellt hat [J. Anal. Math. 56, 231-254, 255-279 (1991; Zbl 0742.11035 und 11036)].
[P.Bundschuh (Köln)]
MSC 2000:
*11J81 Transcendence (general theory)
11J86 Linear forms in logarithms; Baker's method
11J85 Algebraic independence results
Keywords: transcendence method; auxiliary function; Baker's theorem; Gel'fond's method; Schneider's method; duality; analytic functionals; algebraic independence

Citations: Zbl 0716.11035; Zbl 0745.00060; Zbl 0704.11016; Zbl 0742.11035; Zbl 0742.11036

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138

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Zbl 0742.11036
Waldschmidt, Michel
Fonctions auxiliaires et fonctionnelles analytiques. II. (Auxiliary functions and analytical functionals. II). (French)
[J] J. Anal. Math. 56, 255-279 (1991). ISSN 0021-7670

See the joint review of Parts I and II above.
[chuh (Köln)]
MSC 2000:
*11J81 Transcendence (general theory)
11J85 Algebraic independence results
11J86 Linear forms in logarithms; Baker's method
Keywords: Thue-Siegel lemma; exponential polynomial in several variables; Baker's method; Schneider's method; Gel'fond method; Hilbert's seventh problem; Fourier-Borel transform; functionals

Citations: Zbl 0742.11035; Zbl 0454.10020

Cited in: Zbl 0791.11032 Zbl 0774.11036 Zbl 0770.11038 Zbl 0742.11035

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139
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Zbl 0742.11035
Waldschmidt, Michel
Fonctions auxiliaires et fonctionnelles analytiques. I. (Auxiliary functions and analytical functionals. I). (French)
[J] J. Anal. Math. 56, 231-254 (1991). ISSN 0021-7670

(This is a joint review of Parts I and II). \par Gemeinsames Ziel beider Teile der vorliegenden fast 50-seitigen Arbeit ist es, solche Hilfskonstruktionen systematisch zu entwickeln, wie sie im ersten Schritt der meisten Transzendenzbeweise vorkommen. In Teil I benutzt Verf. das Schubfachprinzip (Siegels Lemma), um Hilfsfunktionen in einer oder mehreren Variablen zu gewinnen. Im zweiten Teil wird eine duale Konstruktion vorgestellt, die Hilfsfunktionale liefert.\par Nun zur genaueren Besprechung dieser wichtigen Arbeit. Während $\S 1$ einige hilfreiche generelle Bemerkungen sowie fast zwei Seiten Notationen enthält (der Leser ahnt schon, daß im Referat kaum Details zitiert werden können), werden in $\S 2$ Hilfsfunktionen konstruiert, die eine frühere Arbeit des Verf. [Théorème 3.1 in Invent. Math. 63, 97-127 (1981; Zbl 0454.10020)] verallgemeinert. Dazu werden zwei Hilfsmittel bereitgestellt: Erstens eine geeignete Version des (Thue-)Siegelschen Lemmas, zweitens eine Interpolationsformel. Das Hauptresultat dieses Paragraphen ist Proposition 2.3, in der der Rang einer geeigneten Matrix ins Spiel gebracht wird. In Lemma 2.6 wird dann gezeigt, wie man diesen Rang in geeigneten Fällen nichttrivial nach oben abschätzen kann.\par Satz 3.4, das Hauptergebnis des $\S 3$, liefert die Existenz eines Exponentialpolynoms in mehreren Variablen, das in einer ``großen'' Kreisscheibe betraglich ``klein'' bleibt; man erhält dieses Ergebnis, indem man das aus Lemma 2.6 direkt folgende Korollar 2.7 geeignet spezialisiert. $\S 3$ endet mit zwei konkreten Beispielen, wobei das erste mit der Methode von Baker, das zweite mit einer mehrdimensionalen Verallgemeinerung derjenigen von Schneider zusammenhängt. Während die vorgeführten Konstruktionen besonders für mehrere Variable nützlich sind, wird in $\S 4$ der eindimensionale Spezialfall extra herausgestellt und gezeigt, wie sich die ``klassischen'' Beweise (Hermite-Lindemann, Lindemann-Weierstraß, Gel'fond-Schneider, Sechsexponentensatz, große Transzendenzgrade von Körpern, die von Zahlen der Form $\exp(x\sb jy\sb k)$ erzeugt sind) in das hier entwickelte allgemeine Konzepte einordnen. In $\S 5$, mit dem Teil II beginnt, erläutert Verf. zunächst, wie eng die beiden Wege von Gel'fond bzw. Schneider zur Lösung des 7. Hilbert-Problems miteinander zusammenhängen. Gel'fond benutzte eine Hilfsfunktion $\varphi\sb G(z)=P\sb G(e\sp z,e\sp{\beta z})$, deren Ableitungen er an den Stellen $h\log\alpha$ auswertete; mit dem Ansatz $P\sb G(X,Y)=\sum\sb{(\lambda,\mu)}p(\lambda,\mu)X\sp \lambda Y\sp \mu$ hat man $$(d/dz)\sp t\varphi\sb G(h\log\alpha)=\sum\sb{(\lambda,\mu)}p(\lambda,\mu)(\lambda+\mu \beta)\sp t\alpha\sp{\lambda h}(\alpha\sp \beta)\sp{\mu h}. \leqno (*\sb G):$$ Schneider dagegen arbeitete mit einer Hilfsfunktion $\varphi\sb S(z)=P\sb S(z,\alpha\sp z)$, die er an den Stellen $\lambda+\mu\beta$ auswertete; mit $P\sb S(X,Y)=\sum\sb{(t,h)}q(t,h)=X\sp tY\sp h$ hat man $$\varphi\sb S(\lambda+\mu\beta)=\sum\sb{(t,h)}q(t,h)(\lambda+\mu\beta)\sp t\alpha\sp{\lambda h}(\alpha\sp \beta)\sp{\mu h}. \leqno (*\sb S):$$ Diese Dualität beider Methoden, vgl. $(*\sb G)$ und $(*\sb S)$, war eine der Quellen der vorliegenden Arbeit. So wie die $(d/dz)\sp t\varphi\sb G(h\log\alpha)$ in Gel'fonds Methode die Werte des Funktionals $\eta\sb G(F)=\sum\sb{(\lambda,\mu)}p(\lambda,\mu)F(\lambda+\mu\beta)$ für $F=z\sp t\alpha\sphz$ sind, sind die $\varphi\sb S(\alpha+\mu\beta)$ in Schneiders Methode die Werte des Funktionals $\eta\sb S(F)=\sum\sb{(t,h)}q(t,h)(d/dz)\sp tF(h\log\alpha)$ für $F=\exp((\lambda+\mu\beta)z)$.\par In $\S 6$ konstruiert Verf. allgemeine Hilfsfunktionale (wie die speziellen η soeben), indem er dem im ersten Teil entwickelten Beweisschema genau folgt. Der Grund für die Analogie zwischen beiden Situationen, hier Hilfsfunktionale, dort Hilfsfunktionen, wird in $\S 7$ aufgedeckt: Die Transformation von Fourier-Borel ordnet jedem linearen beschränkten Funktional η eine ganze Funktion $\varphi\sb \eta(\zeta)=\eta(e\sp{\zeta z})$ vom Exponentialtyp zu und umgekehrt entspricht jeder ganzen Funktion $\varphi(\zeta)=\sum\sb \kappa a\sb \kappa\zeta\sp \kappa/\kappa!$ vom Exponentialtyp das beschränkte lineare Funktional $\eta\sb \varphi:\sum\sb \kappa b\sb \kappa z\sp \kappa\mapsto\sum\sb \kappa a\sb \kappa b\sb \kappa$. Die $\S\S 8$ bzw. 9 schließlich sind Beispielen bzw. dem eindimensionalen Fall gewidmet.\par Neue arithmetische Anwendungen der in der vorliegenden Arbeit durchgeführten Konstruktionen von Hilfsfunktionen bzw. -funktionalen werden in Aussicht gestellt.
[P.Bundschuh (Köln)]
MSC 2000:
*11J81 Transcendence (general theory)
11J85 Algebraic independence results
11J86 Linear forms in logarithms; Baker's method
Keywords: Thue-Siegel lemma; exponential polynomial in several variables; Baker's method; Schneider's method; Gel'fond method; Hilbert's seventh problem; Fourier-Borel transform; functionals

Citations: Zbl 0742.11036; Zbl 0454.10020

Cited in: Zbl 0791.11032 Zbl 0774.11036 Zbl 0770.11038 Zbl 0742.11036

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140
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Zbl 0733.11020
Waldschmidt, Michel
Nouvelles méthodes pour minorer des combinaisons linéaires de logarithmes de nombres algébriques. (New methods for lower bounds of linear forms in logarithms of algebraic numbers). (French)
[J] Sémin. Théor. Nombres Bordx., Sér. II 3, No.1, 129-185 (1991). ISSN 0989-5558

Bisher war Bakers Methode im Fall $n\ge 3$ die einzige, die zu effektiven unteren Schranken für nichttriviale Linearformen $\beta\sb

1 \log \alpha\sb 1+...+\beta\sb n \log \alpha\sb n$ in Logarithmen algebraischer $\alpha\sb 1,...,\alpha\sb n$ führte. Während jedoch Bakers Ansatz Gel'fonds Lösung des siebten Hilbert-Problems verallgemeinerte, basiert der Ansatz des Verf. auf Schneiders Lösung des genannten Problems. Verf. benutzt dabei weder eine Extrapolationstechnik noch Kummertheorie; auch sind seine Hilfsfunktionen gänzlich von den Bakerschen verschieden. Seine Methode ist dual (im Sinne seiner Arbeit [J. Anal. Math. 56, 231-254, 255-279 (1991)]) zu der von {\it N. Hirata-Kohno} [Sémin. Théor. Nombres Paris 1988-89, Prog. Math. 91, 117-140 (1990; Zbl 0716.11033)] im elliptischen Fall angewandten und führt in der hier behandelten Situation $\beta\sb 1,...,\beta\sb n\in {\bbfZ}$ zu einem völlig expliziten Resultat (Théorème 1.1), das quantitativ dieselbe Güte erreicht, wie sie Bakers Methode heute herzuleiten gestattet.
[P.Bundschuh (Köln)]
MSC 2000:
*11J86 Linear forms in logarithms; Baker's method
11J82 Measures of irrationality and of transcendence
Keywords: Schneider's method

Citations: Zbl 0716.11033

Cited in: Zbl 0774.11036 Zbl 0752.11029 Zbl 0733.11021

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143

OpenURL

Zbl 0971.11521
Waldschmidt, Michel
Algebraic values of analytic functions. (English)
[J] Doga, Turk. J. Math. 14, No.2, 70-78 (1990). ISSN 1010-7622

The author surveys results and open problems in transcendental number theory highlighting new developments. See also Acta Arith. 47, 97-121 (1986; Zbl 0553.10026).
MSC 2000:
*11J91 Transcendence theory of other special functions
11-03 Historical (number theory)
11J81 Transcendence (general theory)
11J85 Algebraic independence results
Keywords: transcendence

Citations: Zbl 0553.10026

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144

OpenURL

Zbl 0802.11023
Waldschmidt, M.
Transcendence problems connected with Drinfeld modules. (English)
[J] Istanb. Üniv. Fen Fak. Mat. Derg. 49, 57-75 (1990). ISSN 1300-0713

This is a fine exposition on the state of transcendency problems connected with function fields over finite fields and Drinfeld modules circa 1990. We will content ourselves here mentioning a few references. In terms of transcendency of zeta-values for $\bbfF\sb q [t]$ (as discussed on pp. 66-67, N.B.: on p. 67 $p-1$ should be $q-1$) these were first discussed by Thakur in [Th1] for those cases not covered by Wade's results. It was also suggested there that perhaps transcendence of $\zeta(n)/ \pi\sp n$ for $n\not\equiv 0\pmod {q-1}$ might follow from a version of Roth's theorem. However, Roth's result fails in finite characteristic and is now an active area of research. On p. 70, the author mentions the analog of Bessel functions discovered by Carlitz. Recently, these Bessel functions have been put into the context of an amazing theory of hypergeometric functions associated to function fields by {\it D. Thakur} [Hypergeometric functions for function fields, J. Finite Fields Appl. (to appear)]. In particular, there are remarkable connections to higher tensor powers of the Carlitz module. There are also new results that go a long way towards developing the transcendency theory of zeta-values for general A. For these the reader should consult [{\it D. Thakur}, Int. Math. Res. Not., 185-197 (1992; Zbl 0756.11015) and {\it G. Anderson}, Duke Math. J. 73, 491-542 (1994)].
[D.Goss (Columbus / Ohio)]
MSC 2000:
*11G09 Drinfel'd modules, etc.
Keywords: exposition; transcendency; function fields over finite fields; Drinfeld modules; analog of Bessel functions

Citations: Zbl 0756.11015

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146

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Zbl 0733.11021
Waldschmidt, Michel
Sur l'équation de Pillai et la différence entre deux produits de puissances de nombres entiers. (On the Pillai equation and the difference between two products of powers of integers). (French)
[J] C. R. Math. Acad. Sci., Soc. R. Can. 12, No.5, 173-178 (1990). ISSN 0706-1994

Verf. benutzt das Hauptergebnis seiner vorstehend referierten Arbeit, um zu beweisen: Bei $n\ge 2$ seien $a\sb 1,...,a\sb n\in {\bbfN}$ multiplikativ unabhängig und $b\sb 1,...,b\sb n\in {\bbfZ}$ nicht alle Null; gilt $\vert b\sb n\vert \log a\sb n\ge \max\sb{1 \le i<n}\vert b\sb i\vert \log a\sb i,$ $a\sb 0:=\min\sb{1\le i<n}a\sb i$ und $M:=\max (n\sp{4n}e\sp{20n+10},\vert b\sb n\vert / \log a\sb 0),$ so hat man $$ \log \vert a\sb 1\sp{b\sb 1}...a\sb n\sp{b\sb n}-1\vert >-C(n)\log M \log a\sb 1...\log a\sb n\quad mit\quad C(n)<e\sp{3n+9}n\sp{4n+5}. $$ Als Anwendung hiervon wird gezeigt: Genügen x,y,m,n,k$\in {\bbfN}$, $y\ge 2$ der Pillaischen Gleichung $x\sp m-y\sp n=k$ mit $x\sp m\ge k\sp 2$, so gilt $m<137\cdot 10\sp{10} \log y$.
[P.Bundschuh (Köln)]
MSC 2000:
*11J86 Linear forms in logarithms; Baker's method
11D75 Diophantine inequalities
11D61 Exponential diophantine equations
Citations: Zbl 0733.11020

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147

Zbl 0729.11035
Waldschmidt, Michel
Indépendance algébrique de nombres de Liouville. (Algebraic independence of Liouville numbers). (French)
[A] Fifty years of polynomials, Proc. Conf. in Honour of Alain Durand, Paris/Fr. 1988, Lect. Notes Math. 1415, 225-235 (1990).

[For the entire collection see Zbl 0683.00011.] \par This useful survey provides a historical overview of the subject distinguishing results of two kinds: in the first instance those that yield algebraically independent sets of numbers by the value of gap series; and in the second results that arise from the theory of diophantine approximation - in particular from the notion of transcendence measure. There are some 50 references.
[A.J.van der Poorten (North Ryde)]
MSC 2000:
*11J85 Algebraic independence results
11J82 Measures of irrationality and of transcendence
30B10 Power series (one complex variable)
11-02 Research monographs (number theory)
Keywords: survey; historical overview; algebraically independent sets; gap series; transcendence measure

Citations: Zbl 0683.00011

PDF XML ASCII DVI PS BibTeX Online Ordering

149
Zbl 0714.11043
Waldschmidt, M.
Dependence of logarithms of algebraic points. (English)
[A] Number theory. Vol. II. Diophantine and algebraic, Proc. Conf., Budapest/Hung. 1987, Colloq. Math. Soc. János Bolyai 51, 1013-1035 (1990).

[For the entire collection see Zbl 0694.00006.] \par Soit $G={\bbfG}\sb a\sp{d\sb 0}\times {\bbfG}\sb m\sp{d\sb 1}\times G\sb 2$ un groupe algébrique commutatif connexe défini sur le corps ${\bar {\bbfQ}}$ des nombres algébriques, de dimension $d\sb 0+d\sb 1+d\sb 2\ge 2$, on note $\exp\sb G: T\sb G({\bbfC})\to G({\bbfC})$ l'exponentielle de G et Λ l'image réciproque de G(${\bar {\bbfQ}})$ par $\exp\sb G$. Soit V un sous-espace vectoriel de dimension n (sur ${\bbfC})$ de $T\sb G({\bbfC})$. Si V contient une sous-algèbre de Lie algébrique non nulle de $T\sb G({\bbfC})$ définie sur ${\bar {\bbfQ}}$, on sait que le ${\bbfQ}$-espace vectoriel $V\cap \Lambda$ est de dimension infinie. Dans le cas contraire, l'auteur montre que $$ \dim\sb{{\bbfQ}}(V\cap \Lambda) \le (d\sb 1+2d\sb 2-k)(d-t-1), $$ où k est le rang sur ${\bbfZ}$ de $V\cap Ker \exp\sb G$ et t la dimension du ${\bbfC}$-espace vectoriel engendré par $V\cap T\sb G({\bar {\bbfQ}})$. Ce résultat est obtenu à partir du ``théorème du sous-groupe algébrique'' [Théorème 4.1 de {\it M. Waldschmidt}, New advances in transcendence theory, Proc. Symp. Durham 1986, 375-398 (1988; Zbl 0659.10035)], et l'auteur en donne de nombreux corollaires: par exemple, il montre le résultat de {\it G. Wüstholz} sur les logarithmes des groupes algébriques généraux [in: Approximations diophantiennes et nombres transcendants, Colloq. Luminy 1982, Prog. Math. 31, 329-336 (1983; Zbl 0534.10026)], et dans le cas où G est linéaire il retrouve, entre autres, le théorème des 6 exponentielles et les démonstrations du Théorème de Baker par la méthode de Schneider et par la méthode de Gel'fond-Baker. \par L'auteur montre aussi que, si $L=\{x\in {\bbfC} $; $e\sp x\in {\bar {\bbfQ}}\}$, si V est un sous-espace vectoriel de ${\bbfC}\sp d$, avec $d\ge 2$, et si $V\cap {\bbfQ}\sp d=0,$ alors $\dim V\cap L\sp d\le n(n+1),$ améliorant la borne de {\it M. Emsalem} [J. Reine Angew. Math. 382, 181-198 (1987; Zbl 0621.12008). Un exemple montre qu'on ne peut pas espérer une borne $<n(n+1)/2$ et {\it D. Roy} a montré que $n(n+1)/2$ était la borne donnée par la conjecture de Schanuel [Sémin. Théor. Nombres, Paris 1987-88, 273-281 (1990; Zbl 0712.11042)]. Il faut noter que {\it D. Roy}, toujours à partir du ``théorème du sous- groupe algébrique'', a obtenu des majorations analogues pour la dimension sur ${\bar {\bbfQ}}$ de $V\cap {\cal L}\sp d$, où ${\cal L}$ est l'espace des formes linéaires (inhomogènes) de logarithmes à coefficients algébriques [Matrices whose coefficients are linear forms in logarithms, J. Number Theory (to appear)].
[F.Gramain]
MSC 2000:
*11J86 Linear forms in logarithms; Baker's method
14L10 Group varieties
Keywords: commutative algebraic groups; linear forms of logarithms; Schneider's method; Gel'fond's method; Baker's theorem

Citations: Zbl 0694.00006; Zbl 0659.10035; Zbl 0534.10026; Zbl 0621.12008; Zbl 0712.11042

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154
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Zbl 0702.00008
Luck, Jean-Marc (ed.); Moussa, Pierre (ed.); Waldschmidt, Michel (ed.)
Number theory and physics. Proceedings of the winter school, Les Houches, France, March 7-16, 1989. (English)
[B] Springer Proceedings in Physics, 47. Berlin etc.: Springer-Verlag. xiii, 311 p. DM 99.00 (1990). ISBN 3-540-52129-1

Contents: P. Degiovanni, $\Bbb Z/N\Bbb Z$ conformal field theories (2--19); M. Bauer and C. Itzykson, Affine characters and modular transformations (20--32); H. Ooguri, Conformal field theory on a Riemann surface (33--43); A. Shapere, Modular invariance of field theories and string compactifications (44--55); H. J. de Vega, Yang-Baxter algebras and quantum groups (56--67); H. Saleur, Representations of $U_q \textsl(2)$ for q a root of unity (68--76); M. Duneau and C. Oguey, Some quasiperiodic tilings as modulated lattices (78--85); C. Godrèche, Types of order and diffraction spectra for tilings of the line (86--99); A. Katz, A topological constraint on the atomic structure of quasicrystals (100--110); R. Mosseri and F. Bailly, From approximants to quasicrystals: a nonstandard approach (111--117); N. Rivièr, The topological structure of grain boundaries (118--127); J.-L. Verger-Gaugry, Calculation of 6D atomic surfaces from a given approximant crystalline structure using approximate icosahedral periodic tilings (128--137); J. Béllissard, Spectral properties of Schrödinger's operator with a Thue-Morse potential (140--150); P. Cohen, On the noncommutative torus of real dimension two (151--156); H. Kunz, Topological invariants associated with quasi-periodic quantum Hamiltonians (pp. 157--161); A. Sütö, Spectra of some almost periodic operators (162--169); D. J. Thouless, The quantum Hall effect and the Schrödinger equation with competing periods (170--176); J.-P. Allouche, Finite automata in 1-D and 2-D physics (pp. 177--184); J.-M. Dumont, Summation formulae for substitutions on a finite alphabet (185--194); M. Mendès France, The inhomogeneous Ising chain and paperfolding (195--202); F. M. Dekking, A nonlinear evolution with travelling waves (204--208); J. S. Geronimo, Iterating random maps and applications (209--215); S. Golin, A. Knauf and S. Marmi, Hannay angles and classical perturbation theory (216--222); G. Turchetti, Nekhoroshev stability estimates for symplectic maps and physical applications (223--234); D. Verstegen, p-adic dynamical systems (235--242); P. Cassou-Noguès, Dirichlet series associated with a polynomial (244--252); M. L. Mehta, Some remarks on random number generators (253--259); C. J. Smyth, Bounds for nonblocking switch networks (260--263); P. Moussa, The Ising model and the Diophantine moment problem (264--275); B. Julia, Statistical theory of numbers (276--293); F. Vivaldi, Algebraic number theory and Hamiltonian chaos (294--301); J. P. Keating, Semiclassical properties of the cat maps (302--310). The articles of this volume will be reviewed individually under the abbreviation ``Number theory and physics, Proc. Winter Sch., Les Houches/Fr. 1989, Springer Proc. Phys. 47 (1990)''.

