



KPR Institute of
Engineering and
Technology

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Avinashi Road, Arasur, Coimbatore.



Srinivasa Ramanujan

His life and his work

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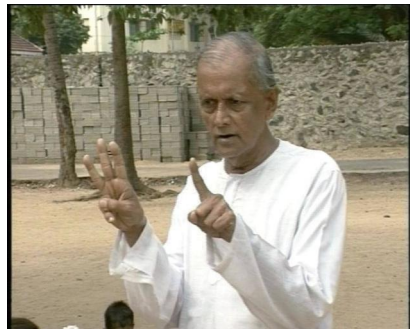
Coimbatore, December 16, 2020.

Abstract

This lecture includes a few biographical informations about [Srinivasan Ramanujan](#). Among the topics which we discuss are [Euler](#) constant, nested roots, divergent series, [Ramanujan – Nagell](#) equation, partitions, [Ramanujan](#) tau function, [Hardy Littlewood](#) and the circle method, highly composite numbers and transcendence theory, the number π , and the lost notebook.

P.K. Srinivasan

(November 4, 1924–June 20, 2005)



PKS was the first biographer of Srinivas Ramanujan.

The Hindu, November 1, 2009
Passion for numbers by
Soudhamini

<http://beta.thehindu.com/education/article41732.ece>

Biography of Srinivasa Ramanujan

(December 22, 1887 — April 26, 1920)

1887: born in Erode (near Tanjore)

1894-1903: school in Kumbakonam

In 1900 he began to work on his own on mathematics summing geometric and arithmetic series.

Gopuram Sarangapani Kumbakonam



Sarangapani Sannidhi Street Kumbakonam



Ramanujan House Kumbakonam



Ramanujan House in Kumbakonam



Ramanujan House Kumbakonam



Town High School Kumbakonam



Town High School Kumbakonam

1903: G.S.Carr - *A synopsis of elementary results — a book on pure mathematics* (1886) 5 000 formulae

$$\sqrt{x} + y = 7, \quad x + \sqrt{y} = 11$$

$$x = 9, \quad y = 4.$$

Biography (*continued*)

1903 (December): exam at Madras University

1904 (January): enters Government Arts College,
Kumbakonam

Sri K. Ranganatha Rao Prize

Subrahmanyam scholarship

MacTutor History of Mathematics

<http://www-history.mcs.st-andrews.ac.uk/>

By 1904 **Ramanujan** had begun to undertake deep research. He investigated the series

$$\sum_n \frac{1}{n}$$

and calculated **Euler's** constant to **15** decimal places.

He began to study the **Bernoulli** numbers, although this was entirely his own independent discovery.

Euler constant

$$S_N = \sum_{n=1}^N \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{N}$$

$$\int_1^N \frac{dx}{x+1} < S_N < 1 + \int_1^N \frac{dx}{x}$$

$$\gamma = \lim_{N \rightarrow \infty} (S_N - \log N).$$

Reference



JEFFREY C. LAGARIAS

*Euler's constant: Euler's work
and modern developments*

Bulletin Amer. Math. Soc.

50 (2013), No. 4, 527–628.

arXiv:1303.1856 [math.NT]

Bibliography: 314 references.

M.W. Is the Euler constant a rational number, an algebraic irrational number or else a transcendental number?

<http://www.imj-prg.fr/~michel.waldschmidt/articles/pdf/EulerConstantVI.pdf>

Euler archives and Eneström index

The Euler Archive

A digital library dedicated to the work and life of Leonhard Euler



<http://eulerarchive.maa.org/>

Gustaf Eneström (1852–1923)

*Die Schriften Euler's
chronologisch nach den Jahren
geordnet, in denen sie verfasst
worden sind*

Jahresbericht der Deutschen
Mathematiker-Vereinigung,
1913.



Gustaf Eneström.
Efter fotografi.

<http://www.math.dartmouth.edu/~euler/index/enestrom.html>

<http://www.eulerarchive.org/>



The screenshot shows the Euler Archive website. At the top, the word "eulerarchive" is written in a large, red, italicized font. Below this is a navigation bar with four tabs: "Home", "About", "News", and "Race". The main content area features a large image of a yellow Formula 1 car. Overlaid on the left side of the car is a text box with Thai text. The text discusses the car's performance and mentions a "VIEW MORE" button.

FI: รถใหม่ ของเฟอร์รารีที่เกือบจะ สมบูรณ์

แม้สำหรับ 2,014 คัน สามารถ กลัง อย่างเป็นทางการ ในการติดตาม
เพื่อ ไม่ก็ วิก่อนที่จะเปิดตัวอย่างเป็นการ ที่ เฟอร์รารี แต่ มี
พรามิเตอร์ ซ้อมุด ค่อนข้างสมบูรณ์ ในการให้บริการของรถ ที่สมบูรณ์
ตามที่ บางคนวิจารณ์ เฟอร์รารี ใหม่ช่วงล่างด้านหน้า จะไม่ใช้ระบบ
แ่ง (fullrod) เป็นรุ่น 2012-2013 กลับเป็น 2011 มีระบบ ก้าน กระหัง
คัน

[VIEW MORE](#)

(Reference [86] of the text by Lagarias)

<http://www.eulerarchive.org/>

MAHA168

Judi Online, Slot Online, Casino Online

DAFTAR LIVECHAT WHATSAPP

dan judi poker online hingga tembak ikan.

Kepopuleran Maha168 membawa setidaknya 25 ribu pemain slot dan casino setiap harinya berkunjung ke situs kami. Kepuasan atas pelayanan dari customer service kami dan minimal deposit yang terjangkau bagi pemain menjadi alasan utama member bergabung bersama kami. Kami memproses setidaknya 5000 ribu transaksi deposit dan withdraw setiap harinya. Rata - rata total penarikan harian di Maha168 mencapai lebih dari 1 miliar. Jumlahnya yang besar untuk membayar kemenangan member hingga membuat Maha168 layak disebut sebagai judi online terpercaya Indonesia. Setiap hari ada sedikitnya 550 member baru yang mendaftar di tempat kami. Member yang mendaftar karena mendapat referensi dari teman juga tidak sedikit. Terbukti member Maha168 dengan referral terbesar memiliki downline lebih dari 500 member.

