Deformations of transfer operators

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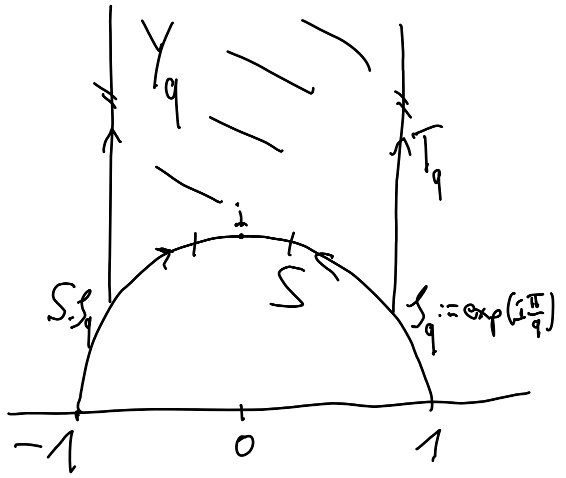
We take the Hecke triangle group family and the Theta group. We study the associated transfer operators arising by the discrete dynamical system induced by the geodesic flow of their corresponding orbifold.



* $S := \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$; $T_q := \begin{bmatrix} 1 & \lambda_q \\ 0 & 1 \end{bmatrix}$; $\lambda_q := \begin{cases} 2\cos\frac{\pi}{q} & q \in \mathbb{N}_{\geq 3} \\ 2 & q = \infty \end{cases}$
* $\Gamma_q := \langle S, T_q \rangle$; $Y_q := \Gamma_q \backslash \mathbb{H}$ ($\Gamma_q \curvearrowright \mathbb{H}$ by Moebius transf.)
* $\mathrm{vol}(Y_q) = \pi(1 - \frac{2}{q})$
* By Phillips-Sarnak conjecture and a result of Judge we expect a violation of Weyl's law for $q \notin \{3, 4, 6, \infty\}$ (No even Maass cusp forms).

1. Setting

$$\begin{array}{ccc} \Gamma_q & \xrightarrow{\quad q \to \infty \quad} & \Gamma_\infty \\ \Big\downarrow F_q & & \Big\downarrow F_\infty \\ \mathcal{L}_{q,s} & \xrightarrow{\quad Q:\ q \to \infty\ ? \quad} & \mathcal{L}_{\infty,s} \end{array}$$

Weighted transfer operator (formally):

$S: \mathbb{R}^*_{>0} \to \mathbb{C}$, $F: \mathbb{R}^*_{>0} \to \mathbb{R}_{>0}$ db, $s \in \mathbb{C}$

$$\mathcal{L}_{F,s} f := \sum_{F(y) = (\cdot)} |F'(y)|^{-s} f(y)$$

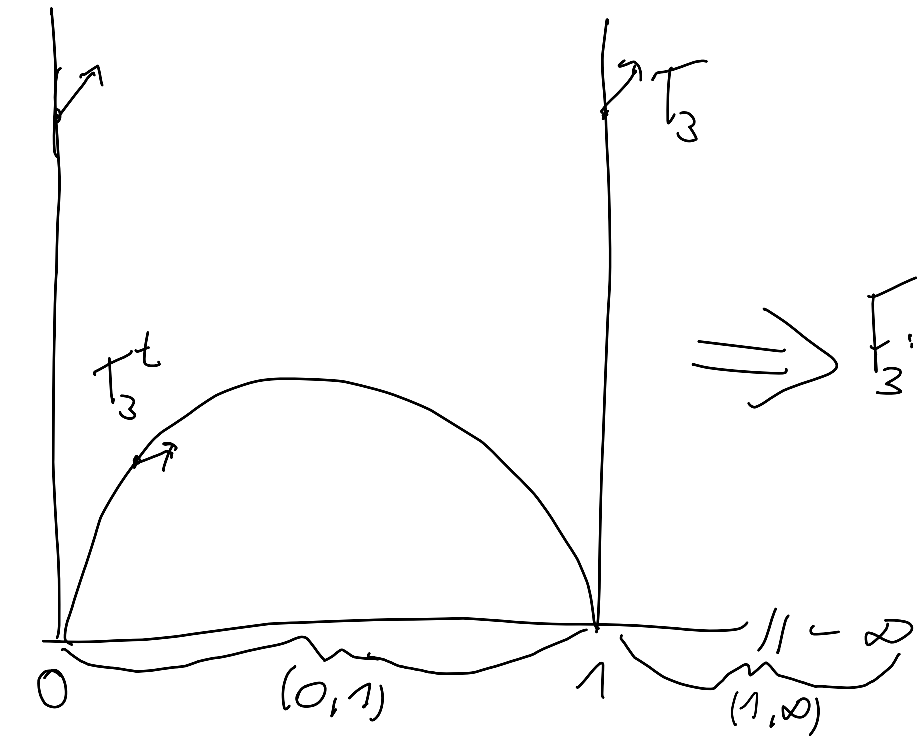
($\mathbb{R}^*_{>0}$ is $\mathbb{R}_{>0}$ with cnt. many points excluded.)