MSC 2000:
*00B25 Proceedings of conferences of miscellaneous specific interest
11-06 Proceedings of conferences (number theory)
11Z05 Miscellaneous appl. of number theory
17-06 Proceedings of conferences (nonassoc. rings and algebras)
37-06 Proceedings of conferences (Dynamical systems and ergodic theory)
52-06 Proceedings of conferences (convex and discrete geometry)
81-06 Proceedings of conferences (quantum theory)
82-06 Proceedings of conferences (statistical mechanics)
Keywords: Proceedings; Number theory; Physics; Winter school; Les Houches (France)

Cited in: Zbl 1104.11003 Zbl 1106.11002 Zbl 0784.00021 Zbl 0731.11015 Zbl 0729.11008 Zbl 0728.11072 Zbl 0727.11033 Zbl 0726.11082 Zbl 0723.60108 Zbl 0723.46047 Zbl 0721.17017 Zbl 0719.58038 Zbl 0718.11009 Zbl 0716.58028 Zbl 0716.52007 Zbl 0716.17030 Zbl 0716.17028 Zbl 0716.17013 Zbl 0714.92030 Zbl 0714.52009 Zbl 0714.52008 Zbl 0714.34134 Zbl 0713.30045 Zbl 0713.11063 Zbl 0713.11020 Zbl 0713.11019 Zbl 0712.65004 Zbl 0712.58024 Zbl 0712.58021 Zbl 0709.94026 Zbl 0709.58024 Zbl 0708.60013 Zbl 0707.70023 Zbl 0706.58039

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155

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Zbl 0697.10029
Waldschmidt, Michel; **Zhu, Yaochen**
Algebraic independence of certain numbers related to Liouville numbers. (English)
[J] Sci. China, Ser. A 33, No.3, 257-268 (1990). ISSN 1006-9283

Reprenant une méthode de D. Mordoukhay-Boltovskoy les auteurs développent divers résultats d'indépendance algébrique basés sur la propriété de bonne approximation des nombres de Liouville. Les nombres de Liouville apparaissant ici, sont des nombres complexes η tels qu'il existe une infinité de rationnels p/q satisfaisant: $$ 0<\vert \eta q-p\vert <\exp (-q^{a(\log q)^b \tau (q)}) $$ où a,b≥ 0 et τ est une fonction croissante. \par Considérons $\alpha_1,...,\alpha_m\in {\bar {\bf Q}}$ multiplicativement indépendant et α,$\beta\in {\bar {\bf Q}}$ différents de 0,1. Alors si η est de Liouville (avec $a=4m+4$, $b=0)$ les nombres $\eta,\alpha^{\eta}_1,...,\alpha^{\eta}_m,\alpha^{\beta^{\eta}}$ sont algébriquement indépendants sur ${\bf Q}.$ \par Si $\wp$ est une fonction elliptique de Weierstrass définie sur ${\bar {\bf Q}}$ et $u\ne 0$ est un point algébrique de $\wp$ tel que $\beta^{\eta}u$ ne soit pas pôle de $\wp$, alors les nombres $\eta,\alpha^{\eta}_1,..., \alpha^{\eta}_m,\wp (\beta^{\eta}u)$ sont algébriquement indépendants sur ${\bf Q}.$ \par Si $\eta_1,...,\eta_s$ sont des nombres de Liouville ${\bf Q}$- linéairement indépendants, alors les nombres $\alpha_1^{\eta_1},...,\alpha_m^{\eta_s}$, $e^{\eta_1},...,e^{\eta_s}$ sont algébriquement indépendants sur ${\bf Q}(\eta_1,...,\eta_s).$ \par Les démonstrations reposent sur un résultat élémentaire de théorie de Kummer ou sur des mesures de transcendance.
[P.Philippon]
MSC 2000:
*11J85 Algebraic independence results
11J04 Homogeneous approximation to one number
11J69
Keywords: Liouville number; Kummer theory; transcendence measure; algebraic independence; Weierstrass elliptic function; exponential function

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157

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Zbl 0683.00011
Langevin, M. (ed.); **Waldschmidt, M. (ed.)**
(Durand, Alain)
Cinquante ans de polynômes. Fifty years of polynomials. Proceedings of a conference held in honour of Alain Durand at the Institut Henri Poincaré, Paris, France, May 26-27, 1988. (English)
[B] Lecture Notes in Mathematics, 1415. Berlin etc.: Springer-Verlag. vii, 235 p. DM 45.00 (1990). ISBN 3-540-52190-9

The articles of this volume will be reviewed individually under the abbreviation ``Fifty years of polynomials, Proc. Conf. in Honour of Alain Durand, Paris/Fr. 1988, Lect. Notes Math. 1415 (1990)''.
MSC 2000:
*00B25 Proceedings of conferences of miscellaneous specific interest
12-06 Proceedings of conferences (field theory)
30-06 Proceedings of conferences (functions of a complex variable)
11-06 Proceedings of conferences (number theory)
Keywords: Polynomials; Proceedings; Conference; Paris (France)

Cited in: Zbl 0729.11035 Zbl 0723.12001 Zbl 0709.11038 Zbl 0707.11074 Zbl 0703.30004 Zbl 0703.30003 Zbl 0702.30006 Zbl 0702.11042 Zbl 0702.11041 Zbl 0701.11023 Zbl 0699.30005 Zbl 0699.30004 Zbl 0699.12035 Zbl 0697.10016 Zbl 0693.30003

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159

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Zbl 0702.11044
Mignotte, Maurice; **Waldschmidt, Michel**
Linear forms in two logarithms and Schneider's method. III. (English)
[J] Ann. Fac. Sci. Toulouse, V. Sér., Math. 1989, Suppl., 43-75 (1989).

Verf. verfeinern ihre in [Acta Arith. 53, No.3, 251-287 (1989; Zbl 0642.10034)] erhaltene untere Abschätzung für $\vert b_1 \log \alpha_1-b_2 \log \alpha_2\vert \ne 0$ bei algebraischen $\alpha_j\ne 0$ und ganzrationalen b_j. Dazu kombinieren sie ihre a.a.O. entwickelte Methode mit einer Technik, die sie bereits in [Math. Ann. 231, 241-267 (1978; Zbl 0349.10029)] vorgestellt haben. Die numerischen Resultate sind jetzt erheblich verbessert, was für eine Reihe von Anwendungen von großer Bedeutung ist, z.B. bei Problem der Klassenzahl 1 imaginär-quadratischer Zahlkörper oder bei exponentiellen diophantischen Gleichungen des Typs $a^x-b^y=c.$ Gesondert bearbeitet ist schließlich der Spezialfall, wo ein α_j eine Einheitswurzel ist.
[P.Bundschuh]
MSC 2000:
*11J86 Linear forms in logarithms; Baker's method

11J04 Homogeneous approximation to one number
Keywords: lower bounds; logarithms of algebraic numbers

Citations: Zbl 0691.10024; Zbl 0361.10027; Zbl 0642.10034; Zbl 0349.10029

Cited in: Zbl 0820.11023 Zbl 0801.11034 Zbl 0790.11023 Zbl 0783.11011 Zbl 0774.11036 Zbl 0752.11029

PDF XML ASCII DVI PS BibTeX Online Ordering

161
Zbl 0692.10035
Philippon, Patrice; Waldschmidt, Michel
Formes linéaires de logarithmes elliptiques et mesures de transcendance. (Linear forms in elliptic logarithms and transcendence measures). (French)
[A] Théorie des nombres, C. R. Conf. Int., Québec/Can. 1987, 798-805 (1989).

[For the entire collection see Zbl 0674.00008.] \par In their important paper [Ill. J. Math. 32, 281-314 (1988; Zbl 0651.10023)], the authors established a very sharp lower bound for linear forms in logarithms with respect to an arbitrary commutative group variety. In the present note they deduce transcendence measures for numbers associated with a Weierstrass elliptic function. In this way several of the previous results, due mainly to {\it E. Reyssat} [Bull. Soc. Math. Fr. 108, 47-79 (1980; Zbl 0432.10018)] can be improved. With the aid of their more recent paper [Sémin. Théor. Nombres, Paris 1986/87, Prog. Math. 75, 313-347 (1988; Zbl 0681.10024)] on ``simultaneous linear forms'', they improve these measures yet further in the case of complex multiplication.
[D.W.Masser]
MSC 2000:
*11J81 Transcendence (general theory)
Keywords: linear forms in logarithms; commutative group variety; transcendence measures; Weierstrass elliptic function; simultaneous linear forms; complex multiplication

Citations: Zbl 0674.00008; Zbl 0651.10023; Zbl 0432.10018; Zbl 0681.10024

Cited in: Zbl 0702.11043

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162
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Zbl 0689.10049
Balasubramanian, R.; Shorey, T.N.; Waldschmidt, M.
On the maximal length of two sequences of consecutive integers with the same prime divisors. (English)
[J] Acta Math. Hung. 54, No.3/4, 225-236 (1989). ISSN 0236-5294

For each integer $n\ge 2$ let Supp(n) denote the set of prime factors of n. Suppose that for $1\le i\le k$ we have $$ (1)\quad Supp(x +i)=Supp(y+i) $$ where x,y,k are positive integers. A problem of P. Erdös and A. Woods is the following: Does there exist an integer $k\ge 2$ such that (1) implies $x=y?$ The authors prove that (1) implies that $$ (2)\quad \log k\le C, (\log x\quad \log \log x)\sp {1/2}\quad for\quad x\ge 3 $$ $$ (3)\quad y-x>\exp (C\sb 2 k (\log k)\sp 2 (\log \log k)\sp{-1})\quad for\quad k\ge 3 $$ and $$ (4)\quad y-x>(k\quad \log \log y)\sp D\quad for\quad y\ge 27 $$ where $D=C\sb 3 k (\log \log y)$ (log log log y)${}\sp{-1}$. Here $C\sb 1,C\sb 2,C\sb 3$ are effectively computable absolute positive constants. \par A powerful ingredient of the proof is an inequality of A. Baker dealing with the linear forms in the logarithms of rational numbers.
[K.Ramachandra]
MSC 2000:
*11N37 Asymptotic results on arithmetic functions
Keywords: Baker's results; consecutive integers; linear forms in logarithms; prime factors

Cited in: Zbl 0859.11012

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164
OpenURL

Zbl 0676.14005
Serre, Jean-Pierre
(Brown, Martin (ed.); Waldschmidt, Michel)
Lectures on the Mordell-Weil theorem. Transl. and ed. by Martin Brown from notes by Michel Waldschmidt. (English)
[B] Aspects of Mathematics, E 15. Braunschweig etc.: Friedr. Vieweg \& Sohn. x, 218 p. DM 52.00 (1989). ISBN 3-528-08968-7

These notes are based on a course given by J.-P. Serre at the College de France in 1980 and 1981. The notes provide an introduction to and summary of many of the major ideas and results in the theories of rational and integral points on algebraic varieties. \par Beginning with a study of height functions, the book progresses to a proof of the weak Mordell-Weil theorem using the Chevalley-Weil theorem. After a brief discussion of descent arguments the full Mordell-Weil theorem is proved. \par Following this is a description of Mordell's conjecture, and a discussion of the major results (Chabauty's theorem, the Manin-Demjanenko theorem, Mumford's theorem) and applications that preceded Faltings' proof. - Integral points and quasi-integral points on curves are studied via Siegel's theorem and Baker's method. The study includes a discussion of effectivity, and applications to the arithmetic of curves. \par The problem of lifting rational points under morphisms is introduced with the notion of thin sets and Hilbert's irreducibility theorem. Applications to the construction of Galois extensions with certain prescribed Galois groups, and to the construction of elliptic curves of large rank are given. There is also a discussion of the large sieve, and applications to the study of thin sets. \par Finally, an appendix contains some remarks about the class number 1 problem, and connections with elliptic and modular curves.
[S.Kamienny]
MSC 2000:
*14G05 Rationality questions, rational points
14H52 Elliptic curves
14H25 Arithmetic ground fields (curves)
14-02 Research monographs (algebraic geometry)
14G25 Global ground fields
14H45 Special curves and curves of low genus
14K15 Arithmetic ground fields (abelian varieties)
11R23 Iwasawa theory
Keywords: rational points; integral points; height functions; Mordell-Weil theorem; Mordell's conjecture; lifting rational points; Hilbert's irreducibility theorem; class number 1 $problem$

Cited in: Zbl 1078.11029 Zbl 0863.14013 Zbl 0973.11085 Zbl 0891.12004 Zbl 0811.14022 Zbl 0799.11015 Zbl 0778.11019 Zbl 0744.14012

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165

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Zbl 0642.10035
Bundschuh, Peter; Waldschmidt, Michel
Irrationality results for theta functions by Gel'fond-Schneider's method. (English)
[J] Acta Arith. 53, No.3, 289-307 (1989); errata ibid. 78, No.1, 99 (1996). ISSN 0065-1036

The aim of this paper is to investigate the arithmetic nature of the values of entire functions F which are solutions of a Poincaré functional equation $F(az)=F(z)P(z)+Q(z)$. The proof depends on a general result on the algebraic values of analytic functions of one complex variable, which is achieved by means of Gel'fond-Schneider's method. We deduce upper bounds for the number of algebraic points where some theta functions take algebraic values with bounded degree.
[M.Waldschmidt]
MSC 2000:
*11J81 Transcendence (general theory)
30D20 General theory of entire functions
Keywords: irrationality; values of entire functions; Poincaré functional equation; Gel'fond-Schneider's method; upper bounds; theta functions; algebraic values with bounded degree

Cited in: Zbl 0939.11026 Zbl 0714.11040

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166

OpenURL

Zbl 0642.10034
Mignotte, Maurice; Waldschmidt, Michel
Linear forms in two logarithms and Schneider's method. II. (English)
[J] Acta Arith. 53, No.3, 251-287 (1989). ISSN 0065-1036; ISSN 1730-6264

Let $\alpha \sb 1,\alpha \sb 2$ be two multiplicatively independent algebraic numbers, and log α ${}\sb 1$, log α ${}\sb 2$ two non-zero determinations of their logarithms. Denote by D the degree of the field ${\bbfQ}(\alpha \sb 1,\alpha \sb 2)$ over ${\bbfQ}$. Further, let $b\sb 1,b\sb 2$ be two positive rational integers such that $b\sb 1 \log \alpha \sb 1\ne b\sb 2 \log \alpha \sb 2$. Define $B=\max \{b\sb 1,b\sb 2\}$, and choose two positive real numbers $a\sb 1,a\sb 2$ satisfying $$ a\sb j\ge 1,a\sb j\ge h(\alpha \sb j)+\log 2,\quad a\sb j\ge \frac{2e}D\cdot \log \alpha \sb j $$ for $j=1$ and $j=2$. Here, h denotes the absolute logarithmic height. Then $$ \vert b\sb 1 \log \alpha \sb 1-b\sb 2 \log \alpha \sb 2\vert \ge \exp \{-500\cdot D\sp 4a\sb 1a\sb 2(7.5+\log B)\sp 2\}. $$ This is a refinement of the rational case in the result of the authors [Math. Ann. 231, 241-267 (1978; Zbl 0349.10029)], and of the case of two logarithms in the result of {\it J. Blass}, {\it A. M. W. Glass}, {\it D. K. Manski}, {\it D. B. Meronk} and {\it R. P. Steiner} [Constants for lower bounds for linear forms in the logarithms of algebraic numbers, Acta Arith. (to appear)].
[M.Waldschmidt]
MSC 2000:
*11J81 Transcendence (general theory)
Keywords: transcendence; linear forms in logarithms; Schneider's method; algebraic numbers; lower bounds

Citations: Zbl 0349.10029; Zbl 0361.10027

Cited in: Zbl 0752.11029 Zbl 0716.11016 Zbl 0702.11044

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169

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Zbl 0714.11042
Waldschmidt, Michel
Sur les méthodes de Schneider, Gel'fond et Baker. (On the methods of Schneider, Gel'fond and Baker). (French)
[J] Sémin. Théor. Nombres, Univ. Bordeaux I 1987/1988, Exp. No.30, 13 p. (1988).

Man kennt derzeit drei verschiedene Beweissätze für das qualitative Bakersche Resultat über die lineare Unabhängigkeit von Logarithmen algebraischer Zahlen, die im Titel genannt und im Text kurz vorgestellt werden. Sodann wird erläutert, wie jede zu quantitativen (und effektiven) Resultaten verfeinert werden kann. Dabei führen die Methoden von Schneider bzw. Gel'fond im wesentlichen zu denselben unteren Abschätzungen; man kann beide Methoden als dual (in einem hier präzisierten Sinne) betrachten. Bakers Methode, eine Verfeinerung der Gel'fondschen, liefert dagegen eine bessere Abschätzung. Verf. merkt an, es sei von Interesse, auch Schneiders Methode so zu verfeinern, daß man eine zu Bakers Methode duale erhält. \par (Man vergleiche hierzu auch die Aufsätze ``Nouvelles méthodes pour minorer des combinaisons linéaires de logarithmes de nombres algébriques", I bzw. II des Verf. [Sémin. Théorie Nombres, Bordeaux, to appear] bzw. [Publ. Math. Univ. P. et M. Curie Paris, Probl. Dioph. 93, No.8, 36 p. (1989/90)].)
[P.Bundschuh]
MSC 2000:
*11J86 Linear forms in logarithms; Baker's method
Keywords: Schneider's method; Gel'fond's method; Baker's method; linear independence; logarithms of algebraic numbers

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170

OpenURL

Zbl 0714.11040
Waldschmidt, Michel
Fonctions theta et nombres transcendants. (Theta functions and transcendental numbers). (French)
[J] Sémin. Théor. Nombres, Univ. Bordeaux I 1987/1988, Exp. No.15, 7 p. (1988).