Maha168 Situs Judi Online Terpercaya di Indonesia

1	TOTAL MEMBER AKTIF : 18.000 MEMBER / BULAN
2	TOTAL TRANSAKSI : 7.200 TRANSAKSI / HARI
3	TOTAL WITHDRAWAL : IDR 1.200.000.000 / HARI
4	RATA - RATA PENDAFTARAN MEMBER BARU : 150 MEMBER / HARI
5	RATA - RATA KEMUNGKINAN TERSEKUT MEMBER : IDR 225.000.000,-
6	GAME DENGAN JUMLAH PENKIAN TERBANYAK : SLOT GAMES
7	MEMBER DENGAN REFERRAL TERBANYAK : 400+ (401 MEMBER)

Menjadi situs judi online Indonesia yang terpercaya merupakan kebanggaan Maha168 selama beroperasi selama hampir 7 tahun. Setiap bulannya ada ratusan member kami yang menjadi miliarder baru lahir dari Maha168. Kemenangan terbesar yang pernah dibayar oleh kami adalah sebesar 10 miliar, ketika salah satu pemain Maha168 mendapatkan jackpot progresive di judi slot online Maha168. Tak heran brand kami menjadi begitu populer dan terkenal Indonesia hingga Asia sebagai situs taruhan online. Terbukti menurut data dari Google, setiap bulannya terdapat lebih dari 100 ribu pencarian dengan kata kunci "Maha168". Selain itu brand kami juga sangat mudah ditemukan dimana saja seperti media social Facebook, Line hingga di berbagai kata kunci di Google seperti casino online Maha168.

(Reference [86] of the text by Lagarias)

Harmonic numbers

$$H_1 = 1, \quad H_2 = 1 + \frac{1}{2} = \frac{3}{2}, \quad H_3 = 1 + \frac{1}{2} + \frac{1}{3} = \frac{11}{6},$$

$$H_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} = \sum_{j=1}^n \frac{1}{j}.$$

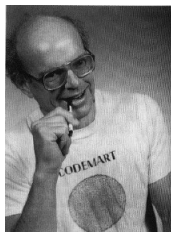
Sequence:

$$1, \quad \frac{3}{2}, \quad \frac{11}{6}, \quad \frac{25}{12}, \quad \frac{137}{60}, \quad \frac{49}{20}, \quad \frac{363}{140}, \quad \frac{761}{280}, \quad \frac{7129}{2520}, \dots$$

The online encyclopaedia of integer sequences

<https://oeis.org/>

Neil J. A. Sloane



Numerators and denominators

Numerators: <https://oeis.org/A001008>

1, 3, 11, 25, 137, 49, 363, 761, 7129, 7381, 83711, 86021, 1145993,

1171733, 1195757, 2436559, 42142223, 14274301, 275295799,

55835135, 18858053, 19093197, 444316699, 1347822955, ...

Denominators: <https://oeis.org/A002805>

1, 2, 6, 12, 60, 20, 140, 280, 2520, 2520, 27720, 27720, 360360,

360360, 360360, 720720, 12252240, 4084080, 77597520,

15519504, 5173168, 5173168, 118982864, 356948592, ...

Euler (1731)

De progressionibus harmonicis observationes

The sequence

$$H_n - \log n$$

has a limit $\gamma = 0,577\,218\dots$
when n tends to infinity.

Leonhard Euler
(1707–1783)



Moreover,

$$\gamma = \sum_{m=2}^{\infty} (-1)^m \frac{\zeta(m)}{m}.$$

Riemann zeta function



$$\begin{aligned}\zeta(s) &= \sum_{n \geq 1} \frac{1}{n^s} \\ &= \prod_p \frac{1}{1 - p^{-s}}\end{aligned}$$



Euler: $s \in \mathbb{R}$.

Riemann: $s \in \mathbb{C}$.

Numerical value of Euler's constant

The online encyclopaedia of integer sequences

<https://oeis.org/A001620>

Decimal expansion of Euler's constant
(or Euler–Mascheroni constant) gamma.

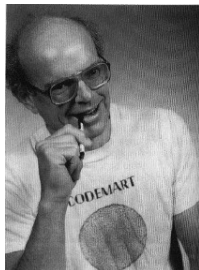
Yee (2010) computed 29 844 489 545 decimal digits of gamma.

$\gamma = 0,577\,215\,664\,901\,532\,860\,606\,512\,090\,082\,402\,431\,042\,\dots$

Euler constant

Euler–Mascheroni constant

$$\gamma = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} - \log n \right) = 0.577\,215\,664\,9\dots$$



Neil J. A. Sloane's encyclopaedia

<http://www.research.att.com/~njas/sequences/A001620>

Bernoulli numbers



$$B_0 = 1, \quad \sum_{k=0}^{n-1} \binom{n}{k} B_k = 0 \quad \text{for } n > 1.$$

Jacob Bernoulli (1654 – 1705)

$$B_0 + 2B_1 = 0$$

$$B_1 = -\frac{1}{2}$$

$$B_0 + 3B_1 + 3B_2 = 0$$

$$B_2 = \frac{1}{6}$$

$$B_0 + 4B_1 + 6B_2 + 4B_3 = 0$$

$$B_3 = 0$$

$$B_0 + 5B_1 + 10B_2 + 10B_3 + 5B_4 = 0$$

$$B_4 = -\frac{1}{30}$$

⋮

Kumbakonam

1905: Fails final exam

1906: Enters Pachaiyappa's College, Madras
III, goes back to Kumbakonam

1907 (December): Fails final exam.

