INVARIANT RINGS $\mathbb{C}[X, Y]^G$ FOR FINITE SUBGROUPS OF $\mathrm{SL}_2(\mathbb{C})$

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The famous classification of finite subgroups of $\mathrm{SO}_3(\mathbb{R})$ (cyclic, dihedral, tetrahedral, octahedral and icosahedral) together with the natural surjection $\mathrm{SU}_2(\mathbb{C}) \to \mathrm{SO}_3(\mathbb{R})$ gives rise to a classification of finite subgroups of $\mathrm{SL}_2(\mathbb{C})$ into five types

- $\mathbb{A}_n$ type (for cyclic subgroups),
- $\mathbb{D}_n$ type (for binary dihedral subgroups),
- $\mathbb{E}_6$ type (for binary tetrahedral subgroups of order 24),
- $\mathbb{E}_7$ type (for binary octahedral subgroups of order 48),
- $\mathbb{E}_8$ type (for binary icosahedral subgroups of order 120).

Let G be a finite subgroup of $\mathrm{SL}_2(\mathbb{C})$. The representation theory of G has many specificities which are illustrated by the Mckay correspondence and by the presentation of the invariant ring $\mathbb{C}[X, Y]^G$ as an isolated hypersurface singularity $\mathbb{C}[u, v, w]/(F)$.

On one hand, the Mckay correspondence explains part of the theory of characters of G by displaying the irreducible representations of G into an extended Dynkin diagram according to the type of G. On the other hand, the computation of characters of *Grundformen* allow one to get explicit generators u, v, w of the algebra $\mathbb{C}[X, Y]^G$ together with a unique generating relation $F(u, v, w) = 0$ with

- $F = uv + w^n$ in type $\mathbb{A}_n$,
- $F = u^{n+1} + uv^2 + w^2$ in type $\mathbb{D}_n$,
- $F = u^3 + v^3 + w^2$ in type $\mathbb{E}_6$,
- $F = u^3 + uv^3 + w^2$ in type $\mathbb{E}_7$,
- $F = u^2 + y^3 + z^5$ in type $\mathbb{E}_8$.

The objective of the course is to present these results and some ideas of their proofs based on the theory of characters of G.