In dem der Note zugrundeliegenden Vortrag wurden zwei Aspekte des im Titel anklingenden Themas diskutiert. 1) Arithmetische Fragestellungen nach Transzendenz oder wenigstens Irrationalität der Werte von Thetafunktionen. Dabei bringt man beweismethodisch entweder die mit elliptischen Integralen verknüpften algebraischen Gruppen ins Spiel oder man benützt direkt die Funktionalgleichung der Thetafunktionen. Als Beispiel für den zuletzt genannten Zugang wird das Hauptergebnis des Verf. und des Ref. [Acta Arith. 53, 289-307 (1989; Zbl 0642.10035)] vorgestellt. 2) Bei den Beweisen gewisser diophantischer Eigenschaften algebraischer Gruppen benötigt man Präzisierungen bezüglich quasi- projektiver Einbettungen und hierzu wiederum neue Eigenschaften von Thetafunktionen. Diesbezüglich werden einige der Resultate von {\it S. David} [C. R. Acad. Sci., Paris, Sér. I 305, 211-214 (1987; Zbl 0628.14035)] erwähnt.

[P.Bundschuh]
MSC 2000:
*11J81 Transcendence (general theory)
11J72 Irrationality
11J91 Transcendence theory of other special functions
14K25 Theta-functions
14L10 Group varieties
Keywords: transcendence of values of theta functions; diophantine properties of algebraic groups; quasi-projective embeddings

Citations: Zbl 0691.10025; Zbl 0642.10035; Zbl 0628.14035

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171
Zbl 0696.10029
Waldschmidt, Michel
Some transcendental aspects of Ramanujan's work. (English)
[A] Proc. Ramanujan Cent. Int. Conf., Annamalainagar/India 1987, RMS Publ. 1, 67-76 (1988).

[For the entire collection see Zbl 0694.00007.] \par This is an expository account of some of the aspects of Ramanujan's work connected with transcendental number theory and diophantine approximations.
[T.M.Apostol]
MSC 2000:
*11J81 Transcendence (general theory)
11-03 Historical (number theory)
Keywords: expository account; Ramanujan; transcendental number theory; diophantine approximations

Citations: Zbl 0694.00007

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173
Zbl 0681.10024
Philippon, P.; Waldschmidt, M.
Formes linéaires de logarithmes simultanées sur les groupes algébriques commutatifs. (Simultaneous linear forms in logarithms on commutative algebraic groups). (French)
[A] Sémin. Théor. Nombres, Paris 1986-87, Prog. Math. 75, 313-347 (1988).

[For the entire collection see Zbl 0653.00005.] \par Let G be a commutative connected algebraic group defined over the field ${\bar {\bbfQ}}$ of algebraic numbers, and let $T\sb G$ be its tangent space at the origin. Let v be an element of $T\sb G$ whose image $ \exp\sb G v$ under the exponential map lies in G(${\bar {\bbfQ}}$), and let W be a subspace of $T\sb G$ defined over ${\bar {\bbfQ}}$. Suppose that for every connected algebraic subgroup H of G we have (*) either $v\not\in T\sb H$ or $T\sb h\varsubsetneq W$. Then a fundamental transcendence result of {\it G. Wüstholz} [Approximations diophantiennes et nombres transcendants, Luminy 1982, Prog. Math. 31, 329-336 (1983; Zbl 0534.10026)] asserts that $v\not\in W.$ \par In their earlier paper [Ill. J. Math. 32, No.2, 281-314 (1988; Zbl 0651.10023)] the authors proved a very strong quantitative version of this, in the form of a positive lower bound for the distance of v from W, in the case that W has codimension 1 in $T\sb G$. This amounts to a lower bound for a single linear form in generalized logarithms. In the present paper they extend their results to arbitrary W under a hypothesis slightly stronger than (*). This corresponds to lower bounds for several linear forms simultaneously. \par Their full result is stated (as in the above cited paper) for products $G\sb 0\times G\sb 1\times... \times G\sb k,$ but for simplicity we stick to a single factor G, of dimension d, say. Let K be a number field of degree at most $D\ge 2$ such that G, $\gamma =\exp\sb G v$ and W are defined over K, and suppose there is a projective embedding i: $G\to {\bbfP}\sb N$ also defined over K. Choose a norm $\Vert \Vert$ on $T\sb G ({\bbfC})$ and let $V\ge e$ be any real number satisfying $\log V\ge \max \{h(i(\gamma)),\quad \Vert v\Vert\sp 2\},$ where h denotes the absolute logarithmic Weil height on ${\bbfP}\sb n(K)$. Choose a basis B of $T\sb G(K)$, thus identifying it with $K\sp d $, and suppose W is given by the vanishing of $t\ge 1$ independent linear forms $L\sb{\tau}(z)=\sum\spd\sb{i=1}\beta\sb{i\tau}z \sb i$ (1$\le \tau \le t)$ defined over K. Let $B\ge e$ be any real number satisfying log $B\ge D \log V$, log $B\ge h(\beta\sb{i\tau}) $ (1$\le i\le d$, $1\le \tau \le t)$, where this time the height is on K. Instead of (*) suppose that either $v\not\in T\sb H$ or $T\sb H +W\ne T\sb G.$ Then again $v\not\in W$, and the $authors'$ result is that $$ \max\sb{1\le \tau \le t}\log \vert L\sb{\tau}(v)\vert \quad \ge \quad -CD\sp{1+(2d+1)/t}\cdot (\log B)\sp{1+(d/t)}\cdot (\log V)\sp{d/t}\cdot (\log D)\sp{\quad -2d/t} $$ for some C depending only on G, the embedding i, the norm $\Vert \Vert$, and the basis B. \par For $t=1$ this reduces to one of the results of their earlier paper (loc.cit.). \par As mentioned above, the authors prove a more precise version which distinguishes between linear group varieties and abelian varieties occurring as factors of G. For linear varieties alone a similar lower bound was obtained by {\it J. H. Loxton} [Acta Arith. 46, 113-123 (1986; Zbl 0549.10012)]. For abelian varieties the authors deduce a lower bound for a single linear form in abelian logarithms, in just the shape needed for application to Siegel's Theorem on integral points on curves. The exponent of log B in this lower bound turns out to depend on the number of logarithms being considered. However, the authors also indicate how, in the case of complex multiplication, this exponent can be reduced down to $1+\epsilon$ for any $\epsilon >0$, which is close to best possible. This latter estimate improves some results of the reviewer [Invent. Math. 45, 61-82 (1978; Zbl 0375.10022)].
[D.W.Masser]
MSC 2000:
*11J81 Transcendence (general theory)
14L10 Group varieties
Keywords: linear forms in generalized logarithms; connected algebraic group; linear group varieties; abelian varieties; single linear form in abelian logarithms; complex multiplication

Citations: Zbl 0581.10007; Zbl 0653.00005; Zbl 0534.10026; Zbl 0651.10023; Zbl 0549.10012; Zbl 0375.10022

Cited in: Zbl 0776.11038 Zbl 0692.10035

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174
Zbl 0659.10037
Philippon, P.; Waldschmidt, M.
Lower bounds for linear forms in logarithms. (English)
[A] New advances in transcendence theory, Proc. Symp., Durham/UK 1986, 280-312 (1988).

[For the entire collection see Zbl 0644.00005.] \par This paper contains some new, rather precise, lower bounds for linear forms in logarithms. Let D be a positive integer, let $\alpha\sb 1,...\alpha\sb n$ be non-zero algebraic numbers, and let $\beta\sb 0,\beta \sb 1,...,\beta\sb n$ be algebraic numbers all in a number field of degree at most D. Suppose the $\alpha\sb i$ have ``naive'' height at most $A\sb i\ge e\sp n$ (1$\le i\le n)$ and the $\beta\sb i$ have ``naive'' height at most $B\ge e$ (0$\le i\le n)$, and let $A= \max (A\sb 1,...,A\sb n)$. Finally let log $\alpha\sb 1,...,\log \alpha\sb n$ be determinations of the logarithms satisfying $\vert \log

\alpha\sb i\vert \le \log A\sb i$ (1$\le i\le n)$, and such that $\Lambda =\beta\sb 0+\beta\sb 1 \log \alpha\sb 1+...+\beta\sb n \log \alpha\sb n\ne 0$. Then it is proved that $$ (*)\quad \log \vert \Lambda \vert \quad \ge \quad - 2\sp{8n+53} n\sp{2n} D\sp{n+2} \Omega (\log B+\log \log A) $$ where $\Omega =\log A\sb 1... \log A\sb n.$ \par If in addition $\beta\sb 0=0$ and $\beta\sb 1=b\sb 1,...,\beta\sb n=b\sb n$ are rational integers such that $\alpha\sb 1\sp{b\sb 1}... \alpha\sb n\sp{b\sb n}\ne 1,$ then it is shown that $$ (**)\quad \vert \alpha\sb 1\sp{b\sb 1}... \alpha\sb n\sp{b\sb n}-1\vert \quad \ge \quad B\sp{- c\Omega} $$ for some positive effective c depending only on D and n. \par Actually the authors prove more detailed results, but their essential features are already in the above examples. The second author [Acta Arith. 37, 257-283 (1980; Zbl 0357.10017)] had previously proved a result similar to (*) with Ω multiplied by an extra log log term, and the corresponding version of (**) was first proved by {\it A. J. van der Poorten} [Math. Proc. Camb. Philos. Soc. 80, 233-248 (1976; Zbl 0341.10030)]; see also {\it A. Baker} [Transcend. Theory Adv. Appl., Cambridge 1976, 1-27 (1977; Zbl 0361.10028)]. \par The new tool here consists of the ``multiplicity estimates'' of the first author [Bull. Soc. Math. Fr. 114, 355-383 (1986; Zbl 0617.14001)], together with the ``inductive descent'' method of both authors [Ill. J. Math. 32, No.2, 281-314 (1988; Zbl 0651.10023)]. \par For an alternative approach see the paper of {\it G. Wüstholz} in the same Proceedings (see the preceding review Zbl 0659.10036).
[D.W.Masser]
MSC 2000:
*11J81 Transcendence (general theory)
Keywords: inductive descent method; lower bounds for linear forms in logarithms; multiplicity estimates

Citations: Zbl 0439.10020; Zbl 0644.00005; Zbl 0357.10017; Zbl 0341.10030; Zbl 0361.10028; Zbl 0617.14001; Zbl 0651.10023; Zbl 0659.10036

Cited in: Zbl 0911.11019 Zbl 0774.11034 Zbl 0709.11037 Zbl 0659.10036

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176
Zbl 0659.10035
Waldschmidt, M.
On the transcendence methods of Gelfond and Schneider in several variables. (English)
[A] New advances in transcendence theory, Proc. Symp., Durham/UK 1986, 375-398 (1988).

[For the entire collection see Zbl 0644.00005.] \par This paper deals with transcendence theorems for values of functions associated to commutative algebraic groups. Since the main theorem (Thm. 4.1) is a bit too technical to quote here, we content ourselves by stating the most interesting Corollary (Theorem 1.1). Let G be a commutative algebraic group of dimension d, which is defined over ${\bar {\bbfQ}}$. Let $T\sb G({\bbfC})$ be the tangent space of G at 0 and let $\exp\sb G: T\sb G({\bbfC})\to G({\bbfC})$ be the exponential map. Let $d\sb 0$ (resp. $d\sb 1$) be the dimension of the maximal unipotent (resp. multiplicative) factor of G. Write $d\sb 2=d-d\sb 0-d\sb 1$. Let V be a hyperplane of $T\sb G({\bbfC})$, W a subspace of V of dimension t over ${\bbfC}$, and $Y={\bbfZ}y\sb 1+...+{\bbfZ}y\sb m$ a finitely generated subgroup of V of rank m over ${\bbfZ}$. Assume that W is defined over ${\bar {\bbfQ}}$ in $T\sb G({\bbfC})$, and that $\exp\sb G(Y)$ is contained in G(${\bar {\bbfQ}}$). Then, if $m>(d\sb 1+2d\sb 2)(d-1-t),$ the space V contains a non-trivial algebraic Lie subalgebra of $T\sb G({\bbfC})$ defined over ${\bar {\bbfQ}}.$ \par Among the corollaries of this result are a refinement of the six exponential theorem $(G={\bbfG}\sp 2\sb a\times {\bbfG}\sp 2\sb m)$, Wüstholz' transcendence theorem $(t=d-1)$ and Baker's theorem on ${\bar {\bbfQ}}$-linear independence of logarithms of algebraic numbers. Although Baker's theorem could have been derived from Wüstholz' theorem, the author's Theorem 1.1 offers an alternative approach via Schneider's method. \par The proof of Theorem 4.1 is by an interesting combination of Gel'fond's and Schneider's approach, which can be thought of as being represented by the extreme cases $t=d-1$ (Gel'fond) and $t=0$ (Schneider) respectively. In this paper a sketch of the main result, Theorem 4.1 is given, thereby referring to other papers, and a deduction of Theorem 1.1 from Theorem 4.1 is presented.
[F.Beukers]
MSC 2000:
*11J81 Transcendence (general theory)
Keywords: Gelfond method; transcendence theorems; commutative algebraic groups; six exponential theorem; Wüstholz' transcendence theorem; Baker's theorem; linear independence of logarithms of algebraic numbers; Schneider's method

Citations: Zbl 0644.00005

Cited in: Zbl 0791.11032 Zbl 0762.11027 Zbl 0714.11043 Zbl 0719.11042

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177
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Zbl 0651.10023
Philippon, Patrice; Waldschmidt, Michel
Formes linéaires de logarithmes sur les groupes algébriques commutatifs. (Linear forms in logarithms on commutative algebraic groups). (French)
[J] Ill. J. Math. 32, No.2, 281-314 (1988). ISSN 0019-2082

This article contains some very precise lower bounds for linear forms in logarithms with respect to an arbitrary commutative group variety. They generalize the classical results for conventional logarithms due to A. Baker and others. \par Suppose G is defined over a number field K and is of dimension $d+1$. Select basis elements over K of the tangent space to G at its origin, and write exp for the resulting map from ${\bbfC}\sp{d+1}$ to G(${\bbfC}$). Let $L(z)=\beta\sb 0z\sb 0+...+\beta\sb dz\sb d$ be a non-zero linear form on ${\bbfC}\sp{d+1}$ with coefficients in K, and denote by W the subspace of ${\bbfC}\sp{d+1}$ defined by $L(z)=0$. Let v be in ${\bbfC}\sp{d+1}$ such that exp(v) is in G(K). Suppose finally that for every connected algebraic subgroup G' of G whose tangent space lies in W this tangent space does not contain v. Then the Analytic Subgroup Theorem of {\it G. Wüstholz} [see for example, Prog. Math. 31, 329-336 (1983; Zbl 0534.10026)] shows that L(v)$\ne 0.$ \par The present article gives a positive lower bound for $\vert L(v)\vert$ in these circumstances. In order to include the classical results on ordinary logarithms, the group G is assumed to have the form $G=G\sb 0\times G\sb 1\times...\times G\sb k$, where $G\sb 0$ is the additive group and $G\sb 1=...=G\sb{d\sb 1}$ is the multiplicative group for some $d\sb 1$ with $0\le d\sb 1\le k$. Write $\delta\sb i$ for the dimension of $G\sb i$ (0$\le i\le k)$, and define $\rho\sb i=0$ $(i=0)$, $\rho\sb i=1$ $(1\le i\le d\sb 1)$ and $\rho\sb i=2$ otherwise. Break the tangent space into corresponding factors, select basis elements still defined over K, and let $\exp\sb i$ be the map on the corresponding factor of ${\bbfC}\sp{d+1}$ (0$\le i\le k)$. Choose norms $\Vert \Vert$ on these latter factors. Also assume each $G\sb i$ embedded in projective space (0$\le i\le k)$. Then the main result of the authors is the existence of a constant c, depending only on the quantities introduced in the present paragraph, with the following property. \par Suppose the point v above can be written as $v=(1,v\sb 1,...,v\sb k)$ with $v\sb i\ne 0$ and $\gamma\sb i=\exp\sb i(v\sb i)$ in $G\sb i(K)$ (1$\le i\le k)$. Write h for the absolute logarithmic Weil height on projective space over K. Let $D=[K : {\bbfQ}]$, and let $B,E,V\sb 1,...,V\sb k$ be real numbers satisfying $\log V\sb i \ge \max \{h(\gamma\sb i),\Vert v\sb i\Vert\sp{\rho\sb i}/D\},\quad B\ge D \log V\sb i$ (1$\le i\le k)$ and $\log B\ge h(\beta\sb j)$ (0$\le j\le d)$ as well as $E\le \min \{B\quad D, e(D \log V\sb i)\sp{1/\rho\sb i}/\Vert v\sb i\Vert \}\quad (1\le i\le k).$ Then $$\log \vert L(v)\vert \quad >\quad -cD\sp{d+d\sb 2+2}(\log B)\sp{d\sb 2+1} \log (DE)(\log E)\sp{-d- d\sb 2-1}V, $$ with $V=\prod \spk\sb{i=1}(\log V\sb i)\sp{\delta\sb i}$. For purposes of comparison, note that if $d\sb 2=0$ (the classical situation of ordinary

logarithms) this estimate compares very well with the sharpest results to date [see for example, {\it M. Waldschmidt}, Acta Arith. 37, 257-283 (1980; Zbl 0357.10017)]. But if $d\sb 2>0$ the results are essentially completely new, except for certain cases involving complex multiplication. \par The method of proof is rather ingenious. Of course it relies on the full power of multiplicity estimates, as established first by {\it G. Wüstholz} [Habilitationsschrift, Wuppertal 1982] and sharpened by {\it P. Philippon} [(*): Bull. Soc. Math. Fr. 114, 355-383 (1986; Zbl 0617.14001)]. But the new idea is an inductive ``descent'' that succeeds only because the estimates of (*) are nearly best possible. Another ingredient of the proof is an interesting inequality of {\it D. Bertrand} and {\it P. Philippon} [Ill. J. Math. 32, 263-280 (1988; Zbl 0618.14020)].
[D.W.Masser]
MSC 2000:
*11J85 Algebraic independence results
14L10 Group varieties
14A05 Relevant commutative algebra
Keywords: linear independence; Baker's method; lower bounds; linear forms in logarithms; commutative group variety; multiplicity estimates

Citations: Zbl 0439.10020; Zbl 0639.14029; Zbl 0534.10026; Zbl 0357.10017; Zbl 0617.14001; Zbl 0618.14020

Cited in: Zbl 0957.11030 Zbl 0692.10035 Zbl 0681.10024 Zbl 0659.10037

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185

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Zbl 0623.10023
Loxton, J.H.; Mignotte, M.; van der Poorten, A.J.; Waldschmidt, M.
A lower bound for linear forms in the logarithms of algebraic numbers. (English)
[J] C. R. Math. Acad. Sci., Soc. R. Can. 11, 119-124 (1987). ISSN 0706-1994

This paper contains some lower bounds for a "homogeneous rational" linear form in logarithms. Let $\alpha\sb 1,...,\alpha\sb n$ be non-zero algebraic numbers, let $K={\bbfQ}(\alpha\sb 1,...,\alpha\sb n)$, and suppose the "strong independence condition" $[K(\alpha\sb 1\sp{1/2},..., \alpha\sb n\sp{1/2}) : K]=2\sp n$ holds. Let D, $V\sb 1,...,V\sb n$ satisfy $$ V\sb i\ge \max \{h(\alpha\sb i), D\sp{-1}\vert \log \alpha\sb i\vert, D\sp{-1}\}\quad (1\le i\le n),\quad V\sb 1\le...\le V\sb n,\quad D\ge [K : {\bbfQ}]\quad, $$ where $h(\alpha\sb i)$ is the absolute logarithmic height and log α ${}\sb i$ is any determination of the logarithm. Further for rational integers $b\sb 1,...,b\sb n$ let B, $B\sb n$ and E satisfy $$ B\ge \max (\vert b\sb 1\vert,...,\vert b\sb{n- 1}\vert),\quad B\sb n\ge \vert b\sb n\vert,\quad e\le E\le \min \{e\sp{DV\sb 1}, eDn(\sum\spn\sb{i=1}V\sb i\sp{-1} \vert \log \alpha\sb i\vert) \sp{-1}\}\quad. $$ Finally let W satisfy $W\ge \log (V\sb 1\sp{-1} B\sb n+V\sb n\sp{-1} B+1)+\log E$. Then there is an absolute constant $c>0$ such that if $\Lambda =b\sb 1 \log \alpha\sb 1+... +b\sb n \log \alpha\sb n\ne 0$ then $$ \log \vert \Lambda \vert \le -c\sp n n\sp n D\sp{n+2} \log (EDV\sp+\sb{n-1})V\sb 1... V\sb n W(\log E)\sp{-n-1}\quad, $$ where $V\sp+\sb{n-1}= \max (1,V\sb{n-1})$. The main improvement over earlier results consists of reducing the value of W; before, W was defined as $\log (B\sb n+B)+\log E.$ A proof is briefly sketched by indicating how to modify the arguments of {\it M. Waldschmidt} [Acta Arith. 37, 257-283 (1980; Zbl 0357.10017)]. \par The authors also state a similar result without the strong independence condition, but no proof is sketched.
[D.W.Masser]
MSC 2000:
*11J81 Transcendence (general theory)
Keywords: lower bounds; linear form in logarithms; algebraic numbers; strong independence condition

Citations: Zbl 0439.10020; Zbl 0357.10017

Cited in: Zbl 0774.11036

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186

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Zbl 0621.10022
Waldschmidt, Michel
(Bertrand, Daniel; Serre, Jean-Pierre)
Nombres transcendants et groupes algébriques. (Transcendental numbers and algebraic groups). Complété par deux appendices de Daniel Bertrand et Jean-Pierre Serre. 2e éd. (French)
[B] Astérisque, 69/70. Publié avec le concours du Centre National de la Recherche Scientifique. Paris: Société Mathématique de France. 218 p.; FF 145.00; {\$} 24.00 (1987).

