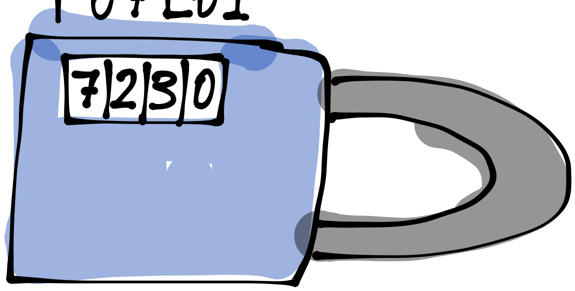
Feuille Proba 1 (F07)

F07 E01



Quadruplet de chiffres (a_1, a_2, a_3, a_4) où $a_i \in \{0, \dots, 9\}$

Arrangement de chiffres (a_1, a_2, a_3, a_4) où $a_i \in \{0, \dots, 9\}$
tels que $a_i \neq a_j$ pour $i \neq j$

Combinaison de chiffres $\{a_1, a_2, a_3, a_4\} \subseteq \{0, 1, \dots, 9\}$
où $a_i \neq a_j$ pour $i \neq j$

NB !
$\{7, 2, 3, 0\} = \{0, 2, 3, 7\}$ mais $(7,2,3,0) \neq (0,2,3,7)$
et $(0,0,0,0)$ est un quadruplet mais pas un arrangement.

Reponse : Le code est un quadruplet

$(a_1, a_2, a_3, a_4) \in \{0, 1, \dots, 9\}^4$

Question combien de codes ?
$\mathrm{Card}(A \times B) = \mathrm{Card}\, A \times \mathrm{Card}\, B$ $\qquad \#(A \times B) = \#A \times \#B$

Reponse:

$\#\{0,\dots,9\}^4 = (\#\{0,\dots,9\})^4 = 10^4$

Question : Combien de code (quadruplets) sans repetition ?

$\Updownarrow$

Arrangement

#Arrangements de 4 elements parmi 10 elements $= A_{10}^4$

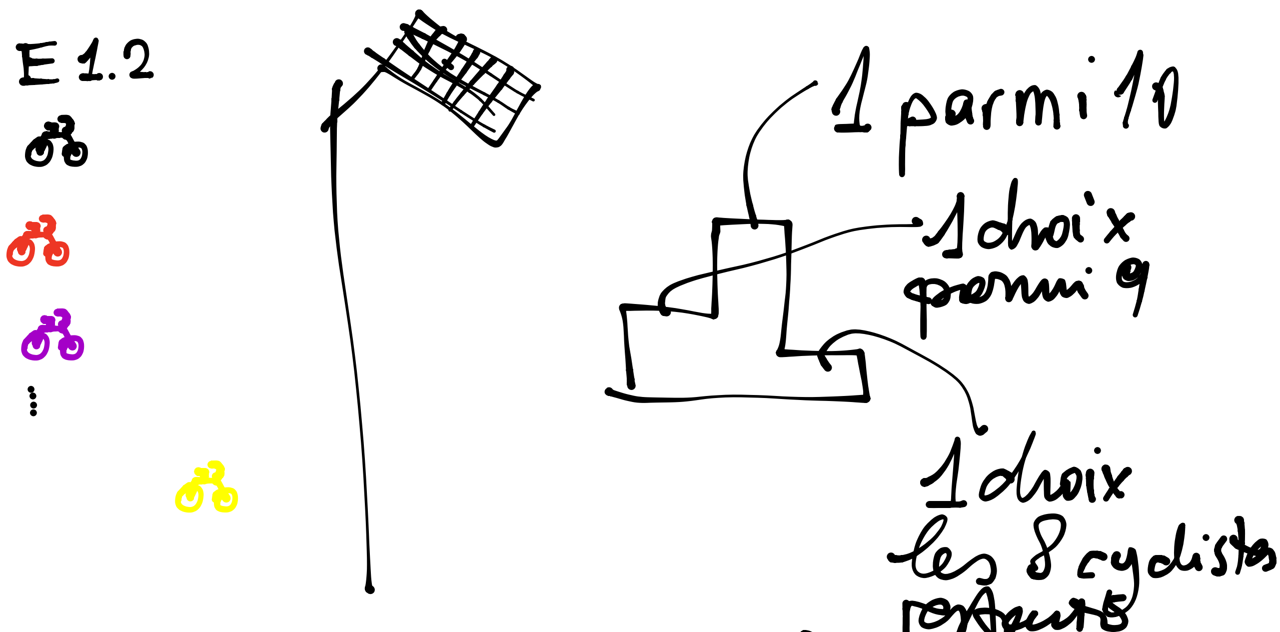
$$A_{10}^4 = \frac{10!}{(10-4)!} = 10\times 9\times 8\times 7 = 5040 \qquad A_n^k = \frac{n!}{(n-k)!}$$