$\mathcal{L}_{\infty,s} \to$ (i) Well-definedness of $\mathcal{L}_{\infty,s}$?

$\mathcal{L}_{\infty,s} \to$ (ii) Convergence of $\mathcal{L}_{q,s} \to \mathcal{L}_{\infty,s}$?

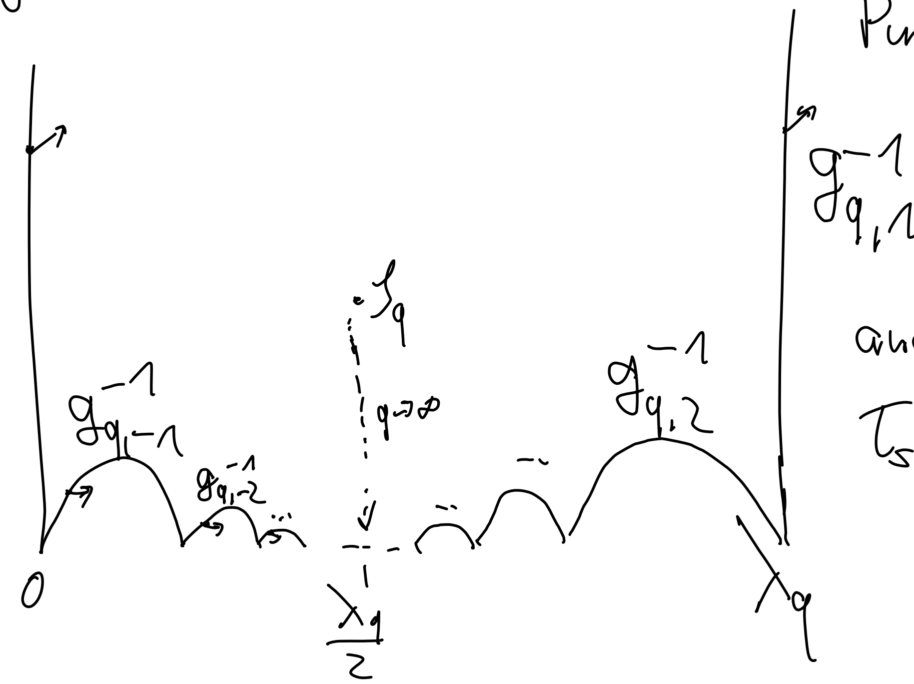
F_q is the discrete dynamical system arising by the discretization of the geodesic flow on the unit tangent bundle SY_q.

For $q=3$:



$$\Longrightarrow F_3 : \mathbb{R}^*_{>0} \to \mathbb{R}_{>0} : x \mapsto \begin{cases} (T_3^t)^{-1}.x & x \in (0,1] \\ T_3^{-1}.x & x \in (1,\infty) \end{cases}$$

General case:



Put $g_{q,k} := \left((T_q S)^k S\right)^{-1}$

and set

$$\tau_s(g_{q,k}) f(x) := \left(g_{q,k}^{-1\,\prime}(x)\right)^s f(g_{q,k}^{-1} \cdot x).$$

$$\Longrightarrow \mathcal{L}_{q,s} f = \begin{cases} \sum_{k=1}^{q-1} \tau_s(g_{q,k}) & q \in \mathbb{N}_{\geq 3} \\ \sum_{k=1}^{\infty} \tau_s(g_{q,k}) + \tau_s(g_{q,-k}) & q = \infty \end{cases}$$

2. Results

* Regarding (i) we let $\mathcal{L}_{\infty,s}$ act on real analytic fcts. $C^\omega(\mathbb{R}_{>0},\mathbb{C})$. For every $s\in\mathbb{C}$, $\mathrm{Re}\, s>\frac{1}{2}$, every $f\in C^\omega(\mathbb{R}_{>0},\mathbb{C})$ $\mathcal{L}_{\infty,s}f$ converges uniformly on compact subsets of $\mathbb{R}_{>0}$.
* Regarding (ii) we have the following result:

Thm (A, Pohl): If $f\in C^\omega(\mathbb{R}_{>0},\mathbb{C})$ and $f(x)\ll|x-1|^{2\delta}$ for $x\sim 1$ and some $\delta>0$ then for every $s\in\mathbb{C}$, $\mathrm{Re}\, s>\frac{1}{2}-\delta$ we have convergence of

$$\mathcal{L}_{q,s}\to\mathcal{L}_{\infty,s}\quad\text{for } q\to\infty$$

* uniform $\begin{cases}\text{on compact sets on } \mathbb{R}_{>0}.\\ \text{on } \mathbb{R}_{>0} \text{ under stronger conditions on } f.\end{cases}$
* uniform in small changes of s.
* uniform for certain subsets $\mathcal{F}\subset C^\omega(\mathbb{R}_{>0},\mathbb{C})$ where $f\in\mathcal{F}$.