1908: [continued fractions](#) and divergent series

1909 (April): underwent an operation

1909 (July 14): marriage with [S Janaki Ammal](#) (1900—1994)

S Janaki Ammal



Madras

1910: meets [Ramaswami Aiyar](#)

1911: first mathematical paper

1912: clerk office, Madras Port Trust — [Sir Francis Spring](#) and [Sir Gilbert Walker](#) get a scholarship for him from the University of Madras starting May 1913 for 2 years.

Nested roots

Journal of the Indian Mathematical Society (1912) – problems solved by Ramanujan

$$\sqrt{1 + 2\sqrt{1 + 3\sqrt{1 + 4\sqrt{1 + \dots}}}} = ?$$

$$\sqrt{6 + 2\sqrt{7 + 3\sqrt{8 + 4\sqrt{9 + \dots}}}} = ?$$

Answers from Ramanujan

$$\sqrt{1 + 2\sqrt{1 + 3\sqrt{1 + 4\sqrt{1 + \cdots}}}} = 3$$

$$\sqrt{6 + 2\sqrt{7 + 3\sqrt{8 + 4\sqrt{9 + \cdots}}}} = 4$$

“Proofs” $n(n + 2)$

$$(n + 2)^2 = 1 + (n + 1)(n + 3)$$

$$n(n + 2) = n\sqrt{1 + (n + 1)(n + 3)}$$

$$f(n) = n(n + 2)$$

$$f(n) = n\sqrt{1 + f(n + 1)}$$

$$f(n) = n\sqrt{1 + (n + 1)\sqrt{1 + f(n + 2)}}$$

“Proofs” $n(n + 3)$

$$(n + 3)^2 = n + 5 + (n + 1)(n + 4)$$

$$n(n + 3) = n\sqrt{n + 5 + (n + 1)(n + 4)}$$

$$g(n) = n(n + 3)$$

$$g(n) = n\sqrt{n + 5 + g(n + 1)}$$

$$g(n) = n\sqrt{n + 5 + (n + 1)\sqrt{n + 6 + g(n + 2)}}$$

Letter of S. Ramanujan to M.J.M. Hill in 1912

$$1 + 2 + 3 + \cdots + \infty = -\frac{1}{12}$$

$$1^2 + 2^2 + 3^2 + \cdots + \infty^2 = 0$$

$$1^3 + 2^3 + 3^3 + \cdots + \infty^3 = \frac{1}{120}$$



Answer of M.J.M. Hill in 1912

$$1 + 2 + 3 + \cdots + n = \frac{1}{2}n(n + 1)$$

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(2n + 1)(n + 1)}{6}$$

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \left(\frac{n(n + 1)}{2} \right)^2$$

Renormalisation of divergent series



Leonhard Euler

(1707 – 1783)

Introductio in analysin infinitorum

(1748)

Euler

Values of Riemann zeta function at negative integers :

$$\zeta(-k) = -\frac{B_{k+1}}{k+1} \quad (n \geq 1)$$

$$\zeta(-2n) = 1^{2n} + 2^{2n} + 3^{2n} + 4^{2n} + \dots = 0 \quad (n \geq 1)$$

$$\zeta(-1) = 1 + 2 + 3 + 4 + \dots = -\frac{1}{12}$$

$$\zeta(-3) = 1^3 + 2^3 + 3^3 + 4^3 + \dots = \frac{1}{120}$$

$$\zeta(-5) = 1^5 + 2^5 + 3^5 + 4^5 + \dots = -\frac{1}{252}$$

G.H. Hardy: Divergent Series (1949)



Niels Henrik Abel
(1802 – 1829)

Divergent series are the invention of the devil, and it is shameful to base on them any demonstration whatsoever.

Letters to H.F. Baker and E.W. Hobson in 1912

No answer to his letters to H.F. Baker and E.W. Hobson in 1912...

Letter of Ramanujan to Hardy

(January 16, 1913)

I have had no university education but I have undergone the ordinary school course. After leaving school I have been employing the spare time at my disposal to work at mathematics. I have not trodden through the conventional regular course which is followed in a university course, but I am striking out a new path for myself. I have made a special investigation of divergent series in general and the results I get are termed by the local mathematicians as “startling”.

Godfrey Harold Hardy (1877 – 1947)



John Edensor Littlewood (1885 – 1977)



Hardy and Littlewood



Letter from Ramanujan to Hardy

(January 16, 1913)

$$1 - 2 + 3 - 4 + \dots = \frac{1}{4}$$

$$1 - 1! + 2! - 3! + \dots = .596\dots$$

Answer from Hardy

(February 8, 1913)

I was exceedingly interested by your letter and by the theorems which you state. You will however understand that, before I can judge properly of the value of what you have done, it is essential that I should see proofs of some of your assertions. Your results seem to me to fall into roughly three classes:

(1) there are a number of results that are already known, or easily deducible from known theorems;

(2) there are results which, so far as I know, are new and interesting, but interesting rather from their curiosity and apparent difficulty than their importance;

(3) there are results which appear to be new and important. . .

1913–1920

1913, February 27: New letter from Ramanujan to Hardy

1913: Visit of Neville to India

1914, March 17 to April 14: travel to Cambridge.

1918: (May) Fellow of the Royal Society
(November) Fellow of Trinity College, Cambridge.

1919, February 27 to March 13: travel back to India.

An interesting street number

The puzzle itself was about a street in the town of Louvain in Belgium, where houses are numbered consecutively. One of the house numbers had the peculiar property that the total of the numbers lower than it was exactly equal to the total of the numbers above it. Furthermore, the mysterious house number was greater than 50 but less than 500.



Prasanta Chandra Mahalanobis
1893 – 1972



Srinivasa Ramanujan
1887 – 1920

Balancing numbers

Answer: house number 204 in a street with 288 houses.

Sequence of balancing numbers (number of the house)

<https://oeis.org/A001109>

0, 1, 6, 35, 204, 1189, 6930, 40391, 235416, 1372105, 7997214...

This is a linear recurrence sequence

$$u_{n+1} = 6u_n - u_{n-1}$$

with the initial conditions $u_0 = 0$, $u_1 = 1$.