For a review of the first edition (1979) see Zbl 0428.10017.
MSC 2000:
*11J81 Transcendence (general theory)
11-02 Research monographs (number theory)
14L05 Formal groups
Keywords: transcendental numbers; algebraic points; graph of analytic homomorphism; commutative algebraic groups; generalized Dirichlet exponent; linear groups; one-parameter groups; Schneider-Lang criterion; Schwarz lemma; abelian varieties of complex multiplication type; zero estimates; exponential function; elliptic function; abelian function

Citations: Zbl 0302.10030; Zbl 0428.10017

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191

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Zbl 0616.10029
Waldschmidt, Michel
Algebraic independence of values of exponential and elliptic functions. (English)
[J] J. Indian Math. Soc., New Ser. 48(1986), 215-228 (1984). ISSN 0019-5839

The main result of this article is an algebraic independence statement for mixed values of exponential and elliptic functions. Let $\wp(z)$ be a Weierstrass function with invariants $g\sb 2,g\sb 3$. For $d\sb 1\ge 0$ let $x\sb 1,...,x\sb{d\sb 1}$ be complex numbers linearly independent over ${\bbfQ}$, and for $m\ge 1$ let $y\sb 1,...,y\sb m$ be complex numbers also linearly independent over ${\bbfQ}$. For $d\sb 2\ge 0$ let $u\sb 1,...,u\sb{d\sb 2}$ be complex numbers linearly independent over the field of complex multiplications of $\wp (z)$. Define K as the field generated over ${\bbfQ}(g\sb 2,g\sb 3)$ by the $\exp (x\sb i y\sb j)$ $(1\le i\le d\sb 1$, $1\le j\le m)$ together with the finite values of the $\wp (u\sb k y\sb j)$ $(1\le k\le d\sb 2$, $1\le j\le m)$. Then it is proved that the transcendence degree of K over ${\bbfQ}$ is at least $$ ((d\sb 1+d\sb 2)m-m-d\sb 1-2d\sb 2)/(m+d\sb 1+2d\sb

2), $$ provided certain (familiar) technical hypotheses are satisfied. Similar results are also given for the fields $K(y\sb 1,...,y\sb m),$ $L=K(x\sb 1,...,x\sb{d\sb 1},u\sb 1,...,u\sb{d\sb 2})$ and $L(y\sb 1,...,y\sb m).$ For $d\sb 2=0$ (no elliptic function) these were proved by {\it P. Philippon} [Publ. Math., Inst. Hautes Étud. Sci. 64, 5-52 (1986; Zbl 0615.10044)], and for $d\sb 1=0$ (no exponential function) by the author himself [(*) Acta Math. 156, 253-302 (1986; Zbl 0592.10028)]. The full result is here deduced from a sharpening of the more general theorem in (*) about algebraic groups, and a sketch is given as to how such a sharpening can be proved.
[D.W.Masser]
MSC 2000:
*11J85 Algebraic independence results
11J81 Transcendence (general theory)
Keywords: algebraic independence; mixed values of exponential and elliptic functions; Weierstrass function; transcendence degree

Citations: Zbl 0615.10044; Zbl 0592.10028

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196

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Zbl 0592.10028
Waldschmidt, Michel
(Fresnel, J.)
Groupes algébriques et grands degrés de transcendance. Appendice: Déformation d'un groupe algébrique par J. Fresnel. (Algebraic groups and large transcendence degrees). (French)
[J] Acta Math. 156, 253-302 (1986). ISSN 0001-5962; ISSN 1871-2509

L'A. développe dans ce texte une nouvelle approche des démonstrations d'indépendance algébrique pour les grands degrés de transcendance. Il généralise ainsi aux groupes algébriques quelconques des résultats jusqu'alors connus seulement en une variable pour la fonction exponentielle et sous une forme plus faible pour certaines fonctions elliptiques (cf. référence [MW2] due texte). L'auteur explicite dans les {\S}{\S} 12 à 15 différents corollaires intéressants de son théorème principal, d'abord sur les valeurs de fonctions exponentielle et elliptiques en plusieurs variables puis sur les valeurs de fonctions zétas et sigma et enfin sur les valeurs de fonctions abéliennes. \par Une des originalités de la démonstration de ces résultats est l'utilisation d'une méthode de petites perturbations ({\S} 5) pour éliminer les zéros de la fonction auxiliaire au voisinagge du point considéré. Ceci permet d'éviter le recours à un lemme de petites valeurs comme le lemme de Tijdeman apparaissant auparavant dans la démonstration du cas exponentiel en une variable et aussi de ne pas faire d'hypothèse sur les endomorphismes du groupe. La méthode de l'auteur opère même lorsque le groupe algébrique est lui-même défini sur une extension transcendante de ${\bbfQ}$. La mise en oeuvre est alors un peu délicate et fait l'objet de l'important {\S} 10. Les lecteurs préférant la géométrie des schémas à l'algèbre trouverons en appendice une autre démonstration de la proposition 10.1, par {\it J. Fresnel}, reposant sur EGA4.
[P.Philippon]
MSC 2000:
*11J85 Algebraic independence results
14H52 Elliptic curves
14L10 Group varieties
Keywords: algebraic independence; large transcendence degrees; algebraic; groups; deformations; values of exponential function; elliptic; functions in several variables; zeta-functions; sigma function; abelian function

Cited in: Zbl 0616.10029 Zbl 0615.10044

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198

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Zbl 0553.10026
Gramain, François; Mignotte, Maurice; Waldschmidt, Michel
Valeurs algébriques de fonctions analytiques. (Algebraic values of analytic functions). (French)
[J] Acta Arith. 47, 97-121 (1986). ISSN 0065-1036; ISSN 1730-6264

We give two general theorems on the algebraicity of the values of analytic functions of one complex variable, the first one using Schneider's method, and the second one using Mahler's method. \par Our first result states that analytic functions (either in a disc or in the whole complex plane) which take often simultaneously algebraic values are algebraically independent. This statement includes most of the earlier results depending on Schneider's method. Moreover we do not restrict the values in a fixed number field, but we merely require bounds for their degree and height. \par Next we show that in Mahler's method, the only use of functional equations is to ensure the existence of a sequence of algebraic points where the considered function takes algebraic values, together with bounds for the heights. Finally, we study the functional equation $f(z\sp k)=af(z)\sp k+bz\sp h,$ (a,b$\in {\bbfC}$, h,k$\in {\bbfZ})$, and we give a transcendence result for the values of such functions (for algebraic a,b).
MSC 2000:
*11J81 Transcendence (general theory)
11J85 Algebraic independence results
Keywords: analytic functions; algebraic values; transcendental numbers; Schneider's method; Mahler method; functional equations; algebraic independence

Cited in: Zbl 0971.11521

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204

OpenURL

Zbl 0587.10018
Waldschmidt, Michel
Indépendance algébrique de nombres transcendants: Quelques résultats récents. (Algebraic independence of transcendental numbers: Some recent results). (French)
[J] Sémin. Théor. Nombres, Univ. Bordeaux I 1984/1985, Exp. No.15, 5 p. (1985).

Dans cet exposé l'A. complète et met à jour sa revue de la méthode de Gel'fond pour l'indépendance algébrique parue précédemment dans Perspectives in mathematics, Anniv. Oberwolfach 1984, 551-571 (1984; Zbl 0556.10023). Une des clefs des progrès réalisés est un critère d'indépendance algébrique de récens. que l'A. utilise conjointement à un argument de déformation de groupe algébrique pour obtenir de nombreuses minorations de degrés de transcendance sur ${\bbfQ}$ [voir Acta Math. 156, 253-302 (1986)]. R. Tubbs a, lui, développé systématiquement la méthode de Gel'fond pour les petits degrés de transcendance. L'aspect quantitatif, ou mesure d'indépendance algébrique, a aussi été étudié par Yu. V. Nesterenko, le récens. et E. M. Jabbouri qui a établi les versions quantitatives des analogues elliptiques et abéliens du théorème de Lindemann-Weierstrass.

[P.Philippon]
MSC 2000:
*11J81 Transcendence (general theory)
11J85 Algebraic independence results
11-02 Research monographs (number theory)
14H52 Elliptic curves
Keywords: survey; Gelfond method; algebraic independence; algebraic groups

Citations: Zbl 0556.10023

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205

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Zbl 0571.10034
Waldschmidt, Michel
Petits degrés de transcendance par la méthode de Schneider en une variable. (Small transcendence degrees obtained by Schneider's method in one variable). (French)
[J] C. R. Math. Acad. Sci., Soc. R. Can. 7, 143-148 (1985). ISSN 0706-1994

Let K be a subfield of ${\bbfC}$ of transcendence degree t over ${\bbfQ}$, and G a commutative algebraic group defined over K. Let Y be a finitely generated subgroup of the tangent space at the origin $T\sb G({\bbfC})$ such that $\exp\sb G(Y)$ is contained in G (K). Finally, let V be the vector subspace of $T\sb G({\bbfC})$ generated by Y. A general problem is to give sufficient conditions, involving the dimension of G, the dimension of V, and the rank of Y, which ensure that T is large [see the author, Groupes algébriques et grands degrés de transcendance, Acta Math. (to appear)]. Here, we assume that V is of dimension 1, and we give sufficient conditions which yield $t\ge 1$ or $t\ge 2.$ \par Recently, P. Philippon proved a sharp zero estimate [see {\it D. Bertrand}, Lemmes de zéros et nombres transcendants, Sémin. Bourbaki 1985, No.652)] which enables one to generalize the results of the present paper to several variables. Also, {\it R. Tubbs}, by using Gel'fond's method in one variable, produced further results when V is defined over K [Indépendance algébrique et groupes algébriques, C. R. Acad. Sci., Paris, Sér. I 302, 91-94 (1986)].
MSC 2000:
*11J85 Algebraic independence results
11J81 Transcendence (general theory)
Keywords: algebraic independence; transcendental numbers; algebraic groups; Schneider's method; transcendence degree

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207

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Zbl 0658.01007
Waldschmidt, Michel
Origins of transcendental number theory. On the centennial of the proof of transcendence of the number π. (Bulgarian. French original)
[J] Fiz.-Mat. Spis. 26(59), 167-181 (1984); translation from Cah. Sémin. Hist. Math. 4, 93-115 (1983). ISSN 0015-3265

See the review in Zbl 0507.01004.
MSC 2000:
*01A55 Mathematics in the 19th century
11J81 Transcendence (general theory)
11-03 Historical (number theory)
Keywords: transcendental number; algebraic numbers; Ch. Hermite; J. Liouville; J. Cantor; F. v. Lindemann; axiom of choice; constructive proofs

Citations: Zbl 0507.01004

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211
Zbl 0556.10023
Waldschmidt, Michel
Algebraic independence of transcendental numbers. Gel'fond's method and its developments. (English)
[A] Perspectives in mathematics, Anniv. Oberwolfach 1984, 551-571 (1984).

[For the entire collection see Zbl 0548.00010.] \par The present survey paper gives an excellent account to the method of {\it A. O. Gel'fond} [Usp. Mat. Nauk 4, No.5(33), 14-48 (1949; Zbl 0039.044)] which until now is one of the very few more general methods for algebraic independence proofs. The author traces the most important developments of this method during the last 15 years, when its most significant generalizations, refinements, applications etc. were discovered. More precisely the text is subdivided as follows. \par I. Small transcendence degree: How to produce fields generated by values of exponential or elliptic functions, for which the transcendence degree is at least two or three? II. Large transcendence degree: Same question as in I, but with transcendence degree $\ge n$, where n is large; the three known approaches due to Chudnovsky, Masser-Wüstholz, and Philippon are discussed. III. Criteria of algebraic independence: Gel'fond's original criterion for one variable, its generalization to two variables, the criteria of Chudnovsky-Reyssat and Philippon. IV. Further conjectures (on exponential and elliptic functions as well as on algebraic groups). \par \{Reviewer's remarks: In line 20 on p. 554 [${\bbfQ}(\beta):{\bbfQ}]$ has to be 3, not 2. As D. W. Masser has indicated, the effective version of Schanuel's conjecture on p. 556 contradicts results of {\it A. Bijlsma} [Compos. Math. 35, 99-111 (1977; Zbl 0355.10025)].\}
[P.Bundschuh]
MSC 2000:
*11J85 Algebraic independence results
11-02 Research monographs (number theory)
Keywords: Gel'fond's method; transcendental numbers; bibliography; exponential function; survey; Small transcendence degree; Large transcendence degree; algebraic independence; elliptic functions

Citations: Zbl 0548.00010; Zbl 0039.044; Zbl 0355.10025

Cited in: Zbl 0587.10018

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215

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Zbl 0548.10021
Bertrand, D.; Emsalem, M.; Gramain, F.; Huttner, M.; Langevin, M.; Laurent, M.; Mignotte, M.; Moreau, J.-C.; Philippon, P.; Reyssat,

E.; Waldschmidt, M.
Les nombres transcendants. (French)
[J] Mém. Soc. Math. Fr., Nouv. Sér. 13, 60 p. (1984). ISSN 0037-9484

This tract is a witty introduction to the work of French school at the Institut Henri Poincaré. Its modest 60 pages expose a wealth of topics in transcendence theory. These include the zeros theorem of Masser and Wüstholz, the growth of entire arithmetic functions, the transcendence criteria of Gel'fond, Schneider and Lang, linear forms in logarithms, approaches to Schanuel's conjecture, and work on the E- and G-functions of Siegel. In addition, there are sketches of the application of transcendence theory to diophantine equations, class numbers of imaginary quadratic fields, Lehmer's problem on algebraic integers close to the unit circle, Kummer theory and p-adic regulators. This is a fascinating chapter of mathematics and a well-prepared, if somewhat racy, introduction to the state of the art.
[J.H.Loxton]
MSC 2000:
*11J81 Transcendence (general theory)
11-02 Research monographs (number theory)
Keywords: zero estimates; E-functions; applications; transcendence theory; growth of entire arithmetic functions; transcendence criteria; linear forms in logarithms; Schanuel's conjecture; G-functions

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216
Zbl 0541.12003
Waldschmidt, M.
A lower bound for the p-adic rank of the units of an algebraic number field. (English)
[A] Topics in classical number theory, Colloq. Budapest 1981, Vol. II, Colloq. Math. Soc. János Bolyai 34, 1617-1650 (1984).

[For the entire collection see Zbl 0541.00002.] \par The results of this article are also published in Sémin. Théor. Nombres, Univ. Grenoble I 1980-1981, Exp. No.5 (1981; Zbl 0506.12006).
MSC 2000:
*11R27 Units and factorization
11J81 Transcendence (general theory)
11R80 Totally real fields, etc.
Keywords: p-adic rank of units; transcendence result; exponential polynomials; Leopoldt conjecture

Citations: Zbl 0454.10019; Zbl 0454.10020; Zbl 0171.011; Zbl 0436.32005; Zbl 0541.00002; Zbl 0506.12006

Cited in: Zbl 0717.11049

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217
Zbl 0541.10031
Waldschmidt, Michel
Indépendance algébrique et exponentielles en plusieurs variables. (French)
[A] Number theory, Proc. Journ. arith., Noordwijkerhout/Neth. 1983, Lect. Notes Math. 1068, 268-279 (1984).

