

February 19, 2021

University of the Philippines Diliman

College of Science, Institute of Mathematics

I-Math Webinar dedicated to the 60th birthday of Chancellor Fidel R. Nemenzo
(organized by the Coding and Number Theory Academic Group)

With Fidel, from Phnom Penh to Diliman: some personal reminiscences

Michel Waldschmidt

Professeur Émérite, Sorbonne Université,
Institut de Mathématiques de Jussieu, Paris

<http://www.imj-prg.fr/~michel.waldschmidt/>

Interview by CIMPA, December 9, 2020



Interview de Michel Waldschmidt (Professeur émérite - Sorbonne Université, France)

https://www.youtube.com/watch?v=ukfrf2W_BSw

Royal University of Phnom Penh (Cambodia)



RUPP Class Number Theory 2012



Anise hotel, Phnom Penh, March 2012



IMU Volunteer Lecturer Program

Lecturing | Volunteer Lecturer Program | Country Reports | VLP Cambodia



<https://www.mathunion.org/cdc/lecturing/volunteer-lecturer-program/country-reports/vlp-cambodia>

With Harold Stark in 1993, Hong Kong



Number Theory meeting in Hong Kong

First conference talk by Fidel on *Gauss' genus theorem*.

With Faqir Bhatti, San Ling Guo and Charng Rang



Number Theory meeting in Hong Kong, 1993

Mathematics Genealogy Project

Mathematics Genealogy Project

Fidel R. Nemenzo

[MathSciNet](#)

D.Sc. [Sophia University](#) 1998



Dissertation: *Congruent Numbers and the Tate-Shafarevich Group of the Elliptic Curve*
 $y^2 = x^3 - n^2x$

Mathematics Subject Classification: 11—Number theory

Advisor: [Hideo Wada](#)

Student:

Name	School	Year	Descendants
Manongsong, Saraleen Mae	University of the Philippines Diliman	2019	

According to our current on-line database, Fidel Nemenzo has 1 [student](#) and 1 [descendant](#).

<https://www.genealogy.math.ndsu.nodak.edu/id.php?id=92600>

Mathematics Genealogy Project

Mathematics Genealogy Project

Hideo Wada

Ph.D. [University of Tokyo](#)



Dissertation:

Advisor: [Shokichi Iyanaga](#)

Students:

Click [here](#) to see the students listed in chronological order.

Name	School	Year	Descendants
Basilla, Julius Magalona	Sophia University	2005	
Nemenzo, Fidel	Sophia University	1998	1

According to our current on-line database, Hideo Wada has 2 [students](#) and 3 [descendants](#).

We welcome any additional information.

<https://www.genealogy.math.ndsu.nodak.edu/id.php?id=76784>

Mathematics Genealogy Project

Mathematics Genealogy Project

Shokichi Iyanaga

[Biography MathSciNet](#)

Ph.D. [Tokyo Imperial University](#) 1931



Dissertation:

Advisor: [Teiji Takagi](#)

According to our current on-line database, Shokichi Iyanaga has 23 [students](#) and 935 [descendants](#).

<https://www.genealogy.math.ndsu.nodak.edu/id.php?id=45944>

Mathematics Genealogy Project

Mathematics Genealogy Project

Shokichi Iyanaga

[Biography MathSciNet](#)

Ph.D. Tokyo Imperial University 1931 

Dissertation:

Advisor: [Teiji Takagi](#)

Students:

Click [here](#) to see the students listed in chronological order.

Name	School	Year	Descendants
Azumava, Goro	Nagoya University	1949	20
Furuwa, Shigeru	University of Tokyo	1961	
Hattori, Akio	University of Tokyo	1959	18
Ihara, Yasufaka	University of Tokyo	1967	144
Ito, Kiyosi	Tokyo Imperial University	1945	48
Ito, Seizō			35
Iwahori, Nagayoshi	University of Tokyo	1961	89
Iwasawa, Kenkichi	University of Tokyo	1945	235
Kawada, Yujiyoshi	University of Tokyo	1945	8
Kodaira, Kunihiko	Tokyo Imperial University	1949	105
Kuga, Michio	University of Tokyo	1960	41
Ono, Takashi	Nagoya University	1958	50
Satake, Ichiro	University of Tokyo	1959	17
Sato, Mikio	University of Tokyo	1963	21
Suzuki, Michio	University of Tokyo	1952	93
Takasu, Satoru	University of Tokyo	1958	17
Takeuchi, Masanu	University of Tokyo	1965	
Takeuti, Gaisi	University of Tokyo	1956	37
Tamagawa, Tsuneo	University of Tokyo	1954	65
Tamura, Ichiro	University of Tokyo	1960	19
Wada, Hideo	University of Tokyo		3
Yasugi, Mariko	University of Tokyo	1966	
Yoneda, Nobuo	University of Tokyo	1952	1

According to our current on-line database, Shokichi Iyanaga has 23 [students](#) and 935 [descendants](#).