The number of houses is <https://oeis.org/A001108>

0, 1, 8, 49, 288, 1681, 9800, 57121, 332928, 1940449, ...

M.W. *The square root of 2, the Golden ratio and the Fibonacci sequence.* <http://www.imj-prg.fr/~michel.waldschmidt/articles/pdf/sqrt2.pdf>

YouTube: <https://www.youtube.com/watch?v=CIQ0w3SRtE&t=247s> 

Ramanujan – Taxi Cab Number 1729

Hardy's obituary of Ramanujan:

I had ridden in taxi-cab No 1729, and remarked that the number ($7 \cdot 13 \cdot 19$) seemed to me a rather dull one. . .

$$1729 = 1^3 + 12^3 = 9^3 + 10^3$$

$$12^3 = 1728, \quad 9^3 = 729$$

Narendra Jadhav — Taxi Cab Number 1729

Narendra Jadhav (born 1953), Member of Rajya Sabha, the upper house of Indian Parliament.



Former Vice-Chancellor (from 24 August 2006 to 15 June 2009) of Savitribai Phule Pune University.

Author of *Outcaste – A Memoir, Life and Triumphs of an Untouchable Family In India* (2003).

<http://www.drnarendrajadhav.info>

Ramanujan – Taxi Cab Number 1729

$$12^3 = 1728, \quad 9^3 = 729$$

$$50 = 7^2 + 1^2 = 5^2 + 5^2$$

$$\begin{aligned} 4104 &= 2^3 + 16^3 = 9^3 + 15^3 \\ 13832 &= 2^3 + 24^3 = 18^3 + 20^3 \\ 40033 &= 9^3 + 34^3 = 16^3 + 33^3 \\ &\vdots \end{aligned}$$

Leonhard Euler (1707 – 1783)



$$59^4 + 158^4 = 133^4 + 134^4 = 635\,318\,657$$

Diophantine equations

$$x^3 + y^3 + z^3 = w^3$$

$$(x, y, z, w) = (3, 4, 5, 6)$$

$$3^3 + 4^3 + 5^3 = 27 + 64 + 125 = 216 = 6^3$$

Parametric solution:

$$x = 3a^2 + 5ab - 5b^2$$

$$z = 5a^2 - 5ab - 3b^2$$

$$y = 4a^2 - 4ab + 6b^2$$

$$w = 6a^2 - 4ab + 4b^2$$

Ramanujan – Nagell Equation

Trygve Nagell (1895 – 1988)

$$x^2 + 7 = 2^n$$

$$\begin{aligned} 1^2 + 7 &= 2^3 = 8 \\ 3^2 + 7 &= 2^4 = 16 \\ 5^2 + 7 &= 2^5 = 32 \\ 11^2 + 7 &= 2^7 = 128 \\ 181^2 + 7 &= 2^{15} = 32\,768 \end{aligned}$$

$$x^2 + D = 2^n$$

Nagell (1948): for $D = 7$, no further solution

R. Apéry (1960): for $D > 0$, $D \neq 7$, the equation $x^2 + D = 2^n$ has at most 2 solutions.

Examples with 2 solutions:

$$D = 23 : \quad 3^2 + 23 = 32, \quad 45^2 + 23 = 2^{11} = 2048$$

$$D = 2^{\ell+1} - 1, \ell \geq 3: \quad (2^\ell - 1)^2 + 2^{\ell+1} - 1 = 2^{2\ell}$$

$$x^2 + D = 2^n$$

F. Beukers (1980): at most one solution otherwise.

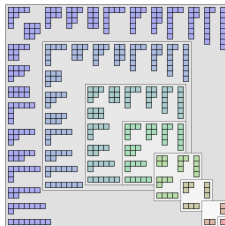


M. Bennett (1995): considers the case $D < 0$.

Partitions

$$\begin{aligned}1 & \\2 & = 1 + 1 \\3 & = 2 + 1 = 1 + 1 + 1 \\4 & = 3 + 1 = 2 + 2 = 2 + 1 + 1 \\ & = 1 + 1 + 1 + 1\end{aligned}$$

$$\begin{aligned}p(1) &= 1 \\p(2) &= 2 \\p(3) &= 3 \\p(4) &= 5\end{aligned}$$



$$p(5) = 7, \quad p(6) = 11, \quad p(7) = 15, \dots$$

MacMahon: table of the first 200 values

Neil J. A. Sloane's encyclopaedia

<http://www.research.att.com/~njas/sequences/A000041>

Ramanujan

$p(5n + 4)$ is a multiple of 5

$p(7n + 5)$ is a multiple of 7

$p(11n + 6)$ is a multiple of 11

$p(25n + 24)$ is a multiple of 25

$p(49n + 47)$ is a multiple of 49

$p(121n + 116)$ is a multiple of 121

Ramanujan conjecture

If $m = 5^a 7^b 11^c$ and $24\ell - 1$ is a multiple of m , then $p(nm + \ell)$ is a multiple of m for all n .

Hansraj Gupta : $p(243) = 133\,978\,259\,344\,888$ is not a multiple of 7^3 .

S. Chowla : As $24 \cdot 243 - 1$ is divisible by 7^3 , the above conjecture is false.

S. Chowla, . "Congruence Properties of Partitions." J. London Math. Soc. **9**, 247, 1934.

<https://doi.org/10.1112/jlms/s1-9.4.247a>



Sarvadaman Chowla
(1907–1995)

CONGRUENCE PROPERTIES OF PARTITIONS

S. CHOWLA*.

Let $p(n)$ denote the number of unrestricted partitions of n . Ramanujan conjectured† that

If $\delta = 5^a 7^b 11^c$ and $24\lambda \equiv 1 \pmod{\delta}$, then

$$p(\lambda), p(\lambda + \delta), p(\lambda + 2\delta), \dots \equiv 0 \pmod{\delta}.$$

This result has been proved in a number of special cases, e.g. when‡

$$\delta = 5, 7, 11, 5^2, 7^2, 11^2, 5^3.$$

A table of partitions recently calculated by H. Gupta§ shows that *the above conjecture is false for $\delta = 7^3$* . In fact,

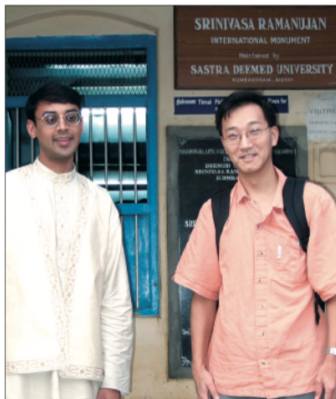
$$24 \cdot 243 = 24(343 - 100) \equiv -2400 = -7^4 + 1 \equiv 1 \pmod{7^3},$$

but

$$p(243) = 133978259344888,$$

and it is easily verified that $p(243) \equiv 0 \pmod{7^2}$ but $p(243) \not\equiv 0 \pmod{7^3}$.