[For the entrie collection see Zbl 0535.00008.] \par Seien $x\sb 1,...,x\sb d$, $y\sb 1,...,y\sb{\ell}$ aus ${\bbfC}\sp n$ und es bezeichne $< , >$ das Standardskalarprodukt auf ${\bbfC}\sp n$; sei $t:=Transzendenz\textgrad\sb{{\bbfQ}}\quad {\bbfQ}(\exp<x\sb i,y\sb i>;\quad 1\le i\le d;\quad 1\le j\le \ell).$ Dann gilt (eine schärfere, aber schwerer zu formulierende Ungleichung als) $(*): \quad 2\sp t(\mu(X)+\mu(Y))\ge \mu(X)\mu(Y)$ für $X:={\bbfZ}x\sb 1+...+{\bbfZ}x\sb d,$ wenn μ (X) das Minimum aller Quotienten $(Rang\sb{{\bbfZ}} X/X\cap W)/(\dim\sb{{\bbfC}} {\bbfC}\sp n/W), W(\ne {\bbfC}\sp n)$ Unterraum von ${\bbfC}\sp n$, bezeichnet und Y,μ (Y) analog erklärt sind. Außerdem muß (**): $<\sum\spd\sb{i=1}\alpha\sb ix\sb i,\sum\sp{\ell}\sb{j=1}\alpha\sb{d+j}y\sb j>$ für $(\alpha\sb 1,...,\alpha\sb{d+\ell})\in {\bbfZ}\sp{d+\ell}$ entweder 0 oder aber, verglichen mit $Max \vert \alpha\sb k\vert,$ nicht zu klein sein. \par Im Fall $n=1$ verschärft (*) (und erst recht dessen Verfeinerung) das eingangs des vorstehenden Referats zitierte Ergebnis von Chudnovsky- Endell. Im allgemeinen Fall vermutet man, daß $\mu(X)\mu(Y)>\mu(X)+\mu(Y)$ bereits $t+1>\mu(X)\mu(Y)/(\mu(X)+\mu(Y))$ impliziert und zwar ohne Approximationsvoraussetzung (**). \par Wesentliche Beweishilfsmittel sind ein geeignetes Kriterium für algebraische Unabhängigkeit und ein Nullstellensatz von {\it D. W. Masser} und {\it G. Wüstholz} [Invent. Math. 72, 407-464 (1983; Zbl 0516.10027)]. Schließlich wird das Hauptergebnis dieser letztgenannten Arbeit auf mehrere Variable ausgedehnt.
[P.Bundschuh]
MSC 2000:
*11J85 Algebraic independence results
Keywords: algebraic independence; values of exponential function; inequality

Citations: Zbl 0541.10030; Zbl 0535.00008; Zbl 0516.10027

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219
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Zbl 0579.14039
Waldschmidt, Michel
Sous-groupes analytiques de groupes algébriques. (French)
[J] Ann. Math. (2) 117, 627-657 (1983). ISSN 0003-486X

Soient G un groupe algébrique défini sur la clôture algébrique ${\bar {\bbfQ}}$ de ${\bbfQ}$ dans ${\bbfC}$, et ϕ : ${\bbfC}\sp n\to G({\bbfC})$ un homomorphisme analytique. En général, $\phi ({\bbfC}\sp n)$ n'est pas un sous-groupe algébrique de G($ {\bbfC})$, mais s'enroule autour de son adhérence de Zariski. Nous allons donner des conditions arithmétiques qui assurent que l'image est fermée. Ce problème a été étudié d'abord par {\it S. Lang} ["Introduction to transcendental numbers" (1966; Zbl 0144.041); chapter II] pour les sous-groupes à un paramètre. \par Dans le cas général (n$\ge 1)$, les premiers résultats sur ce sujet ont été obtenus par {\it E. Bombieri} et {\it S. Lang} [Invent. Math. 11, 1-14 (1970; Zbl 0237.14015)]. \par Les énoncés ici imposaient des conditions sévères sur l'approximation diophantienne des logarithmes. Ces conditions provenaient d'une estimation analytique, le lemme de Schwarz. \par Nous utilisons ici une approche différente, qui évite le lemme de Schwarz, mais qui utilise un résultat puissant, le "lemme de zéros" de {\it D. W. Masser} et {\it G. Wüstholz} [Invent. Math. 64, 489-516 (1981; Zbl 0467.10025)]. \par Nous introduisons un nombre $\rho =\rho (G)$ défini par $\rho =1$ si G est linéaire, $\rho =2$ sinon. \par Voici l'énoncé précis: Théorème. Soient G une groupe algébrique défini sur ${\bar {\bbfQ}}$ de dimension d, ϕ : ${\bbfC}\sp n\to G({\bbfC})$ un sous-groupe à n paramètres de G, de dimension d, et Y un sous-groupe de ${\bbfC}\sp n$ de rang m sur ${\bbfZ}$ tel que ϕ (Y)$\subset G({\bar {\bbfQ}})$. On suppose $md>n(m+d\rho).$ Alors il existe un sous-espace vectoriel V de ${\bbfC}\sp n$ tel que, si H désigne l'adhérence de Zariski sur ${\bar {\bbfQ}}$ de ϕ (V), et si on note $n\sb 1=\dim\sb{{\bbfC}}({\bbfC}\sp n/V)$, $d \sb 1=\dim (G/H)$, et $m\sb 1=rang\sb{{\bbfZ}}(Y/Y\cap V)$, on ait $n\sb 1>0$, $d\sb 1/n\sb 1>d/n$, et $m\sb 1d\sb 1\le n\sb

1(m\sb 1+d\sb 1\rho)$.
MSC 2000:
*14L10 Group varieties
11J85 Algebraic independence results
32H99 Holomorphic mappings on analytic spaces
14G25 Global ground fields
Keywords: image under analytic homomorphism; n-parameter subgroup of; algebraic group; rank of quotient space; dimension of quotient; space

Citations: Zbl 0144.041; Zbl 0237.14015; Zbl 0467.10025

Cited in: Zbl 0689.12003

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224
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Zbl 0531.10037
Waldschmidt, Michel; Zhu, Yaochen
Une généralisation en plusieurs variables d'un critère de transcendance de Gel'fond. (French)
[J] C. R. Acad. Sci., Paris, Sér. I 297, 229-232 (1983). ISSN 0764-4442

For $P\in {\bbfC}[X\sb 1,...,X\sb n]$, $P\ne 0$, let H(P) denote the maximum modulus of the coefficients of P and d(P) the maximum of the degrees of P with respect to $X\sb 1,...,X\sb n$. Let $t(P)=\max(\log H(P),\quad i+d(P)).$ The following theorem with $c\sb 0 (1)=1$ and $c\sb 0(n)=\prod\spn\sb{\nu =2}(7\nu)\sp{2\sp{\nu -n}}$ ($n\ge 2)$ is proved: For every integer $n\ge 1$, there exists a real number $c\sb 0=c\sb 0(n)\ge 1$ having the following property. Let $(\theta\sb 1,...,\theta\sb n)$ be in ${\bbfC}\sp n$ and $ \eta\ge 1$ a real number. Suppose that there exist two real numbers $N\sb 0$ and c with $N\sb 0>0$ and $c>6c\sb 0\sp{\eta}$, such that for every real number $N\ge N\sb 0$ and every (n- 1)-tuple $(z\sb 2,...,z\sb n)\in {\bbfC}\sp{n-1}$ satisfying $$ \vert z \sb 1-\theta\sb i\vert<\exp(-2cN\sp{\eta}),\quad 2\le i\le n, $$ there exists a non-zero polynomial $\quad F\in {\bbfZ}[X\sb 1,...,X \sb n]$ satisfying t(F)$\le N$ and $$ 0<\vert F(\theta\sb 1,z\sb 2,...,z\sb n)\vert<\exp(-cN\sp{\eta}). $$ Then $\eta<2\sp n.$ \par The particular case $n=1$ of the theorem gives the transcendence criterion of {\it A. O. Gel'fond} [see Transcendental and algebraic numbers (Moscow 1952; Zbl 0048.033); New York, 1960]. From the theorem one can also deduce the criteria for the algebraic independence of several numbers of {\it G. V. Chudnovsky} [Top. Number Theory, Debrecen 1974, Colloq. Math. Soc. János Bolyai 13, 19-30 (1976; Zbl 0337.10023, p. 23] and {\it E. Reyssat} [J. Reine Angew. Math. 329, 66-81 (1981; Zbl 0459.10023)], but the proof of the authors is much simpler.
[K.Yu]
MSC 2000:
*11J85 Algebraic independence results
11J81 Transcendence (general theory)
Keywords: several complex variables; Gelfond transcendence criterion; algebraic independence; Chudnovsky semi-resultant; ZFM 90, 261

Citations: Zbl 0048.033; Zbl 0337.10023; Zbl 0459.10023

Cited in: Zbl 0662.10025 Zbl 0615.10044 Zbl 0558.10031 Zbl 0549.10024

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225
Zbl 0519.32004
Waldschmidt, M.
Un lemme de Schwarz pour des intersections d'hyperplans. (French)
[A] Studies in pure mathematics, Mem. of P. Turan, 751-759 (1983).

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MSC 2000:
*32A10 Holomorphic functions (several variables)
14M10 Complete intersections
30C80 Maximum principle, etc. (one complex variable)
32P05 Non-Archimedean complex analysis
32A30 Generalizations of function theory to several variables
12J15 Ordered fields
Keywords: complete intersection of hyperplanes; Schwarz function; ultrametric; ideal of analytic functions

Citations: Zbl 0512.00007

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226
Zbl 0513.14028
Waldschmidt, M.
Dependance de logarithmes dans les groupes algébriques. (French)
[A] Approximations diophantiennes et nombres transcendants, Colloq. Luminy/Fr. 1982, Prog. Math. 31, 289-328 (1983).

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MSC 2000:
*14L10 Group varieties
14K15 Arithmetic ground fields (abelian varieties)
11J81 Transcendence (general theory)
Keywords: algebraic groups; Leopoldt conjecture; p-adic rank of units

Citations: Zbl 0504.00005

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227
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Zbl 0507.01004
Waldschmidt, Michel
Les debuts de la théorie des nombres transcendants (a l'occasion du centenaire de la transcendance de pi). (French)
[J] Cah. Semin. Hist. Math. 4, 93-115 (1983).

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MSC 2000:
*01A55 Mathematics in the 19th century
01A50 Mathematics in the 18th century
11J81 Transcendence (general theory)
11-03 Historical (number theory)
Keywords: transcendental numbers; algebraic numbers; Ch. Hermite; J. Liouville; J. Cantor; F. v. Lindemann; axiom of choice; constructive proofs

Cited in: Zbl 0658.01007

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228

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Zbl 0504.00005
Bertrand, D. (ed.); Waldschmidt, M. (ed.)
Approximations diophantiennes et nombres transcendants. Colloque de Luminy, 1982. (French)
[B] Progress in Mathematics, Vol. 31. Boston - Basel - Stuttgart: Birkhäuser. VII, 336 p. SFr. 56.00; DM 65.00 (1983).

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MSC 2000:
*00Bxx Conference proceedings and collections of papers
11-06 Proceedings of conferences (number theory)
14-06 Proceedings of conferences (algebraic geometry)
Keywords: Approximations diophantiennes; Nombres transcendants; Colloque; Luminy/France; transcendental numbers; Diophantine approximation

Cited in: Zbl 0745.00060 Zbl 0584.14020 Zbl 0579.14038 Zbl 0578.10040 Zbl 0549.32002 Zbl 0541.10029 Zbl 0534.10026 Zbl 0529.10032 Zbl 0526.10029 Zbl 0522.10023 Zbl 0522.10012 Zbl 0521.10027 Zbl 0518.10040 Zbl 0518.10039 Zbl 0514.10028 Zbl 0513.14028 Zbl 0513.14015 Zbl 0513.10036 Zbl 0513.10035 Zbl 0513.10034

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230
Zbl 0505.10017
Waldschmidt, Michel
Diophantine properties of the periods of the Fermat curve. (English)
[A] Number theory related to Fermat's last theorem, Proc. Conf., Prog. Math. 26, 79-88 (1982).

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MSC 2000:
*11J81 Transcendence (general theory)
11J85 Algebraic independence results
11-02 Research monographs (number theory)
Keywords: transcendence; linear independence; algebraic independence; expository article; arithmetic properties; values of gamma function; beta function; abelian functions

Citations: Zbl 0491.00009

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231
Zbl 0498.10020
Waldschmidt, Michel
Nombres transcendants: Quelques résultats recents. (French)
[A] Journees arithmetiques, Metz 1981, Asterisque 94, 187-196 (1982).
MSC 2000:
*11J81 Transcendence (general theory)
11J85 Algebraic independence results
11-02 Research monographs (number theory)
14Kxx Abelian varieties and schemes
30D15 Special classes of entire functions
33E05 Elliptic functions and integrals
Keywords: survey; algebraic independence; transcendence theory

Citations: Zbl 0492.00004; Zbl 0461.10028; Zbl 0491.10026; Zbl 0488.10031; Zbl 0475.10031; Zbl 0425.10041; Zbl 0452.10035; Zbl 0486.10024; Zbl 0459.10024; Zbl 0459.10023; Zbl 0467.10025; Zbl 0481.10034; Zbl 0498.10021; Zbl 0454.10020

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232
Zbl 0494.12007
Waldschmidt, Michel
Sur certains characteres du groupe des classes d'ideles d'un corps du nombres. (French)
[A] Theorie des nombres, Semin. Delange-Pisot-Poitou, Paris 1980-81, Prog. Math. 22, 323-335 (1982).
MSC 2000:
*11R45 Density theorems
11R23 Iwasawa theory
11J81 Transcendence (general theory)
11R56 Adele rings and groups
Keywords: Hecke characters; Größencharacter; idele class group; Hecke L-series; transcendence; exponential polynomials in several complex variables; finitely generated subgroup of multiplicative group

Citations: Zbl 0483.00002; Zbl 0073.263; Zbl 0454.10020; Zbl 0436.32005; Zbl 0454.10019; Zbl 0434.14019; Zbl 0186.257

Cited in: Zbl 0780.11060 Zbl 0523.12014 Zbl 0498.12012

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Zbl 0506.12006
Waldschmidt, Michel
Minorations du rang p-adique du groupe des unites. (French)

[J] Semin. Theor. Nombres, Univ. Grenoble I 1980-1981, Expose No.5, 20 p. (1981).

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MSC 2000:
*11R27 Units and factorization
11J81 Transcendence (general theory)
11R80 Totally real fields, etc.
Keywords: p-adic rank of units; transcendence result; exponential polynomials; Leopoldt's conjecture

Citations: Zbl 0454.10019; Zbl 0454.10020; Zbl 0171.011; Zbl 0436.32005

Cited in: Zbl 0541.12003 Zbl 0528.12006

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234
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Zbl 0462.10023
Bertrand, Daniel; Waldschmidt, Michel
On meromorphic functions of one complex variable having algebraic Laurent coefficients. (English)
[J] Bull. Aust. Math. Soc. 24, 247-267 (1981). ISSN 0004-9727

Display scanned Zentralblatt-MATH page with this review.
MSC 2000:
*11J81 Transcendence (general theory)
30D15 Special classes of entire functions
Keywords: meromorphic functions; algebraic coefficients; Laurent expansions; transcendence criteria; new proof of Siegel's results on E-functions

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235
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Zbl 0454.10020
Waldschmidt, Michel
Transcendance et exponentielles en plusieurs variables. (French)
[J] Invent. Math. 63, 97-127 (1981). ISSN 0020-9910; ISSN 1432-1297

Display scanned Zentralblatt-MATH page with this review.
MSC 2000:
*11J81 Transcendence (general theory)
11R04 Algebraic numbers
11S99 Algebraic number theory over local and p-adic fields
Keywords: generalization of six exponentials theorem; p-adic theorem; lower bound for p-adic rank of the unit group

Cited in: Zbl 0780.11060 Zbl 0742.11036 Zbl 0742.11035 Zbl 0712.11042 Zbl 0555.10017 Zbl 0541.12003 Zbl 0523.12014 Zbl 0498.10020 Zbl 0494.12007 Zbl 0488.10030 Zbl 0506.12006

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236
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Zbl 0454.12002
Waldschmidt, Michel
Sur le produit des conjugues exterieurs au cercle unite d'un entier algébrique. (French)
[J] Enseign. Math., II. Sér. 26, 201-209 (1980). ISSN 0013-8584

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MSC 2000:
*11R06 Special algebraic numbers
Keywords: Lehmer problem; measure of algebraic integer

Cited in: Zbl 0504.12003

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237
Zbl 0444.10028
Waldschmidt, Michel
Propriétés arithmétiques de fonctions de plusieurs variables. III. (French)
[A] Semin. P. Lelong - H. Skoda, Analyse, Annees 1978/79, Lect. Notes Math. 822, 332-356 (1980).

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MSC 2000:
*11J81 Transcendence (general theory)
11H60 Mean value and transfer theorems
11R56 Adele rings and groups
32A15 Entire functions (several variables)
Keywords: arithmetic properties; functions of several variables; theorem of six exponentials; idele class group of algebraic number field; Schwarz lemma; transference theorem

Citations: Zbl 0336.32007; Zbl 0363.32003; Zbl 0428.10017

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Zbl 0444.10026
Bertrand, D.; Waldschmidt, M.
Quelques travaux recents en théorie des nombres transcendants. (French)
[J] Mém. Soc. Math. Fr., Nouv. Sér. 2, 107-119 (1980). ISSN 0249-633X

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MSC 2000:
*11J81 Transcendence (general theory)
11-02 Research monographs (number theory)
11J85 Algebraic independence results
14K15 Arithmetic ground fields (abelian varieties)
32A99 Holomorphic functions of several variables
Keywords: transcendence; algebraic independence; elliptic function; abelian function

Citations: Zbl 0407.10025; Zbl 0439.10022; Zbl 0432.10018; Zbl 0431.10019; Zbl 0428.10017; Zbl 0419.10034

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239

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Zbl 0441.00008
Bertrand, D. (ed.); Waldschmidt, M. (ed.)
Fonctions abeliennes et nombres transcendants. Colloque tenu du 23 au 26 mai 1979 à l'Ecole Polytechnique, Palaiseau. (French)
[B] Memoire de la Societe Mathematique de France, Nouvelle Serie No.2. Supplement au Bulletin de la Societe Mathematique de France, Tome 108, Fasc. 2. Paris: Gauthier-Villars. 119 p. (1980).

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MSC 2000:
*00Bxx Conference proceedings and collections of papers
11-06 Proceedings of conferences (number theory)
14-06 Proceedings of conferences (algebraic geometry)
Keywords: FOnctions abeliennes; Nombres transcendants; Colloque; Ecole Polytechnique; Palaiseau; transcendental numbers; abelian functions

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Zbl 0357.10017
Waldschmidt, Michel
A lower bound for linear forms in logarithms. (English)
[J] Acta Arith. 37, 257-283 (1980). ISSN 0065-1036; ISSN 1730-6264

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MSC 2000:
*11J81 Transcendence (general theory)
Keywords: lower bound; linear forms in logarithms

Citations: Zbl 0357.10017; Zbl 0301.10030; Zbl 0361.10028

Cited in: Zbl 0787.11008 Zbl 0751.11017 Zbl 0709.11037 Zbl 0704.11006 Zbl 0703.11017 Zbl 0657.10016 Zbl 0659.10037 Zbl 0651.10023 Zbl 0657.10015 Zbl 0625.10013 Zbl 0623.10023 Zbl 0623.10012 Zbl 0623.10011 Zbl 0549.10007 Zbl 0523.10008 Zbl 0357.10017

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Zbl 0462.10022
Waldschmidt, Michel
Applications de la théorie de Kummer à des problèmes diophantiens. (French)
[J] Publ. Math. Fac. Sci. Besancon, Theor. Nombres, Annee 1978-1979, Exp. No.1, 12 p. (1979).

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MSC 2000:
*11J81 Transcendence (general theory)
11R18 Cyclotomic extensions
14H52 Elliptic curves
14G25 Global ground fields
Citations: Zbl 0432.10016; Zbl 0388.10001; Zbl 0432.10017

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Zbl 0432.10017
Waldschmidt, Michel
Applications de la théorie de Kummer à des problèmes de transcendance. (French)
[J] Semin. Theor. Nombres 1978-1979, Expose No.22, 14 P. (1979).

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MSC 2000:
*11J81 Transcendence (general theory)
11R18 Cyclotomic extensions
14H52 Elliptic curves
14G25 Global ground fields
Keywords: Kummer theory; multiplicative group; algebraic groups; logarithms; division

Citations: Zbl 0388.10001

Cited in: Zbl 0462.10022

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Zbl 0432.10016
Waldschmidt, Michel
Transcendence methods. (English)
[B] Queen's Pap. Pure Appl. Math. 52, 132 p. (1979).

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MSC 2000:
*11J81 Transcendence (general theory)
11-01 Textbooks (number theory)
11J85 Algebraic independence results
11-02 Research monographs (number theory)
Keywords: Gelfond's method; Schneider's method; Baker's method; Kummer's theory; linear independence of elliptic logarithms; transcendence and linear independence of periods; algebraic independence of periods; Schneider's method in several variables; Gel'fond's method in several variables

Cited in: Zbl 0462.10022

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Zbl 0428.10017
Waldschmidt, Michel
(Bertrand, D.; Serre, Jean-Pierre)
Nombres transcendants et groupes algébriques. Complete par deux appendices de Daniel Bertrand et Jean-Pierre Serre. (French)
[B] Asterisque 69-70, 218 p. (1979).

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MSC 2000:
*11J81 Transcendence (general theory)
11-02 Research monographs (number theory)
14L05 Formal groups
Keywords: generalized Dirichlet exponent; linear groups; one-parameter subgroups; Schneider-Lang criterion; many-parameter subgroups; abelian varieties of complex multiplication type; Schwarz lemmas; exponential function; elliptic function; systematic exposition; transcendence theory; abelian functions

Citations: Zbl 0302.10030

Cited in: Zbl 0616.14036 Zbl 0621.10022 Zbl 0597.10033 Zbl 0564.10023 Zbl 0543.14029 Zbl 0497.41001 Zbl 0496.14024 Zbl 0495.10023 Zbl 0479.14020 Zbl 0472.10034 Zbl 0467.10025 Zbl 0455.32004 Zbl 0436.32005 Zbl 0445.14020 Zbl 0444.10028 Zbl 0444.10026

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Zbl 0402.10037
Waldschmidt, Michel
Nombres transcendants et fonctions sigma de Weierstrass. (French)
[J] Math. Rep. Acad. Sci., R. Soc. Can. 1, 111-114 (1979).

