Representation Theory. College of Science, University of Sulaimani, Sulaymaniyah. CIMPA West Asian Mathematical School.

Introduction to Galois Theory

Michel Waldschmidt

Professeur Émérite, Sorbonne Université,

Institut de Mathématiques de Jussieu, Paris

http://www.imj-prg.fr/~michel.waldschmidt/

Abstract

Field extensions. Degree of extension. Algebraic numbers. Geometric constructions with ruler and compasses. The Galois group of an extension. The Galois correspondence between subgroups and intermediate fields. Splitting field for a polynomial. Transitivity of the Galois group on the zeros of an irreducible polynomial in a normal extension. Properties equivalent to normality. Galois groups of normal separable extensions. Properties of Galois correspondence for normal separable extensions. Normal subgroups and normal intermediate extensions. The Fundamental Theorem of Galois Theory.

Fields : $\mathbb{C}$, $\overline{\mathbb{Q}}$, $\mathbb{R}$,

$\mathbb{Q}$

$\mathbb{F}_q$

Function fields

Differential fields

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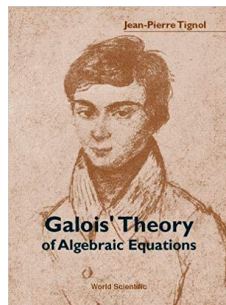
$\mathbb{Q}$

$\mathbb{F}_q$

Function fields

Differential fields

Galois theory of algebraic equations

Jean–Pierre Tignol

Quadratic equations

Simon Stevin
1548–1620

François Viète
1540–1603

http://www-history.mcs.st-and.ac.uk/Biographies/Stevin.html
http://www-history.mcs.st-and.ac.uk/Biographies/Viete.html

$x^2 - x = b$

Plimpton 322
1800BC

Emmanuel Peyre. Les Points Rationnels. Gazette SMF N° 159, janvier 2019, 13–22.

$x + y = a, xy = b$

Euclid of Alexandria

325 BC–265 BC

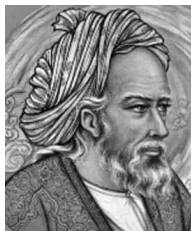$$x^2 + ax = b, \; x^2 + b = ax, \; x^2 = ax + b$$

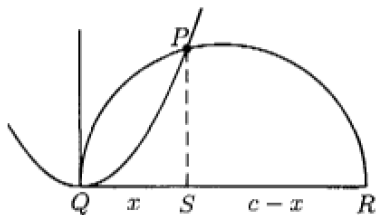

Abu Ja'far Muhammad ibn Musa Al-Khwarizmi

780–850

Hisab al-jabr w'al-muqabala

Geometric solutions to cubic equations

Omar Khayyam
1048–1131

$$x^3 + b^2x = b^2c$$

$x^2 = by$, the triangles QSP and PSR are similar,

$$\frac{x}{PS} = \frac{PS}{c-x}, \quad \frac{b}{x} = \frac{x}{y} = \frac{y}{c-x} == \frac{x^2}{b(c-x)}.$$

Leonardo da Pisa (Fibonacci)

Fibonacci

1170–1250

Liber Abaci (1202), Flos (1225)

In Flos, Fibonacci proves that the root of the equation $10x + 2x^2 + x^3 = 20$ (from Omar Khayyam's algebra book) is neither an integer nor a fraction, nor the square root of a fraction and gives the approximation $1.368\,808\,107\,5$, which is correct to nine decimal places.

"The solutions of $x^3 + mx = n$ and $x^3 + n = mx$ are as impossible as the quadrature of the circle"

Luca Pacioli

1445–1517

$x^3 + mx = n$ – unpublished

Scipione del Ferro

1465–1526

Nicolo Fontana alias Tartaglia

Nicolo Tartaglia

1500–1557

$$x^3 + mx = n$$

Girolamo Cardano

1501–1576

$x^3 + mx = n$, $t - u = n$, $tu = (m/3)^3$, $x = \sqrt[3]{t} - \sqrt[3]{u}$.

