



Differential Equations

Assignment #1

Exercise 1. Consider the following equation:

$$x'(t) = 2tx(t) + \frac{1}{t} \exp(t^2)$$

defined on $\mathbb{R}_{>0}$, with initial condition $x(1) = 0$.

1. Write the so-called homogeneous equation (without any initial condition). Give the general solution (of the homogeneous equation).
2. Give one special solution of the original equation. Then give the general solution (of the original equation).
3. Now give the only solution satisfying the initial condition.

Exercise 2. We consider the following differential equation:

$$(\mathcal{E}_H) : \quad x^{(4)} = x^{(3)} + 7x^{(2)} - 13x' + 6x.$$

1. Provide a basis of the space solution S_H .
2. Give the explicit solution with the initial conditions

$$x(0) = 2; \quad x'(0) = 0; \quad x''(0) = -2 \quad \text{and} \quad x^{(3)}(0) = -4$$

Exercise 3. Consider the following matrix:

$$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

Prove that:

$$\exp(A) = \begin{pmatrix} \cos(1) & -\sin(1) \\ \sin(1) & \cos(1) \end{pmatrix}$$