Update: 24/03/2021

April 12 - 23, 2021: Hanoi (Vietnam) (online)
CIMPA School on Functional Equations: Theory, Practice and Interaction.

Introduction to transcendental numbers
Michel Waldschmidt

Content of the course

This course consists of the 8 items [1] to [8] below. On top of that, the participants are advised to watch the lectures [9] to [11], especially the first part of [10].

The courses [1] to [5] are given on the blackboard; for each of them, an associated pdf file provides the detailed content, and sometimes more.

The courses [6] to [8] are given with slides.

The lectures [9] to [11] are given on the blackboard, there is no associated pdf file.

[1] Criteria for irrationality and transcendence.
- 1.1 Irrationality criterion
- 1.2 A transcendence criterion
- 1.3 Criteria for linear independence, for algebraic independence

• Link to the video (48')
• Link to the pdf file (p. 1 – 7)

[2] Liouville type estimates
- 2.1 Liouville inequality
- 2.2 Heights
- 2.3 Explicit Liouville estimates

• Link to the video (54')
• Link to the pdf file (p. 8 – 13)

[3] Thue Siegel Roth
• Link to the video (1:12')
• Link to the pdf file (p. 14 – 19)

[4] Schmidt's Subspace Theorem
• Link to the video (1:17')
• Link to the pdf file (p. 20 – 27)

[5] Effective methods
- 5.1 Linear combinations of logarithms
- 5.2 Baker's transcendence results
- 5.3 The S–unit equation
- 5.4 An explicit lower bound
- 5.5 Conjectures

• Link to the video (59')
• Link to the pdf file (p. 28 – 33)

[6] Early history of transcendental number theory
- Link to the video (50')
- Link to the slides (52 p.)

[7] Transcendence of values of special functions
- Link to the video (1:40')
- Link to the slides (116 p.)

[8] Conjectures. Algebraic independence of transcendental numbers.
- Link to the video (1:07')
- Link to the slides (132 p.)

Beware: this file is a slightly expanded version of the file which was used to record the video (132 pages vs 87 pages).

The three items below are videos of talks given in New Delhi in 2018 under the title *three lectures on the unity of mathematics: examples from transcendental number theory.*

[9] ISID_SMU Seminar, 1/3 – October 3, 2018, ISI Delhi.

Content: Continued fractions, Liouville, Hermite, Lindemann, Padé, Siegel, Schneider–Lang, sketch of proof.
- Link to the video (1:03)

[10] JNU Seminar, 2/3 – October 4, 2018 JNU Delhi

Content: The first part is devoted to a sketch of a transcendence proof (Schneider Lang Criterion), introducing the following tools: the construction of an auxiliary function, the zero estimate, Schwarz Lemma, Liouville type estimate. The second part discusses an algebraic invariant arising from a generalization of Schwarz Lemma in several variables and its connection with symbolic powers.
- Link to the video (1:08')

[11] ISID_SMU Seminar, 3/3 – October 5, 2018, ISI Delhi.

Content: Laurent interpolation determinant. Several variables: Bombieri generalization of the Schneider-Lang Theorem, the 4 exponentials Theorem, the Four exponentials conjecture, rank of matrices with entries logarithms of algebraic numbers, algebraic independence of logarithms, Leopoldt's Conjecture, topology, dynamical systems, multiple zeta values, work of Adamczewski and Bugeaud using Schmidt Subspace Theorem, automata.
- Link to the video (1:14')

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