$x^3 = 15x + 4 : x = 4$. Introduction complex numbers.

$x^2 + 2x = 48 : $ 1 quad p :2 pos aeq 48

http://www-history.mcs.st-and.ac.uk/Biographies/Cardan.html

Algebra

Rafael Bombelli

1526–1572

Solution of quartic equations

Lodovico Ferrari
1522–1565

Published by Cardano : "Ars Magna".
Resolvent cubic equation.

http://www-history.mcs.st-and.ac.uk/Biographies/Ferrari.html

Negative numbers

Simon Stevin

1548–1620

http://www-history.mcs.st-and.ac.uk/Biographies/Stevin.html

Letters for unknown quantities (positive numbers)

François Viète

1540–1603

http://www-history.mcs.st-and.ac.uk/Biographies/Viete.html

La Géométrie, 1637

René Descartes

1596–1650

1567 : $X - a$ divides $P(X)$ if and only if $P(a) = 0$.

$X^4 + pX^2 + qX + r = (X^2 + aX + b)(X^2 + cX + d)$

$a^6 + 2pa^4 + (p^2 - 4r)a^2 - q^2 = 0$

1629 : invention nouvelle en algèbre

Albert Girard

1595–1632

The number of roots is the degree (including impossible solutions).

Relation between roots and coefficients.

Sir Isaac Newton

1643–1727

$\sum {\alpha_i}^k$.

Numerical methods.

Tschirnhaus

Ehrenfried Walter von Tschirnhaus
1651–1708

$$X^n + a_{n-1}X^{n-1} + \cdots + a_1 X + a_0$$
$$Y = X^m + b_{m-1}X^{m-1} + \cdots + b_1 X + b_0$$

http://www-history.mcs.st-and.ac.uk/Biographies/Tschirnhaus.html

How to solve Tschirnhaus system ?

Gottfried Wilhelm von Leibniz

1646–1716

Réflexions sur la résolution algébrique des équations : 1772

Joseph-Louis Lagrange
1736–1813

Permutations of the roots. Number of permutations leaving invariant the polynomial. Lagrange theorem on the order of a subgroup. Lagrange resolvent.

The fondamental theorem of algebra

Albert Girard

1595–1632

1629 : invention nouvelle en algèbre

http://www-history.mcs.st-and.ac.uk/Biographies/Girard_Albert.html

The fondamental theorem of algebra : 1746

Jean Le Rond d'Alembert
1717–1813

Analytic proof

http://www-history.mcs.st-and.ac.uk/Biographies/DAlembert.html

The fondamental theorem of algebra : 1749

Leonhard Euler

1707–1783

Any irreducible polynomial over $\mathbb{R}$ has degree 1 or 2.

Réflexions sur la résolution algébrique des équations : 1772

Joseph-Louis Lagrange

1736–1813

Assumes there are d "imaginary" roots to a polynomial of degree d, proves that they are in $\mathbb{C}$.

The fondamental theorem of algebra : 1799, 1815

Johann Carl Friedrich Gauss

1777-1855

Corrects the proofs of d'Alembert and Lagrange, later produces two other proofs.

http://www-history.mcs.st-and.ac.uk/Biographies/Gauss.html

Mémoire sur la résolution des équations 1774

Alexandre-Théophile Vandermonde
1735–1796

Abbé Henri Grégoire
1750–1831

Determinant
Cyclotomic polynomials

http://www-history.mcs.st-and.ac.uk/Biographies/Vandermonde.html
https://fr.wikipedia.org/wiki/Henri_Gregoire

Cyclotomic polynomials 1799

Johann Carl Friedrich Gauss

1777-1855

1837

Pierre Laurent Wantzel

1814–1848

Fermat primes

Pierre de Fermat

1601 ?–1665

$F_n = 2^{2^n} + 1$:
$F_0 = 3,\ F_1 = 5,\ F_2 = 17,\ F_3 = 257,\ F_4 = 65\,537$

$F_5 = 2^{32} + 1 \equiv 0 \pmod{641}$.

http://www-history.mcs.st-and.ac.uk/Biographies/Fermat.html

1799 : general equation of degree 5

Paolo Ruffini

1765–1822

516 pages - non solvability of the general equation of degree 5.
Gauss : "It appears more and more likely that this resolution is
impossible".