Notation : $n! = n(n-1)\dots(2)\times 1$

$3! = 6$

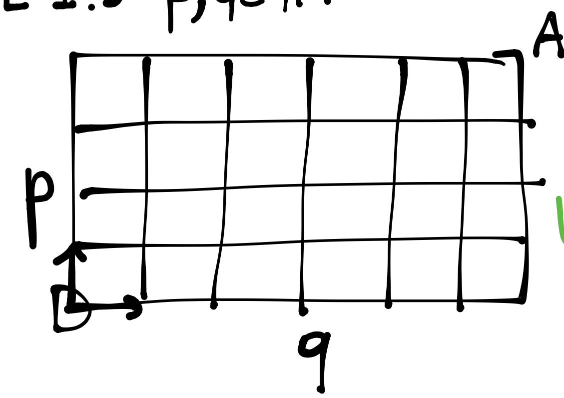
$4! = 24$

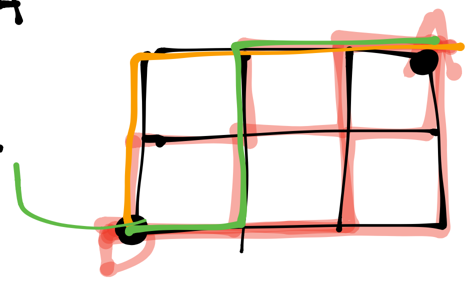
E 1.2



Question : Combien de podium possibles : $A_{10}^3 = 10\times 9\times 8 =$

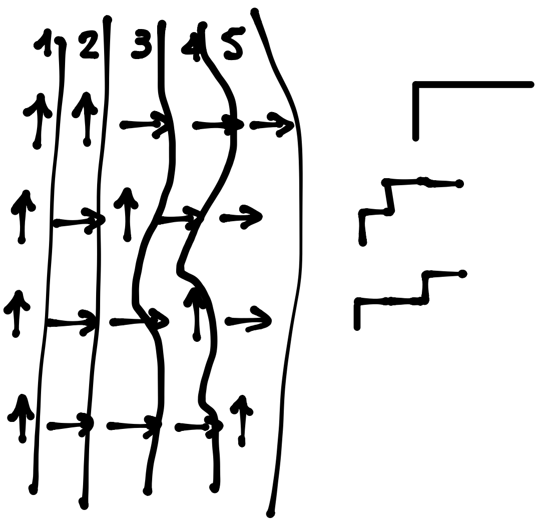
E 1.3 $p, q \in \mathbb{N}^*$





chemins = 10

deplacements = $p+q$



le nombre de Combinaisons de 2 parmi 5 $= C_5^2$

$A_5^2 = 5 \times 4 = 20 \Rightarrow C_5^2 = \frac{20}{2}$

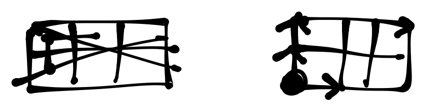
Rappel $C_n^k = \frac{n!}{(n-k)!\,k!} = \binom{n}{k}$ coeff binomial