<https://www.genealogy.math.ndsu.nodak.edu/id.php?id=45944>

Shokichi Iyanaga



Shokichi Iyanaga

1906–2006

Born 2 April 1906 Tokyo, Japan

Died 1 June 2006

<https://mathshistory.st-andrews.ac.uk/Biographies/Iyanaga/>

Jean Delsarte



Jean Delsarte

1903–1968

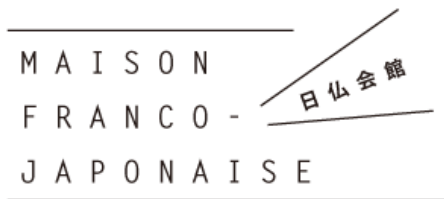
Born 19 October 1903 Fourmies (Nord), France

Died 28 November 1968 Nancy, France

<https://mathshistory.st-andrews.ac.uk/Biographies/Delsarte/>

Maison Franco-Japonaise

Pierre Kaplan (Univ. de Nancy I, mathématiques)
Directeur juillet 1968 - septembre 1970.



Institut français de recherche sur le Japon
à la Maison franco-japonaise
日仏会館, *Nichi-Futsu kaikan*

https://www.mfj.gr.jp/recherche/equipe/anciens_personnels/
http://wikimonde.com/article/Maison_franco-japonaise

Congruent numbers

Fidel R. Nemenzo

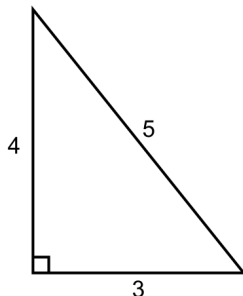
[MathSciNet](#)

D.Sc. [Sophia University](#) 1998



Dissertation: Congruent Numbers and the Tate-Shafarevich Group of the Elliptic Curve
 $y^2 = x^3 - n^2x$

A congruent number is a positive integer that is the area of a right triangle with three rational number sides. For instance [6](#) is a congruent number, the area of a triangle of sides [3](#), [4](#), [5](#).



5 is a congruent number

Sides

$$\frac{20}{3}, \quad \frac{3}{2}, \quad \frac{41}{6}.$$

$$\left(\frac{20}{3}\right)^2 + \left(\frac{3}{2}\right)^2 = \frac{400}{9} + \frac{9}{4} = \frac{1681}{36} = \left(\frac{41}{6}\right)^2.$$

Area :

$$\frac{1}{2} \cdot \frac{20}{3} \cdot \frac{3}{2} = 5.$$

Jean Lagrange

Table 4

Nombres non classés

113	263	282	317	337	367	373	389	409	482	503	521
543	557	569	573	577	593	597	599	607	613	623	627
647	653	677	701	706	717	727	733	743	757	797	802
809	823	829	853	857	863	877	881	887	893	897	898
911	917	933	938	939	941	953	965	967	983	997	

JEAN LAGRANGE. *Nombres congruents et courbes elliptiques*. Séminaire Delange-Pisot-Poitou. Théorie des nombres, tome 16, no.1 (1974–1975), exp. no.16, p. 1–17

http://www.numdam.org/article/SDPP_1974-1975__16_1_A11_0.pdf

JEAN LAGRANGE. *Construction d'une table de nombres congruents*. Mémoires de la S. M. F., tome 49-50 (1977), p. 125–130