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MSC 2000:
*11J81 Transcendence (general theory)
33E05 Elliptic functions and integrals
14H52 Elliptic curves
Keywords: Elliptic Function; Transcendency

Cited in: Zbl 0472.10033

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249
OpenURL

Zbl 0388.10023
Waldschmidt, Michel
Simultaneous approximation of numbers connected with the exponential function. (English)
[J] J. Aust. Math. Soc., Ser. A 25, 466-478 (1978). ISSN 0263-6115

Display scanned Zentralblatt-MATH page with this review.
MSC 2000:
*11J81 Transcendence (general theory)
11J69
Cited in: Zbl 0431.10020

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250
OpenURL

Zbl 0388.10022
Waldschmidt, Michel
Transcendence measures for exponentials and logarithms. (English)
[J] J. Aust. Math. Soc., Ser. A 25, 445-465 (1978). ISSN 0263-6115

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MSC 2000:
*11J81 Transcendence (general theory)
11J04 Homogeneous approximation to one number

Cited in: Zbl 0785.11039 Zbl 0649.10023 Zbl 0459.10022
PDF XML ASCII DVI PS BibTeX Online Ordering Link to Serial
251
OpenURL

Zbl 0381.10029
Waldschmidt, M.
Polya's theorem by Schneider's method. (English)
[J] Acta Math. Acad. Sci. Hung. 31, 21-25 (1978). ISSN 0001-5954

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MSC 2000:
*11J81 Transcendence (general theory)
30D15 Special classes of entire functions
30D20 General theory of entire functions
30D30 General theory of meromorphic functions
30D35 Distribution of values (one complex variable)
Cited in: Zbl 0541.10029 Zbl 0482.10035 Zbl 0461.10028
PDF XML ASCII DVI PS BibTeX Online Ordering
252
OpenURL

Zbl 0349.10029
Mignotte, Maurice; Waldschmidt, Michel
Linear forms in two logarithms and Schneider's method. (English)
[J] Math. Ann. 231, 241-267 (1978). ISSN 0025-5831; ISSN 1432-1807

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MSC 2000:
*11J81 Transcendence (general theory)
11J69
Cited in: Zbl 0752.11029 Zbl 0702.11044 Zbl 0642.10034
PDF XML ASCII DVI PS BibTeX Online Ordering Link to Serial
253
Zbl 0366.10029
Waldschmidt, Michel
On functions of several variables having algebraic Taylor coefficients. (English)
[A] Transcend. Theory, Proc. Conf., Cambridge 1976, 169-186 (1977).

Display scanned Zentralblatt-MATH page with this review.
MSC 2000:
*11J81 Transcendence (general theory)
32A20 Meromorphic functions (several variables)
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255
Zbl 0363.32003
Waldschmidt, Michel
Propriétés arithmétiques de fonctions de plusieurs variables. II. (French)
[A] Semin. Pierre Lelong, Anal., Annee 1975/76, Journ. Fonct. anal., Toulouse 1976, Lect. Notes Math. 578, 108-133 (1977).

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MSC 2000:
*32A15 Entire functions (several variables)
32A20 Meromorphic functions (several variables)
11J81 Transcendence (general theory)
Cited in: Zbl 0444.10028 Zbl 0408.10021 Zbl 0396.10022
PDF XML ASCII DVI PS BibTeX Online Ordering
256
OpenURL

Zbl 0357.10018
Brownawell, W.D.; Waldschmidt, M.
The algebraic independence of certain numbers to algebraic powers. (English)
[J] Acta Arith. 32, 63-71 (1977). ISSN 0065-1036; ISSN 1730-6264

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MSC 2000:
*11J81 Transcendence (general theory)
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257
Zbl 0355.10026
Waldschmidt, Michel
Transcendance dans les variétés de groupe. II. (French)
[A] Semin. Delange-Pisot-Poitou, 17e Annee 1975/76, Theor. des Nombres, Fasc. 1, Expose 2, 7 p. (1977).

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MSC 2000:
*11J81 Transcendence (general theory)
14L10 Group varieties
PDF XML ASCII DVI PS BibTeX Online Ordering
259
Zbl 0353.10026

Waldschmidt, Michel
Une mesure de transcendance de e^π. (French)
[A] Semin. Delange-Pisot-Poitou, 17e Annee 1975/76, Theor. des Nombres, Groupe d'Etude; Fasc. 2, Expose G4, 5 p. (1977).

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MSC 2000:
*11J81 Transcendence (general theory)
11J04 Homogeneous approximation to one number
Cited in: Zbl 0608.10040
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260
Zbl 0348.10024
Waldschmidt, Michel
Suites colorees (d'apres G. V. Cudnovskij). (Italian)
[A] Semin. Delange-Pisot-Poitou, 17e Annee 1975/76, Theor. des Nombres, Groupe d'Etude; Fasc. 2, Expose G 21, 11 p. (1977).

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MSC 2000:
*11J81 Transcendence (general theory)
11J04 Homogeneous approximation to one number
Cited in: Zbl 0566.10027 Zbl 0549.10023 Zbl 0406.10026 Zbl 0442.10024
PDF XML ASCII DVI PS BibTeX Online Ordering

262
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Zbl 0345.10022
Mignotte, Maurice; Waldschmidt, Michel
Approximation simultanee de valeurs de la fonction exponentielle. (French)
[J] Compos. Math. 34, 127-139 (1977). ISSN 0010-437X; ISSN 1570-5846

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MSC 2000:
*11J81 Transcendence (general theory)
11J69
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263
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Zbl 0345.10021
Cijsouw, Pieter L.; Waldschmidt, Michel
Linear forms and simultaneous approximations. (English)
[J] Compos. Math. 34, 173-197 (1977). ISSN 0010-437X; ISSN 1570-5846

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MSC 2000:
*11J81 Transcendence (general theory)
11J69
11J17 Approximation by numbers from a fixed field
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264
Zbl 0345.10020
Waldschmidt, Michel
(Chudnovs'kij, G.V.)
Les travaux de G. V. Cudnovskii sur les nombres transcendants. (French)
[A] Semin. Bourbaki 1975/76, Lect. Notes Math. 567, Expose 488, 19 p. (1977).

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MSC 2000:
*11J81 Transcendence (general theory)
Cited in: Zbl 0594.10024
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265
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Zbl 0345.10019
Waldschmidt, Michel
Rapport sur la transcendance. (French)
[J] Astérisque 41-42, 127-134 (1977).

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MSC 2000:
*11J81 Transcendence (general theory)
11D61 Exponential diophantine equations
11-02 Research monographs (number theory)
PDF XML ASCII DVI PS BibTeX Online Ordering
266
Zbl 0339.14023
Waldschmidt, Michel
Zeros de fonctions entières et hypersurfaces algébriques. (French)
[A] Semin. Theor. Nombres 1975-1976, Univ. Bordeaux, Expose No.19, 8 p. (1976).

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MSC 2000:
*14J15 Analytic moduli, classification (surfaces)
32A15 Entire functions (several variables)
11J81 Transcendence (general theory)
PDF XML ASCII DVI PS BibTeX Online Ordering

267
Zbl 0336.32007
Waldschmidt, Michel
Propriétés arithmétiques de fonctions de plusieurs variables. I. (French)
[A] Semin. Pierre Lelong, Anal., Annee 1974/75, Lect. Notes Math. 524, 106-129 (1976).

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MSC 2000:
*32A20 Meromorphic functions (several variables)
11J81 Transcendence (general theory)
14K20 Analytic theory; abelian integrals and differentials
33B15 Gamma-functions, etc.
Cited in: Zbl 0444.10028
PDF XML ASCII DVI PS BibTeX Online Ordering

268
Zbl 0335.10037
Waldschmidt, M.
Some topics in transcendental number theory. (English)
[A] Top. Number Theory, Debrecen 1974, Colloq. Math. Soc. Janos Bolyai 13, 417-427 (1976).

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MSC 2000:
*11J81 Transcendence (general theory)
11-02 Research monographs (number theory)
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269
Zbl 0324.10032
Mignotte, Maurice
Independance algébrique de certains nombres de la forme α^β et α^{β^2} (d'apres W. Dale Brownawell et Michel Waldschmidt). (French)
[A] Semin. Delange-Pisot-Poitou, 16e annee 1974/75, Theorie des Nombres, Fasc. 2, Expose G 9, 5 p. (1975).

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MSC 2000:
*11J81 Transcendence (general theory)
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270
Zbl 0324.10031
Waldschmidt, Michel
Independance algébrique par la méthode de G. V. Cudnovskij. (French)
[A] Semin. Delange-Pisot-Poitou, 16e annee 1974/75, Theorie des Nombres, Fasc. 2, Expose G 8, 18 p. (1975).

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MSC 2000:
*11J81 Transcendence (general theory)
Cited in: Zbl 0566.10027 Zbl 0549.10023
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271
Zbl 0324.10030
Waldschmidt, Michel
Independance algébrique de puissances algébriques de nombres algébriques. (French)
[A] Semin. Theor. Nombres 1974-1975, Univ. Bordeaux, Expose No.5, 12 p. (1975).

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MSC 2000:
*11J81 Transcendence (general theory)
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272
Zbl 0324.10028
Waldschmidt, Michel
Minorations effectives de formes linéaires de logarithmes. (Apercu historique). (French)
[A] Semin. Delange-Pisot-Poitou, 15e annee 1973/74, Theorie des Nombres, Fasc. 2, Expose G 3, 8 p. (1975).

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MSC 2000:
*11J81 Transcendence (general theory)
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273
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Zbl 0314.14014
Waldschmidt, Michel
Images de points algébriques par un sous-groupe analytique d'une variété de groupe. (French)
[J] C. R. Acad. Sci., Paris, Sér. A 281, 855-858 (1975).

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MSC 2000:
*14L10 Group varieties
11J81 Transcendence (general theory)
14G99 Special ground fields
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Zbl 0305.10028
Mignotte, Maurice; Waldschmidt, Michel
Approximation des valeurs de fonctions transcendantes. (French)
[J] Astérisque 24-25, 183-186 (1975).

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MSC 2000:
*11J17 Approximation by numbers from a fixed field
11J81 Transcendence (general theory)
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275
OpenURL

Zbl 0305.10027
Mignotte, Maurice; Waldschmidt, Michel
Approximation des valeurs de fonctions transcendantes. (French)
[J] Nederl. Akad. Wet., Proc., Ser. A 78, 213-223 (1975).

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MSC 2000:
*11J17 Approximation by numbers from a fixed field
11J81 Transcendence (general theory)
Cited in: Zbl 0992.11045
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277
Zbl 0314.14013
Waldschmidt, Michel
Sous-groupes analytiques de variétés de groupe. (French)
[A] Semin. Theor. Nombres 1973-1974, Univ. Bordeaux, Expose 11, 9 p. (1974).

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MSC 2000:
*14L10 Group varieties
11J81 Transcendence (general theory)
32A20 Meromorphic functions (several variables)
33B10 Elementary functions
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278
OpenURL

Zbl 0302.10030
Waldschmidt, Michel
Nombres transcendants. (French)
[B] Lecture Notes in Mathematics. 402. Berlin-Heidelberg-New York: Springer-Verlag. VIII, 277 p. DM 25.00 (1974).

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MSC 2000:
*11J81 Transcendence (general theory)
11-01 Textbooks (number theory)
11J61 Approximation in non-Archimedean valuations
30C15 Zeros of polynomials, etc. (one complex variable)
Cited in: Zbl 0924.11059 Zbl 0621.10022 Zbl 0637.12009 Zbl 0428.10017
PDF XML ASCII DVI PS BibTeX Online Ordering

279
OpenURL

Zbl 0289.10024
Waldschmidt, Michel
Approximation par des nombres algébriques des zéros de séries entières à coefficients algébriques. (French)
[J] C. R. Acad. Sci., Paris, Sér. A 279, 793-796 (1974).

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MSC 2000:
*11J81 Transcendence (general theory)
11J04 Homogeneous approximation to one number
30B10 Power series (one complex variable)
PDF XML ASCII DVI PS BibTeX Online Ordering

280
OpenURL

Zbl 0287.10021
Waldschmidt, Michel
À propos de la méthode de Baker. (French)
[J] Bull. Soc. Math. Fr., Suppl., Mém. 37 (Journées arithmétiques, Grenoble 1973), 181-192 (1974). ISSN 0037-9484

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MSC 2000:
*11J81 Transcendence (general theory)
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281
OpenURL

Zbl 0284.10012
Waldschmidt, Michel
Initiation aux nombres transcendants. (French)
[J] Enseign. Math., II. Sér. 20, 53-85 (1974). ISSN 0013-8584

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MSC 2000:
*11J81 Transcendence (general theory)
PDF XML ASCII DVI PS BibTeX Online Ordering

282
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Zbl 0275.14022
Waldschmidt, Michel
Dimension algébrique de sous-groupes analytiques de variétés de groupe. (French)
[J] Ann. Inst. Fourier 25, No.1, 23-33 (1975). ISSN 0373-0956

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*14L10 Group varieties
11J81 Transcendence (general theory)
32A20 Meromorphic functions (several variables)
33B10 Elementary functions
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283
Zbl 0348.10023
Waldschmidt, Michel
La conjecture de Schanuel. (French)
[A] Semin. Nombres transc., Paris 1972-73, Expose No.8, 13 p. (1973).

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MSC 2000:
*11J81 Transcendence (general theory)
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284
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Zbl 0335.00004
Waldschmidt, Michel (ed.)
Seminaire sur les nombres transcendants 1972-1973. Exposes 1 à 8. (French)
[B] Publication Mathematique d'Orsay. No.57. Orsay, France: Universite Paris XI, U.E.R. Mathematique. Pagination non continue (1973).

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MSC 2000:
*00Bxx Conference proceedings and collections of papers
11-06 Proceedings of conferences (number theory)
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285
Zbl 0325.10022
Waldschmidt, Michel
Transcendance dans les variétés de groupe. (French)
[A] Semin. Delange-Pisot-Poitou, 14e annee 1972/73, Theorie des Nombres, Fasc. 1, 2, Expose 23, 16 p. (1973).

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MSC 2000:
*11J81 Transcendence (general theory)
14L10 Group varieties
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286
Zbl 0325.10021
Waldschmidt, Michel
Une méthode 'élémentaire' dans la théorie des nombres transcendants. II. (French)
[A] Semin. Delange-Pisot-Poitou, 14e annee 1972/73, Theorie des Nombres, Fasc. 1, 2, Expose G5, 5 p. (1973).

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MSC 2000:
*11J81 Transcendence (general theory)
PDF XML ASCII DVI PS BibTeX Online Ordering

287
Zbl 0325.10020
Waldschmidt, Michel
Une méthode 'élémentaire' dans la théorie des nombres transcendants. I. (French)
[A] Semin. Delange-Pisot-Poitou, 14e annee 1972/73, Theorie des Nombres, Fasc. 1, 2, Expose G1, 5 p. (1973).

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*11J81 Transcendence (general theory)

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288
Zbl 0319.10038
Waldschmidt, Michel
Transcendance et independance algébrique des valeurs de fonctions méromorphes. (French)
[A] Semin. Delange-Pisot-Poitou, 13e annee 1971/72, Theorie des Nombres, Fasc. 1, 2, Expose 5, 9 p. (1973).

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MSC 2000:
*11J81 Transcendence (general theory)
30D30 General theory of meromorphic functions
PDF XML ASCII DVI PS BibTeX Online Ordering

289
Zbl 0292.12010
Waldschmidt, Michel
Problèmes de nombres de classes de corps quadratiques imaginaires (apercu historique). (French)
[A] Sem. Theorie Nombres 1972-1973, Univ. Bordeaux, Expose No.12, 15 p. (1973).

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MSC 2000:
*11R23 Iwasawa theory
12-03 Historical (field theory)
12-02 Research monographs (field theory)
PDF XML ASCII DVI PS BibTeX Online Ordering

290
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Zbl 0262.10021
Waldschmidt, Michel
Solution du huitieme problème de Schneider. (French)
[J] J. Number Theory 5, 191-202 (1973). ISSN 0022-314X

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MSC 2000:
*11J81 Transcendence (general theory)
Cited in: Zbl 1011.11049 Zbl 0860.11040
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291
OpenURL

Zbl 0227.10022
Waldschmidt, Michel
Propriétés arithmétiques des valeurs de fonctions méromorphes algébriquement indépendantes. (Arithmetic properties of the values of algebraically independent meromorphic functions.). (French)
[J] Acta Arith. 23, 19-88 (1973). ISSN 0065-1036; ISSN 1730-6264

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MSC 2000:
*11J81 Transcendence (general theory)
11J72 Irrationality
PDF XML ASCII DVI PS BibTeX Online Ordering Link to Serial Link to Serial

292
Zbl 0276.10017
Waldschmidt, Michel
Fonctions elliptiques et nombres transcendants. (French)
[A] Sem. Theorie Nombres 1971-1972, Univ. Bordeaux, No.4, 21 p. (1972).

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MSC 2000:
*11J81 Transcendence (general theory)
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293
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Zbl 0244.10033
Waldschmidt, Michel
Utilisation de la méthode de Baker dans des problèmes d'independance algébrique. (French)
[J] C. R. Acad. Sci., Paris, Sér. A 275, 1215-1217 (1972).

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MSC 2000:
*11J81 Transcendence (general theory)
PDF XML ASCII DVI PS BibTeX Online Ordering

294
Zbl 0244.10032
Waldschmidt, Michel
Independance algébrique des valeurs de la fonction exponentielle. (French)
[A] Sem. Delange-Pisot-Poitou 12 (1970/71), Theorie Nombres, No.6, 8 p. (1972).

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MSC 2000:
*11J81 Transcendence (general theory)
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295
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Zbl 0236.14020
Waldschmidt, Michel
Dimension algébrique de sous-groupes analytiques de variétés abeliennes. (French)
[J] C. R. Acad. Sci., Paris, Sér. A 274, 1681-1683 (1972).

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MSC 2000:
*14K99 Abelian varieties and schemes
14L10 Group varieties
11J81 Transcendence (general theory)
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296
Zbl 0297.10019
Waldschmidt, Michel
Majoration du nombre de zéros d'une somme polynomiale d'exponentielles p- adiques. (French)
[A] Sem. Theorie Nombres 1969-1970/1970-1971, Additif, Univ. Bordeaux, No.7 bis, 9 p. (1971).

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MSC 2000:
*11J99 Diophantine approximation
30C15 Zeros of polynomials, etc. (one complex variable)
11S05 Polynomials over local fields
11M99 Analytic theory of zeta and L-functions
11J81 Transcendence (general theory)
PDF XML ASCII DVI PS BibTeX Online Ordering
297
Zbl 0297.10018
Waldschmidt, Michel
Répartition des valeurs d'une somme de fonctions exponentielles. (French)
[A] Sem. Theorie Nombres 1969-1970/1970-1971, Additif, Univ. Bordeaux, No.11 bis, 19 p. (1971).

Display scanned Zentralblatt-MATH page with this review.
MSC 2000:
*11J99 Diophantine approximation
30C15 Zeros of polynomials, etc. (one complex variable)
11M99 Analytic theory of zeta and L-functions
11J81 Transcendence (general theory)
PDF XML ASCII DVI PS BibTeX Online Ordering
298
Zbl 0297.10016
Waldschmidt, Michel
Transcendance et independance algébrique dans les groupes linéaires. (French)
[A] Sem. Theorie Nombres 1970-1971, Univ. Bordeaux, No.20, 10 p. (1971).

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MSC 2000:
*11J81 Transcendence (general theory)
14L10 Group varieties
20G20 Linear algebraic groups over the reals
11E57 Arithmetic properties of classical groups
PDF XML ASCII DVI PS BibTeX Online Ordering
299
Zbl 0297.10015
Waldschmidt, Michel
Independance algébrique d'exponentielles. (French)
[A] Sem. Theorie Nombres 1970-1971, Univ. Bordeaux, No.2, 18 p. (1971).