1824 : non solvability of equations of degree 5

Niels Henrik Abel

1802–1829

http://www-history.mcs.st-and.ac.uk/Biographies/Abel.html

Évariste Galois

1811–1832

Necessary and sufficient condition for an equation to be solvable by radicals.

http://www-history.mcs.st-and.ac.uk/Biographies/Galois.html

Referee of Galois's note

Jean Baptiste Joseph Fourier

1768–1830

1846 : publication of Galois's work

Joseph Liouville

1809–1882

Permutations which preserves the relations among the roots.
Behavior under the extension of the base field.

http://www-history.mcs.st-and.ac.uk/Biographies/Liouville.html

Field theory : constructivist
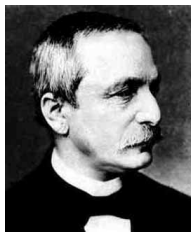
Leopold Kronecker

1823–1891

Field theory : axiomatic

Julius Wilhelm Richard Dedekind

1831–1916

http://www-history.mcs.st-and.ac.uk/Biographies/Dedekind.html

Galois correspondence - published 1942
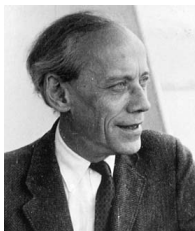
Emil Artin

1898–1962

http://www-history.mcs.st-and.ac.uk/Biographies/Artin.html

Moderne Algebra 1930
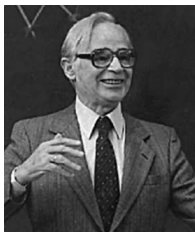
Bartel Leendert van der Waerden

1903–1996

Field theory

Field extensions K/k.

Algebraic elements, transcendental elements.

Algebraic extensions , finite extensions , degree of an extension $[K : k]$. Number field.
A finite extension is algebraic.
The set of algebraic numbers (over $\mathbb{Q}$) is a field : $\overline{\mathbb{Q}}$.

Stem field of a polynomial. Splitting field for a polynomial.

Conjugates, normal extensions

Conjugate of an element. Unicity of the stem field up to isomorphism.

Normal extensions. Properties equivalent to normality : finite normal extension = splitting field of a polynomial.

Separable irreducible polynomial. Separable polynomial. Separable algebraic extension.

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Galois extensions

The subgroup $G(K/k)$ of $\mathrm{Aut}(K)$. Fixed field K^H of a subgroup H of $G(K/k)$.

Galois extension = finite, normal, separable.

Transitivity of the Galois group on the zeros of an irreducible polynomial in a normal extension.

Proposition. *If K/k is Galois, then $G(K/k)$ is a finite group of order $[K : k]$ and k is the fixed field of $G(K/k)$.*

Theorem. *If H is a finite subgroup of $\mathrm{Aut}(K)$ and $k = K^H$ the fixed field of H, then K/k is Galois and $H = G(K/k)$.*

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The Fundamental Theorem of Galois Theory

The Galois correspondence between subgroups and intermediate fields.

K/k a Galois extension, $G = G(K/k)$

$$
\begin{aligned}
S(K/k) &= \{\text{subfields } E \text{ of } K \text{ containing } k\} & k \subset E \subset K \\
S(G) &= \{\text{subgroups } H \text{ of } G\} & H \subset G
\end{aligned}
$$

$$
\begin{aligned}
S(K/k) &\rightarrow S(G) & S(G) &\rightarrow S(K/k) \\
E &\mapsto G(K/E) & H &\mapsto K^H
\end{aligned}
$$

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$$
\begin{array}{ccc}
S(K/k) & \to & S(G) \\
E & \mapsto & G(K/E)
\end{array}
\qquad\qquad
\begin{array}{ccc}
S(G) & \to & S(K/k) \\
H & \mapsto & K^H
\end{array}
$$