http://www.numdam.org/item?id=MSMF_1977__49-50__125_0

Table 2

Nombres congruents

5	6	7	13	14	15	21	22	23	29	30	31
34	37	38	39	41	46	47	53	55	61	62	65
69	70	71	77	78	79	85	86	87	93	94	95
101	102	103	109	110	111	118	119	127	133	134	137
138	141	142	143	145	149	151	154	157	158	159	161
165	166	167	173	174	181	182	183	190	191	194	197
199	205	206	210	213	214	215	219	221	222	223	226
229	230	231	237	238	239	246	247	253	254	255	257
262	265	269	271	277	278	285	286	287	291	293	295
299	301	302	303	309	310	311	313	318	319	323	326
327	330	334	335	341	349	353	357	358	359	365	366
371	374	381	382	383	386	390	391	395	397	398	399
406	407	410	413	415	421	422	426	429	430	431	434
437	438	439	442	445	446	447	453	454	455	457	461
462	463	465	469	470	471	478	479	485	487	493	494
501	502	505	509	510	511	514	517	518	519	526	527
533	534	535	541	542	546	551	559	561	565	566	574
581	582	583	589	590	591	598	602	606	609	614	615
622	629	631	638	645	646	651	654	655	658	661	662
663	669	670	671	674	678	679	685	687	689	694	695
703	709	710	718	719	721	723	731	734	741	742	749
751	758	759	761	766	767	773	777	781	782	789	790
791	793	798	799	805	806	807	813	814	815	821	822
830	831	838	839	854	861	862	866	869	870	871	878
879	885	886	889	890	894	895	901	902	903	905	910
915	919	926	934	935	942	943	949	951	957	958	959
966	973	974	982	985	987	989	991	995	998		

Table 3

Nombres non congruents

1	2	3	10	11	17	19	26	33	35	42	43
51	57	58	59	66	67	73	74	82	83	89	91
97	105	106	107	114	115	122	123	129	130	131	139
146	155	163	170	177	178	179	185	186	187	193	195
201	202	203	209	211	217	218	227	233	235	241	249
251	258	259	266	267	273	274	281	283	290	298	305
307	314	321	322	329	331	339	345	346	347	354	355
362	370	377	379	385	393	394	401	402	403	411	417
418	419	427	433	435	443	449	451	458	466	467	473
474	481	483	489	491	497	498	499	506	515	523	530
537	538	545	547	553	554	555	562	563	570	571	579
586	587	595	601	610	611	617	618	619	626	633	634
635	641	642	643	649	659	665	667	673	681	682	683
690	691	697	698	699	705	707	713	714	715	730	737
739	745	746	753	754	755	762	763	769	770	771	778
779	785	786	787	794	795	803	811	817	818	826	827
834	835	842	843	849	851	858	859	865	874	883	899
906	907	913	914	921	922	923	929	930	937	946	947
955	962	969	970	971	977	978	979	986	993	994	

Don Zagier : 157 is a congruent number

Sides :

$$6803298487826435051217540$$
$$\hline 411340519227716149383203$$
$$411340519227716149383203$$
$$\hline 21666555693714761309610$$
$$224403517704336969924557513090674863160948472041$$
$$\hline 8912332268928859588025535178967163570016480830$$

<https://images.math.cnrs.fr/Les-nombres-congruents.html>

<http://people.mpim-bonn.mpg.de/zagier/files/mpim/89-23/fulltext.pdf>

p.5 : Für $n = 157$ führt diese Methode nach einigen Schritten zu einer Lösung. Diese ist, wie Fermat vielleicht gesagt haben würde, ganz wunderbar, aber die Seite ist leider zu schmal, um sie zu enthalten : die drei rationalen Quadratzahlen im Zähler und Nenner jeweils fast 100 Dezimalziffern.

Elliptic curves : Tunnell's contribution

A squarefree positive integer n is congruent if and only if there are rational numbers (x, y) with $y \neq 0$ such that

$$y^2 = x^3 - n^2x.$$

JERROLD B. TUNNELL, *A classical Diophantine problem and modular forms of weight 3/2*, *Inventiones Mathematicae*, **72** (2) (1983), 323–334.

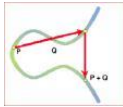
<https://doi.org/10.1007/BF01389327>

NEAL KOBLITZ, *Introduction to elliptic curves and modular forms*, *Graduate Texts in Mathematics*, **97** (1993), Springer-Verlag, New York.

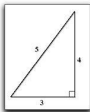
<https://doi.org/10.1007/978-1-4612-0909-6>

PIERRE COLMEZ, *Le problème des nombres congruents*, 2005.