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MSC 2000:
*11J81 Transcendence (general theory)
11-02 Research monographs (number theory)
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300
Zbl 0297.10014
Waldschmidt, Michel
La méthode de Gel'fond en théorie des nombres transcendants. (French)
[A] Sem. Theorie Nombres 1970-1971, Univ. Bordeaux, No.1, 20 p. (1971).

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MSC 2000:
*11J81 Transcendence (general theory)
11-02 Research monographs (number theory)
PDF XML ASCII DVI PS BibTeX Online Ordering
211
Zbl 0556.10023
Waldschmidt, Michel
Algebraic independence of transcendental numbers. Gel'fond's method and its developments. (English)
[A] Perspectives in mathematics, Anniv. Oberwolfach 1984, 551-571 (1984).

[For the entire collection see Zbl 0548.00010.] \par The present survey paper gives an excellent account to the method of {\it A. O.

Gel'fond} [Usp. Mat. Nauk 4, No.5(33), 14-48 (1949; Zbl 0039.044)] which until now is one of the very few more general methods for algebraic independence proofs. The author traces the most important developments of this method during the last 15 years, when its most significant generalizations, refinements, applications etc. were discovered. More precisely the text is subdivided as follows. \par I. Small transcendence degree: How to produce fields generated by values of exponential or elliptic functions, for which the transcendence degree is at least two or three? II. Large transcendence degree: Same question as in I, but with transcendence degree $ \ge n$, where n is large; the three known approaches due to Chudnovsky, Masser-Wüstholz, and Philippon are discussed. III. Criteria of algebraic independence: Gel'fond's original criterion for one variable, its generalization to two variables, the criteria of Chudnovsky-Reyssat and Philippon. IV. Further conjectures (on exponential and elliptic functions as well as on algebraic groups). \par \{Reviewer's remarks: In line 20 on p. 554 [${\bbfQ}(\beta):{\bbfQ}]$ has to be 3, not 2. As D. W. Masser has indicated, the effective version of Schanuel's conjecture on p. 556 contradicts results of {\it A. Bijlsma} [Compos. Math. 35, 99-111 (1977; Zbl 0355.10025)].\}
[P.Bundschuh]
MSC 2000:
*11J85 Algebraic independence results
11-02 Research monographs (number theory)
Keywords: Gel'fond's method; transcendental numbers; bibliography; exponential function; survey; Small transcendence degree; Large transcendence degree; algebraic independence; elliptic functions

Citations: Zbl 0548.00010; Zbl 0039.044; Zbl 0355.10025

Cited in: Zbl 0587.10018

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215

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Zbl 0548.10021
Bertrand, D.; Emsalem, M.; Gramain, F.; Huttner, M.; Langevin, M.; Laurent, M.; Mignotte, M.; Moreau, J.-C.; Philippon, P.; Reyssat, E.; Waldschmidt, M.
Les nombres transcendants. (French)
[J] Mém. Soc. Math. Fr., Nouv. Sér. 13, 60 p. (1984). ISSN 0037-9484

This tract is a witty introduction to the work of French school at the Institut Henri Poincaré. Its modest 60 pages expose a wealth of topics in transcendence theory. These include the zeros theorem of Masser and Wüstholz, the growth of entire arithmetic functions, the transcendence criteria of Gel'fond, Schneider and Lang, linear forms in logarithms, approaches to Schanuel's conjecture, and work on the E- and G-functions of Siegel. In addition, there are sketches of the application of transcendence theory to diophantine equations, class numbers of imaginary quadratic fields, Lehmer's problem on algebraic integers close to the unit circle, Kummer theory and p-adic regulators. This is a fascinating chapter of mathematics and a well-prepared, if somewhat racy, introduction to the state of the art.
[J.H.Loxton]
MSC 2000:
*11J81 Transcendence (general theory)
11-02 Research monographs (number theory)
Keywords: zero estimates; E-functions; applications; transcendence theory; growth of entire arithmetic functions; transcendence criteria; linear forms in logarithms; Schanuel's conjecture; G-functions

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216
Zbl 0541.12003
Waldschmidt, M.
A lower bound for the p-adic rank of the units of an algebraic number field. (English)
[A] Topics in classical number theory, Colloq. Budapest 1981, Vol. II, Colloq. Math. Soc. János Bolyai 34, 1617-1650 (1984).

[For the entire collection see Zbl 0541.00002.] \par The results of this article are also published in Sémin. Théor. Nombres, Univ. Grenoble I 1980-1981, Exp. No.5 (1981; Zbl 0506.12006).
MSC 2000:
*11R27 Units and factorization
11J81 Transcendence (general theory)
11R80 Totally real fields, etc.
Keywords: p-adic rank of units; transcendence result; exponential polynomials; Leopoldt conjecture

Citations: Zbl 0454.10019; Zbl 0454.10020; Zbl 0171.011; Zbl 0436.32005; Zbl 0541.00002; Zbl 0506.12006

Cited in: Zbl 0717.11049

PDF XML ASCII DVI PS BibTeX Online Ordering

217
Zbl 0541.10031
Waldschmidt, Michel
Indépendance algébrique et exponentielles en plusieurs variables. (French)
[A] Number theory, Proc. Journ. arith., Noordwijkerhout/Neth. 1983, Lect. Notes Math. 1068, 268-279 (1984).

[For the entrie collection see Zbl 0535.00008.] \par Seien $x\sb 1,...,x\sb d$, $y\sb 1,...,y\sb{\ell}$ aus ${\bbfC}\sp n$ und es bezeichne $< , >$ das Standardskalarprodukt auf ${\bbfC}\sp n$; sei $t:=Transzendenz\textgrad\sb{{\bbfQ}}\quad {\bbfQ}(\exp<x \sb i,y\sb i>;\quad 1\le i\le d;\quad 1\le j\le \ell).$ Dann gilt (eine schärfere, aber schwerer zu formulierende Ungleichung als) $(*): \quad 2\sp t(\mu(X)+\mu(Y))\ge \mu(X)\mu(Y)$ für $X:={\bbfZ}x\sb 1+...+{\bbfZ}x\sb d,$ wenn μ (X) das Minimum aller Quotienten $(Rang\sb{{\bbfZ}} X/X\cap W)/(\dim\sb{{\bbfC}} {\bbfC}\sp n/W), W(\ne {\bbfC}\sp n)$ Unterraum von ${\bbfC}\sp n$, bezeichnet und Y,μ (Y) analog erklärt sind. Außerdem muß (**): $<\sum\spd\sb{i=1}\alpha\sb ix\sb i,\sum\sp{\ell}\sb{j=1} \alpha\sb{d+j}y\sb j>$ für $(\alpha\sb 1,...,\alpha\sb{d+\ell})\in {\bbfZ}\sp{d+\ell}$ entweder 0 oder aber, verglichen mit $Max \vert \alpha\sb k\vert,$ nicht zu klein sein. \par Im Fall $n=1$ verschärft (*) (und erst recht dessen Verfeinerung) das eingangs des vorstehenden Referats zitierte Ergebnis von Chudnovsky- Endell. Im allgemeinen Fall vermutet man, daß $\mu(X)\mu(Y)>\mu(X)+ \mu(Y)$ bereits $t+1>\mu(X)\mu(Y)/(\mu(X)+\mu(Y))$ impliziert und zwar ohne Approximationsvoraussetzung (**). \par Wesentliche Beweishilfsmittel sind ein geeignetes Kriterium für algebraische Unabhängigkeit und ein Nullstellensatz von {\it D. W. Masser} und {\it G. Wüstholz} [Invent. Math. 72, 407-464 (1983; Zbl 0516.10027)]. Schließlich wird das Hauptergebnis dieser letztgenannten Arbeit auf mehrere Variable ausgedehnt.
[P.Bundschuh]
MSC 2000:
*11J85 Algebraic independence results

Keywords: algebraic independence; values of exponential function; inequality

Citations: Zbl 0541.10030; Zbl 0535.00008; Zbl 0516.10027

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219

OpenURL

Zbl 0579.14039

Waldschmidt, Michel

Sous-groupes analytiques de groupes algébriques. (French)

[J] Ann. Math. (2) 117, 627-657 (1983). ISSN 0003-486X

Soient G un groupe algébrique défini sur la clôture algébrique ${\bar {\bbfQ}}$ de ${\bbfQ}$ dans ${\bbfC}$, et ϕ : ${\bbfC}\sp n\to G({\bbfC})$ un homomorphisme analytique. En général, $\phi ({\bbfC}\sp n)$ n'est pas un sous-groupe algébrique de G(${\bbfC})$, mais s'enroule autour de son adhérence de Zariski. Nous allons donner des conditions arithmétiques qui assurent que l'image est fermée. Ce problème a été étudié d'abord par {\it S. Lang} ["Introduction to transcendental numbers" (1966; Zbl 0144.041); chapter II] pour les sous-groupes à un paramètre. \par Dans le cas général (n$\ge 1)$, les premiers résultats sur ce sujet ont été obtenus par {\it E. Bombieri} et {\it S. Lang} [Invent. Math. 11, 1-14 (1970; Zbl 0237.14015)]. \par Les énoncés ici imposaient des conditions sévères sur l'approximation diophantienne des logarithmes. Ces conditions provenaient d'une estimation analytique, le lemme de Schwarz. \par Nous utilisons ici une approche différente, qui évite le lemme de Schwarz, mais qui utilise un résultat puissant, le "lemme de zéros" de {\it D. W. Masser} et {\it G. Wüstholz} [Invent. Math. 64, 489-516 (1981; Zbl 0467.10025)]. \par Nous introduisons un nombre $\rho =\rho (G)$ défini par $\rho =1$ si G est linéaire, $\rho =2$ sinon. \par Voici l'énoncé précis: Théorème. Soient G une groupe algébrique défini sur ${\bar {\bbfQ}}$ de dimension d, ϕ : ${\bbfC}\sp n\to G({\bbfC})$ un sous-groupe à n paramètres de G, de dimension d, et Y un sous-groupe de ${\bbfC}\sp n$ de rang m sur ${\bbfZ}$ tel que ϕ (Y)$\subset G({\bar {\bbfQ}})$. On suppose $md>n(m+d\rho).$ Alors il existe un sous-espace vectoriel V de ${\bbfC}\sp n$ tel que, si H désigne l'adhérence de Zariski sur ${\bar {\bbfQ}}$ de ϕ (V), et si on note $n\sb 1=\dim\sb{{\bbfC}}({\bbfC}\sp n/V)$, $d\sb 1=\dim (G/H)$, et $m\sb 1=rang\sb{{\bbfZ}}(Y/Y\cap V)$, on ait $n\sb 1>0$, $d\sb 1/n\sb 1>d/n$, et $m\sb 1d\sb 1\le n\sb 1(m\sb 1+d\sb 1\rho)$.

MSC 2000:

*14L10 Group varieties

11J85 Algebraic independence results

32H99 Holomorphic mappings on analytic spaces

14G25 Global ground fields

Keywords: image under analytic homomorphism; n-parameter subgroup of; algebraic group; rank of quotient space; dimension of quotient; space

Citations: Zbl 0144.041; Zbl 0237.14015; Zbl 0467.10025

Cited in: Zbl 0689.12003

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224

OpenURL

Zbl 0531.10037

Waldschmidt, Michel; Zhu, Yaochen

Une généralisation en plusieurs variables d'un critère de transcendance de Gel'fond. (French)

[J] C. R. Acad. Sci., Paris, Sér. I 297, 229-232 (1983). ISSN 0764-4442

For $P\in {\bbfC}[X\sb 1,...,X\sb n]$, $P\ne 0$, let H(P) denote the maximum modulus of the coefficients of P and d(P) the maximum of the degrees of P with respect to $X\sb 1,...,X\sb n$. Let $t(P)=\max(\log H(P),\quad i+d(P)).$ The following theorem with $c\sb 0(1)=1$ and $c\sb 0(n)=\prod\spn\sb{\nu =2}(7\nu)\sp{2\sp{\nu -n}}$ ($n\ge 2)$ is proved: For every integer $n\ge 1$, there exists a real number $c\sb 0=c\sb 0(n)\ge 1$ having the following property. Let $(\theta\sb 1,...,\theta\sb n)$ be in ${\bbfC}\sp n$ and $\eta\ge 1$ a real number. Suppose that there exist two real numbers $N\sb 0$ and c with $N\sb 0>0$ and $c>6c\sb 0\sp{\eta}$, such that for every real number $N\ge N\sb 0$ and every (n- 1)-tuple $(z\sb 2,...,z\sb n)\in {\bbfC}\sp{n-1}$ satisfying $$ \vert z\sb 1-\theta\sb i\vert<\exp(-2cN\sp{\eta}),\quad 2\le i\le n, $$ there exists a non-zero polynomial $\quad F\in {\bbfZ}[X\sb 1,...,X\sb n]$ satisfying t(F)$\le N$ and $$ 0<\vert F(\theta\sb 1,z\sb 2,...,z\sb n)\vert<\exp(-cN\sp{\eta}). $$ Then $\eta<2\sp n.$ \par The particular case $n=1$ of the theorem gives the transcendence criterion of {\it A. O. Gel'fond} [see Transcendental and algebraic numbers (Moscow 1952; Zbl 0048.033); New York, 1960]. From the theorem one can also deduce the criteria for the algebraic independence of several numbers of {\it G. V. Chudnovsky} [Top. Number Theory, Debrecen 1974, Colloq. Math. Soc. János Bolyai 13, 19-30 (1976; Zbl 0337.10023, p. 23] and {\it E. Reyssat} [J. Reine Angew. Math. 329, 66-81 (1981; Zbl 0459.10023)], but the proof of the authors is much simpler.

[K.Yu]

MSC 2000:

*11J85 Algebraic independence results

11J81 Transcendence (general theory)

Keywords: several complex variables; Gelfond transcendence criterion; algebraic independence; Chudnovsky semi-resultant; ZFM 90, 261

Citations: Zbl 0048.033; Zbl 0337.10023; Zbl 0459.10023

Cited in: Zbl 0662.10025 Zbl 0615.10044 Zbl 0558.10031 Zbl 0549.10024

PDF XML ASCII DVI PS BibTeX Online Ordering

225

Zbl 0519.32004

Waldschmidt, M.

Un lemme de Schwarz pour des intersections d'hyperplans. (French)

[A] Studies in pure mathematics, Mem. of P. Turan, 751-759 (1983).

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MSC 2000:

*32A10 Holomorphic functions (several variables)

14M10 Complete intersections

30C80 Maximum principle, etc. (one complex variable)

32P05 Non-Archimedean complex analysis

32A30 Generalizations of function theory to several variables

12J15 Ordered fields

Keywords: complete intersection of hyperplanes; Schwarz function; ultrametric; ideal of analytic functions

Citations: Zbl 0512.00007

PDF XML ASCII DVI PS BibTeX Online Ordering

226
Zbl 0513.14028
Waldschmidt, M.
Dependance de logarithmes dans les groupes algébriques. (French)
[A] Approximations diophantiennes et nombres transcendants, Colloq. Luminy/Fr. 1982, Prog. Math. 31, 289-328 (1983).

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MSC 2000:
*14L10 Group varieties
14K15 Arithmetic ground fields (abelian varieties)
11J81 Transcendence (general theory)
Keywords: algebraic groups; Leopoldt conjecture; p-adic rank of units

Citations: Zbl 0504.00005

PDF XML ASCII DVI PS BibTeX Online Ordering

227
OpenURL

Zbl 0507.01004
Waldschmidt, Michel
Les debuts de la théorie des nombres transcendants (a l'occasion du centenaire de la transcendance de pi). (French)
[J] Cah. Semin. Hist. Math. 4, 93-115 (1983).

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MSC 2000:
*01A55 Mathematics in the 19th century
01A50 Mathematics in the 18th century
11J81 Transcendence (general theory)
11-03 Historical (number theory)
Keywords: transcendental numbers; algebraic numbers; Ch. Hermite; J. Liouville; J. Cantor; F. v. Lindemann; axiom of choice; constructive proofs

Cited in: Zbl 0658.01007

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Zbl 0504.00005
Bertrand, D. (ed.); Waldschmidt, M. (ed.)
Approximations diophantiennes et nombres transcendants. Colloque de Luminy, 1982. (French)
[B] Progress in Mathematics, Vol. 31. Boston - Basel - Stuttgart: Birkhäuser. VII, 336 p. SFr. 56.00; DM 65.00 (1983).

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MSC 2000:
*00Bxx Conference proceedings and collections of papers
11-06 Proceedings of conferences (number theory)
14-06 Proceedings of conferences (algebraic geometry)
Keywords: Approximations diophantiennes; Nombres transcendants; Colloque; Luminy/France; transcendental numbers; Diophantine approximation

Cited in: Zbl 0745.00060 Zbl 0584.14020 Zbl 0579.14038 Zbl 0578.10040 Zbl 0549.32002 Zbl 0541.10029 Zbl 0534.10026 Zbl 0529.10032 Zbl 0526.10029 Zbl 0522.10023 Zbl 0522.10012 Zbl 0521.10027 Zbl 0518.10040 Zbl 0518.10039 Zbl 0514.10028 Zbl 0513.14028 Zbl 0513.14015 Zbl 0513.10036 Zbl 0513.10035 Zbl 0513.10034

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Zbl 0505.10017
Waldschmidt, Michel
Diophantine properties of the periods of the Fermat curve. (English)
[A] Number theory related to Fermat's last theorem, Proc. Conf., Prog. Math. 26, 79-88 (1982).

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MSC 2000:
*11J81 Transcendence (general theory)
11J85 Algebraic independence results
11-02 Research monographs (number theory)
Keywords: transcendence; linear independence; algebraic independence; expository article; arithmetic properties; values of gamma function; beta function; abelian functions

Citations: Zbl 0491.00009

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231
Zbl 0498.10020
Waldschmidt, Michel
Nombres transcendants: Quelques résultats recents. (French)
[A] Journees arithmetiques, Metz 1981, Asterisque 94, 187-196 (1982).
MSC 2000:
*11J81 Transcendence (general theory)
11J85 Algebraic independence results
11-02 Research monographs (number theory)
14Kxx Abelian varieties and schemes
30D15 Special classes of entire functions
33E05 Elliptic functions and integrals
Keywords: survey; algebraic independence; transcendence theory

Citations: Zbl 0492.00004; Zbl 0461.10028; Zbl 0491.10026; Zbl 0488.10031; Zbl 0475.10031; Zbl 0425.10041; Zbl 0452.10035; Zbl 0486.10024; Zbl 0459.10024; Zbl 0459.10023; Zbl 0467.10025; Zbl 0481.10034; Zbl 0498.10021; Zbl 0454.10020

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Zbl 0494.12007
Waldschmidt, Michel
Sur certains characteres du groupe des classes d'ideles d'un corps du nombres. (French)
[A] Theorie des nombres, Semin. Delange-Pisot-Poitou, Paris 1980-81, Prog. Math. 22, 323-335 (1982).
MSC 2000:
*11R45 Density theorems
11R23 Iwasawa theory
11J81 Transcendence (general theory)
11R56 Adele rings and groups
Keywords: Hecke characters; Größencharacter; idele class group; Hecke L-series; transcendence; exponential polynomials in several complex variables; finitely generated subgroup of multiplicative group

Citations: Zbl 0483.00002; Zbl 0073.263; Zbl 0454.10020; Zbl 0436.32005; Zbl 0454.10019; Zbl 0434.14019; Zbl 0186.257

Cited in: Zbl 0780.11060 Zbl 0523.12014 Zbl 0498.12012

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Zbl 0506.12006
Waldschmidt, Michel
Minorations du rang p-adique du groupe des unites. (French)
[J] Semin. Theor. Nombres, Univ. Grenoble I 1980-1981, Expose No.5, 20 p. (1981).