Galois correspondence

$H \in S(G) \Rightarrow$
$K^H \in S(K/k)$

$E \in S(K/k) \Rightarrow$
$G(K/E) \in S(G)$

$$G \left\{ \begin{array}{c} K \\ | \\ K^H \\ | \\ k \end{array} \right. \quad H$$

$$G \left\{ \begin{array}{c} K \\ | \\ E \\ | \\ k \end{array} \right. \quad G(K/E)$$

Normal subgroups and normal intermediate extensions

$$G \begin{cases} K \\ \;\;| \quad\quad H \\ K^H \\ \;\;| \quad\quad G/H \\ k \end{cases}$$

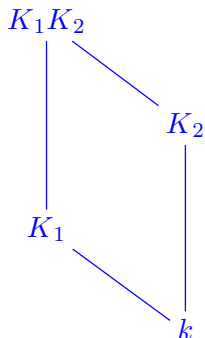H is normal in G if and only if K^H/k is a Galois extension. In this case, $G(K^H/k) = G/H$.

Compositum

Assume K_1/k is Galois. Then K_1K_2/K_2 is Galois and

$$G(K_1K_2/K_2) \subset G(K_1/k)$$

hence

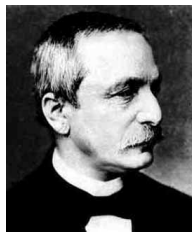$[K_1K_2 : K_2]$ divides $[K_1 : k]$.

K_1K_2

K_2

K_1

k

Remark : $k = \mathbb{Q}$, $K_1 = \mathbb{Q}(\sqrt[3]{2})$, $K_2 = \mathbb{Q}(j\sqrt[3]{2})$, $[K_1 : \mathbb{Q}] = 3$, $[K_1K_2 : K_2] = 2$.

Cyclotomic fields. Abelian extensions of $\mathbb{Q}$

Let μ_n be the cyclic group of n–th roos of unity in $\mathbb{C}$. The cyclotomic field $\mathbb{Q}(\mu_n)$ is a Galois extension of $\mathbb{Q}$ with Galois group $(\mathbb{Z}/n\mathbb{Z})^{\times}$.

Kronecker–Weber Theorem. *Every finite abelian extension of $\mathbb{Q}$ is contained in some cyclotomic field.*
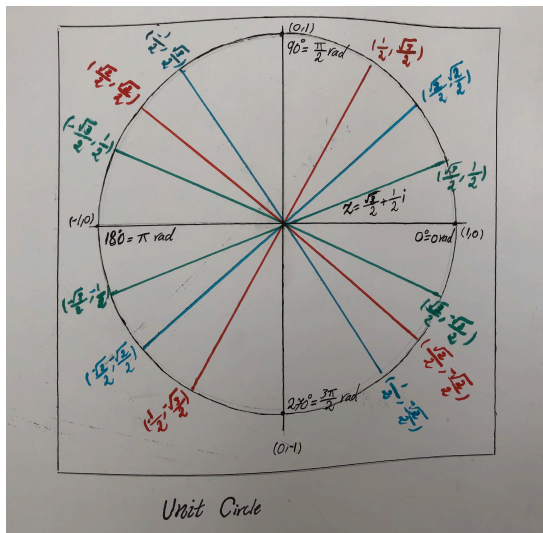