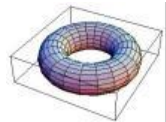
<http://www.math.jussieu.fr/~colmez/congruents.pdf>



a mathematical vignette: numbers, triangles and curves



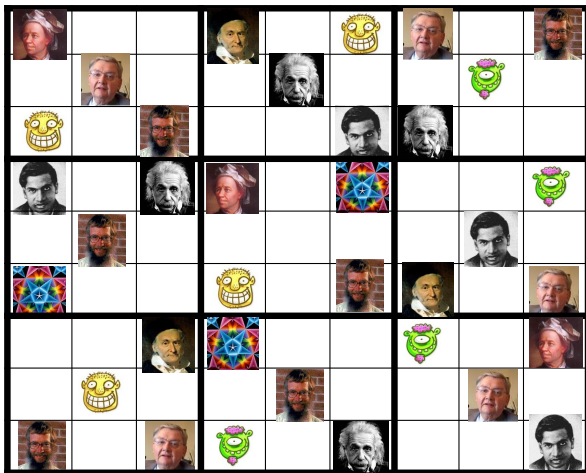
Fidel Nemenzo
Professor, Institute of Mathematics
University of the Philippines

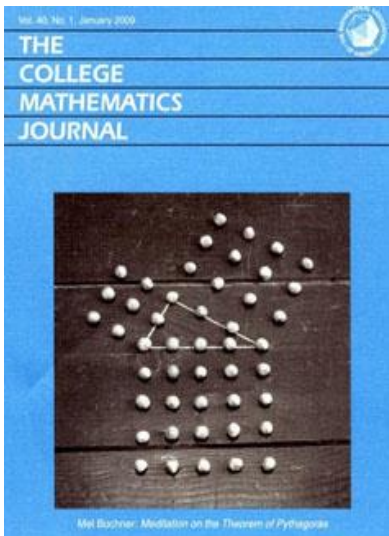


International Conference on Science and Mathematics Education in Developing Countries,
Mandalay, Myanmar 2013

Sudoku

math without numbers





from
**The College
Mathematics Journal,
(January 2009 issue)**

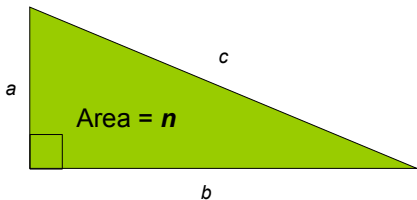
What is wrong
with the diagram?



Another old question, asked by Arab geometers
(Al-Karaji, c.953-c.1029)

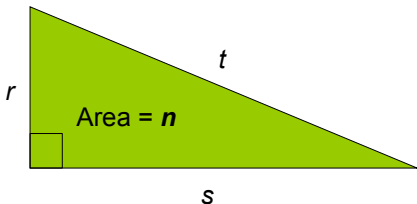
Let n be a positive integer.

Is there a rational
triangle with area = n ?



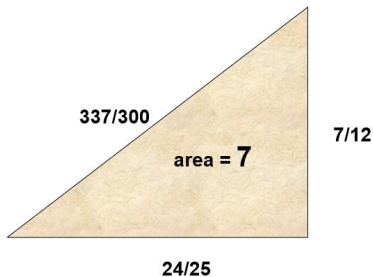
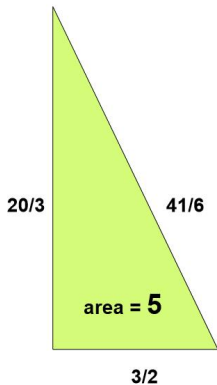
Another old question, asked by Arab geometers
(Al-Karaji, c.953-c.1029)

Let n be a positive integer.
Are there rational numbers
 r , s , and t such that $r^2 + s^2 = t^2$
and $rs=2n$?



Fibonacci (1225):

5 and 7 are congruent numbers.



Theorem (Fermat, 1650)

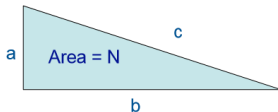
1 is **not** a congruent number.



Pierre de Fermat

Pierre de Fermat was a French lawyer who did mathematics as a hobby. He made important contributions to many areas of mathematics, including number theory, analytic geometry and probability theory.