Partitions - Ken Ono



Manjul Bhargava, left, and Ken Ono in front of Ramanujan's house.

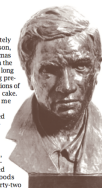
Honoring a Gift from Kumbakonam

Ken Ono

Today was an absolutely glorious day in Madison, Wisconsin. It is Christmas 2005, and everyone in the house is asleep after a long day of enjoying family, opening presents, and eating enormous portions of mashed potatoes and yule log cake. Yet powerful images keep me awake.

Thirty-six hours ago I returned from a six-day whirlwind journey to a far-off place. I spent forty hours on airplanes, and I endured fourteen hours in cars dodging bicycles, rickshaws, cows, goats, and masses of people on roads severely damaged by recent flooding. These floods would be blamed for at least forty-two deaths. Despite these hardships and bad luck, this adventure exceeded my lofty expectations.

I ostensibly travelled to Kumbakonam with the purpose of giving a



Bust of Srinivasa Ramanujan by artist Paul Crawford.

This adventure was a pilgrimage to pay homage to Srinivasa Ramanujan, the Indian legend whose congruences, formulas, and identities have inspired much of my own work. This fulfilled a personal journey, one with an unlikely beginning in 1984.

The Story of Ramanujan

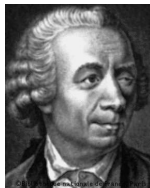
Ramanujan was born on December 22, 1887, in Erode, a small town about 250 miles southwest of Chennai (formerly known as Madras). He was a Brahmin, a member of India's priestly caste, and as a consequence he lived his life as a strict vegetarian.

When Ramanujan was one year old, he moved to Kumbakonam, a small town about 170 miles south of Chennai, where his father Srinivasa was a cloth merchant's clerk. Kumbakonam, which is situated on the banks of the sacred Kaveri River,

Notices of the AMS, 53 N°6 (July 2006), 640–651

<http://www.ams.org/notices/200606/fea-ono.pdf>

Leonhard Euler (1707 – 1783)



$$1 + p(1)x + p(2)x^2 + \cdots + p(n)x^n + \cdots$$
$$= \frac{1}{(1-x)(1-x^2)(1-x^3)\cdots(1-x^n)\cdots}$$

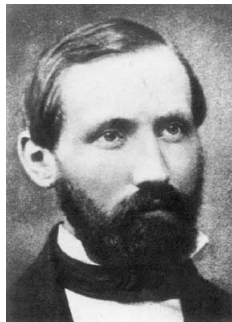
$$1 + \sum_{n=1}^{\infty} p(n)x^n = \prod_{n=1}^{\infty} (1-x^n)^{-1}$$

Eulerian products

Riemann zeta function

For $s > 1$,

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_p (1 - p^{-s})^{-1}$$



Georg Friedrich Bernhard
Riemann (1826 - 1866)

Circle method



Srinivasa Ramanujan
(1887 – 1920)



G.H. Hardy
(1877 – 1947)



J.E. Littlewood
(1885 – 1977)

Hardy, ICM Stockholm, 1916

Hardy and Ramanujan (1918): partitions

Hardy and Littlewood (1920 – 1928):

Some problems in Partitio Numerorum

Circle method

This method was further developed by Hardy, Littlewood, Rademacher, Vinogradov, Davenport, . . .

This gave rise to:

- Large Sieve
- Ternary Goldbach Conjecture
- Progress on binary Goldbach Conjecture
- Waring's problem

All these problems looked beyond reach before the birth of Circle method.

Ramanujan tau function

$$x(1-x)^{-1} = \sum_{n=1}^{\infty} x^n$$

$$x \prod_{n=1}^{\infty} (1-x^n)^{24} = \sum_{n=1}^{\infty} \tau(n)x^n.$$

$$\sum_{n=1}^{\infty} \frac{\tau(n)}{n^s} = \prod_p (1 - \tau(p)p^{-s} + p^{11-2s})^{-1}$$

Ramanujan's Congruences

$\tau(pn)$ is divisible by p for $p = 2, 3, 5, 7, 23$.

also: congruences modulo 691
(numerator of Bernoulli number B_{12})

Pierre Deligne

Ramanujan's Conjecture,
proved by Deligne in 1974

$$|\tau(p)| < 2p^{11/2}$$



Hardy–Ramanujan

For *almost all* integers n , the number of prime factors of n is $\log \log n$.

$$A_\epsilon(x) = \{n \leq x ; (1 - \epsilon) \log \log n < \omega(n) < (1 + \epsilon) \log \log n\}.$$

$$\frac{1}{x} A_\epsilon(x) \rightarrow 1 \quad \text{when } x \rightarrow \infty.$$

Highly composite numbers

(Proc. London Math. Soc. 1915)

$$\begin{array}{rcccccccc} n = & 2 & 4 & 6 & 12 & 24 & 36 & 48 & 60 & 120 \dots \\ d(n) = & 2 & 3 & 4 & 6 & 8 & 9 & 10 & 12 & 16 \dots \end{array}$$

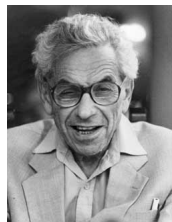
Question: For $t \in \mathbf{R}$, do the conditions $2^t \in \mathbf{Z}$ and $3^t \in \mathbf{Z}$ imply $t \in \mathbf{Z}$?