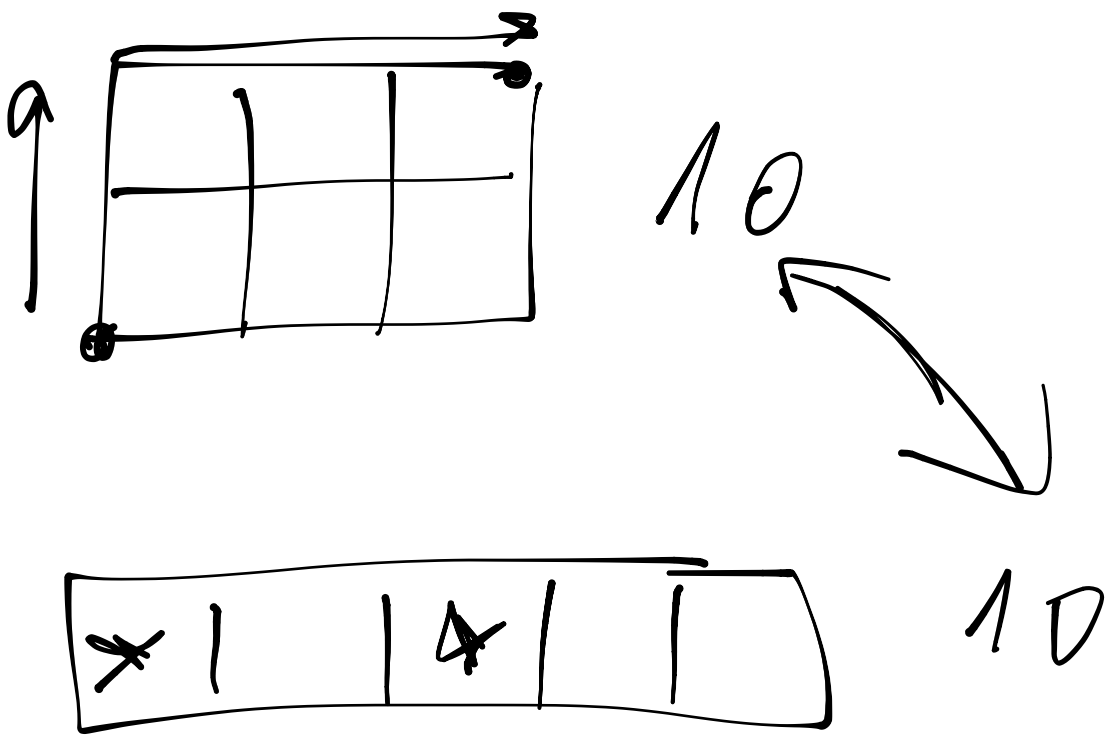
$$\frac{15!}{2!\,13!} = C_{15}^{2} = C_{15}^{13} = \frac{15!}{13!\,2!}$$

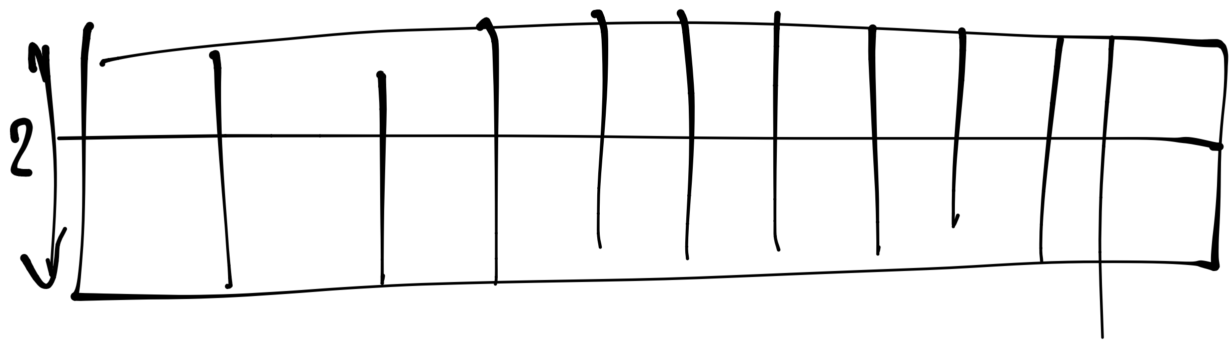
Reponse : $\binom{p+q}{p} = \frac{(p+q)!}{p!\,q!}$ est le nombre de chemins

car on a p+q deplacements (haut et droite confondus)

et exactement p vers le haut.