Rational sides of congruent number triangles



N	Rational numbers a, b, c
5	$3/2, 20/3, 41/6$
6	$3, 4, 5$
7	$24/5, 35/12, 337/60$
13	$780/323, 323/30, 106921/9690$
14	$8/3, 63/6, 65/6$
15	$15/2, 4, 17/2$
20	$3, 40/3, 41/3$

All congruent numbers less than 40 000

MR1617754 (99e:11070) 11G05 (11D25 11Y40 11Y50)

Nemenzo, Fidel Ronquillo (J-SOPHE)

All congruent numbers less than 40000.

Proc. Japan Acad. Ser. A Math. Sci. **74** (1998), *no. 1*, 29–31.

The author applies computational techniques for bounding the rank of an elliptic curve to the well-known problem of determining whether or not n is a congruent number. A square-free positive integer n is a congruent number if it is the area of a right triangle with rational sides; equivalently, the elliptic curve $y^2 = x^3 - nx$ has positive rank. He analyzes each square-free n less than 100 000. Assuming the Birch and Swinnerton-Dyer conjecture and the accuracy of some numerical estimates for L -series, he finds 26 729 curves of rank 0, 30 220 of rank 1, 3656 of rank 2, 185 of rank 3, and 4 of rank 4. Using 2-descent and explicit searches for points, the question of whether the rank is positive or not is settled for most n , and in particular, this question is definitively settled for all $n < 42\,553$.

Reviewed by *Joe P. Buhler*

© Copyright American Mathematical Society 1999, 2011

A mathematical magic trick

Pick a one or two digit number and raise it to the fifth power
(using a calculator)

Tell me the large number you get as an answer

I can tell you the number you started with.

$$1^5 = 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 1$$

$$2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$$

$$3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243$$

$$4^5 = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 1\,024$$

$$5^5 = 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 3\,125$$

$$6^5 = 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 = 7\,776$$

$$7^5 = 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 = 16\,807$$

$$8^5 = 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 = 32\,768$$

$$9^5 = 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 = 59\,049$$

Mathematical Magic Tricks and why they work

$$10^5 = 100\,000$$

$$20^5 = 3\,200\,000$$

$$30^5 = 24\,300\,000$$

$$40^5 = 102\,400\,000$$

$$50^5 = 312\,500\,000$$

$$60^5 = 777\,600\,000$$

$$70^5 = 1\,680\,700\,000$$

$$80^5 = 3\,276\,800\,000$$

$$90^5 = 5\,904\,900\,000$$

BRUCE BUKIET *Mathematical Magic Tricks and why they work*

<https://web.njit.edu/~bukiet/Tricks.pdf>

The first digit

If the large number is 229 345 007 :

$$10^5 = 100\ 000$$

$$20^5 = 3\ 200\ 000$$

$$30^5 = 24\ 300\ 000$$

$$40^5 = 102\ 400\ 000$$

$$50^5 = 312\ 500\ 000$$

229 345 007

$$60^5 = 777\ 600\ 000$$

$$70^5 = 1\ 680\ 700\ 000$$

$$80^5 = 3\ 276\ 800\ 000$$

$$90^5 = 5\ 904\ 900\ 000$$

then the initial number is between 40 and 50, hence the first digit is 4.

The last digit

When a number is raised to the fifth power, the result has the original digit as its final digit.

$$1^5 = 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 1$$

$$2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$$

$$3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243$$

$$4^5 = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 1\,024$$

$$5^5 = 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 3\,125$$

$$6^5 = 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 = 7\,776$$

$$7^5 = 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 = 16\,807$$

$$8^5 = 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 = 32\,768$$

$$9^5 = 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 = 59\,049$$

Hence

$$229\,345\,007 = 47^5.$$

BRUCE BUKIET *Mathematical Magic Tricks and why they work*

<https://web.njit.edu/~bukiet/Tricks.pdf>

Fermat's little Theorem



Pierre de Fermat
1601(?)–1665

If p is a prime number and a a positive integer, then $a^p - a$ is a multiple of p .

For instance $a^5 - a$ is a multiple of 5, but also a multiple of 2, hence a multiple of 10.

<https://mathshistory.st-andrews.ac.uk/Biographies/Fermat/>

Fermat's little Theorem – Applications

Finite fields.

Coding theory

(this is the *Coding and Number Theory Academic Group*.)