Leopold Kronecker
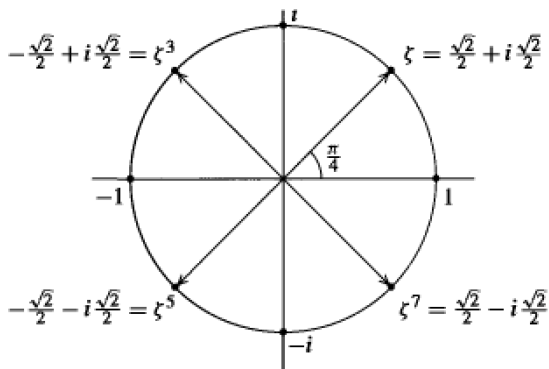1823–1891

Heinrich Weber
1842–1913

Sixteenth roots of unity : $\Phi_{16}(X) = X^8 + 1$, $\zeta^{16} = 1$

Unit Circle

Eighth roots of unity : $\Phi_8(X) = X^4 + 1$, $\zeta^8 = 1$

$[\mathbb{Q}(\zeta_8) : \mathbb{Q}] = \varphi(8) = 4$, $G(\mathbb{Q}(\zeta_8)/\mathbb{Q}) = (\mathbb{Z}/8\mathbb{Z})^\times \simeq (\mathbb{Z}/2\mathbb{Z}) \times (\mathbb{Z}/2\mathbb{Z})$.

David S. Dummit & Richard M. Foote, Abstract Algebra, Prentice Hall, 1991 2nd Ed. 1999.

Splitting field of $X^8 - 2$ and subfields

$\theta = \sqrt[8]{2}$

David S. Dummit & Richard M. Foote, Abstract Algebra, Prentice Hall, 1991
2nd Ed. 1999.
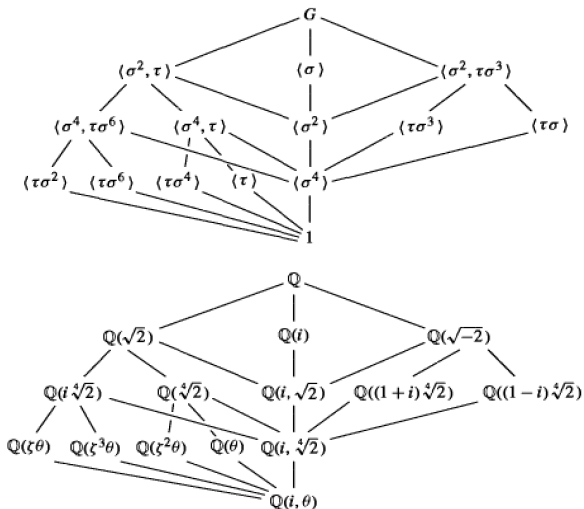
Galois theory of finite fields

If F is a finite extension of $\mathbb{F}_q$, then $F/\mathbb{F}_q$ is a Galois extension and $G(F/\mathbb{F}_q)$ is a cyclic group generated by the Frobenius $\mathrm{Frob}_q : x \to x^q$.

Ferdinand Georg Frobenius

1849–1917

http://www-history.mcs.st-and.ac.uk/Biographies/Frobenius.html

Solvability of algebraic equations by radicals

Extension solvable by radicals.

In characteristic $\neq 2$ and $\neq 3$, an extension of degree ≤ 4 is solvable by radicals.

Existence of extensions of degree p with Galois group $\mathfrak{S}_p$.

Example : $X^5 + 2X^3 - 24X - 2$

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Geometric constructions with ruler and compasses

Stable subsets of $\mathbb{R} \times \mathbb{R}$. Stable closure $S(E)$ of a subset E of $\mathbb{R} \times \mathbb{R}$. Stable closure $S(k)$ of a subfield k of $\mathbb{R}$.

Let k be a subfield of $\mathbb{R}$.
An element $x \in \mathbb{R}$ is in $C(k)$ if and only if there exists a Galois extension K/k such that $[K : k] = 2^m$ and $x \in K$.

- Trisection of an angle

- Squaring the circle

- Doubling the cube

- Construction of regular polygons

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https://math24.files.wordpress.com/2013/02/algebra-serge-lang.pdf

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Representation Theory. College of Science, University of Sulaimani, Sulaymaniyah. CIMPA West Asian Mathematical School.

Introduction to Galois Theory

Michel Waldschmidt

Professeur Émérite, Sorbonne Université,

Institut de Mathématiques de Jussieu, Paris

http://www.imj-prg.fr/~michel.waldschmidt/