Example: For $t = \frac{\log 1729}{\log 2}$, we have $2^t = 1729 \in \mathbf{Z}$, but

$$3^t = \exp((\log 3)(\log 1729)/\log 2) = 135451.447153 \dots$$

is not an integer.

Pàl Erdős



Carl Ludwig Siegel



Alaoglu and Erdős: *On highly composite and similar numbers*, 1944.

C.L. Siegel: For $t \in \mathbf{R}$, the conditions $2^t \in \mathbf{Z}$, $3^t \in \mathbf{Z}$ and $5^t \in \mathbf{Z}$ imply $t \in \mathbf{Z}$.

Serge Lang, K. Ramachandra: *six exponentials theorem, four exponentials conjecture*.

Five exponentials Theorem and generalizations

1985: Five exponentials
Theorem.

1993, [Damien Roy](#), Matrices
whose coefficients are linear
forms in logarithms.
J. Number Theory **41** (1992),
no. 1, 22–47.



Approximation for π due to Ramanujan

$$\frac{63}{25} \left(\frac{17 + 15\sqrt{5}}{7 + 15\sqrt{5}} \right) = 3.141592653\ 80568820189839000630\dots$$

$$\pi = 3.141592653\ 58979323846264338328\dots$$

Another formula due to Ramanujan for π

$$\pi = \frac{9\,801}{\sqrt{8}} \left(\sum_{n=0}^{\infty} \frac{(4n)!(1\,103 + 26\,390n)}{(n!)^4 396^{4n}} \right)^{-1}$$

$n = 0$: 6 exact digits for 3.141592...

$n \rightarrow n + 1$: 8 more digits

Ramanujan's formula for $1/\pi$

$$\frac{1}{\pi} = \sum_{m=0}^{\infty} \binom{2m}{m} \frac{42m + 5}{2^{12m+4}}$$

Decimals of π

Ramanujan's formulae were used in 1985: $1.7 \cdot 10^7$ digits for π
(1.7 crores)

In 1999: $2 \cdot 10^{10}$ digits (2 000 crores)

18 Aug 2009: Pi Calculation Record Destroyed: 2.5 Trillion
Decimals ($2.5 \cdot 10^{12}$).

2,576,980,377,524 decimal places in 73 hours 36 minutes

Massive parallel computer called : T2K Tsukuba System.

Team leader professor Daisuke Takahashi.

Ramanujan Notebooks

Written from 1903 to 1914

First: 16 chapters, 134 pages

Second: 21 chapters, 252 pages

Third: 33 pages

B.M. Wilson, G.N. Watson

Edited in 1957 in Bombay

The lost notebook

George Andrews, 1976



Bruce Berndt, 1985–87 (5 volumes)

Last work of Ramanujan

Mock theta functions



S. ZWEGERS – “Mock ϑ -functions and real analytic modular forms.”, in *Berndt, Bruce C. (ed.) et al., q-series with applications to combinatorics, number theory, and physics. Proceedings of a conference, University of Illinois, Urbana-Champaign, IL, USA, October 26-28, 2000. Providence, RI: American Mathematical Society (AMS). Contemp. Math. 291, 269-277, 2001.*

SASTRA Ramanujan Prize

The SASTRA Ramanujan Prize, founded by Shanmugha Arts, Science, Technology & Research Academy (SASTRA) located near Kumbakonam, India, Srinivasa Ramanujan's hometown, is awarded every year to a young mathematician judged to have done outstanding work in Ramanujan's fields of interest. The age limit for the prize has been set at 32 (the age at which Ramanujan died), and the current award is \$10,000.

<https://sas.sastra.edu/ramanujan/Ramanujan-Awards.php>

SASTRA Ramanujan Prize

Year	Name	Institution
2005	Manjul Bhargava	Princeton University
	Kannan Soundararajan	University of Michigan
2006	Terence Tao	University of California at Los Angeles
2007	Ben Green	Cambridge University
2008	Akshay Venkatesh	Stanford University
2009	Kathrin Bringmann	University of Cologne, Germany, and University of Minnesota, USA
2010	Wei Zhang	Harvard University
2011	Roman Holowinsky ^[1]	Ohio State University
2012	Zhiwei Yun ^[2]	Stanford University
2013	Peter Scholze ^[3]	University of Bonn
2014	James Maynard ^[4]	Oxford University, England, and University of Montreal, Canada
2015	Jacob Tsimerman	University of Toronto, Canada
2016	Kaisa Matomäki	University of Turku, Finland;
	Maksym Radziwiłł ^[5]	McGill University, Canada, and Rutgers University, USA
2017	Maryna Viazovska ^[6]	École Polytechnique Fédérale de Lausanne, Switzerland
2018	Yifeng Liu	Yale University, USA;
	Jack Thorne ^[7]	Cambridge University, United Kingdom
2019	Adam Harper ^[8]	University of Warwick, England (UK)
2020	Shai Evra ^[9]	Princeton University, USA, and Hebrew University of Jerusalem, Israel

https://en.wikipedia.org/wiki/SASTRA_Ramanujan_Prize

ICTP Ramanujan Prize

The Ramanujan Prize

For young mathematicians from developing countries

The Ramanujan Prize for young mathematicians from developing countries has been awarded annually since 2005. It was originally instituted by ICTP, the Niels Henrik Abel Memorial Fund, and the **International Mathematical Union (IMU)**. The participation of the Abel Fund ended in 2012; the Department of Science and Technology of the Government of India (DST) has now agreed to fund the Prize for a 5 year period, starting with the 2014 Prize.

The Prize is awarded annually to a researcher from a developing country who is less than 45 years of age on 31 December of the year of the award, and who has conducted outstanding research in a developing country. Researchers working in any branch of the mathematical sciences are eligible. The Prize carries a \$15,000 cash award. The winner will be invited to ICTP to receive the Prize and deliver a lecture. The Prize is usually awarded to one person, but may be shared equally among recipients who have contributed to the same body of work.