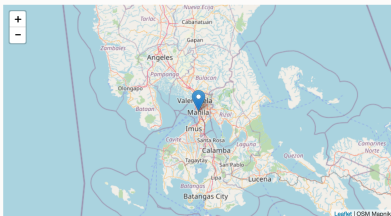
Cryptography.

Elliptic curves.

Diophantine geometry.

July 22 - August 2, 2013; CIMPA-ICTP-UNESCO-MESR-MINECO-PHILIPPINES research school:

Algebraic Curves over Finite Fields and Applications



Location

Manila, Philippines

Dates

22/07/2013 to 02/08/2013

Administrative and scientific coordinators

Fidel Nemenzo (University of the Philippines Diliman, Philippines, fidel@math.upd.edu.ph)
Michel Waldschmidt (Université Pierre et Marie Curie - Paris 6, France, miw@math.jussieu.fr)

Scientific committee

Fidel Nemenzo (University of the Philippines Diliman, Philippines)
Francesco Pappalardi (Università "Roma Tre", Italy)
Valerio Talamanca (Università "Roma Tre", Italy)
Jorge Jimenez Urroz (Universitat Politècnica de Catalunya, Spain)
Michel Waldschmidt (Université Pierre et Marie Curie - Paris 6, France)

Course 10: "Finite fields", Michel Waldschmidt (Université Pierre et Marie Curie, France)

UP Diliman 2013 CIMPA research school



Administrative and scientific coordinators

Fidel Nemenzo (University of the Philippines Diliman, Philippines, fidel@math.upd.edu.ph)

Michel Waldschmidt (Université Pierre et Marie Curie - Paris 6, France, miw@math.jussieu.fr)

SEAMS School 2017. Topics on elliptic curves



Institute of Mathematics of the University of the Philippines,
Diliman, Quezon City, Metro Manila.

July 17-25, 2017

My course :
Basic theory of elliptic curves.

CIMPA School 2022, Manila

Title:

Introduction to Galois representations and modular forms
and their computational aspects

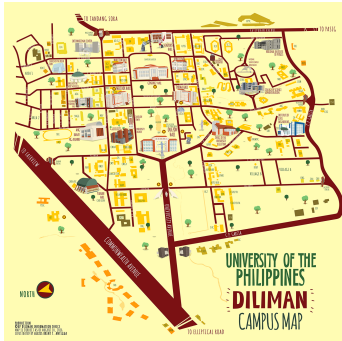
Dates:

January 10-21, 2022

Courses and Lecturers:

2. Elliptic curves with complex multiplication (I-II)
Michel Waldschmidt and Jerome T. Dimabayao

Diliman Campus





The author and Fidel Nemenzo: While Paul has been based in the US for years, "there is a bond that can't be broken between us."

CULTURE

SPOTLIGHT

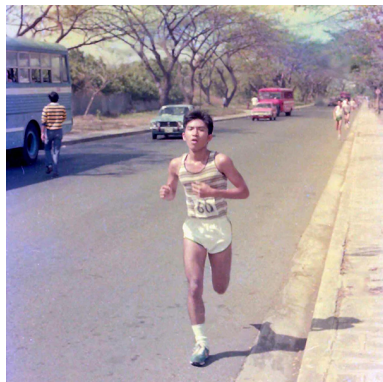
The Fidel Nemenzo I know: Growing up with the recently-appointed UP Chancellor

A friend since childhood recalls jam sessions, Math classes, and kite-flying trips with his fellow Child of Faculty, the recently appointed UP Chancellor – with the Diliman campus as their playground, their world, and universe.

<https://news.abs-cbn.com/ancx/culture/spotlight/02/07/20/the-fidel-nemenzo-i-know>

The Academic Oval

Fidel running along the Academic Oval during a race on campus in the late 70s.



10 000 m in 33'

800 m in 1'57

Two laps of the Academic
Oval in 15'

One lap (2,2 km) in less than
7'

February 19, 2021

University of the Philippines Diliman

College of Science, Institute of Mathematics

I-Math Webinar dedicated to the 60th birthday of Chancellor Fidel R. Nemenzo
(organized by the Coding and Number Theory Academic Group)

With Fidel, from Phnom Penh to Diliman: some personal reminiscences

Michel Waldschmidt

Professeur Émérite, Sorbonne Université,
Institut de Mathématiques de Jussieu, Paris

<http://www.imj-prg.fr/~michel.waldschmidt/>