13 cm

#chemin C_{15}^{2}

n^k

A_n^k

$C_n^k = \binom{n}{k}$

$C_0^k \rightarrow 1$

$C_1^k \rightarrow 1 \quad 1$

$C_2^k \rightarrow 1 \quad 2 \quad 1$

$C_3^k \rightarrow 1 \quad 3 \quad 3 \quad 1$

$1 \quad 4 \quad 6 \quad 4 \quad 1$

$C_5^k \rightarrow 1 \quad 5 \quad \boxed{10} \quad 10 \quad 5 \quad 1$

$$\boxed{10} = C_5^2 = \binom{5}{2}$$

Voir $\boxed{\text{E5.2}}$

$$C_5^3 = C_5^2$$

$$\frac{n!}{k!(n-k)!}$$

$$\overset{1}{C_0^0}$$

$$\overset{1}{C_1^0} \qquad \overset{1}{C_1^1}$$

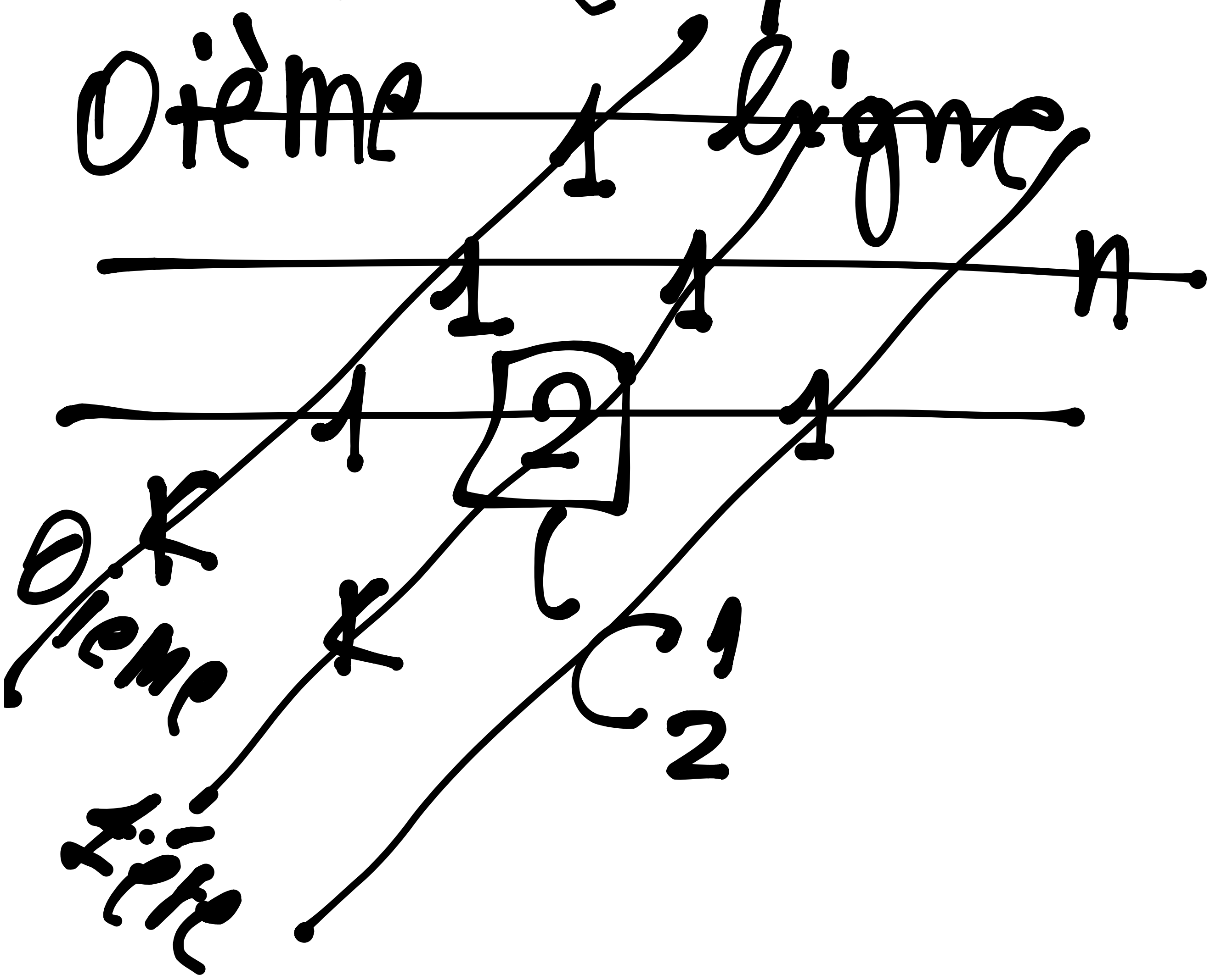
$$\overset{1}{C_2^0} - \overset{2}{C_2^1} + \overset{1}{C_2^2} = 0$$

$$\overset{1}{C_3^0} + \overset{}{C_3^2} - \overset{}{C_3^2} + \overset{1}{C_3^3} = 0$$

E 5.2. (c)

$$\sum_{k=0}^{n} (-1)^k \binom{n}{k} = 0$$

$$C_n^k = \binom{n}{k} = \frac{n!}{k!(n-k)!}$$



$$C_0^0$$

$$C_1^0 \quad C_1^1$$

$$C_2^0 \quad C_2^1 \quad C_2^2$$

$$C_3^0 \quad C_3^1 \quad C_3^2 \quad C_3^3$$

$$\#\{0,\dots,n-1\}^k = n^k$$

$$A_n^k = \frac{n!}{(n-k)!}$$

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

$$(x+y)^2 = x^2 + 2xy + y^2$$

$$\rightarrow 1 \quad 2 \quad 1$$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

$$(1+x)^n = \sum_{k=0}^{n} \binom{n}{k} x^k$$

$\|$ si $x=-1$

0

$\Downarrow$

E5.2.c $$\sum_{k=0}^{n} \binom{n}{k} (-1)^k = 0$$