The Selection Committee takes into account not only the scientific quality of the research, but also the background of the candidate and the environment in which the work was carried out. The Committee in particular favours candidates who have overcome adversity to achieve distinction in mathematics.

The Committee consists of eminent mathematicians appointed in consultation between ICTP, the IMU, and the DST.

ICTP Ramanujan Prize

The Ramanujan Prize Winners

2020	Carolina Araujo
2019	Hoàng Hiệp Phạm
2018	Ritabrata Munshi (India)
2017	Eduardo Teixeira (Brazil)
2016	Chenyang Xu (People's Republic of China)
2015	Amalendu Krishna (India)
2014	Miguel Walsh (Argentina)
2013	Ye Tian (People's Republic of China)
2012	Fernando Codá Marques (Brazil)
2011	Philibert Nang (Gabon)
2010	Yuguang Shi (People's Republic of China)
2009	Ernesto Lupercio (Mexico)
2008	Enrique R. Pujals (Brazil/Argentina)
2007	Jorge Lauret (Argentina)
2006	Sujatha Ramdorai (India)
2005	Marcelo Viana (Brazil)

References (continued)

Don Zagier (March 16, 2005, BNF/SMF) :

"Ramanujan to Hardy, from the first to the last letter..."

<http://smf.emath.fr/VieSociete/Rencontres/BNF/2005/>

MacTutor History of Mathematics

<http://www-groups.dcs.st-and.ac.uk/~history/>

Eric Weisstein worlds of mathematics, Wolfram Research

<http://scienceworld.wolfram.com/>

Wikipedia, the free encyclopedia.

<http://en.wikipedia.org/wiki/Ramanujan>

Ramanujan according to Wikipedia



Erode December 22, 1887 —
Chetput, (Madras), April 26,
1920

Landau–Ramanujan constant
Mock theta functions
Ramanujan prime
Ramanujan–Soldner constant
Ramanujan theta function
Ramanujan's sum
Rogers–Ramanujan identities

http://en.wikipedia.org/wiki/Srinivasa_Ramanujan

Landau–Ramanujan constant

In mathematics, the **Landau–Ramanujan** constant occurs in a number theory result stating that the number of positive integers less than x which are the sum of two square numbers, for large x , varies as

$$x / \sqrt{\ln(x)}.$$

The constant of proportionality is the **Landau–Ramanujan** constant, which was discovered independently by **Edmund Landau** and **Srinivasa Ramanujan**.

More formally, if $N(x)$ is the number of positive integers less than x which are the sum of two squares, then

$$\lim_{x \rightarrow \infty} \frac{N(x) \sqrt{\ln(x)}}{x} \approx 0.76422365358922066299069873125.$$

Landau–Ramanujan constant

M.W. *Representation of integers by cyclotomic binary forms.*

Number Theory Web Seminar, Tuesday, May 12, 2020.

Link to recording and slides

<https://sites.google.com/view/ntwebseminar/previous-talks>



Étienne Fouvry



Claude Levesque

EF+CL+MW, *Representation of integers by cyclotomic binary forms.* Acta Arithmetica, **184**.1 (2018), 67 – 86.

EF+MW, *Sur la représentation des entiers par des formes cyclotomiques de grand degré.* Bull. Soc. Math. France, **148** (2020), 253–282.

International Conference on Number theory and Discrete Mathematics. Cochin

Web conference using Google Meet December 11–14, 2020.
Organized by the [Ramanujan](#) Mathematical Society (RMS)
and hosted by the Rajagiri School of Engineering &
Technology (RSET), Kakkannad.

<https://www.rajagiritech.ac.in/icntdm/index.asp>

[M.W.](#) Lecture on *On some families of binary forms and the integers they represent.*

Monday, December 14, 12:00 IST = 7:30 am CST.

[http://www.imj-prg.fr/~michel.waldschmidt/
articles/pdf/FamiliesBinaryForms.pdf](http://www.imj-prg.fr/~michel.waldschmidt/articles/pdf/FamiliesBinaryForms.pdf)

Ramanujan primes

In mathematics, a **Ramanujan prime** is a prime number that satisfies a result proven by **Srinivasa Ramanujan** relating to the prime-counting function.

$$\pi(x) - \pi(x/2) \geq 1, 2, 3, 4, 5, \dots \quad \text{for all } x \geq 2, 11, 17, 29, 41, \dots$$

respectively, where $\pi(x)$ is the prime-counting function, that is, the number of primes less than or equal to x .

Ramanujan primes

2, 11, 17, 29, 41, 47, 59, 67, 71, 97, 101, 107, 127, 149,
151, 167, 179, 181, 227, 229, 233, 239, 241, 263, 269, ...

$a(n)$ is the smallest number such that if $x \geq a(n)$, then $\pi(x) - \pi(x/2) \geq n$, where $\pi(x)$ is the number of primes $\leq x$.

$\pi(2) - \pi(1) = 1$, $a(1) = 2$ (Bertrand's Postulate).

$\pi(10) - \pi(5) = 1$, a single prime (namely 7) in $(5, 10]$;

$\pi(n) - \pi(n/2) \geq 2$ for $n \geq 11$, hence $a(2) = 11$.

Neil J. A. Sloane's encyclopaedia

<http://www.research.att.com/~njas/sequences/A104272>

Ramanujan–Soldner constant



In mathematics, the Ramanujan–Soldner constant is a mathematical constant defined as the unique positive zero of the logarithmic integral function. It is named after Srinivasa Ramanujan and Johann Georg von Soldner (16 July 1776 - 13 May 1833).

Its value is approximately

1.451369234883381050283968485892027449493 ...