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MSC 2000:
*11R27 Units and factorization
11J81 Transcendence (general theory)
11R80 Totally real fields, etc.
Keywords: p-adic rank of units; transcendence result; exponential polynomials; Leopoldt's conjecture

Citations: Zbl 0454.10019; Zbl 0454.10020; Zbl 0171.011; Zbl 0436.32005

Cited in: Zbl 0541.12003 Zbl 0528.12006

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Zbl 0462.10023
Bertrand, Daniel; Waldschmidt, Michel
On meromorphic functions of one complex variable having algebraic Laurent coefficients. (English)
[J] Bull. Aust. Math. Soc. 24, 247-267 (1981). ISSN 0004-9727

Display scanned Zentralblatt-MATH page with this review.
MSC 2000:
*11J81 Transcendence (general theory)
30D15 Special classes of entire functions
Keywords: meromorphic functions; algebraic coefficients; Laurent expansions; transcendence criteria; new proof of Siegel's results on E-functions

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235
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Zbl 0454.10020
Waldschmidt, Michel
Transcendance et exponentielles en plusieurs variables. (French)
[J] Invent. Math. 63, 97-127 (1981). ISSN 0020-9910; ISSN 1432-1297

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MSC 2000:
*11J81 Transcendence (general theory)
11R04 Algebraic numbers
11S99 Algebraic number theory over local and p-adic fields
Keywords: generalization of six exponentials theorem; p-adic theorem; lower bound for p-adic rank of the unit group

Cited in: Zbl 0780.11060 Zbl 0742.11036 Zbl 0742.11035 Zbl 0712.11042 Zbl 0555.10017 Zbl 0541.12003 Zbl 0523.12014 Zbl 0498.10020 Zbl 0494.12007 Zbl 0488.10030 Zbl 0506.12006

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Zbl 0454.12002
Waldschmidt, Michel
Sur le produit des conjugues exterieurs au cercle unite d'un entier algébrique. (French)
[J] Enseign. Math., II. Sér. 26, 201-209 (1980). ISSN 0013-8584

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MSC 2000:
*11R06 Special algebraic numbers
Keywords: Lehmer problem; measure of algebraic integer

Cited in: Zbl 0504.12003

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237
Zbl 0444.10028
Waldschmidt, Michel
Propriétés arithmétiques de fonctions de plusieurs variables. III. (French)
[A] Semin. P. Lelong - H. Skoda, Analyse, Annees 1978/79, Lect. Notes Math. 822, 332-356 (1980).

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MSC 2000:
*11J81 Transcendence (general theory)
11H60 Mean value and transfer theorems
11R56 Adele rings and groups
32A15 Entire functions (several variables)
Keywords: arithmetic properties; functions of several variables; theorem of six exponentials; idele class group of algebraic number field; Schwarz lemma; transference theorem

Citations: Zbl 0336.32007; Zbl 0363.32003; Zbl 0428.10017

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Zbl 0444.10026
Bertrand, D.; Waldschmidt, M.
Quelques travaux recents en théorie des nombres transcendants. (French)
[J] Mém. Soc. Math. Fr., Nouv. Sér. 2, 107-119 (1980). ISSN 0249-633X

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MSC 2000:
*11J81 Transcendence (general theory)
11-02 Research monographs (number theory)
11J85 Algebraic independence results
14K15 Arithmetic ground fields (abelian varieties)
32A99 Holomorphic functions of several variables
Keywords: transcendence; algebraic independence; elliptic function; abelian function

Citations: Zbl 0407.10025; Zbl 0439.10022; Zbl 0432.10018; Zbl 0431.10019; Zbl 0428.10017; Zbl 0419.10034

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Zbl 0441.00008
Bertrand, D. (ed.); Waldschmidt, M. (ed.)
Fonctions abeliennes et nombres transcendants. Colloque tenu du 23 au 26 mai 1979 à l'Ecole Polytechnique, Palaiseau. (French)
[B] Memoire de la Societe Mathematique de France, Nouvelle Serie No.2. Supplement au Bulletin de la Societe Mathematique de France, Tome 108, Fasc. 2. Paris: Gauthier-Villars. 119 p. (1980).

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MSC 2000:
*00Bxx Conference proceedings and collections of papers
11-06 Proceedings of conferences (number theory)
14-06 Proceedings of conferences (algebraic geometry)
Keywords: FOnctions abeliennes; Nombres transcendants; Colloque; Ecole Polytechnique; Palaiseau; transcendental numbers; abelian functions

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Zbl 0357.10017
Waldschmidt, Michel
A lower bound for linear forms in logarithms. (English)
[J] Acta Arith. 37, 257-283 (1980). ISSN 0065-1036; ISSN 1730-6264

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MSC 2000:
*11J81 Transcendence (general theory)
Keywords: lower bound; linear forms in logarithms

Citations: Zbl 0357.10017; Zbl 0301.10030; Zbl 0361.10028

Cited in: Zbl 0787.11008 Zbl 0751.11017 Zbl 0709.11037 Zbl 0704.11006 Zbl 0703.11017 Zbl 0657.10016 Zbl 0659.10037 Zbl 0651.10023 Zbl 0657.10015 Zbl 0625.10013 Zbl 0623.10023 Zbl 0623.10012 Zbl 0623.10011 Zbl 0549.10007 Zbl 0523.10008 Zbl 0357.10017

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243
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Zbl 0462.10022
Waldschmidt, Michel
Applications de la théorie de Kummer à des problèmes diophantiens. (French)
[J] Publ. Math. Fac. Sci. Besancon, Theor. Nombres, Annee 1978-1979, Exp. No.1, 12 p. (1979).

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MSC 2000:
*11J81 Transcendence (general theory)
11R18 Cyclotomic extensions

14H52 Elliptic curves
14G25 Global ground fields
Citations: Zbl 0432.10016; Zbl 0388.10001; Zbl 0432.10017

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Zbl 0432.10017
Waldschmidt, Michel
Applications de la théorie de Kummer à des problèmes de transcendance. (French)
[J] Semin. Theor. Nombres 1978-1979, Expose No.22, 14 P. (1979).

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MSC 2000:
*11J81 Transcendence (general theory)
11R18 Cyclotomic extensions
14H52 Elliptic curves
14G25 Global ground fields
Keywords: Kummer theory; multiplicative group; algebraic groups; logarithms; division

Citations: Zbl 0388.10001

Cited in: Zbl 0462.10022

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Zbl 0432.10016
Waldschmidt, Michel
Transcendence methods. (English)
[B] Queen's Pap. Pure Appl. Math. 52, 132 p. (1979).

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MSC 2000:
*11J81 Transcendence (general theory)
11-01 Textbooks (number theory)
11J85 Algebraic independence results
11-02 Research monographs (number theory)
Keywords: Gelfond's method; Schneider's method; Baker's method; Kummer's theory; linear independence of elliptic logarithms; transcendence and linear independence of periods; algebraic independence of periods; Schneider's method in several variables; Gel'fond's method in several variables

Cited in: Zbl 0462.10022

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Zbl 0428.10017
Waldschmidt, Michel
(Bertrand, D.; Serre, Jean-Pierre)
Nombres transcendants et groupes algébriques. Complete par deux appendices de Daniel Bertrand et Jean-Pierre Serre. (French)
[B] Asterisque 69-70, 218 p. (1979).

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MSC 2000:
*11J81 Transcendence (general theory)
11-02 Research monographs (number theory)
14L05 Formal groups
Keywords: generalized Dirichlet exponent; linear groups; one-parameter subgroups; Schneider-Lang criterion; many-parameter subgroups; abelian varieties of complex multiplication type; Schwarz lemmas; exponential function; elliptic function; systematic exposition; transcendence theory; abelian functions

Citations: Zbl 0302.10030

Cited in: Zbl 0616.14036 Zbl 0621.10022 Zbl 0597.10033 Zbl 0564.10023 Zbl 0543.14029 Zbl 0497.41001 Zbl 0496.14024 Zbl 0495.10023 Zbl 0479.14020 Zbl 0472.10034 Zbl 0467.10025 Zbl 0455.32004 Zbl 0436.32005 Zbl 0445.14020 Zbl 0444.10028 Zbl 0444.10026

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Zbl 0402.10037
Waldschmidt, Michel
Nombres transcendants et fonctions sigma de Weierstrass. (French)
[J] Math. Rep. Acad. Sci., R. Soc. Can. 1, 111-114 (1979).

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MSC 2000:
*11J81 Transcendence (general theory)
33E05 Elliptic functions and integrals
14H52 Elliptic curves
Keywords: Elliptic Function; Transcendency

Cited in: Zbl 0472.10033

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Zbl 0388.10023
Waldschmidt, Michel
Simultaneous approximation of numbers connected with the exponential function. (English)
[J] J. Aust. Math. Soc., Ser. A 25, 466-478 (1978). ISSN 0263-6115

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MSC 2000:
*11J81 Transcendence (general theory)
11J69
Cited in: Zbl 0431.10020
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250
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Zbl 0388.10022
Waldschmidt, Michel
Transcendence measures for exponentials and logarithms. (English)
[J] J. Aust. Math. Soc., Ser. A 25, 445-465 (1978). ISSN 0263-6115

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MSC 2000:
*11J81 Transcendence (general theory)
11J04 Homogeneous approximation to one number
Cited in: Zbl 0785.11039 Zbl 0649.10023 Zbl 0459.10022
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251
OpenURL

Zbl 0381.10029
Waldschmidt, M.
Polya's theorem by Schneider's method. (English)
[J] Acta Math. Acad. Sci. Hung. 31, 21-25 (1978). ISSN 0001-5954

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MSC 2000:
*11J81 Transcendence (general theory)
30D15 Special classes of entire functions
30D20 General theory of entire functions
30D30 General theory of meromorphic functions
30D35 Distribution of values (one complex variable)
Cited in: Zbl 0541.10029 Zbl 0482.10035 Zbl 0461.10028
PDF XML ASCII DVI PS BibTeX Online Ordering
252
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Zbl 0349.10029
Mignotte, Maurice; Waldschmidt, Michel
Linear forms in two logarithms and Schneider's method. (English)
[J] Math. Ann. 231, 241-267 (1978). ISSN 0025-5831; ISSN 1432-1807

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MSC 2000:
*11J81 Transcendence (general theory)
11J69
Cited in: Zbl 0752.11029 Zbl 0702.11044 Zbl 0642.10034
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Zbl 0366.10029
Waldschmidt, Michel
On functions of several variables having algebraic Taylor coefficients. (English)
[A] Transcend. Theory, Proc. Conf., Cambridge 1976, 169-186 (1977).

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MSC 2000:
*11J81 Transcendence (general theory)
32A20 Meromorphic functions (several variables)
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255
Zbl 0363.32003
Waldschmidt, Michel
Propriétés arithmétiques de fonctions de plusieurs variables. II. (French)
[A] Semin. Pierre Lelong, Anal., Annee 1975/76, Journ. Fonct. anal., Toulouse 1976, Lect. Notes Math. 578, 108-133 (1977).

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MSC 2000:
*32A15 Entire functions (several variables)
32A20 Meromorphic functions (several variables)
11J81 Transcendence (general theory)
Cited in: Zbl 0444.10028 Zbl 0408.10021 Zbl 0396.10022
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Zbl 0357.10018
Brownawell, W.D.; Waldschmidt, M.
The algebraic independence of certain numbers to algebraic powers. (English)
[J] Acta Arith. 32, 63-71 (1977). ISSN 0065-1036; ISSN 1730-6264

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MSC 2000:
*11J81 Transcendence (general theory)
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257
Zbl 0355.10026
Waldschmidt, Michel
Transcendance dans les variétés de groupe. II. (French)
[A] Semin. Delange-Pisot-Poitou, 17e Annee 1975/76, Theor. des Nombres, Fasc. 1, Expose 2, 7 p. (1977).

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MSC 2000:
*11J81 Transcendence (general theory)
14L10 Group varieties
PDF XML ASCII DVI PS BibTeX Online Ordering
259
Zbl 0353.10026
Waldschmidt, Michel
Une mesure de transcendance de e^π. (French)
[A] Semin. Delange-Pisot-Poitou, 17e Annee 1975/76, Theor. des Nombres, Groupe d'Etude; Fasc. 2, Expose G4, 5 p. (1977).

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MSC 2000:
*11J81 Transcendence (general theory)
11J04 Homogeneous approximation to one number
Cited in: Zbl 0608.10040
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260
Zbl 0348.10024
Waldschmidt, Michel
Suites colorees (d'apres G. V. Cudnovskij). (Italian)
[A] Semin. Delange-Pisot-Poitou, 17e Annee 1975/76, Theor. des Nombres, Groupe d'Etude; Fasc. 2, Expose G 21, 11 p. (1977).

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MSC 2000:
*11J81 Transcendence (general theory)
11J04 Homogeneous approximation to one number
Cited in: Zbl 0566.10027 Zbl 0549.10023 Zbl 0406.10026 Zbl 0442.10024
PDF XML ASCII DVI PS BibTeX Online Ordering
262
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Zbl 0345.10022
Mignotte, Maurice; Waldschmidt, Michel
Approximation simultanee de valeurs de la fonction exponentielle. (French)
[J] Compos. Math. 34, 127-139 (1977). ISSN 0010-437X; ISSN 1570-5846

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MSC 2000:
*11J81 Transcendence (general theory)
11J69
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263
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Zbl 0345.10021
Cijsouw, Pieter L.; Waldschmidt, Michel
Linear forms and simultaneous approximations. (English)
[J] Compos. Math. 34, 173-197 (1977). ISSN 0010-437X; ISSN 1570-5846

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MSC 2000:
*11J81 Transcendence (general theory)
11J69
11J17 Approximation by numbers from a fixed field
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264
Zbl 0345.10020
Waldschmidt, Michel
(Chudnovs'kij, G.V.)
Les travaux de G. V. Cudnovskii sur les nombres transcendants. (French)
[A] Semin. Bourbaki 1975/76, Lect. Notes Math. 567, Expose 488, 19 p. (1977).

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MSC 2000:
*11J81 Transcendence (general theory)

Cited in: Zbl 0594.10024

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Zbl 0345.10019

Waldschmidt, Michel

Rapport sur la transcendance. (French)

[J] Astérisque 41-42, 127-134 (1977).

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MSC 2000:

*11J81 Transcendence (general theory)

11D61 Exponential diophantine equations

11-02 Research monographs (number theory)

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266

Zbl 0339.14023

Waldschmidt, Michel

Zeros de fonctions entières et hypersurfaces algébriques. (French)

[A] Semin. Theor. Nombres 1975-1976, Univ. Bordeaux, Expose No.19, 8 p. (1976).

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MSC 2000:

*14J15 Analytic moduli, classification (surfaces)

32A15 Entire functions (several variables)

11J81 Transcendence (general theory)

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267

Zbl 0336.32007

Waldschmidt, Michel

Propriétés arithmétiques de fonctions de plusieurs variables. I. (French)

[A] Semin. Pierre Lelong, Anal., Annee 1974/75, Lect. Notes Math. 524, 106-129 (1976).

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MSC 2000:

*32A20 Meromorphic functions (several variables)

11J81 Transcendence (general theory)

14K20 Analytic theory; abelian integrals and differentials

33B15 Gamma-functions, etc.

Cited in: Zbl 0444.10028

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268

Zbl 0335.10037

Waldschmidt, M.

Some topics in transcendental number theory. (English)

[A] Top. Number Theory, Debrecen 1974, Colloq. Math. Soc. Janos Bolyai 13, 417-427 (1976).

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MSC 2000:

*11J81 Transcendence (general theory)

11-02 Research monographs (number theory)

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270

Zbl 0324.10031

Waldschmidt, Michel

Independance algébrique par la méthode de G. V. Cudnovskij. (French)

[A] Semin. Delange-Pisot-Poitou, 16e annee 1974/75, Theorie des Nombres, Fasc. 2, Expose G 8, 18 p. (1975).

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MSC 2000:

*11J81 Transcendence (general theory)

Cited in: Zbl 0566.10027 Zbl 0549.10023

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271

Zbl 0324.10030

Waldschmidt, Michel

Independance algébrique de puissances algébriques de nombres algébriques. (French)

[A] Semin. Theor. Nombres 1974-1975, Univ. Bordeaux, Expose No.5, 12 p. (1975).

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MSC 2000:

*11J81 Transcendence (general theory)

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Zbl 0324.10028

Waldschmidt, Michel

Minorations effectives de formes linéaires de logarithmes. (Apercu historique). (French)

[A] Semin. Delange-Pisot-Poitou, 15e annee 1973/74, Theorie des Nombres, Fasc. 2, Expose G 3, 8 p. (1975).

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MSC 2000:

*11J81 Transcendence (general theory)
11-02 Research monographs (number theory)
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273
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Zbl 0314.14014
Waldschmidt, Michel
Images de points algébriques par un sous-groupe analytique d'une variété de groupe. (French)
[J] C. R. Acad. Sci., Paris, Sér. A 281, 855-858 (1975).

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MSC 2000:
*14L10 Group varieties
11J81 Transcendence (general theory)
14G99 Special ground fields
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274
OpenURL

Zbl 0305.10028
Mignotte, Maurice; Waldschmidt, Michel
Approximation des valeurs de fonctions transcendantes. (French)
[J] Astérisque 24-25, 183-186 (1975).

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MSC 2000:
*11J17 Approximation by numbers from a fixed field
11J81 Transcendence (general theory)
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Zbl 0305.10027
Mignotte, Maurice; Waldschmidt, Michel
Approximation des valeurs de fonctions transcendantes. (French)
[J] Nederl. Akad. Wet., Proc., Ser. A 78, 213-223 (1975).

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*11J17 Approximation by numbers from a fixed field
11J81 Transcendence (general theory)
Cited in: Zbl 0992.11045
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Zbl 0314.14013
Waldschmidt, Michel
Sous-groupes analytiques de variétés de groupe. (French)
[A] Semin. Theor. Nombres 1973-1974, Univ. Bordeaux, Expose 11, 9 p. (1974).

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MSC 2000:
*14L10 Group varieties
11J81 Transcendence (general theory)
32A20 Meromorphic functions (several variables)
33B10 Elementary functions
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Zbl 0302.10030
Waldschmidt, Michel
Nombres transcendants. (French)
[B] Lecture Notes in Mathematics. 402. Berlin-Heidelberg-New York: Springer-Verlag. VIII, 277 p. DM 25.00 (1974).

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MSC 2000:
*11J81 Transcendence (general theory)
11-01 Textbooks (number theory)
11J61 Approximation in non-Archimedean valuations
30C15 Zeros of polynomials, etc. (one complex variable)
Cited in: Zbl 0924.11059 Zbl 0621.10022 Zbl 0637.12009 Zbl 0428.10017
PDF XML ASCII DVI PS BibTeX Online Ordering

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OpenURL

Zbl 0289.10024
Waldschmidt, Michel
Approximation par des nombres algébriques des zéros de séries entières à coefficients algébriques. (French)
[J] C. R. Acad. Sci., Paris, Sér. A 279, 793-796 (1974).

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MSC 2000:
*11J81 Transcendence (general theory)

11J04 Homogeneous approximation to one number
30B10 Power series (one complex variable)
PDF XML ASCII DVI PS BibTeX Online Ordering

280
OpenURL

Zbl 0287.10021
Waldschmidt, Michel
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*14L10 Group varieties
11J81 Transcendence (general theory)
32A20 Meromorphic functions (several variables)
33B10 Elementary functions
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30D30 General theory of meromorphic functions
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*11R23 Iwasawa theory
12-03 Historical (field theory)
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*14K99 Abelian varieties and schemes
14L10 Group varieties
11J81 Transcendence (general theory)
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*11J99 Diophantine approximation
30C15 Zeros of polynomials, etc. (one complex variable)
11S05 Polynomials over local fields
11M99 Analytic theory of zeta and L-functions
11J81 Transcendence (general theory)
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30C15 Zeros of polynomials, etc. (one complex variable)
11M99 Analytic theory of zeta and L-functions
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*11J81 Transcendence (general theory)
14L10 Group varieties
20G20 Linear algebraic groups over the reals
11E57 Arithmetic properties of classical groups
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11-02 Research monographs (number theory)
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*20C15 Ordinary representations and characters of groups
11E95 p-adic theory of forms
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11J91 Transcendence theory of other special functions
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