CORRECTION TO 'ON THE CYCLIC HOMOLOGY OF EXACT CATEGORIES'

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1. Mistakes pointed out by R.-O. Buchweitz

I thank R.-O. Buchweitz for his letter [1], where he points out and corrects the following mistakes in [2]:

In 1.11 of [2], it should be $HH_*(mod\, k) = k$ (rather than $k[u]$).

Paragraph 2.5 [2] ends with a claim about the structure of $HH_*(B)$, where $B = k[T]$ viewed as a DG algebra with T of degree 1 and trivial differential. This claim is false (unless k is of characteristic 2). The correct result is given in R.-O. Buchweitz' letter [1]: $\ldots B$ has to be viewed not as the symmetric but as the tensor algebra over T, and the correct answer is somewhat smaller. One has, e.g. from Prop. 5.3.10 in Loday's book [3]

$$\begin{array}{rcl} HH_*(k[T]) & = & (k[T^2]/(2T^2) \oplus k[T^2]dT) \oplus (Tk[T^2] \oplus Ann_2 Tk[T^2]dT) \\ & \cong & HH_0 \oplus (HH_1 \oplus HH_2)[T^2], \\ HH_0 & = & k \oplus kdT \\ HH_1 & = & kT \oplus Ann_2(kdT) \\ HH_2 & = & (k/2k)T^2 \oplus kT^2 dT \end{array}$$

where the brackets in the first expansion correspond to even and odd part resp., as T is of homological degree -1 and dT is of homological degree 0.

Finally, the readers of [2] should be warned about a poor choice of notation in 2.5 and 2.6: In both paragraphs, we consider the differential graded endomorphism algebra $Hom_A(P, P)$ of a complex P. In 2.5, the symbol B denotes the homology of this algebra, whereas in 2.6, it denotes the algebra itself (note that in both paragraphs, B denotes a differential graded algebra quasi-isomorphic to the endomorphism algebra; this was in fact the reason for the choice of the notation).

2. Mistake concerning homotopy

In section 3.1 of [2], we consider homotopies between morphisms between DG algebras and erroneously claim that the homotopy relation

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is an equivalence relation. Instead we should consider the equivalence relation generated by the homotopy relation. The error has no incidence on what follows. Indeed, what we use in the sequel is homotopy between DG functors, treated in 3.3 of [loc. cit.] and in this section, we correctly consider the equivalence relation generated by all 'one-step homotopies'.

References

[1] R.-O. Buchweitz, *Letter to B. Keller*, May 23, 2000.
[2] B. Keller, *On the cyclic homology of exact categories*, JPAA **136** (1999), 1–56.
[3] J.-L. Loday, *Cyclic homology*, Grundlehren 301, Second edition, Springer-Verlag Berlin Heidelberg, 1998.

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