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On a question of B. Mazur and the density of rational points on Abelian varieties

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A question raised by B. Mazur suggests the following conjecture: *Let A be a simple Abelian variety defined over a real number field K. Denote by $A(\mathbf{R})$ the Lie group of its real points and by $A(\mathbf{R})^0$ the connected component of the origin. Then the group $\mathbf{Z}P$ generated by any point P of infinite order in $A(K) \cap A(\mathbf{R})^0$ is dense in $A(\mathbf{R})^0$.*

Transcendence methods yield weaker statements like the following: *Let A be a simple Abelian variety of dimension d defined over a number field K embedded in $\mathbf{R}$. Let Γ be a subgroup of $A(K) \cap A(\mathbf{R})^0$ of rank $\geq d^2 - d + 1$. Then Γ is dense in $A(\mathbf{R})^0$.*

Some results can also be obtained on the density in $A(\mathbf{C})$ of subgroups of $A(K)$, when K is any number field embedded into the field $\mathbf{C}$ of complex numbers.

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