Ramanujan's sum (1918)

In number theory, a branch of mathematics, **Ramanujan's sum**, usually denoted $c_q(n)$, is a function of two positive integer variables q and n defined by the formula

$$c_q(n) = \sum_{\substack{1 \leq a \leq q \\ (a,q)=1}} e^{2\pi i \frac{a}{q} n}.$$

Ramanujan's sums are used in the proof of **Vinogradov's theorem** that every sufficiently-large odd number is the sum of three primes.

Ramanujan theta function

$$f(a, b) = \sum_{n=-\infty}^{\infty} a^{n(n+1)/2} b^{n(n-1)/2}$$



Carl Gustav Jacob Jacobi
(1804–1851)

In mathematics, the Ramanujan theta function generalizes the form of the Jacobi theta functions, while capturing their general properties. In particular, the Jacobi triple product takes on a particularly elegant form when written in terms of the Ramanujan theta.

Rogers–Ramanujan identities



Leonard James Rogers
1862 - 1933

In mathematics, the **Rogers–Ramanujan** identities are a set of identities related to basic hypergeometric series. They were discovered by **Leonard James Rogers** (1894) and subsequently rediscovered by **Srinivasa Ramanujan** (1913) as well as by **Issai Schur** (1917).

G.H. Hardy: Divergent Series



In

$$\frac{1}{1-z} = 1 + z + z^2 + z^3 + \dots$$

set $z = -1$, as Euler does:

$$1 - 1 + 1 - 1 + \dots = \frac{1}{2}$$

Similarly, from the derivative of the previous series

$$\frac{1}{(1-z)^2} = 1 + 2z + 3z^2 + 4z^3 + \dots$$

deduce

$$1 - 2 + 3 - 4 + \dots = \frac{1}{4}$$

$$s = 1 - 2 + 3 - 4 + \dots$$

There are further reasons to attribute the value $1/4$ to s . For instance

$$s = 1 - (1 - 1 + 1 - 1 + \dots) - (1 - 2 + 3 - 4 + \dots) = 1 - \frac{1}{2} - s$$

gives $2s = 1/2$, hence $s = 1/4$.

Also computing the square by expanding the product

$$(1 - 1 + 1 - 1 + \dots)^2 = (1 - 1 + 1 - 1 + \dots)(1 - 1 + 1 - 1 + \dots)$$

yields to

$$1 - 2 + 3 - 4 + \dots = \left(\frac{1}{2}\right)^2 = \frac{1}{4}.$$

Cesaro convergence

For a series

$$a_0 + a_1 + \cdots + a_n + \cdots = s$$

converging (in the sense of **Cauchy**), the partial sums

$$s_n = a_0 + a_1 + \cdots + a_n$$

have a mean value

$$\frac{s_0 + \cdots + s_n}{n + 1}$$

which is a sequence which converges (in the sense of **Cauchy**) to s .

For the diverging series

$$1 - 1 + 1 - 1 + \cdots$$

the limit exists and is $1/2$.

Cesaro convergence

Ernesto Cesàro
(1859 – 1906)



For the series

$$1 + 0 - 1 + 1 + 0 - 1 + \dots$$

the Cesaro limit

$$\lim_{n \rightarrow \infty} \frac{s_0 + \dots + s_n}{n + 1}$$

exists and is $2/3$.

Rules for summing divergent series

$$a_0 + a_1 + a_2 + \cdots = s \quad \text{implies} \quad ka_0 + ka_1 + ka_2 + \cdots = ks.$$

$$a_0 + a_1 + a_2 + \cdots = s \quad \text{and} \quad b_0 + b_1 + b_2 + \cdots = t \quad \text{implies} \\ a_0 + b_0 + a_1 + b_1 + a_2 + b_2 + \cdots = s + t.$$

$$a_0 + a_1 + a_2 + \cdots = s \quad \text{if and only if} \quad a_1 + a_2 + \cdots = s - a_0.$$

$$1^2 - 2^2 + 3^2 - 4^2 + \dots$$

Recall

$$\frac{1}{(1-z)^2} = 1 + 2z + 3z^2 + 4z^3 + \dots$$

Take one more derivative, you find also

$$1 \cdot 2 - 2 \cdot 3 + 3 \cdot 4 - 4 \cdot 5 + \dots = \frac{1}{4}$$

from which you deduce

$$1^2 - 2^2 + 3^2 - 4^2 + \dots = 0.$$

Further examples of divergent Series

$$1 + 1 + 1 + \dots = 0$$

$$1 - 2 + 4 - 8 + \dots = \frac{1}{3}$$

$$1 + 2 + 4 + 8 + \dots = -1$$

$$1^{2k} + 2^{2k} + 3^{2k} + 4^{2k} + 5^{2k} + \dots = 0 \quad \text{for } k \geq 1.$$

Euler:

$$1 - 1! + 2! - 3! + 4! + \dots = -e\left(\gamma - 1 + \frac{1}{2 \cdot 2!} - \frac{1}{3 \cdot 3!} + \dots\right)$$

gives the value **0.5963...** also found by **Ramanujan**.

Ramanujan's method (following Joseph Oesterlé)

Here is Ramanujan's method for computing the value of divergent series and for accelerating the convergence of series.



The series

$$a_0 - a_1 + a_2 - a_3 + \cdots$$

can be written

$$\frac{1}{2} (a_0 + (b_0 - b_1 + b_3 - b_4 + \cdots))$$

where $b_n = a_n - a_{n+1}$.

Acceleration of convergence

For instance in the case $a_n = 1/n^s$ we have $b_n \sim s/n^{s+1}$.
Repeating the process yields the analytic continuation of the **Riemann** zeta function.

For $s = -k$ where k is a positive integer, **Ramanujan's** method yields the **Bernoulli** numbers.

In the case of convergent series like

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots = \log 2,$$

or for **Euler** constant, **Ramanujan's** method gives an efficient way of accelerating the convergence.



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Srinivasa Ramanujan

His life and his work

Michel Waldschmidt

Sorbonne University
Institut de Mathématiques de Jussieu
<http://www.imj-prg.fr/~michel.waldschmidt/>

Coimbatore, December 16, 2020.